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Three Essays on Macro-Finance

A Dissertation submitted in partial satisfaction of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

SungJun Huh

June 2018

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To my family
This dissertation investigates the relationship between the mechanism of limited borrowing capacity of financial intermediaries and the equity premium in a production economy. A medium-scale New Keynesian model is proposed, featuring an agency problem between financial intermediaries and their private creditors, and generalized recursive preferences. The model considers not only the linkages between banking frictions with the macroeconomy, but also with financial markets. The findings are that banking frictions associated with the agency problem generate a plausible and novel enhancing mechanism for risk premia. In the benchmark setting, banking frictions increase the level of the equity premium substantially and the model produces a fourfold greater response to shocks compared to the case of no banking frictions. The paper also finds that the interaction between monetary policy and banking frictions plays a crucial role in determining the dynamics of the equity premium.

Recent financial crises show that the housing market, financial markets, and the rest of the economy are closely linked to each other. The present dissertation also examines
the impact of credit limit fluctuations on the equity premium through the financial accelerator channel in a production economy. To do so, this chapter introduces the Kiyotaki-Moore (1997) type collateral constraint and Epstein-Zin-Weil preferences into a medium-scale New Keynesian DSGE model with nominal rigidity. My findings are twofold. First, the endogenous fluctuations of credit limit, a key ingredient of the financial accelerator channel, have only minimal effects on the equity premium both quantitatively and qualitatively. Second, liquidity and housing demand shocks that are closely related to credit limit fluctuations also have a very small impact on the equity premium, while technology shocks help generate the observed equity premium.

Also, recent financial crises indicate that a significant decline in house prices can reduce confidence of economic agents and cause bank runs and a fire sale. Accordingly, this dissertation investigates the role of housing on the households attitudes toward risk and derives the closed-form expressions for risk aversion with generalized recursive preferences. This chapter finds that including housing in the utility function lowers risk aversion because housing partially absorbs aggregate shocks to consumption and labor.
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Chapter 1

Introduction

The impacts of financial frictions on the macroeconomy are intriguing research topics. Recent financial crises show that financial markets and the macroeconomy are closely related to each other. The fluctuations of borrowing/lending capacity affect economic activity, which further decreases the borrowers credit limit and raises less funds to finance investment. Many macroeconomic models, however, do not take into account asset prices and risk premia, and are not particularly good at matching financial market variables. This can be a sign of fundamental flaws in the models as Cochrane’s (2008) critique tells us the models should be consistent with basic asset pricing facts.

The present dissertation examines the role of financial frictions on asset pricing. In addition to this, this dissertation exploits structural macroeconomic models with generalized recursive preferences to resolve many puzzles in finance. In particular, I focus here on the relation between limited borrowing capacity of financial intermediaries and the equity premium in a medium-scale New Keynesian dynamic stochastic general equilibrium
(henceforth, DSGE) production economy.

My attempt has important implications for the macro-finance literature. Traditionally, there are two different approaches in the asset pricing literature to capturing sufficiently large risk premia: one is increasing risk in the model by introducing uncertainty (e.g., Weitzman, 2007; and Barillas, Hansen, and Sargent, 2009), long-run risk (e.g., Bansal and Yaron, 2004; and Rudebusch and Swanson, 2012), or rare disaster (e.g., Rietz, 1988; Barro, 2006; and Gourio, 2012) in the model. The other is using heterogeneous agents (e.g., Constantinides and Duffie, 1996; and Schmidt, 2015). I therefore make an attempt to expand the understanding of the interaction between the macroeconomy and financial markets by analyzing the role of financial frictions on the risk premium.

In the second chapter, Limited Borrowing Capacity of Financial Intermediaries and the Equity Premium, I develop a medium-scale New Keynesian model with an agency problem in financial intermediaries for analyzing asset pricing. The proposed model considers not only the linkages between financial frictions with the macroeconomy, but also with financial markets. I find that limited borrowing capacity of financial intermediaries increases risk in the model. In particular, banking frictions increase the equity premium both quantitatively and qualitatively. This finding has important implications for the macro-finance literature, since the equity premium is higher in this framework even without using other approaches mentioned above.

The third chapter I wrote in this area, Implications of Credit Limit Fluctuations for the Equity Premium, examines implications from credit limit fluctuations on the equity premium. The financial friction in this chapter represents the borrower’s credit limit as an
additional constraint (as in Kiyotaki and Moore, 1997) rather than modeling an explicit financial system as in my job market paper (as in Bernanke, Gertler, and Gilchrist, 1999). Moreover, debt capacity mainly relies on housing value rather than physical capital stock, which implies that the housing market can cause business cycle fluctuations. I find that credit limit fluctuations in this framework have a negligible impact on the equity premium. The result indicates that these two different kinds of financial frictions have similar impact on the macroeconomy, but not on asset pricing.

The fourth chapter, Housing and Relative Risk Aversion with Generalized Recursive Preferences, measures risk aversion in a standard macroeconomic model when housing service is included in the utility function. In addition, this chapter uses generalized recursive preferences since they allow standard macroeconomic models to match basic asset pricing facts. The results show that a household’s attitudes toward risk can be very different if the household has the ability to partially insure itself from asset price fluctuations by varying its housing service demand.

The structure of this dissertation is as follows. Chapter 2 develops a New Keynesian DSGE model with banking frictions and generalized recursive preferences. The model investigates how the model explains the equity premium dynamics. Chapter 3 explores how credit limit fluctuations accompanied with house prices affect the equity premium and evaluates their impact on both macroeconomy and financial markets. Chapter 4 derives the closed form expressions of relative risk aversion with generalized recursive preferences and discusses the contribution of housing. Finally, I conclude the main findings of my dissertation.
Chapter 2

Limited Borrowing Capacity of Financial Intermediaries and the Equity Premium

2.1 Introduction

Macroeconomic models with financial frictions have received substantial attention after recent financial crises. Financial frictions introduce a wedge between lenders and borrowers that amplifies business cycle fluctuations in macroeconomic models. With negative shocks, the amplification mechanism is driven by the disruption of asset value that reduces the borrowing capacity of financial intermediaries. As the producers’ financing reduces, economic production further declines, creating a vicious cycle that intensify the recession. Although it is widely agreed that limited borrowing capacity of financial intermediaries could have a
significant impact on business cycle fluctuations, there are very few studies toward understanding the influence on asset prices and risk premia.

This paper investigates the links between limited borrowing capacity of financial intermediaries due to an agency problem and the equity premium in a production economy. A medium-scale New Keynesian model is proposed with the agency problem in financial intermediaries for asset pricing. The model includes generalized recursive preferences, which enables distinction between high risk aversion from the intertemporal elasticity of substitution and, thus, resolving the risk-free rate puzzle of Weil (1989). As far as I know, this is the first paper to study the dynamics of banking frictions and asset pricing.

The main findings are as follows. First, banking frictions make a significant contribution to the size of the equity premium. The equity premium rises by 46 basis points with banking frictions, which accounts for a sizable fraction of the observed data. Second, the model produces a fourfold greater response of the equity premium to a negative technology shock compared to the case of no banking frictions. Third, the dynamics of the equity premium are affected by the interaction between monetary policy and banking frictions. Finally, the equity premium rises with interest rate smoothing in the model with banking frictions.

The intuition for the amplification mechanism of limited borrowing capacity of financial intermediaries is straightforward. During recessions, the marginal productivity of capital decreases and this leads to a lower capital return. The net worth of financial intermediaries then declines because the return to capital is the only source of profits for

\[\text{1} \text{In this paper, generalized recursive preferences are explored rather than expected utility preferences or habit preferences. As Lettau and Uhlig (2000) and Rudebusch and Swanson (2008) point out, habit-based DSGE models cannot fit the term premium in a production economy because habit preferences generate “super” consumption smoothing.}\]
the bank in the model. A decline in net worth decreases the borrowing capacity of financial intermediaries, reducing security offering and capital. In turn, it leads to a decline in the price of capital, which further reduces capital returns and lowers the net worth of financial intermediaries. Even a small shock in this cycle can have a large impact on the economy, increasing the volatility of consumption and the stochastic discount factor.

Banking frictions not only generate a deeper recession, they also attenuate the rise of inflation in response to a negative technology shock. The deeper recession causes the nominal interest rate to rise less than expected inflation increases, leading to a decline in the real risk-free interest rate. In contrast, the real risk-free rate can rise when banking frictions are absent and monetary policy is conducted by a Taylor rule without interest rate smoothing. Since the real risk-free rate is a key determinant of business cycle fluctuations and the stochastic discount factor, the interaction between monetary policy and banking frictions matters in accounting for the dynamics of the equity premium.\(^2\)

My findings have important implications for the macro-finance literature. Traditionally, to capture sufficiently large risk premia, previous studies increase risk in the model: for example, through model uncertainty (e.g., Weitzman, 2007; and Barillas, Hansen, and Sargent, 2009), long-run risk (e.g., Bansal and Yaron, 2004; and Croce, 2014), rare disasters (e.g., Rietz, 1988; Barro, 2006; and Gourio, 2012), or heterogeneous agents (e.g., Constantinides and Duffie, 1996; and Schmidt, 2015).\(^3\) This paper, therefore, attempts to expand the understanding of the interaction between the macroeconomy and financial markets by

\(^2\)The equity premium is defined as the difference between the real equity return and the real risk-free rate. Accordingly, monetary policy plays an important role in determining the equity premium.

\(^3\)More recently, there have been attempts to solve the equity premium puzzle through various methods such as wage rigidities, price rigidities, and deep habits (e.g., Favilukis and Lin, 2016; Weber, 2015; and van Binsbergen, 2016).
analyzing the effect of banking frictions on the equity premium.\footnote{It is natural to relate rare disasters to banking crises as experienced in the Great Recession and the Great Depression. Bank runs are not modeled in this paper due to multiple equilibria issues, which cannot be solved using perturbation methods. In spite of the fact, my findings show that time-varying limited borrowing capacity of financial intermediaries due to the agency problem can contribute to the equity premium. It is worth mentioning that while the banking friction model here endogenously amplifies the effect of technology shocks, the approach modeling exogenous rare disasters explores a negatively skewed distribution for technology shocks.}

This paper is closely related to two strands of the literature. One strand considers financial frictions in macroeconomic models. Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999, henceforth BGG) focusing on firms’ limited borrowing capacity are the most representatives of this field. More recently, Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) construct quantitative DSGE models incorporating financial intermediaries with limited borrowing capacity that is endogenously determined by balance sheet constraints. The other strand of the literature incorporates generalized recursive preferences into asset pricing models, introduced in the seminal paper of Tallarini (2000). In recent papers, Campanale, Castro, and Clementi (2010) and Swanson (2016) show that generalized recursive preferences allow models to generate substantial risk premia without distorting their ability to match macroeconomic facts. This paper considers these two strands of literature combining banking frictions as in Gertler and Karadi (2011) with generalized recursive preferences. However, in contrast to these papers, the model proposed here considers not only the effect of banking frictions on the macroeconomy, but also on financial markets. Financial market disruptions cause a sharp contraction of the real economy due to limited borrowing capacity of financial intermediaries, which impacts the equity premium.

A number of recent studies examine implications of firms’ limited borrowing ca-
pacity for asset pricing (e.g., Gomes, Yaron, and Zhang, 2003; Nezafat and Slávíc, 2015; and Bigio and Schneider, 2017). Gomes, Yaron, and Zhang (2003) build on Carlstrom and Fuerst (1997) to incorporate financial market imperfections into a macroeconomic model, while Nezafat and Slávíc (2015) and Bigio and Schneider (2017) investigate the importance of a financial shock following the approach in Kiyotaki and Moore (2012). These papers find that models with financial frictions produce a higher equity premium than frictionless models. Nevertheless, they generate counterfactual movements in the equity price or the equity premium because recessions (times of low liquidity) are associated with a low equity premium.\footnote{See Shi (2015) for the detailed reason of the counterfactual response.}

To overcome this difficulty, the proposed model here adopts the conventional asset pricing model as in Mehra and Prescott (1985) and Cochrane (2009) and solve the model nonlinearly to reflect the risk of the model. Therefore, in the model, a shock that drives an economic downturn generates a procyclical response of the equity price and a countercyclical response of the equity premium as observed in the data.

Livdan, Sapriza, and Zhang (2009, henceforth LSZ) report that more financially constrained firms have higher average equity returns. Their approach considers a partial equilibrium model with collateral constraints following Kiyotaki and Moore (1997). The proposed model in this paper complements LSZ in the sense that it analyzes the impact of banking frictions on the equity premium dynamics and on the economy. In particular, the present paper compares the effects of monetary policy on the equity premium, with and without limited borrowing capacity of financial intermediaries.

Lastly, He and Krishnamurthy (2013) build a model that considers financial intermediaries as marginal investors in asset pricing, and a more complex asset pricing structure
to calibrate risk-premia. However, He and Krishnamurthy (2013) use a simple overlapping generation model in an endowment economy for tractability. The model proposed here is, instead, a standard New Keynesian model in a production economy that can be used to investigate the dynamics of household’s stochastic discount factor and asset pricing.

The organization of the paper is as follows. Section 2 describes the baseline model with limited borrowing capacity of financial intermediaries and generalized recursive preferences. Section 3 lays out the calibration results. Sections 4 presents additional discussion and extensions. Section 5 concludes and discusses the direction of the future research. An appendix to the paper provides additional details of how the model is solved.

2.2 The Baseline Model

In this section, I begin by outlining a medium-scale New Keynesian DSGE model and use it to price equity. The model has two important ingredients: limited borrowing capacity of financial intermediaries (as in Gertler and Karadi, 2011) and generalized recursive preferences (as in Tallarini, 2000; and Swanson, 2016). Limited borrowing capacity of financial intermediaries introduces frictions between financial intermediaries and households and allows the model to have the feedback between the financial market and the economy. Generalized recursive preferences allow the model to match the size of the equity premium in the data.

There are four types of agents in the model: households, financial intermediaries, non-financial firms, and capital producers. The latter are required to make the endogenous capital price tractable as suggested by BGG. Figure 2.1 displays the building blocks of the
model. In order to produce output, non-financial firms purchase capital and hire labor from capital producers and households, respectively. Firms issue security claims, $S_t$, to buy capital, $K_{t+1}$, and pay gross return of capital, $R^k_{t+1}$, to financial intermediaries. Households give funds to financial intermediaries as deposits, $D_t$, and receive a risk-free return, $e^{r_{t+1}}$. Finally, the price of capital, $Q_t$, is endogenously determined by capital demand from non-financial firms and supply from capital producers.

### 2.2.1 Households

There is a unit continuum of identical households. Each household is endowed with generalized recursive preferences as in Epstein and Zin (1989) and Weil (1989). For simplicity, the model in the present paper employs the additive separability assumption for period utility
following Woodford (2003).\footnote{Van Binsbergen et al. (2012) uses Cobb-Douglas preferences since they consider consumption and leisure as composite good.}

\[ u(c_t, l_t) \equiv \log c_t - \chi_0 \frac{1^{1+\chi}}{1+\chi}, \quad (2.1) \]

where \( c_t \) is household consumption, \( l_t \) is labor in period \( t \), and \( \chi_0 > 0 \) is the relative weight on labor in the utility function, and \( \chi > 0 \) is the inverse Frisch elasticity of labor supply. Moreover, assuming logarithmic period utility for consumption allows a balanced growth path and unit intertemporal elasticity of substitution as in King and Rebelo (1999).

Households deposit to financial intermediaries to earn the continuously-compounded default-free interest rate, and provide labor to non-financial firms to receive their wages. Using continuous compounding is convenient for equity pricing and comparison with the finance literature. Hence, the household’s budget constraint is given by:

\[ c_t + \frac{d_{t+1}}{P_t} = w_t l_t + e^{it} \frac{d_t}{P_t} + \Pi_t, \quad (2.2) \]

where \( d_t \) is deposits, \( P_t \) is the aggregate price level (to be defined later), \( w_t \) is the real wage, \( e^{it} \) is the nominal gross risk-free return from deposits, and \( \Pi_t \) is the household’s share of profits in the economy.

Following Hansen and Sargent (2001) and Swanson (2016), I assume that households have multiplier preferences.\footnote{Rudebusch and Swanson (2012) use a generalized form of Epstein-Zin-Weil specification with nonnegative period utility:
\[ V_t = u(c_t, l_t) + \beta (E_t V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}} \]
which is similar to expected utility preferences except “twisted” and “untwisted” by the factor \( 1 - \alpha \). Note that the expected utility preferences are the special cases of generalized recursive preferences when \( \alpha = 0 \), and...} In every period, the household faces the budget con-
straint (2.2) and maximizes lifetime utility with the no-Ponzi game constraint. The household’s value function $V^h (d_t; \Theta_t)$ satisfies the Bellman equation:

$$V^h (d_t; \Theta_t) = \max_{c_t, l_t \in \Gamma} (1 - \beta) u (c_t, l_t) - \beta \alpha^{-1} \log \left[ E_t \exp \left( -\alpha V^h (d_{t+1}; \Theta_{t+1}) \right) \right],$$  \hspace{1cm} \text{(2.3)}

where $\Gamma$ is the choice set for $c_t$ and $l_t$, $\Theta_t$ is the state of the economy, $\beta$ is the household’s time discount factor, and $\alpha$ is a parameter. Risk aversion is closely related to the Epstein-Zin parameter $\alpha$ which amplifies risk aversion by including the additional risk for the lifetime utility of households.\(^8\) The household’s stochastic discount factor is given by\(^9\)

$$m_{t+1} = \beta c_t \exp \left( -\alpha V^h (d_{t+1}; \Theta_{t+1}) \right) E_t \exp \left( -\alpha V^h (d_{t+1}; \Theta_{t+1}) \right).$$  \hspace{1cm} \text{(2.4)}

The first order necessary conditions for deposit and labor are given by:

$$d_{t+1} : 1 = E_t \left( m_{t+1} e^{i_{t+1} \pi_{t+1}} \frac{1}{\pi_{t+1}} \right),$$  \hspace{1cm} \text{(2.5)}

$$l_t : \chi_0 l_t \left( \frac{1}{c_t} \right)^{-1} = w_t,$$  \hspace{1cm} \text{(2.6)}

where $\pi_{t+1} \equiv P_{t+1} / P_t$ is the inflation rate.

---

\(^8\)Precisely, $R^e = \alpha + \left( 1 + \frac{1}{\chi_0} \right)^{-1}$ for the case with period utility as (2.1). This closed-form expression considers both consumption and labor which provides additional cushion to the household against the negative shock.

\(^9\)The household’s optimization problem with generalized recursive preferences can be solved using the standard Lagrangian method. See Rudebusch and Swanson (2012) for more detail.
Then, the one-period continuously-compounded risk-free real interest rate, $r_{t+1}$, is

$$e^{-r_{t+1}} = E_t m_{t+1},$$

(2.7)

since $e^{r_{t+1}} = e^{i_{t+1}} \frac{1}{\pi_{t+1}}$.

### 2.2.2 Financial Intermediaries

There is a unit continuum of bankers, and each risk neutral banker runs a financial intermediary.\(^{10}\) The financial intermediaries lend funds to non-financial firms by using their own net worth or issuing deposits to households. As suggested by Gertler and Karadi (2011), I introduce two key assumptions to ensure that there is always friction between financial intermediaries and households. First, financial intermediaries have to borrow from households each period in the form of deposits, implying that $d_{t+1} > 0$ in each period. This assumption prevents the financial intermediary from lending funds to non-financial firms with their own capital alone. Formally, the financial intermediary’s balance sheet constraint is given by:

Balance Sheet Constraint (BC) : $Q_t s_t = n_t + d_{t+1}$,

(2.8)

where $Q_t$ is the relative price of financial claims on firms that the bank holds, $s_t$ is the quantity of claims, and $n_t$ is the banker’s net worth. The asset of the financial intermediary, $Q_t s_t$, is composed of equity capital (or net worth), $n_t$, and positive debt, $d_{t+1}$. To keep the number of bankers stable and to prevent the accumulation of net worth, there is an i.i.d. survival probability $\sigma$ for the fraction who can remain in the financial industry in the next period. So, $(1 - \sigma)$ fraction of bankers retire and consume their net worth when they

\(^{10}\)With a slight abuse of notation, I use the terms “financial intermediaries” and “banks” interchangeably in this paper.
leave. Second, households are willing to deposit in financial intermediaries. There is a moral
hazard problem between depositors and financial intermediaries: a financial intermediary
may divert a portion of its assets after deposits are collected. Consequently, the incentive
constraint must hold in order to avoid households punishing diverting bankers by ceasing
to supply deposits:

\[ \text{Incentive Constraint (IC)}: \quad V_b^t \geq \vartheta Q_t s_t, \quad (2.9) \]

where \( \vartheta \) is a fraction when the banker diverts the assets of the financial intermediary, and
\( V_b^t \) is the bank’s franchise value (defined below). As long as the banker is constrained due
to the agency frictions, the risk neutral banker’s objective is to maximize its consumption
at the exit period:

\[ \max V_b^t = E_t \left[ \sum_{j=1}^{\infty} \beta^j (1 - \sigma) \sigma^{j-1} n_{t+j} \right]. \quad (2.10) \]

Observe that the financial intermediary’s terminal wealth, \( n_{t+j} \), is the banker’s consump-
tion, \( c_{t+j}^b \), in the exit period.\(^{11}\) (2.10) can be written in the first-order recursive form:

\[ V_b^t = E_t \left[ \beta (1 - \sigma) n_{t+1} + \beta \sigma V_b^{t+1} \right]. \quad (2.11) \]

The net worth of a surviving financial intermediary in the next period, \( n_{t+1} \), is
simply the gross return of the asset net of the cost of debts:

\[ n_{t+1} = R_b^k Q_t s_t - e^{r_{t+1}} d_{t+1} \]
\[ = \left( R_b^k - e^{r_{t+1}} \right) Q_t s_t + e^{r_{t+1}} n_t, \quad (2.12) \]

\(^{11}\)For tractability, I assume the financial intermediary is risk neutral following BGG and Gertler and
Kiyotaki (2015). So, the bankers discount net worth with \( \beta \) rather than the household’s stochastic discount
factor \( m_{t+j} \).
where $R_{t+1}^k$ is the gross return of capital. Then, the growth rate of net worth is

$$\frac{n_{t+1}}{n_t} = \left( R_{t+1}^k - e^{r_{t+1}} \right) \phi_t + e^{r_{t+1}}, \tag{2.13}$$

where $\phi_t \equiv \frac{Q_{t+1}}{n_t}$ is the “leverage multiple.” Note that the growth rate of net worth is increasing in the leverage multiple when the spread, $R_{t+1}^k - e^{r_{t+1}}$, is positive.

Since (2.8) and (2.11) are constant returns to scale, (2.11) is equivalent to the following:

$$V_t^b \frac{n_{t+1}}{n_t} = E_t \left[ \beta \left( 1 - \sigma \right) + \sigma \frac{V_{t+1}^b}{n_{t+1}} \right] \frac{n_{t+1}}{n_t} \equiv \mu_t \phi_t + \nu_t, \tag{2.14}$$

where $\mu_t \equiv \beta E_t \Omega_{t+1} (R_{t+1}^k - e^{r_{t+1}})$ is the excess marginal value of assets over deposits, $\nu_t \equiv \beta E_t \Omega_{t+1} e^{r_{t+1}}$ is the marginal cost of deposits, and $\Omega_{t+1} \equiv (1 - \sigma) + \sigma \frac{V_{t+1}^b}{n_{t+1}}$ is the weighted average of the marginal values of net worth to exiting and to continuing bankers at $t + 1$.

Combining (2.9) and (2.14) yields the leverage multiple:

$$\phi_t = \frac{\nu_t}{\vartheta - \mu_t}, \tag{2.15}$$

if and only if $\mu_t \in (0, \vartheta)$ so that the incentive constraint (2.9) binds. Since all financial intermediaries face the same the leverage multiple as in (2.15), the aggregate leverage constraint is

$$Q_t S_t = \phi_t N_t, \tag{2.16}$$

where $S_t$ is the aggregate quantity of claims and $N_t$ is the aggregate net worth.

---

12Gertler and Kiyotaki (2015) calls the franchise value per unit of net worth, $\frac{V_t^b}{n_t}$, as Tobin’s Q.
The aggregate net worth consists of two components. The first is the net worth of surviving financial intermediaries. With the survival probability, $\sigma$, the banker remains in the banking sector, in which case the banker earns the net revenue, $R^k_t Q_{t-1} S_{t-1} - e^{rt} D_t$. The second corresponds to seed money, $\omega Q_t S_{t-1}$, that a new banker receives in every period from their respective household. This seed money is a small fraction, $\omega$, of the value of the exiting financial intermediary’s assets. Accordingly, the aggregate net worth of the entire banking sector is

$$N_t = \sigma \left( R^k_t Q_{t-1} S_{t-1} - e^{rt} D_t \right) + \omega Q_t S_{t-1},$$

where $D_t$ is the aggregate amount of deposits.

Lastly, aggregate consumption of exiting bankers is the fraction $(1 - \sigma)$ of net earnings of assets:

$$C^b_t = (1 - \sigma) \left[ R^k_t Q_{t-1} S_{t-1} - e^{rt} D_t \right],$$

where $C^b_t = C^b_t$ denotes aggregate consumption demanded by bankers.

### 2.2.3 Firms

#### Non-Financial Firms

There is a single final good which is produced using a continuum of intermediate goods indexed by $f \in [0, 1]$ with the following production function:

$$Y_t = \left( \int_0^1 y_t(f) \frac{1}{1+\theta} df \right)^{1+\theta},$$

where $y_t(f)$ is an intermediate good, and $\theta > 0$ is a parameter captures the equilibrium markup. The final goods firms are perfectly competitive and maximize profits subject to the
production function. This implies a downward sloping demand curve for each intermediate
good:

\[ y_t(f) = \left( \frac{p_t(f)}{P_t} \right)^{-\frac{1+\theta}{\theta}} Y_t, \]  

(2.20)

where \( P_t \) is the CES aggregate price of the final good:

\[ P_t = \left( \int_0^1 p_t(f)^{-\frac{1}{\theta}} df \right)^{-\theta}, \]  

(2.21)

which can be derived from the zero profit condition.

The economy contains a continuum of monopolistically competitive intermediate
goods firms indexed by \( f \in [0, 1] \). Firms purchase capital goods from capital producers and
hire labor from households. They also issue claims, \( s_t \), to financial intermediaries in order
to obtain financing. Firms have identical Cobb-Douglas production functions:

\[ y_t(f) = A_t k_t(f)^{1-\eta} l_t(f)^{\eta}, \]  

(2.22)

where \( k_t(f) \) and \( l_t(f) \) are firm \( f \)'s capital and labor inputs, and \( \eta \in (0, 1) \) denotes the firm’s
output elasticity with respect to labor. \( A_t \) is a technology which follows an exogenous AR(1)
process:

\[ \log A_t = \rho_A \log A_{t-1} + \epsilon^A_t, \]  

(2.23)

where \( \rho_A \in (-1, 1] \), and \( \epsilon^A_t \) follows an i.i.d. white noise process with mean zero and variance
\( \sigma^2_A \). I set \( \rho_A = 1 \) for comparability to the asset pricing literature (e.g., Tallarini, 2000;
and Swanson, 2016). Subject to the demand function and the production function, the
intermediate goods firm chooses labor, \( l_t(f) \), and capital, \( k_t(f) \). The first order necessary
conditions are:

\[ l_t(f) : \ w_t P_t = \varphi_t(f) \eta A_t \left( \frac{k_t(f)}{l_t(f)} \right)^{1-\eta}, \tag{2.24} \]

\[ k_t(f) : \ R^k_t P_t Q_{t-1} - Q_t P_t (1-\delta) = \varphi_t(f)(1-\eta) A_t \left( \frac{k_t(f)}{l_t(f)} \right)^{-\eta}, \tag{2.25} \]

where \( \varphi_t(f) \) is the Lagrange multiplier of the cost minimization problem, and \( \delta \) denotes the depreciation rate of capital. Note that \( Q_t P_t (1-\delta) \) in (2.25) is the value of the remained capital stock from the previous period. Combining these conditions yields the capital-labor ratio:

\[ \frac{k_t(f)}{l_t(f)} = \frac{1-\eta}{\eta} \frac{w_t}{R^k_t Q_{t-1} - Q_t (1-\delta)}. \tag{2.26} \]

Since the capital-labor ratio is the same for all firms as in (2.26), it is the same to the aggregate ratio:

\[ \frac{k_t(f)}{l_t(f)} = \frac{K_t}{L_t}, \tag{2.27} \]

where \( K_t \) is aggregate capital and \( L_t \) is the aggregate quantity of labor. Moreover, every firm hires capital and labor in the same way, so marginal cost is also the same across firms.

Let \( mc_t(f) \equiv \frac{\varphi_t(f)}{K_t} \) be the real marginal cost. Then, \( mc_t(f) = MC_t \) for all \( f \) since \( \varphi_t(f) \) is not an individual firm-specific factor either:

\[ MC_t = \frac{1}{A_t} w_t^\eta \left( R^k_t Q_{t-1} - (1-\delta) Q_t \right)^{1-\eta} \left( \frac{1}{\eta} \right)^\eta \left( \frac{1}{1-\eta} \right)^{1-\eta}. \tag{2.28} \]

Therefore, the demand functions for capital and labor are:

\[ R^k_{t+1} = \frac{MC_{t+1} (1-\eta) A_{t+1} \left( \frac{K_{t+1}}{Q_{t+1}} \right)^{-\eta} + (1-\delta) Q_{t+1}}{Q_t}, \tag{2.29} \]
\[ w_t = MC_t \eta A_t \left( \frac{K_t}{L_t} \right)^{1-\eta}. \] (2.30)

Each intermediate goods firm sets the new contract price \( p_t(f) \) to maximize the firm’s lifetime profit according to Calvo contracts: only a fraction, \( 1 - \xi \), can adjust its price each period. Hence, the value of the firm is given by:

\[
\max_{p_t(f)} E_t \sum_{j=0}^{\infty} m_{t,t+j}(P_{t+j}/P_t) \xi^j \left[ p_t(f)e^{\bar{\pi} j} y_{t+j}(f) - mc^n_{t+j}(f)y_{t+j}(f) \right],
\] (2.31)

where \( m_{t,t+j} \equiv \Pi_{i=1}^{t+j} m_{t+i} \) is the stochastic discount factor of household from period \( t \) to \( t+j \), \( \bar{\pi} \) is the steady-state inflation rate, and \( mc^n_t(f) \) is firm-specific nominal marginal cost.

The first order necessary condition of (2.31) with respect to \( p_t(f) \) yields the standard New Keynesian price optimality condition:

\[
p_t^*(f) = \frac{(1 + \theta)E_t \sum_{j=0}^{\infty} m_{t,t+j} P_{t+j}^{1+\theta} Y_{t+j} e^{\bar{\pi} j}}{E_t \sum_{j=0}^{\infty} m_{t,t+j} \xi^j P_{t+j}^{\frac{1}{2}} Y_{t+j} e^{\bar{\pi} j}}.
\] (2.32)

Note that the optimal price \( p_t^*(f) \) is a markup over a weighted average of current and expected future marginal costs.

**Capital Producers**

Lastly, there is a continuum of representative capital producers. They sell new capital to intermediate goods firms at price \( Q_t \), and produce it using the input from the final output at price unity subject to convex (quadratic) investment adjustment cost.\(^{13}\) The capital producer chooses new capital, \( I_t \), in order to maximize expected discounted profits over her lifetime:

\(^{13}\)While there are multiple ways to introduce the investment adjustment cost, this paper follows along the lines of Gertler, Kiyotaki, and Queralto (2012).
\[
\max_{I_t} E_t \sum_{j=0}^{\infty} m_{t,t+j} \left\{ (Q_{t+j} - 1) I_{t+j} - \kappa \left( \frac{I_{t+j+1}}{I_{t+j}} - 1 \right)^2 I_{t+j} \right\},
\]
(2.33)

where \( \kappa \) denotes the elasticity of the investment adjustment costs. Observe that with zero investment adjustment costs, \( \kappa = 0 \), the firms would produce infinite capital if \( Q_t > 1 \).

A large elasticity of the investment adjustment costs \( \kappa \) implies that the capital producer cannot change her supply easily.

The first order necessary condition with respect to \( I_t \) yields:

\[
Q_t = 1 + \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 + \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_t}{I_{t-1}} - 1 \right) - E_t m_{t,t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2,
\]
(2.34)

which is the supply of new capital.

### 2.2.4 Aggregate Resource Constraints and Monetary Policy

As the goal of this paper is illustrating the underlying mechanisms of how limited borrowing capacity of financial intermediaries affects the risk premium, I want to keep the model as simple as possible by considering technology shocks only. This is not an unreasonable assumption: According to Rudebusch and Swanson (2012), the response of the term premium to a technology shock shows a greater response by a factor of 250 and 625 than to a monetary policy shock or government spending shock, respectively. Thus, Tallarini (2000), Gomes, Yaron, and Zhang (2003), and Swanson (2016) also did not consider any exogenous shock other than a technology shock.

The downward sloping demand curve and the production function yields the aggregate output:

\[
Y_t = \Delta_t^{-1} A_t K_t^{1-\eta} L_t^{\eta},
\]
(2.35)

20
where \( \Delta_t \equiv \int_0^1 \left( \frac{p(f)}{P(t)} \right)^{-\frac{1+B}{B}} df \) denotes the cross-sectional price dispersion. A monetary authority in the model determines the one-period nominal interest rate, \( i_t \), by a simple Taylor-type rule with interest-rate smoothing:

\[
i_t = \rho i_{t-1} + (1 - \rho) \left[ r + \log \pi_t + \phi_{\pi} (\log \pi_t - \log \tilde{\pi}) + \frac{\phi_y}{4} (y_t - \bar{y}_t) \right],
\]

where \( \rho \in (0, 1) \) is the smoothing parameter, \( r = \log(1/\beta) \) is the continuously compounded real interest rate in steady state, \( \pi_t \) is the inflation rate, \( \tilde{\pi} \) is the target inflation of the monetary authority, \( y_t \) is the log of output \( Y_t \),

\[
\bar{y}_t = \rho \bar{y}_{t-1} + (1 - \rho_y) y_t,
\]

is a trailing moving average of \( y_t \), and \( \phi_{\pi}, \phi_y \in \mathbb{R} \) and \( \rho_y \in [0, 1) \) are parameters. As suggested by Swanson (2016), the term \( (y_t - \bar{y}_t) \) in (2.36) is an empirically motivated measure of the output gap. In practice, the central bank adjusts the short term nominal interest rate when the output deviates from its recent history. Since monetary policy also affects the real risk-free return according to the Fisher equation, setting the output gap with (2.37) helps to generate the risk premium consistent with the actual data.

Finally, the economy-wide resource constraint is given by:

\[
Y_t = C_t + C^b_t + \left\{ 1 + \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t,
\]

where \( C_t = c_t \) denotes aggregate consumption of households.

### 2.2.5 The Equity Premium

I follow the conventional asset pricing theory using the stochastic discount factor obtained from the model (e.g., Mehra and Prescott, 1985; and Cochrane, 2009). In addition, I model
stocks as a levered claim on the aggregate consumption for simplicity. In every period, the levered equity pays the consumption stream $C^\nu_t$. Note that $\nu$ is the degree of leverage which captures broad leverage in the economy, including operational and financial leverage. Therefore, the price of an equity security in equilibrium is given by:

$$p^e_t = E_t \left( m_{t+1} \left( C^\nu_{t+1} + p^e_{t+1} \right) \right),$$  \hspace{1cm} (2.39)

where $p^e_t$ denotes the ex-dividend price of an equity at time $t$.

Let $R^e_{t+1}$ be the ex-post gross return on equity, $R^e_{t+1} \equiv \frac{C^\nu_{t+1} + p^e_{t+1}}{p^e_t}$. Then, (2.39) is equivalent to

$$1 = E_t \left( m_{t+1} R^e_{t+1} \right),$$  \hspace{1cm} (2.40)

which is the same form as the intertemporal Euler equation.

Defining the equity premium as the difference between the expected return to equity and the risk-free rate, $\psi^e_t \equiv E_t R^e_{t+1} - e^{r_{t+1}}$. By the definition of covariance, (2.40) is equivalent to

$$E_t \left( m_{t+1} R^e_{t+1} \right) = \text{Cov}_t \left( m_{t+1}, R^e_{t+1} \right) + E_t m_{t+1} E_t R^e_{t+1}$$  \hspace{1cm} (2.41)

where $\text{Cov}_t$ denotes the conditional covariance. Using (2.5) and (2.41), and dividing both sides by $E_t m_{t+1}$ yields,

$$\psi^e_t = \frac{1}{E_t m_{t+1}} - \frac{\text{Cov}_t \left( m_{t+1}, R^e_{t+1} \right)}{E_t m_{t+1}} - e^{r_{t+1}}$$

$$= - \frac{\text{Cov}_t \left( m_{t+1}, R^e_{t+1} \right)}{E_t m_{t+1}}$$

$$= - \text{Cov}_t \left( \frac{m_{t+1}}{E_t m_{t+1}}, R^e_{t+1} \right).$$  \hspace{1cm} (2.42)

---

14This interpretation of dividends is simpler and more realistic asset pricing method rather than using a claim on the firm’s profit. This is because firms issue both equity and debt in the real world (e.g., Abel, 1999; Gourio, 2012; Campbell, Pfueger, and Viceira, 2014; and Swanson, 2016).
Intuitively, (3.37) shows why the equity is a very long-lived asset. Recall that the household’s stochastic discount factor is comprised of the consumption and the value function, $V_h^t$, that is the infinite sum of discounted future period utilities. The equity premium is thus sensitive to any changes in the consumption, even at a distant period.

### 2.2.6 Solution Method

I solve the model above using a third-order perturbation method based on the algorithm of Swanson, Anderson, and Levin (2006). I use this solution method for three reasons. First, I have eight state variables: $A_{t-1}$, $\triangle_{t-1}$, $D_{t-1}$, $I_{t-1}$, $i_{t-1}$, $K_{t-1}$, $r_{t-1}$, $\bar{y}_{t-1}$ and one shock $\epsilon_t^A$. Due to high dimensionality, projection methods are computationally not feasible. Second, a third-order perturbation shows almost the same performance as projection methods for models with generalized recursive preferences, but with much faster computing time (Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao, 2012). The model incorporates limited borrowing capacity of financial intermediaries which has the amplification structure containing many state variables. Thus, computation time is also important because a third-order accurate solution may take considerable time to compute. Lastly, a third-order perturbation is necessary to capture the dynamic of the risk premia, such as the impulse-response analysis of the equity premium.

### 2.3 Model Results

I calibrate the model rather than estimate the parameters since the main objective of this study is to illuminate the role of limited borrowing capacity of financial intermediaries on
Table 2.1: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Descriptions</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.992</td>
<td>Discount rate</td>
<td></td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>0.79</td>
<td>Relative utility weight of labor</td>
<td>To normalize $L = 1$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>3</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>Del Negro et al. (2015)</td>
</tr>
<tr>
<td>$R^c$</td>
<td>60</td>
<td>Relative risk aversion</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.6</td>
<td>Labor share</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Monopolistic markup</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.8</td>
<td>Calvo contract parameter</td>
<td>Altit et al. (2011)</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>1</td>
<td>Persistence of technology</td>
<td>Tallarini (2000)</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.007</td>
<td>Standard deviation of technology shocks</td>
<td>King and Rebelo (1999)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>3</td>
<td>Elasticity of investment adjustment cost</td>
<td>Del Negro et al. (2015)</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.19</td>
<td>Seizure rate</td>
<td>Gertler and Kiyotaki (2015)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.002</td>
<td>Proportional transfer to new bank</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.95</td>
<td>Survival probability of bank</td>
<td>Gertler and Kiyotaki (2015)</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.73</td>
<td>Smoothing parameter of monetary policy</td>
<td>Rudebusch (2002)</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>0.53</td>
<td>Response of monetary policy to inflation</td>
<td>Rudebusch (2002)</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.93</td>
<td>Response of monetary policy to output</td>
<td>Rudebusch (2002)</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>0.008</td>
<td>The monetary authority’s inflation target</td>
<td>Swanson (2016)</td>
</tr>
<tr>
<td>$\rho_{\bar{y}}$</td>
<td>0.9</td>
<td>Coefficient of trailing moving average</td>
<td>Swanson (2016)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3</td>
<td>Degree of leverage</td>
<td>Abel (1999)</td>
</tr>
</tbody>
</table>

asset pricing. As can be seen in Table 3.1, the baseline calibration is fairly standard for both macroeconomics and finance variables.

For the household’s discount factor, $\beta$, the depreciation rate, $\delta$, and the elasticity of output with respect to labor, $\eta$, I use conventional values. I also set the relative utility weight of labor, $\chi_0 = 0.79$, to normalize the steady state labor, $L = 1$. I use relatively high risk aversion $R^c = 60$ for simplicity and comparability to the asset pricing literature.

Moreover, this relatively high value is common in the macro-finance literature, and is due...
to the small amount of uncertainty in the simple model.\footnote{Empirical papers using generalized recursive preferences estimate a relatively high risk aversion. For instance, Piazzesi and Schneider (2006) estimate risk aversion to 57, and van Binsbergen et al. (2012) estimate a value of about 65. In addition, Tallarini (2000), Rudebusch and Swanson (2012) and Swanson (2016) use baseline calibration of risk aversion as 100, 75, and 60, respectively.} As Bloom (2009) shows, the real economy has many uncertainties. In contrast, agents in the model perfectly know all parameter values and equations, so the quantity of risk is very small. Barillas, Hansen, and Sargent (2009) document that increasing the uncertainty of the model could lower risk aversion. Another method is to increase the quantity of risk by introducing additional shocks such as long-run risk, heterogeneous agents, or rare disaster.

For the rest of the macroeconomic parameters, I use estimates from previous studies. The inverse Frisch elasticity of labor supply, $\chi$, is set to 3 as in Del Negro, Giannoni, and Schorfheide (2015). The calibrated value of the Calvo contract parameter, $\xi = 0.8$, implies that the lifetime of the contract is five quarters as in Altig, Christiano, Eichenbaum, and Lindé (2011) and Del Negro, Giannoni, and Schorfheide (2015). I also set the elasticity of investment adjustment costs $\kappa = 3$ as the estimate in Del Negro, Giannoni, and Schorfheide (2015) and Gelain and Ilbas (2017). I set firm’s steady state markup, $\theta$, as 10 percent, consistent with the estimates in Smets and Wouters (2007). The persistence of technology, $\rho_A$, is set to 1 as in Tallarini (2000), and the standard deviation of technology shocks, $\sigma_A$, is set to 0.007, consistent with the estimates in King and Rebelo (1999). These calibrated values generate high enough risk in the model so that the equity premium in the model is sufficiently large.

Turning to the financial sector parameters, I set the fraction of capital that can be diverted to $\vartheta = 0.19$, as in Gertler and Kiyotaki (2015). Proportional transfer to a new financial intermediary, $\omega$, is set to 0.002 as in Gertler and Karadi (2011). Lastly, I set
survival probability in banking industry, $\sigma = 0.95$, implying twenty quarters of the expected lifetime for financial intermediary as in Gertler and Kiyotaki (2015).

I set the smoothing parameter of monetary policy, $\rho_t = 0.73$, the response of monetary policy to output, $\phi_y = 0.93$, and the response of monetary policy to inflation, $\phi_\pi = 0.53$, as in Rudebusch (2002). The monetary authority’s inflation target, $\bar{\pi}$, is set to 0.008 which implies 3.2 percent per year in the nonstochastic steady state as in Swanson (2016). I set the coefficient of trailing moving average of output to $\rho_y = 0.9$, which implies that the central bank considers the whole history of past output levels, because $\bar{y}_t$ is an infinite moving average of past $y_t$. Finally, I calibrate the degree of leverage as $\nu = 3$ to match the empirical estimates of dividend growth’s volatility following Abel (1999) and Bansal and Yaron (2004).

2.3.1 Macroeconomic Implications

Figure 3.2 depicts the impulse response functions to a one-standard-deviation (0.7 percent) negative technology shock for the third-order solution of the models. These are computed by the period-by-period difference between two scenarios: (i) given nonstochastic steady state values of state variables, I simulate out the variables in the absence of a shock and (ii) I repeat the same process in the presence of one standard deviation to the shock in the first period. The horizontal axes are periods (quarters) and the vertical axes are percentage deviations from the nonstochastic steady state.

---

16 Note that the average historical lag is about 10 quarters.
17 There are many other alternatives. For example, I draw random numbers for the technology shock $\epsilon_t^A$ from its distribution using a random number generator and use these values for the simulation. There is, however, no large difference in the results between these two methods because agents in the model economy do not have perfect foresight.
Figure 2.2: Impact of Limited Borrowing Capacity of Financial Intermediaries on Macro Variables

Note: The figure plots third-order impulse response functions of the return of capital, $R^k_t$, price of capital, $Q_t$, marginal cost, $MC_t$, inflation rate, $\pi_t$, the net risk-free return, $r_{t+1}$, the spread, $E_tR^k_{t+1} - e^{r_{t+1}}$, consumption, $C_t$, investment, $I_t$, and output, $Y_t$, to a one-standard-deviation (0.7 percent) negative technology shock. The solid blue lines in each panel plot impulse response functions with limited borrowing capacity of financial intermediaries (the baseline model), and the dashed orange lines plot impulse response functions without banking frictions. See text for details.
To better highlight the role of limited borrowing capacity of financial intermediaries, I compare the model responses to those of a model without limited borrowing capacity of financial intermediaries. This alternative model is a medium-scale New Keynesian DSGE model with generalized recursive preferences, and the calibration is the same as the baseline model. Moreover, in the alternative model, the arbitrage condition between the return of capital and the real gross risk-free return holds due to the absence of banking frictions. The solid blue lines in each panel plot impulse response functions to the baseline model, and the dashed orange lines plot the impulse response functions for the alternative model.

Figure 3.2 shows the impulse responses of the key macroeconomic variables to the negative technology shock. The baseline model generates the stronger responses of the aggregate variables compared to the alternative model. The negative shock reduces the marginal productivity of capital and therefore the return on capital. As a consequence, the capital price declines. A reduction in the capital price deteriorates the balance sheets of financial intermediaries, leading to a decline in their borrowing and lending capacity. Thus, the baseline model produces a stronger contraction in economic activity.

As the top right panel illustrates, the response of marginal cost is attenuated in the baseline model with limited borrowing capacity of financial intermediaries. Since the return on capital further declines with banking frictions, the production cost of intermediate goods firms falls. The negative technology shock drives up marginal cost, while the reduction in the return on capital moderately offsets the rise in marginal cost. As a result, inflation rises less in the baseline model with banking frictions.\textsuperscript{18} The real risk-free rate, in turn, declines

\textsuperscript{18} Although the mechanisms are different, the behavior of inflation is similar to that of Gilchrist, Schoenle, Sim, and Zakrajek (2017). The authors find through micro-level data that only intermediary goods firms that are bound by financial constraints raise their prices in recession and explain why inflation has remained
Table 2.2: The Model-Implied Equity Premium

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_t^e$</td>
<td>$\sigma_t(r_{t+1}^e)$</td>
</tr>
<tr>
<td>$\sigma_t$</td>
<td>$\sigma_t(r_{t+1})$</td>
</tr>
<tr>
<td>6.06</td>
<td>6.19</td>
</tr>
</tbody>
</table>

Note: This table reports the model-implied equity premium, $\psi_t^e$, the conditional standard deviation of the net equity return, $\sigma_t(r_{t+1}^e)$, and the conditional standard deviation of the net risk-free return, $\sigma_t(r_{t+1})$, in annualized percentage points. The baseline model is a New Keynesian DSGE model with generalized recursive preferences and limited borrowing capacity of financial intermediaries, while the alternative model has no banking frictions. See the text for more details.

Further in the presence of banking frictions as can be seen in the middle center panel. This is because a stronger contraction in economic activity allows the central bank to lower the nominal interest rate. The spread between $E_t R_{t+1}^k$ and $e^r_{t+1}$ rises in the baseline model, while it declines in the alternative model without banking frictions.\(^{19}\) The bottom panels show a more pronounced fall in consumption, investment, and output in the baseline model. Lastly, the similarity in impulse response functions for key macroeconomic variables in this model and these in the literature allow us to focus on implications for the equity premium.

2.3.2 Equity Premium Results

Table 3.2 reports the model-implied equity premium, $\psi_t^e$, the conditional standard deviation of the net equity return, $\sigma_t(r_{t+1}^e)$, and the conditional standard deviation of the net risk-free return, $\sigma_t(r_{t+1})$, in annualized percentage points. Since the Great Recession.\(^{19}\) In models without banking frictions, the impulse response of the spread is not completely zero, because the figure is the third-order impulse response. On the other hand, in the first-order impulse response, the spread always shows a zero response as in Gertler and Karadi (2011).
free return, $\sigma_t(r_{t+1})$, from the baseline model and the alternative model. The forth column reports the equity premium, $\psi_t^e$, implied by the model with banking frictions, solved to third order, holding other parameters of the model set at their benchmark values. The model-implied equity premium, $\psi_t^e = 6.52$, matches its empirical estimate (typically about 4 to 8.4 percent for quarterly excess returns at an annual rate).\textsuperscript{20} The fifth and sixth columns show that the standard deviation for the net equity return is 7.4 percent and the standard deviation of the risk-free return is 0.57 percent. The volatility predicted by the baseline model are short of fully accounting for its counterpart observed in the data (typically about 1 to 2 percent for the risk-free rate and 15 to 20 percent for the equity return).\textsuperscript{21} Accordingly, the Sharpe ratio, $\psi_t^e/\sqrt{\text{Var}_t(r_{t+1}^e)}$, is 0.88, which is larger than the empirical estimates by Lettau and Ludvigson (2010) at about 0.2 to 0.4. However, this is not surprising. Since the baseline model has one exogenous shock the model-implied standard deviation is relatively small. This problem can be mitigated by adding other shocks to the model. Shocks that are less persistent than a technology shock such as a monetary policy shock or a government spending shock increase the volatility of the equity return without much changing the equity premium (Li and Palomino, 2014).

The first column presents the equity premium from the alternative model without banking frictions, which predicts a high equity premium of 6.03 percent.\textsuperscript{22} The absence of banking frictions reduces the equity premium by about 46 basis points. This difference originated from imperfect borrowing markets accounts for 7.6 percent of the equity pre-

\textsuperscript{20}See, Table 1 in Mehra and Prescott (2003).
\textsuperscript{21}See, Table 2 in Rudebusch and Swanson (2012) and Table 3 in Croce (2014).
\textsuperscript{22}This result implies that generalized recursive preferences play an important role in accounting for the size of the equity premium.
mium. As shown in the second and third columns, the conditional standard deviation of the net equity return is 6.19 percent and the Sharpe ratio is 0.98 in the absence of banking frictions.

One might be interested in the underlying mechanism by which banking frictions increase the equity premium. The presence of banking frictions amplifies business cycle fluctuations, leading to a rise in the volatility of the stochastic discount factor. The standard deviation of the stochastic discount factor is 64.72 percentage points in the presence of financial frictions, while it is 62.22 percentage points in the frictionless model. In response to a negative technology shock, the technology declines, yielding a decline in the return of capital, $R_k^t$. The reduction in the return of capital lowers the aggregate net worth of the financial intermediaries, $N_t$, as the decline in the return of capital reduces the value of assets for financial intermediaries. The reduction in financial intermediaries’ net worth weakens their lending capacity, reducing the quantity of claims, $S_t$, and capital, $K_{t+1}$. In turn, the price of capital, $Q_t$, falls as the demand for capital declines. The lower capital price further pushes down the return of capital, the net worth of financial intermediaries, and the price of capital, respectively. As a consequence, the amplification mechanism of limited borrowing capacity of financial intermediaries increases the volatility of the stochastic discount factor.

Figure 3.3 plots the impulse response functions to a one-standard-deviation (0.7 percent) negative technology shock. The solid blue lines and the orange dashed lines in each panel depict the impulse response functions for the baseline model and the alternative model.

Gomes, Yaron, and Zhang (2003) investigate the implications of costly external finance of firms on asset price fluctuations within a real business cycle framework. They find that the equity premium in their model is very small (0.012 to 0.022 percent) and is procyclical. As they point out, this property of the equity premium is at odds with the data. In contrast to their model, the baseline model predicts a countercyclical equity premium.
Figure 2.3: Impact of Limited Borrowing Capacity of Financial Intermediaries on Financial Variables

*Note:* Third-order impulse response functions for the stochastic discount factor, $m_t$, the equity price, $p_t^e$, and the equity premium, $\psi_t^e$, to a one-standard-deviation (0.7 percent) negative technology shock. The solid blue lines in each panel plot impulse response functions with limited borrowing capacity of financial intermediaries (the baseline model), and the dashed orange lines plot impulse response functions without banking frictions. See the text for more details.
without banking frictions, respectively. The left-hand panel reports the impulse response function for the stochastic discount factor, $m_t$, to the shock. The stochastic discount factor jumps about 65 percent in response to the negative technology shock for the baseline model, while it jumps about 62 percent for the alternative model without banking frictions.

The right panel shows that the equity premium rises more with banking frictions. The initial response of the equity premium is 4 times larger in the baseline model compared to that of the alternative model. This evidence indicates that financial intermediaries’ borrowing constraints generate a substantial contribution to the equity premium. The sign of the equity premium is positive, indicating that the conditional covariance between the stochastic discount factor and the return of the equity is negative. The middle panel shows the impulse response function for the equity price, $p_e$. The baseline model predicts that the equity price plummets about 2.5 percent in response to the shock and gradually converges to its new nonstochastic steady state. In the absence of banking frictions, the equity price drops less and remain higher for a considerable time period than what the baseline model predicts.

### 2.4 Additional Discussion and Extensions

In this section, I discuss whether a particular parameter can increase the impact of banking frictions on the macroeconomy and the equity premium.
2.4.1 Smoothing Parameter of Monetary Policy

This subsection analyzes the impacts of monetary policy on the equity premium by controlling the smoothing parameter, $\rho_i$, rather than only the inflation coefficient or the output gap coefficient separately. Limited borrowing capacity of financial intermediaries affects to setting of the monetary authority’s policy instrument, since it reduces output more and increases inflation less when there is a negative technology shock. With a high degree of policy inertia, the central bank adjusts the nominal risk-free rate gradually in response to changes in economic conditions. This implies that the equity premium can be influenced by the smoothing parameter of monetary policy, which interacts with banking frictions.

Panel A in Table 3.3 reports the model-implied equity premiums calculated with different smoothing parameters, while fixing other parameters at their benchmark calibration as in Table 3.1. In the model with limited borrowing capacity of financial intermediaries, the equity premium increases along with the smoothing parameter of monetary policy. By contrast, the model without banking frictions decreases the equity premium with more persistence in interest rate. The smoothing parameter increases the equity premium by 8 basis points in the baseline model when $\rho_i$ is changed to 0.8 from 0.6, while it decreases the equity premium by 31 basis points in the alternative model without banking frictions.

The model-implied equity premium is more sensitive to the interest rate smoothing in the frictionless model. The risk-free rate increases as the interest rate inertia decreases, since inflation is relatively higher in the model without banking frictions. This reduces the inflation gap but destabilizes output, which induces more compensation for holding equity. As a result, a smaller smoothing parameter increases the equity premium in the frictionless model.
Table 2.3: Comparison of Equity Premium

<table>
<thead>
<tr>
<th>Smoothing parameter $\rho_i$</th>
<th>Seizure rate $\vartheta$</th>
<th>Risk aversion $R^c$</th>
<th>Equity premium without LBC</th>
<th>Equity premium with LBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.19</td>
<td>60</td>
<td>6.24</td>
<td>6.48</td>
</tr>
<tr>
<td>0.8</td>
<td>0.19</td>
<td>60</td>
<td>5.93</td>
<td>6.56</td>
</tr>
<tr>
<td>Panel B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.73</td>
<td>0.38</td>
<td>60</td>
<td>6.06</td>
<td>6.60</td>
</tr>
<tr>
<td>0.73</td>
<td>0.68</td>
<td>60</td>
<td>6.06</td>
<td>6.67</td>
</tr>
<tr>
<td>Panel C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.73</td>
<td>0.19</td>
<td>10</td>
<td>0.98</td>
<td>1.06</td>
</tr>
<tr>
<td>0.73</td>
<td>0.19</td>
<td>30</td>
<td>3.01</td>
<td>3.24</td>
</tr>
<tr>
<td>0.73</td>
<td>0.19</td>
<td>90</td>
<td>9.12</td>
<td>9.80</td>
</tr>
</tbody>
</table>

Note: This table reports the equity premium implied by the model in annualized percentage points with different values of smoothing parameter of monetary policy, $\rho_i$, seizure rate, $\vartheta$, and risk aversion, $R^c$. Model with LBC is a New Keynesian DSGE model with generalized recursive preferences and limited borrowing capacity of financial intermediaries, while Model without is a standard New Keynesian DSGE model without banking frictions. See the text for more details.

The smoothing parameter also affects the dynamics of the equity premium with banking frictions. Figure 3.4 plots the impulse response functions for the risk-free return,
Figure 2.4: Impulse Response Functions with Different Smoothing Parameters

(a) Model with Limited Borrowing Capacity of Financial Intermediaries

Note: The figure plots third-order impulse response functions of the net risk-free return, $r_t$, the equity price, $p^e_t$, and the equity premium, $\psi^e_t$, to a one-standard-deviation (0.7 percent) negative technology shock. The blue solid, the dashed orange, and dash-dot green lines report the impulse response functions from the models when $\rho_i = 0.8$, 0.6, and 0, respectively. See text for details.

(b) Model without Limited Borrowing Capacity of Financial Intermediaries

Note: The figure plots third-order impulse response functions of the net risk-free return, $r_t$, the equity price, $p^e_t$, and the equity premium, $\psi^e_t$, to a one-standard-deviation (0.7 percent) negative technology shock. The blue solid, the dashed orange, and dash-dot green lines report the impulse response functions from the models when $\rho_i = 0.8$, 0.6, and 0, respectively. See text for details.
the equity price, and the equity premium to 0.7 percent negative technology shock with different interest rate inertia values. The solid blue lines in each panel plot impulse response functions for a stronger smoothing parameter ($\rho_i = 0.8$), the dashed orange lines plot the impulse response functions for weaker inertia ($\rho_i = 0.6$), and the dash-dot green lines plot the impulse response functions for no interest rate smoothing. Generally, the central bank concerns the increased inflation gap more because the coefficient of the inflation gap, $\phi_\pi$, is 0.53 and the coefficient of the output gap, $\phi_y$, is about 0.23 per quarter.

In the presence of banking frictions, the risk-free return always responds negatively even if there is no inertia due to a deeper recession and mild inflation. The responses of consumption and equity price are therefore not as volatile as in the frictionless model, and higher interest rate smoothing parameter increases the response of the equity premium as in the right panel at (a). On the other hand, the case is different in the frictionless model. Since inflation is relatively higher and output is less decreased, the central bank sets the risk-free interest rate positively when there is no interest rate inertia as in the left panel at (b). In turn, the responses of consumption and the equity price are further reduced and more volatile due to output destabilization. As a consequence, the equity premium responds more positively with smaller interest rate inertia.

### 2.4.2 Seizure Rate

This subsection analyzes how the seizure rate, $\vartheta$, affects the macroeconomy and the equity premium through limited borrowing capacity of financial intermediaries. The seizure rate is the fraction of capital assets that can be diverted by the financial intermediary. This is one of the most important parameters for banking frictions because the incentive constraint
(2.9) binds when the excess marginal value of assets over deposits, \( \mu_t \), lies between zero and the seizure rate. Accordingly, the multiple leverage, \( \phi_t \), depends on the seizure rate while the amplification effect of banking frictions varies by changing the seizure rate. To analyze the role of the seizure rate, I set it to 0.38 and 0.68 following Gertler and Karadi (2011) and Gelain and Ilbas (2017), holding other parameters of the model set at their benchmark calibration.

Panel B in Table 3.3 presents the equity premium with higher seizure rates. This table suggests that increasing \( \vartheta \) increases the equity premium. For example, when the seizure rate increases to 0.68, the equity premium is 6.67 percent. In this case, the difference with the equity premium from the model without banking frictions is 61 basis points. The reason for this result is that the volatility of the stochastic discount factor increases. The standard deviation of the stochastic discount factor is 66.86 percentage points when the seizure rate is 0.68.

### 2.4.3 Relative Risk Aversion

This subsection analyzes the effect of the coefficient of relative risk aversion on the equity premium along with limited borrowing capacity of financial intermediaries. The coefficient of relative risk aversion, which measures the household’s attitudes toward risk, directly related to the equity premium. Swanson (2012) analytically shows that any risk premium positively depends on risk aversion in a second order approximation. Thus, the risk aversion is one of the most important parameters for the mean of the equity premium because the positive relationship does not change even in a higher order approximation.

Panel C in Table 3.3 presents the equity premium with various values of relative
risk aversion, $R^c$, holding other parameters of the model set at their benchmark calibration. The equity premium increases with risk aversion since it raises the volatility of the stochastic discount factor. At first glance, the difference in the equity premium due to limited borrowing capacity of financial intermediaries seems to be increasing as the risk aversion increases. However, the ratio of increase is quite stable at around 8 percent even if the difference is increasing. This ratio of increase is very stable and barely depend on risk aversion. For example, in the case of $R^c = 90$, the equity premium increases 68 basis points; this difference accounts for 7.5 percent of the equity premium from the model without limited borrowing capacity of financial intermediaries.

### 2.4.4 Investment Adjustment Costs

Jermann (1998) and Gomes, Yaron, and Zhang (2003) astutely note that increasing the adjustment costs of capital improves the asset pricing performance by raising both the volatility of consumption and stock returns. Table 2.4 shows the model-implied equity premium for various investment adjustment costs. The equity premium is not very sensitive to $\kappa$, with or without limited borrowing capacity of financial intermediaries. For example, in the frictionless model, the equity premium increases only 18 basis points, even if the investment adjustment cost is raised unrealistically to 30 from 3.\footnote{This is a big difference from Gomes, Yaron, and Zhang (2003) where the equity premium increases by a factor of four in respect to a similar change.} Therefore, the equity premium is not sensitive to the presence of investment adjustment costs.

The main reason for this is that, as in BGG, my model introduces separate capital producers to easily determine the capital price endogenously. Since households do not own
the capital stock, the inelastic supply of capital has little effect on the volatility of consumption of households and does not raise the volatility of the stochastic discount factor substantially. In contrast, consumption smoothing requires more cost for households with the high elasticity of investment adjustment costs in the standard DSGE model. Accordingly, the stochastic discount factor fluctuates more and this increases the equity premium in Jermann (1998).

Moreover, households do not have habit preferences in the model. If the household owns the capital stock, consumption smoothing can be affected greatly by habit, as in Jermann (1998). At the same time, the equity premium decreases as the investment adjustment costs get larger in the model with limited borrowing capacity of financial intermediaries. For instance, the equity premium is reduced about 12 basis points by increasing the elasticity of investment adjustment costs to 30 from 3. As $\kappa$ increases, the variability of capital decreases, which reduces the volatility of the bank’s net worth. Accordingly, it is less costly to smooth consumption for the households although the impact is very small.

Finally, for a more detailed examination, I decompose the standard deviation of the stochastic discount factor into two parts. The first is a marginal rate of substitution of consumption, so it is the same as the stochastic discount factor from the expected utility preferences. The second is the additional volatility of the stochastic discount factor due to generalized recursive preferences. The result tells us that the volatility of the stochastic discount factor varies due to the additional part, rather than from changes of consumption growth in two models. In the model with limited borrowing capacity of financial intermediaries, the adjustment costs of investment have a small impact on the household’s
Table 2.4: Investment Adjustment Costs

<table>
<thead>
<tr>
<th></th>
<th>Model without LBC</th>
<th>Model with LBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>$\kappa$</td>
<td>$\kappa$</td>
</tr>
<tr>
<td></td>
<td>$= 3$</td>
<td>$= 10$</td>
</tr>
<tr>
<td>$\sigma(m_t)$</td>
<td>62.22</td>
<td>62.29</td>
</tr>
<tr>
<td>$\sigma(m_C^C)$</td>
<td>0.55</td>
<td>0.60</td>
</tr>
<tr>
<td>$\sigma(m_V^V)$</td>
<td>61.68</td>
<td>61.70</td>
</tr>
<tr>
<td>$\psi_e^t$</td>
<td>6.06</td>
<td>6.16</td>
</tr>
</tbody>
</table>

Note: This table reports the model-based unconditional moment of the stochastic discount factor, $\sigma(m_t)$, and the equity premium, $\psi_e^t$. $\sigma(m_C^C)$ denotes the standard deviation of the stochastic discount factor due to consumption growth, and $\sigma(m_V^V)$ denotes the additional standard deviation of the stochastic discount factor due to generalized recursive preferences. Model with LBC is a New Keynesian DSGE model with generalized recursive preferences and limited borrowing capacity of financial intermediaries, while Model without LBC is a standard New Keynesian DSGE model without banking frictions. All numbers are in percentage points. See text for details.

consumption smoothing; therefore, it does not change the equity premium substantially.

2.5 Conclusion

This paper examines the effect of limited borrowing capacity of financial intermediaries on the equity premium with two modifications to a medium-scale New Keynesian DSGE model: first, the model includes banking frictions, which allows the model to have interactions between the financial market and the macroeconomy, and second, generalized recursive preferences, which generate a substantial risk premium without compromising stylized macroeconomic facts. A quantitative analysis of the model shows that limited borrowing capacity of financial intermediaries is a very plausible and new amplification mechanism.
for asset pricing in the model because it increases the volatility of the stochastic discount factor and lowers the risk-free rate. Limited borrowing capacity of financial intermediaries increases the price of the risk in the model since it makes consumption more volatile. Moreover, limited borrowing capacity of financial intermediaries generates more severe recessions but lower inflation in response to a negative technology shock. Due to the lower inflation, the central bank has more incentive to lower the risk-free interest rate in a recession and this increases the equity premium—the difference between the expected return to equity and the risk-free rate.

In a nutshell, the model makes progress on the task of consolidating the analysis of asset prices and macroeconomics with financial frictions. However, there is still substantial potential for improvement because the model has been simplified to understand the underlying mechanism of limited borrowing capacity of financial intermediaries and its impact on the equity premium. In future research, it may be useful to extend the role of lenders to taking into account the collateral of the borrower, because housing finance was a particularly big issue in the Great Recession and household consumption was dampened due to the subprime mortgage crisis. Extending the model to incorporate housing prices such as Iacoviello (2005) and Liu, Wang, and Zha (2013) would be an interesting idea since the financial intermediary could have a greater influence on household consumption and the stochastic discount factor.
Chapter 3

Implications of Credit Limit Fluctuations for the Equity Premium

3.1 Introduction

Recent financial crises show that the housing market, financial markets, and the rest of the economy are closely related to each other. As discussed by Kiyotaki and Moore (1997), the fluctuations of housing price affect the borrower’s credit limit due to the collateral role of housing. A decline in the collateral value of housing prevents credit-constrained firms from raising funds to finance investment. Thus, it has a negative impact on investment and production. A contraction in economic activity further decreases housing prices, causing firms to reduce investment more. This channel is often referred to as the financial accelerator.
mechanism. Time-varying endogenous credit limit is a key ingredient of the financial accelerator mechanism since it amplifies the effect of shocks and propagates them throughout the economy. In this respect, this article studies whether the financial accelerator channel plays an important role in determining the equity premium since it amplifies business cycle fluctuations, leading to an increase in consumption risk.

This article attempts to draw implications from the financial accelerator channel on the equity premium using a New Keynesian dynamic stochastic general equilibrium model with a collateral constraint. I build the model by combining the key elements of the asset pricing model of Swanson (2016) and the collateral constraint models of Kiyotaki and Moore (1997) and Liu, Wang, and Zha (2013). The model includes Epstein-Zin-Weil preferences, a collateral constraint, nominal and real rigidities. Epstein-Zin-Weil preferences separate the intertemporal elasticity of substitution from risk aversion, allowing households to have high risk aversion without extremely low intertemporal elasticity of substitution.\footnote{See Rudebusch and Swanson (2008) for details.}

The model includes a liquidity shock and a technology shock. I also consider a housing demand shock due to the findings of Iacoviello and Neri (2010) and Liu, Wang, and Zha (2013) that it accounts for more than 80 percent of the observed housing price volatility. A housing demand shock, in addition to a liquidity shock, is likely to be a key determinant of credit limit.\footnote{Liu, Wang, and Zha (2013) point out that while the financial accelerator mechanism weakly amplifies the effect of a technology shock on aggregate variables, it rather strongly amplifies the impact of a liquidity shock and a housing preference shock.} This article therefore studies whether the shocks play a crucial role in accounting for the equity premium. I also investigate the contribution of a technology shock to the equity premium for comparison.

The key findings of this article are as follows. First, the financial accelerator
channel has a limited impact on the equity premium. The reason can be found from the fact that the degree of amplification is not large enough to increase the risk of the model so that it has a negligible contribution to the equity premium. Second, a liquidity shock and a housing demand shock, which have a transitory impact on the economy, have a negligible contribution to the equity premium, while a permanent technology shock is able to account for the observed equity premium regardless of the presence of the financial accelerator channel.

Empirical results on the impact of financial constraints on the equity premium are mixed in the literature. Lamont, Polk, and Saá-Requejo (2001) document evidence that more financially constrained firms earn lower equity premium. On the other hand, Whited and Wu (2006) find that more financially constrained firms earn higher equity premium, though the difference is not significant. Livdan, Sapriza, and Zhang (2009, henceforth LSZ) seek to fill the gap between the empirical papers through a structural framework. LSZ study the relationship between the equity premium and financial constraints using a partial equilibrium model in which capital serves as collateral. They find that more financially constrained firms earn higher equity premiums, but that the difference of equity returns between the most and the least constrained firms is small and statistically insignificant after controlling for size. This is because firms with smaller capital stock tend to be more financially constrained.

This paper studies the issue using a different model specification. The model adopted here differs from theirs in four aspects. First, I consider a general equilibrium model with nominal rigidity. Second, I use an endogenous stochastic discount factor based
on Epstein-Zin-Weil preferences for asset pricing, while LSZ use a reduced-form stochastic discount factor. Third, the representative household’s utility depends on housing service. The introduction of housing service into the utility function slightly alters the coefficient of relative risk aversion (Zanetti 2014).\textsuperscript{3} Fourth, firms’ credit limit mainly relies on the value of housing rather than physical capital stock. As highlighted by Liu, Wang, and Zha (2013), the Flow of Funds tables of the Federal Reserve Board show that about 58% of total tangible assets are real estate in the United States, indicating that real estate is more important than equipment in determining firms’ credit limit.\textsuperscript{4} Fifth, I study the role of liquidity and housing preference shocks in accounting for the equity premium along with aggregate technology shocks, while LSZ focus on firm-specific technology shocks and aggregate technology shocks.

This paper is also related to Nezafat and Slavk (2015) and Bigio and Schneider (2017). They study asset pricing implications of the Kiyotaki and Moore (2012) model in which firms are able to sell only a fraction of assets in a given period and are subject to a liquidity shock. These studies find that a liquidity shock increases the equity premium significantly. However, the dynamics of the equity price and the equity premium are puzzling. As pointed out by Bigio and Schneider (2017), the Kiyotaki and Moore (2012) model implies that recessions with low liquidity are associated with a high equity price and a low equity premium. In contrast to the model, this paper shows that the model with a collateral constraint generates the countercyclical dynamics of the equity premium, but the response of the equity premium is small in the face of a liquidity shock. In this respect, the implica-

\textsuperscript{3}In contrast, LSZ do not exploit utility function explicitly.
\textsuperscript{4}Chaney, Sraer, and Thesmar (2012) document evidence that when real estate price increases by a dollar, an average U.S. corporate firm increases investment by six cents.
tions of the Kiyotaki and Moore (1997) type collateral constraint model are different from those of the Kiyotaki and Moore (2012) model with liquidity shocks.

The organization of the paper is as follows. Section 2 describes a dynamic stochastic general equilibrium model featuring generalized recursive preferences and the capitalist’s limited borrowing capacity. Section 3 lays out the calibration results. Section 4 presents additional discussions and extensions. Sections 5 concludes.

3.2 The Baseline Model

The model economy include households, firms, and capitalists. A representative household has Epstein-Zin-Weil preferences, and derives utility from consumption and housing services, and disutility from supplying labor, while a representative capitalist gains utility from consumption only. The capitalist raises funds from the household to finance investment. Following Kiyotaki and Moore (1997), this article assumes that the capitalist faces collateral constraints due to limited enforcement. Since capital and houses serve as collateral, the borrowing capacity of the capitalist depends on the state of the real estate market as well as the capital market. The capitalist lend capital and houses to firms. The production sector has two types of firms: a representative final goods producer and a continuum of intermediate goods producers. The final goods producer faces perfect competition and thus make zero profits. The intermediate goods producers set their prices in a staggered fashion under monopolistic competition.
3.2.1 Households

The economy is populated by a continuum of identical households. Consumption and housing service are assumed to be separable for simplicity.\(^5\)

\[
u(c_{h,t}, h_{h,t}, l_t) \equiv \log c_{h,t} + \vartheta \log h_{h,t} - \eta_h \frac{l_t^{1+\chi}}{1+\chi}, \tag{3.1}
\]

where \(c_{h,t}\) is non-housing consumption, \(h_{h,t}\) is housing service, \(l_t\) is hours of work, \(\vartheta > 0\) is the relative weight on housing, \(\eta_h > 0\) denotes the relative weight on labor, and \(\chi > 0\) is the inverse Frisch elasticity of labor supply.

The budget constraint of the household is given by

\[
c_{h,t} + \frac{b_t}{P_t} + q_t^h (h_{h,t} - h_{h,t-1}) = e^{it-1} \frac{b_{t-1}}{P_{t-1}} + w_t l_t + \Pi_t + T_t, \tag{3.2}
\]

where \(b_t\) is bond, \(q_t^h\) is the real price of housing in units of consumption, \(e^{it}\) is the continuously compounded risk-free rate, \(w_t\) is the real wage rate, \(\Pi_t\) is the household’s share of profits, and \(T_t\) is lump-sum government transfers.

Following Epstein and Zin (1989), Weil (1989), and Tallarini (2000), I assume that the household has a recursive preference defined as follows:\(^6\)

\[
V_h(b_{t-1}, h_{h,t-1}; \Theta_t) = \max_{c_{h,t}, h_{h,t}, l_t \in \Gamma} (1-\beta_h)u(c_{h,t}, h_{h,t}, l_t) - \beta_h \frac{1}{\alpha} \log \left[ E_t \exp \left( -\alpha V_h(b_t, h_{h,t}; \Theta_{t+1}) \right) \right]
\]

Iacoviello (2005) and Liu, Wang, and Zha (2013) use the separability assumption following Bernanke (1984). On the other hand, Piazzesi, Schneider, and Tuzel (2007), and Liu, Miao, and Zha (2016) consider nonseparable preferences between consumption and housing. Lustig and Van Nieuwerburgh (2005) empirically show that the evidence of nonseparability is not statistically significant with the housing collateral ratio for the asset pricing.

This specific version of Epstein-Zin-Weil preferences is known as “multiplier preferences” (e.g., Hansen and Sargent, 2001; and Swanson, 2016).
where $\Gamma$ is the choice set for the household’s consumption, housing, and labor, $\Theta_t$ denotes the state of the economy, $\beta_h \in (0, 1)$ is the household’s discount factor, and $\alpha$ captures the additional curvature of multiplier preferences. The higher the value of $\alpha$, the more the household’s risk aversion is amplified, while the intertemporal elasticity of substitution is the same as expected utility preferences. As $\alpha$ is zero, (3.3) collapses into the usual expected utility preferences. The household chooses $c_{h,t}$, $b_t$, $h_{h,t}$, and $l_t$ to maximize (3.3) subject to (3.2) and a no-Ponzi-game constraint.

The household’s first-order conditions with respect to $l_t$, $b_t$, and $h_{h,t}$ are:

$$l_t : w_t = \eta_l l_t^\gamma c_{h,t} \tag{3.4}$$

$$b_t : 1 = E_t m_{t+1} \frac{1}{\pi_{t+1}} e^{it} \tag{3.5}$$

$$h_{h,t} : q_h^h = E_t m_{t+1} q_{t+1}^h + \frac{c_{h,t}}{p_{h,t}} \tag{3.6}$$

where $m_{t+1} = \beta_h \frac{c_{h,t}}{c_{h,t+1}} \frac{\exp(-\alpha V_h(b_t, h_{h,t}; \Theta_{t+1}))}{E_t \exp(-\alpha V_h(b_t, h_{h,t}; \Theta_{t+1}))}$ denotes the stochastic discount factor with Epstein-Zin-Weil preferences and $\pi_{t+1} \equiv P_{t+1}/P_t$ is the inflation rate. Equation (3.4) relates the real wage to the household’s marginal rate of substitution between consumption and labor. (3.5) is the standard consumption Euler equation that links consumption to the interest rate. The housing Euler equation (3.6) implies that the current house price is the discounted infinite sum of future marginal rates of substitution between housing and consumption.
3.2.2 Capitalists

There is a unit continuum of identical capitalists in the economy. A representative capitalist maximizes a standard utility function given by

$$\max_{c_{c,t}} E_0 \sum_{t=0}^{\infty} \beta_c^t \log c_{c,t}$$

where $c_{c,t}$ is the capitalist’s consumption at $t$, and $\beta_c < \beta_h$ is the capitalist’s subjective discount factor.

The budget constraint of the capitalist is given by

$$c_{c,t} + q_t^h(h_{c,t} - h_{c,t-1}) + I_t + e^{it-1} \frac{b_{t-1}}{P_t} = \frac{b_t}{P_t} + R^k_t k_{t-1} + R^h_t h_{c,t-1}$$

where $h_{c,t}$ is the capitalist’s possession of house, $I_t$ investment, $R^k_t$ the return of capital, and $R^h_t$ the return of housing service.

The capital accumulation equation with investment adjustment costs is given by

$$k_t = \left(1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t + (1 - \delta) k_{t-1}$$

where $\Omega$ is the investment adjustment cost parameter, and $\delta$ denotes the depreciation rate of capital.

Following Kiyotaki and Moore (1997), this article assumes that the capitalist faces a collateral constraint. It limits the amount of debt that the capitalist can borrow from households. Equation (3.10) shows that the amount of real debt is smaller than or equal to a fraction of the total collateral value of house and physical capital stock. This is because
the assets can be sold at devalued prices in the event of liquidation. The constraint is given by

\[
\frac{b_t}{P_t} \leq \zeta E_t \left( \omega_1 q^h_{t+1} h_{c,t} \frac{\pi_{t+1}}{e^t} + \omega_2 q^k_{t+1} k_t \frac{\pi_{t+1}}{e^t} \right),
\]

(3.10)

where \( \zeta \in (0, 1) \) is a constant parameter that captures the loan-to-value (LTV) ratio, \( q^k_t \) is the shadow price of capital (Tobin’s q), which is the marginal value of an additional one unit of capital, and \( \omega_1 \) and \( \omega_2 \) are relative weights on house and capital, respectively. I assume that the collateral constraint (3.10) binds as in the literature (e.g., Kiyotaki and Moore, 1997; Iacoviello, 2005; and Liu, Wang, and Zha, 2013).

The capitalist’s first-order conditions with respect to \( c_{c,t}, b_t, h_{c,t}, k_t, \) and \( I_t \) are:

\[
c_{c,t} : \quad \lambda_t = \frac{1}{c_{c,t}}
\]

(3.11)

\[
b_t : \quad \lambda_t = E_t \left( \beta \lambda_{t+1} e^t \left( \frac{1}{\pi_{t+1}} \right) + \nu_t \right)
\]

(3.12)

\[
h_{c,t} : \quad \lambda_t q^h_{t} = E_t \left[ \beta \lambda_{t+1} \left( R^h_{t+1} + q^h_{t+1} \right) + \nu_t \zeta \omega_1 q^h_{t+1} \frac{\pi_{t+1}}{e^t} \right]
\]

(3.13)

\[
k_t : \quad \mu_t = E_t \left[ \beta \left( \lambda_{t+1} R^k_{t+1} + \mu_{t+1} (1 - \delta) \right) + \nu_t \zeta \omega_2 q^k_{t+1} \frac{\pi_{t+1}}{e^t} \right]
\]

(3.14)

\[
I_t : \quad \lambda_t = \mu_t \left\{ 1 - \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_{t+1}}{I_{t-1}} \right\} + \beta E_t \left( \mu_{t+1} \Omega \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right)
\]

(3.15)

where \( \lambda_t, \mu_t, \) and \( \nu_t \) are the Lagrangian multipliers associated with the budget constraint, the capital accumulation equation, and the collateral constraint, respectively. Equation (3.12) and (3.5) show that the capitalist’s consumption is determined by the tightness of the borrowing constraint as well as the real interest rate. The capital price can be defined as
because this ratio implies how much consumption one is willing to give up to install some additional future capital.\textsuperscript{7} Equation (3.13) describes the relationship between the price of house and its rental rate, while (3.14) illustrates the relationship of the price of capital with its rental rate. Due to the collateral constraint, the term $\nu_t$ plays a role in determining the prices of capital and house. Lastly, equation (3.15) describes the dynamics of investment.

3.2.3 Firms

Final goods sector

A perfectly competitive final goods sector aggregates intermediate goods using a CES production function

$$Y_t = \left( \int_0^1 y_t(f)^{\frac{1}{1+\theta}} df \right)^{1+\theta} \quad (3.16)$$

where $Y_t$ is the quantity of the final goods, $y_t(f)$ is an intermediate good of firm $f$, and $\theta > 0$ is the steady state markup. Each final goods producing firm maximizes its profit given the production function (3.16) and the prices of intermediate and final goods. An intermediate goods producing firm $f$ accordingly faces a downward-sloping demand curve

$$y_t(f) = \left( \frac{p_t(f)}{P_t} \right)^{-\frac{1+\theta}{\theta}} Y_t \quad (3.17)$$

where $P_t \equiv \left( \int_0^1 p_t(f)^{-\frac{1}{\theta}} df \right)^{-\theta}$ is the CES aggregate price of final goods and $p_t(f)$ is the intermediate goods price.

\textsuperscript{7} The term $\mu_t$ is the marginal utility of new capital and $\lambda_t$ is the marginal utility of consumption, so their ratio can be interpreted as the relative price of capital normalized by consumption.
Intermediate goods sector

The economy also contains a continuum of intermediate goods firms indexed by $f \in [0, 1]$. They sell slightly differentiated goods under monopolistic competition. Firms produce output using housing, capital, and labor as inputs. The production function is given by

$$y_t(f) = A_t \left( h_{c,t-1}(f)^\phi k_{t-1}(f)^{1-\phi} \right)^{1-\eta} l_t(f)^\eta$$

(3.18)

where $A_t$ is an aggregate productivity shock and $k_t(f)$, $h_{c,t}(f)$ and $l_t(f)$ are firm $f$'s capital, housing, and labor inputs, respectively. The parameters $\eta \in (0, 1)$ and $\phi \in (0, 1)$ denote the elasticity of output with respect to labor and housing.

The technology shock follows an AR(1) process in logs,

$$\log A_t = \rho_A \log A_t + \epsilon_A^t$$

(3.19)

where $\rho_A \in (0, 1]$, and $\epsilon_A^t$ is drawn from an i.i.d. white noise process with zero mean and variance $\sigma_A^2$. In the baseline calibration, I assume that the autoregressive parameter $\rho_A$ is one.\(^8\)

Cost minimization yields the following first-order conditions for capital, housing, and labor:

$$k_{t-1}(f) : R_t^k = \frac{q_t(f)}{P_t} \left[ (1 - \eta)A_t \left( h_{c,t-1}(f)^\phi k_{t-1}(f)^{1-\phi} \right)^{-\eta} l_t(f)^\eta (1 - \phi) h_{c,t-1}(f)^\phi k_{t-1}(f)^{-\phi} \right]$$

(3.20)

\(^8\)For simplicity, I assume that the steady state growth rate of technology is zero since the aggregate housing supply is fixed in the model economy.
h_{c,t-1}(f) : \begin{align*} R_h^t = & \frac{\varphi_t(f)}{P_t} \left[ (1 - \eta) A_t \left( h_{c,t-1}(f)^{\phi} k_{t-1} (f)^{1-\phi} \right)^{-\eta} l_t(f)^{\eta} \phi h_{c,t-1}(f)^{\phi-1} k_{t-1} (f)^{1-\phi} \right] \\
\end{align*} (3.21)

l_t(f) : \begin{align*} w_t = & \frac{\varphi_t(f)}{P_t} \left[ \eta A_t \left( h_{c,t-1}(f)^{\phi} k_{t-1} (f)^{1-\phi} \right) \right]^{1-\eta} \left[ l_t(f)^{\eta} \right]^{-1} \\
\end{align*} (3.22)

where $\varphi_t(f)$ is the Lagrange multiplier of the cost minimization problem.

Dividing (3.20) by (3.21), (3.21) by (3.22), and (3.22) by (3.20) yields the housing-capital, the labor-housing, and the capital-labor ratios, respectively:

\begin{align*}
\frac{R_k^t}{R_h^t} &= \frac{1 - \phi}{\phi} \frac{H_{c,t-1}}{K_{t-1}} \\
\frac{R_h^t}{w_t} &= \frac{\phi (1 - \eta)}{\eta} \frac{L_t}{H_{c,t-1}} \\
\frac{w_t}{R_k^t} &= \frac{\eta}{(1 - \phi)(1 - \eta)} \frac{K_{t-1}}{L_t} \\
\end{align*} (3.23-3.25)

where $H_{c,t}$ is the aggregate housing of the capitalist, $K_t$ is the aggregate capital stock, and $L_t$ is the aggregate quantity of labor. The ratios are common across firms because they face the same rental rates. As a result, the demand functions for capital, housing, and labor are:

\begin{align*}
R_k^t &= M C_t (1 - \phi)(1 - \eta) A_t \left( \frac{H_{c,t-1}}{K_{t-1}} \right)^{\phi(1-\eta)} \left( \frac{L_t}{K_{t-1}} \right)^{\eta} \\
R_h^t &= M C_t \phi (1 - \eta) A_t \left( \frac{K_{t-1}}{H_{c,t-1}} \right)^{1-\phi(1-\eta)} \left( \frac{L_t}{K_{t-1}} \right)^{\eta} \\
\end{align*} (3.26-3.27)
\[ w_t = MC_t \eta A_t \left( \frac{H_{c,t-1}}{K_{t-1}} \right)^{\phi(1-\eta)} \left( \frac{K_{t-1}}{L_t} \right)^{1-\eta} \]  

(3.28)

where \( MC_t \equiv \frac{\phi_t}{P_t} = \left( \frac{1}{1-\phi} \right)^{(1-\phi)(1-\eta)} \left( \frac{1}{\phi} \right)^{(1-\eta)} \left( \frac{1}{1-\eta} \right)^{\eta} \left( R_h^k \right)^{(1-\phi)(1-\eta)} \left( R_h^k \phi^{(1-\eta)} w_t^\eta \right) \) denotes the real marginal cost at time \( t \).

The intermediate goods firm sets the optimal price \( p_t^* (f) \) to maximize its discounted sum of current and future cash flows in a staggered fashion as in Calvo (1983). Only a fraction \( 1 - \xi \) of firms update their prices at every period while others adjust them by indexing their previous prices to the steady state inflation rate. The objective function of the firm adjusting its price at time \( t \) is given by

\[
\max_{p_t(f)} E_t \sum_{j=0}^{\infty} m_{t,t+j}(P_t/P_{t+j}) \xi^j \left[ p_t(f) e^{j\bar{\pi}} y_{t+j}(f) - mc^n_{t+j}(f) y_{t+j}(f) \right]
\]  

(3.29)

where \( m_{t,t+j} \equiv \prod_{i=1}^j m_{t+i} \) is the household’s stochastic discount factor from period \( t \) to \( t+j \), \( \bar{\pi} \) is the nonstochastic steady-state inflation rate, and \( mc^n_{t+j}(f) \) is the nominal marginal cost for firm \( f \). Given (3.17), differentiating (3.29) with respect to \( p_t^*(f) \) yields the optimal price

\[
p_t^*(f) = \frac{(1 + \theta) E_t \sum_{j=0}^{\infty} m_{t,t+j} \xi^j MC_{t+j} P_{t+j}^{1+\theta} Y_{t+j}}{E_t \sum_{j=0}^{\infty} m_{t,t+j} \xi^j P_{t+j}^{1+\theta} Y_{t+j} e^{j\bar{\pi}}}
\]  

(3.30)

Notice that the optimal price \( p_t^*(f) \) is a markup over a weighted average of present and expected future real marginal costs.

### 3.2.4 Aggregate Resource Constraints and Policy Rule

The aggregate production function can be written as
\[ Y_t = \Delta_t^{-1} A_t \left( H_{c,t-1}^{1-\phi} K_{t-1}^{1-\phi} \right)^{1-\eta} L_t^\eta \]  \hspace{1cm} (3.31)

where \( L_t \equiv \int_0^1 l_t(f) df \) and \( \Delta_t \equiv \int_0^1 \left( \frac{p_t(f)}{\bar{p}} \right)^{\frac{1+\theta}{\theta}} df \) captures the cross-sectional dispersion of prices across firms due to staggered price contracts.

In a competitive equilibrium, the market clearing condition for the final goods implies

\[ Y_t = C_{h,t} + C_{c,t} + I_t \]  \hspace{1cm} (3.32)

where \( C_{h,t} \) is the aggregate consumption of the household, and \( C_{c,t} \) is the aggregate consumption of the capitalist. The market clearing condition for housing is given by

\[ H_{h,t} + H_{c,t} = \bar{H} \]  \hspace{1cm} (3.33)

where \( H_{h,t} \) is the aggregate housing demand of the household and \( H_{c,t} \) is the aggregate housing demand of the capitalist. I assume that the aggregate housing supply is fixed at unitary as in the literature (e.g., Iacoviello, 2005; and Liu, Miao, and Zha, 2016).

Lastly, I consider a Taylor (1993)-type monetary policy rule that is analogous to that of Swanson (2016):

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ \ln \frac{1}{\beta} + \log \pi_t + \phi_{\pi} \left( \log \pi_t - \log \bar{\pi} \right) + \frac{\phi_y}{4} (y_t - \bar{y}_t) \right] \]  \hspace{1cm} (3.34)

where \( \rho_i \in (0, 1) \) is the smoothing parameter, \( y_t \equiv \log Y_t \) is log output, \( \bar{y}_t \equiv \rho_y \bar{y}_{t-1} + (1 - \rho_y) y_t \) is a trailing moving average of \( y_t \) with a parameter \( \rho_y \in (0, 1) \), and the coefficients
φ_π and φ_y measure the Fed’s response to inflation and economic activity, respectively. The central bank sets short term nominal interest rates in response to the deviation of output from the recent average of output. The term y_t − ȳ_t in (3.34) is an empirically motivated output gap.

3.2.5 The Equity Premium

The stochastic discount factor derived from the model in Section 2.1 can be used to evaluate the price of any asset. In particular, the price of equity is as follows:

\[ p^e_t = E_t (m_{t+1} (C^u_{h,t+1} + p^e_{t+1})) , \]

(3.35)

where \( p^e_t \) is the ex-dividend price of an equity, and \( C^u_{h,t+1} \) is a levered claim on the aggregate consumption of the household which can be considered as a dividend for simplicity (e.g., Abel, 1999; Bansal and Yaron, 2004; and Campbell, Pflueger, and Viceira, 2014). The degree of leverage, \( v \), can be interpreted as broad leverage in the economy, which includes operational and financial leverage.

The return of equity, \( R^e_{t+1} \), is defined to be the sum of the future dividend and equity price divided by the current equity price,

\[ R^e_{t+1} = \frac{C^u_{h,t+1} + p^e_{t+1}}{p^e_t}. \]

(3.36)

The equity premium is defined as the difference between the expected return to equity and the risk-free rate, \( \psi^e_t \equiv E_t R^e_{t+1} - e^{r_t} \). Using the fact that \( E_t (m_{t+1} R^e_{t+1}) = Cov_t (m_{t+1}, R^e_{t+1}) + E_t m_{t+1} E_t R^e_{t+1} \), the equity premium can be written as
\[ \psi^e_t = 1 - \frac{Cov_t (m_{t+1}, R^e_t)}{E_t m_{t+1}} - e^{r_t} \]
\[ \psi^e_t = -\frac{Cov_t (m_{t+1}, R^e_t)}{E_t m_{t+1}} \]
\[ \psi^e_t = -Cov_t \left( \frac{m_{t+1}}{E_t m_{t+1}}, R^e_{t+1} \right) \] (3.37)

where the first equality of (3.37) is due to \( e^{r_t} = 1/E_t m_{t+1} \). The equity premium is positive when the stochastic discount factor is negatively related with the equity return.

### 3.3 Model Results

The model is calibrated to study the behavior of the equity premium. I use a third order perturbation method as in Swanson, Anderson, and Levin (2006) since the perturbation method makes it easy to study the dynamics of the risk premium.\(^9\) In this section, all the results, including the impulse response functions, are obtained from the third order perturbation method.

#### 3.3.1 The Baseline Calibration

The baseline calibration is summarized in Table 3.1, and is consistent with estimated parameter values from the literature (e.g., Smets and Wouters, 2007; Liu, Wang, and Zha, 2013; and Liu, Miao, and Zha, 2016). The baseline calibration is fairly standard in both macroeconomics and finance.

The household’s discount factor, \( \beta_h \), is set to 0.992, implying a steady state real...
interest rate of 3 percent per year as in Iacoviello and Neri (2010). The relative utility
weight of labor, $\eta_h$, is set to 0.84 to normalize $L = 1$ in steady state. Following Del Negro,
Giannoni, and Schorfheide (2015), the inverse Frisch elasticity of labor supply, $\chi$, is set to
3. The relative utility weight of house, $\vartheta$, is set to 0.045, consistent with estimates of Liu,
Wang, and Zha (2013). I choose the value of $\alpha$ to pin down the coefficient of relative risk
aversion $R^c = 30$. As in the literature, the risk aversion coefficient is set to be high since
the model economy has less uncertainty than reality.\footnote{See Barillas, Hansen, and Sargent (2009) for a detailed discussion. Tallarini (2000), Rudebusch and
Swanson (2012) and Swanson (2016) consider 100, 110, and 60 for the value of risk aversion, respectively.
The risk aversion parameter is estimated to be 57 by Piazzesi and Schneider (2006) and 66 by Binsbergen,
Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012).}

The parameters pertaining to the production sector are as follows. Following Liu,
Miao, and Zha (2016), I choose the capitalist’s time discount factor, $\beta_c$, to 0.945 which allows
the collateral constraint (3.10) to hold with equality around the nonstochastic steady state.
The elasticity of output to labor, $\eta$, is set to 0.6, while the elasticity of output to housing, $\phi$, is set to 0.07 as in Liu, Wang, and Zha (2013). Abstracting housing as a collateral asset from
macroeconomic models, the investment adjustment cost parameter is estimated from about
3 to 10 in the literature (e.g., Christiano, Eichenbaum and Evans, 2005; and Christiano,
Motto and Rostagno, 2014). However, the investment adjustment cost parameter is often
found to be very small when the model economy has housing as collateral. I therefore set $\Omega = 0.114$, consistent with estimates in Liu, Miao, and Zha (2016).\footnote{My choice for the investment adjustment cost parameter does not affect the results.}
The depreciation rate of capital, $\delta$, is set to 0.025 as in Iacoviello and Neri (2010). Following Smets and Wouters
(2007), I fix the parameter $\theta$ to 0.1, implying a 10 percent markup over marginal cost in
steady state. The Calvo pricing parameter, $\xi$, is set to 0.8, following estimates of Altig,
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_h$</td>
<td>0.9925</td>
<td>Discount rate for the household</td>
</tr>
<tr>
<td>$\eta_h$</td>
<td>0.84</td>
<td>Relative utility weight of labor</td>
</tr>
<tr>
<td>$\chi$</td>
<td>3</td>
<td>Inverse Frisch elasticity of labor supply</td>
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<tr>
<td>$\theta$</td>
<td>0.045</td>
<td>Relative utility weight of house</td>
</tr>
<tr>
<td>$R^c$</td>
<td>30</td>
<td>Relative risk aversion</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>0.945</td>
<td>Discount rate for the capitalist</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.6</td>
<td>Elasticity of output to labor</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.07</td>
<td>Elasticity of output to housing</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.114</td>
<td>Elasticity of investment adjustment costs</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Monopolistic markup</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.8</td>
<td>Calvo contract parameter</td>
</tr>
<tr>
<td>$\zeta$</td>
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<td>Loan-to-value ratio</td>
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<td>Weight of house in the collateral constraint</td>
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<td>$\omega_2$</td>
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<td>Weight of capital in the collateral constraint</td>
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<td>$\rho_i$</td>
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<td>Smoothing parameter of monetary policy</td>
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<td>$\phi_\pi$</td>
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<td>Response of monetary policy to inflation</td>
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<td>$\phi_y$</td>
<td>0.93</td>
<td>Response of monetary policy to output</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>0.008</td>
<td>The monetary authority’s inflation target</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.9</td>
<td>Coefficient of trailing moving average</td>
</tr>
<tr>
<td>$\rho_A$</td>
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<td>Persistence of technology</td>
</tr>
<tr>
<td>$\sigma_A$</td>
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<td>Standard deviation of technology shocks</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3</td>
<td>Degree of leverage</td>
</tr>
</tbody>
</table>

Christiano, Eichenbaum, and Lind (2011), which indicates that price contracts expire after five quarters on average.

Turning to the collateral constraint, I fix the LTV ratio at 0.75, suggesting that the capitalist can borrow 75 percent of the collateral value. The relative weights on housing and capital in the collateral constraint are set equal to 1 and 0.1, respectively. All these
parameters in the collateral constraint are consistent with estimates in Liu, Wang, and Zha (2013) and Liu, Miao, and Zha (2016).

Next, the monetary policy parameters governing the degree of interest rate smoothing and the response of monetary policy to inflation and output, $\rho$, $\phi_{\pi}$, and $\phi_{y}$, are calibrated to 0.73, 0.53, and 0.93, respectively. These values are consistent with estimates in Rudebusch (2002). The monetary authority’s inflation target, $\bar{\pi}$, is fixed at 0.008 which indicates a 3.2 percent of the steady state inflation rate per year. Although inflation target is known to be around 2 percent, the historical average is around 3 percent (Swanson, 2016). I fix the coefficient $\rho_{y}$ at 0.9, which implies that the monetary authority responds to the deviation of output from its moving average level over the last 10 quarters.

Finally, I calibrate the persistence of technology, $\rho_{A}$, to 1, following Tallarini (2000) and Swanson (2016). The standard deviation of a technology shock, $\sigma_{A}$, is set to 0.007 as in King and Rebelo (1999). Finally, I fix the degree of leverage at $\upsilon = 3$ to match the observed volatility of dividend growth following Abel (1999) and Bansal and Yaron (2004).

### 3.3.2 Macroeconomic Implications

This section studies whether credit limit fluctuations driven by time-varying housing price play a crucial role in accounting for the observed equity premium between 3 to 6.\textsuperscript{12} To investigate this issue, I consider an alternative model with fixed credit limit (FCL) for comparison.\textsuperscript{13} The collateral constraint amplifies business cycle fluctuations since credit

\textsuperscript{12}See Campbell (1999) and Fama and French (2002) for details.\textsuperscript{13}Kocherlakota (2000) also compares a model with a collateral constraint to its counterpart with fixed credit limit to investigate whether the financial accelerator channel sufficiently amplifies the impact of a technology shock and propagates it throughout the economy. Kocherlakota (2000) focuses on business cycle amplifications of the financial accelerator channel, but not on the equity premium.
limit varies with economic conditions. For example, in a recession, borrowers have difficulty in financing investment as the value of collateral falls. This reduction in investment decreases production and the collateral value further, accelerating the recession. On the other hand, if borrowers are able to raise funds stably regardless of economic conditions, the economy would be rather stable. Accordingly, in the alternative model I assume that credit limit is always fixed at its nonstochastic steady state value. Otherwise, the parameter values and model equations are the same as the baseline model.

The impulse response functions of the variables of interest to a one-standard-deviation (0.7 percent) negative technology shock are reported in Figure 3.1. These are obtained by calculating the difference between the case where there is a negative one-standard-deviation technology shock and the case where there is no shock, while the state variables are set to their nonstochastic steady state values as an initial point. The top right panel of Figure 3.1 shows the impulse response of bond, $B_t$. As expected, the bond does not react in the FCL model, while it declines in the baseline model. This difference accounts for why the representative capitalist’s consumption in the top middle panel declines less in the FCL model. The assumption of the capitalist’s constant borrowing implies that the household purchases the same amount of bond even though a negative shock lowers income. Therefore, household consumption initially declines more and capitalist consumption drops less in the FCL model compared to the baseline model.

The household’s demand on housing declines in response to the negative technology shock, while the capitalist’s demand increases. Notice that the aggregate stock of housing is fixed. The figure shows that the housing price drops further with endogenous credit
Figure 3.1: Impulse Response Functions of Macro Variables

Note: The figure plots the impulse response functions of household consumption, $C_{h,t}$, capitalist consumption, $C_{c,t}$, bond, $B_t$, housing price, $q_{h,t}^h$, housing stock owned by households, $H_{h,t}$, housing stock owned by capitalists, $H_{c,t}$, capital price, $q_{k,t}^k$, capital, $K_t$, investment, $I_t$, net real risk-free return, $r_t$, inflation rate, $\pi_t$, and output, $Y_t$, to a one-standard-deviation (0.7 percent) negative technology shock. The solid blue lines are from the baseline model, while the dashed orange lines are from the alternative model with fixed credit limit (FCL). See text for details.
limit fluctuations. Capital price, capital stock, investment, and output decline more in the baseline model due to the financial accelerator mechanism. As shown in the bottom middle panel, inflation initially increases by 1.06% in the baseline model and 1.20% in the alternative model. Inflation rises less in the baseline model since economic activity contracts further with endogenous credit limit. The real interest rate also declines more due to the financial accelerator mechanism.

### 3.3.3 Equity Premium Implications

The model-implied equity premium is reported in Table 3.2. In order to compute the equity premium, we need a closed form solution for the coefficient of relative risk aversion. The coefficient of relative risk aversion can be written as

\[
R^c(b, h; \Theta) = \frac{1}{1 + \frac{\nu h}{\chi} + \vartheta} + \alpha
\]  

(3.38)

under Epstein-Zin-Weil preferences when (dis)utility is derived by consumption, labor, and housing service. As discussed in Zanetti (2014), the introduction of housing service into the household’s utility function lowers the coefficient of relative risk aversion. This is because households are able to reduce consumption risk by rebalancing spending on consumption and housing service in response to an income shock.\(^{14}\) (4.46) is equivalent to the relative risk aversion derived by Swanson (2015) when housing is absent in the model \((\vartheta = 0)\).

The last column of Panel A presents the equity premium generated from the baseline model. The model-implied equity premium is 3.62 percent, consistent with the

\(^{14}\)The closed-form expression of risk aversion is derived by using the household’s period utility or budget constraint. The coefficient of relative risk aversion might be changed when the budget constraint contains additional factors such as transaction costs or adjustment costs. I do not investigate this issue since it is out of the scope of this article. I focus on the role of financial frictions in determining the equity premium given the coefficient of relative risk aversion.
Table 3.2: Comparison of Equity Premium

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>Loan-to-value</th>
<th>Elasticity of output to house</th>
<th>Equity premium $\psi^e_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^c$</td>
<td>$\zeta$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.75</td>
<td>0.07</td>
<td>1.82</td>
</tr>
<tr>
<td>30</td>
<td>0.75</td>
<td>0.07</td>
<td>3.65</td>
</tr>
<tr>
<td>45</td>
<td>0.75</td>
<td>0.07</td>
<td>5.48</td>
</tr>
<tr>
<td>60</td>
<td>0.75</td>
<td>0.07</td>
<td>7.30</td>
</tr>
<tr>
<td>Panel B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.65</td>
<td>0.07</td>
<td>3.65</td>
</tr>
<tr>
<td>30</td>
<td>0.85</td>
<td>0.07</td>
<td>3.65</td>
</tr>
<tr>
<td>Panel C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.75</td>
<td>0.14</td>
<td>3.29</td>
</tr>
<tr>
<td>30</td>
<td>0.75</td>
<td>0.5</td>
<td>2.28</td>
</tr>
</tbody>
</table>

Note: The model-implied annual equity premium with different values of the risk aversion parameter, $R^c$, loan-to-value ratio, $\zeta$, and elasticity of output to house, $\phi$. The baseline model has endogenous credit limit, while the alternative model has fixed credit limit (FCL). See the text for more details.

The fourth column of Panel A presents the equity premium produced from the FCL model in which the capitalist raises a constant amount of fund to finance investment every period. Interestingly, the last two columns reveal that the equity premium of the FCL model is very similar to that of the baseline model, indicating that endogenous credit limit fluctuations do not have a sizable contribution to the equity premium. It shows that the financial accelerator channel plays a negligible role in accounting for the equity premium.
To investigate whether the results are sensitive to alternative steady state LTV values, I consider $\zeta = 0.65$ and $\zeta = 0.85$. Panel B shows that the alternative LTV ratios do not change the finding that the financial accelerator channel associated with endogenous credit limit fluctuations plays a negligible role in accounting for the equity premium. I also consider alternative values for the elasticity of output to house, $\phi$, in the production function. Kocherlakota (2000) points out that the amplification impact of collateral constraints changes with the parameter value of $\phi$. As shown in Panel C, the equity premium declines when the parameter $\phi$ increases. However, there is a negligible difference in the equity premium between the baseline model and the FCL model regardless of the value of $\phi$. Overall, the table reveals the finding that the financial accelerator has no role in accounting for the equity premium is not sensitive to alternative parameter values.

In order to scrutinize why the financial accelerator mechanism has a negligible impact on the equity premium, I plot the third-order impulse response functions of the stochastic discount factor and the equity price in Figure 3.2. Again, the blue solid lines indicate the impulse responses to the one-standard-deviation negative technology shock in the baseline model. The orange dotted lines show the impulse responses generated from the FCL model. The left panel of Figure 3.2 shows that the response of the stochastic discount factor in the baseline model is slightly greater than that of the FCL model even though the difference is negligible. The unconditional standard deviation of stochastic discount factor is 32.55 in the baseline model, and 32.47 in the FCL model. The middle panel of Figure 3.2 plots the response of the equity price. It shows that the equity price drops further with

---

15I computed the unconditional standard deviation using logarithmic deviation of the stochastic discount factor from a Hodrick-Prescott filter. The unit for the unconditional standard deviation is percentage points.
Figure 3.2: Impulse Response Functions of Financial Variables

Note: The third-order impulse response functions for the stochastic discount factor, $m_t$, the equity price, $p^e_t$, and the equity premium, $\psi^e_t$, to a one-standard-deviation (0.7 percent) negative technology shock. The solid blue lines in each panel plot the impulse response functions of the baseline model, and the dashed orange lines plot the impulse response functions of the alternative model with fixed credit limit (FCL). See the text for more details.

endogenous credit limit fluctuations. However, the contribution of the financial accelerator channel to the equity premium is very limited as shown in the right panel of Figure 3.2. The reason for this result may be attributed to the fact that the amplification effect is not large enough to increase the equity premium.\textsuperscript{16}

\textsuperscript{16}As discussed in Table 3.2, the permanent technology shock is able to account for the observed equity premium regardless of the presence of the accelerator channel. Notice that the nonstationary technology shock leads to substantial fluctuations of the equity price, contributing to the equity premium. The technology shock increases the model’s quantity of risk, so that it helps generate the observed equity premium (e.g., Rudebusch and Swanson 2012 and Li and Palomino 2014).
3.3.4 Implications of the Loan-to-Value Ratio on the Equity Premium

This subsection examines whether the equity premium is sensitive to alternative the LTV ratios. Figure 3.3 shows the impulse response functions to a negative technology shock when the LTV ratio is less than or greater than the baseline calibration ($\zeta = 0.75$), while the other parameter values remain the same as the baseline calibration. For comparison, the impulse response functions of the baseline calibration are plotted in the middle column. In the left column, the LTV ratio is 0.65, so the collateral constraint becomes tighter than the baseline calibration. The right column reports the impulse response functions when the capitalist faces a looser collateral constraint of $\zeta = 0.85$ compared to the baseline calibration. The first row presents the impulse responses of output. The variations of the LTV ratio in the FCL model do not change the response of output, while they alter the response of output in the baseline model. The highest LTV ratio yields the deepest contraction in economic activity among the cases considered in the figure. Notice that the volatility of the capitalist’s credit limit increases with the LTV ratio since the capitalist’s credit limit is equal to the LTV ratio times collateral value. A rise in the LTV ratio increases credit limit’s risk, yielding a contraction in economic activity.

The second row shows that the deepest drop in the equity price is associated with the baseline model with $\zeta = 0.85$. When the LTV ratio is 0.85, the equity price drops to about 3.4 percent. The last row presents the impulse response functions of the equity premium. The equity premium rises by 3 basis points when the LTV ratio rises from 0.65 to 0.85 in the baseline model, indicating that the LTV ratio has a very limited contribution to the equity premium. Overall, the results indicate that the equity premium is not sensitive
Figure 3.3: Impulse Response Functions with Different LTV

Note: The third-order impulse response functions for output, the equity price, and the equity premium with various loan-to-value ratios. The left column is for a LTV ratio of 0.65, the middle column is for a LTV ratio of 0.75, and the right column is for a LTV ratio of 0.85. In the face of a one-standard-deviation negative technology shock, the blue solid lines plot the impulse responses of the baseline model. The orange dotted lines plot the impulse responses to the same shock in the alternative model with fixed credit limit (FCL). See the text for more details.
to alternative LTV ratios.

3.4 Implications of Liquidity and Housing Demand Shocks on Equity Premium

This section introduces additional shocks into the model—in particular, a housing demand shock and a liquidity shock—and discuss its impact on the equity premium. These two shocks are chosen since they have a considerable effect on the housing price and the capitalist’s debt capacity. A liquidity shock directly affects the capitalist’s credit limit, while a housing preference shock indirectly affects the capitalist’ debt capacity through its impact on housing price fluctuations. According to Iacoviello and Neri (2010) and Liu, Wang, and Zha (2013), a housing preference shock accounts for more than 80 percent of the observed housing price volatility. In this respect, I consider a housing preference shock along with a liquidity shock.

To investigate the effect of the credit market’s tightness on the equity premium, I extend the model to include the liquidity shock, $\zeta_t$, as follows

$$\frac{b_t}{P_t} \leq \zeta_t E_t \left( \omega_1 q_{t+1}^h h_{c,t} \frac{\pi_{t+1}}{e^{\pi_t}} + \omega_2 q_{t+1}^k k_t \frac{\pi_{t+1}}{e^{\pi_t}} \right),$$

(3.39)

where $\log \zeta_t = (1 - \rho_\zeta) \log \zeta + \rho_\zeta \log \zeta_{t-1} + \epsilon_\zeta^t$ and $\epsilon_\zeta^t$ follows an i.i.d. white noise process with mean zero and variance $\sigma_\zeta^2$. I set $\rho_\zeta = 0.98$ and $\sigma_\zeta = 0.01$ following Liu, Wang, and Zha (2013).

The middle panel in Figure 3.4 displays the impulse response functions for the housing price, the equity price, and the equity premium to an adverse one-standard-
Figure 3.4: Impulse Response Functions with Additional Shocks

Note: The impulse responses of the housing price, the equity price, and the equity premium to a negative one-standard-deviation shock to technology, housing demand and liquidity. The blue solid lines indicate the responses to a one-standard-deviation negative shock in the baseline model, and the orange dotted lines plot the responses in the alternative model with fixed credit limit (FCL). See the text for more details.

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deviation liquidity shock in the models. For comparison, I plot the impulse response functions associated with a negative one-standard-deviation technology shock in the first column. The liquidity shock is less persistent than the permanent technology shock so that the impact of the liquidity shock on the housing price, the equity price, and the equity premium is much smaller than that of the technology shock.\footnote{These results are associated with the fact that the liquidity shock has a limited contribution to the consumption of households compared to the technology shock.}

To introduce a housing demand shock, $\vartheta_t$, into the model, I assume that the period utility of the household is given by

$$ u(c_{h,t}, h_{h,t}, l_t) \equiv \log c_{h,t} + \vartheta_t \log h_{h,t} - \eta_h l_t^{1+\chi}, $$

(3.40)

where $\log \vartheta_t = (1 - \rho_{\vartheta}) \log \bar{\vartheta} + \rho_{\vartheta} \log \vartheta_{t-1} + \epsilon_t^{\vartheta}$ and $\epsilon_t^{\vartheta}$ follows an i.i.d. white noise process with mean zero and variance $\sigma^2_{\vartheta}$. I calibrate $\rho_{\vartheta} = 0.99$ and $\sigma_{\vartheta} = 0.046$ in line with estimates in Liu, Wang, and Zha (2013).

The right panel in Figure 3.4 displays the third-order impulse response functions for the housing price, the equity price, and the equity premium to a negative one-standard-deviation housing demand shock. The housing demand shock has a substantial impact on the housing price. However, the impact of a housing preference shock on the equity price and premium is substantially smaller than that of the technology shock.\footnote{These results are in line with the findings of Rudebusch and Swanson (2012) and Li and Palomino (2014) that nonstationary technology shocks play a major role in generating sizable risk premia.}

Table 3.3 reports the contribution of technology, liquidity, and housing demand shocks to the equity premium. Technology shocks generate an equity premium of 3.62 in the baseline model and 3.65 in the model with fixed credit limit. Technology shocks are able to
Table 3.3: Equity Premium and Shocks

<table>
<thead>
<tr>
<th></th>
<th>FCL</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology shocks only</td>
<td>3.65</td>
<td>3.62</td>
</tr>
<tr>
<td>Liquidity shocks only</td>
<td>-0.004</td>
<td>-0.00007</td>
</tr>
<tr>
<td>Housing demand shocks only</td>
<td>0.01</td>
<td>0.07</td>
</tr>
<tr>
<td>All shocks</td>
<td>3.67</td>
<td>3.68</td>
</tr>
</tbody>
</table>

Note: The equity premium implied by the models in annualized percentage points.

account for the observed equity premium. As shown in the table, liquidity shocks have an extremely small impact on the equity premium regardless the assumptions of credit limit. The table also shows that housing demand shocks play a limited role in accounting for the equity premium. With only housing demand shocks, the baseline model generates an equity premium of 0.07 and the FCL model generates an equity premium of 0.01. Overall, the findings show that the financial accelerator mechanism plays a limited role in accounting for the equity premium regardless of types of shock.

3.5 Conclusions

Macroeconomic models with financial frictions have attracted a lot of attention among economists after the recent financial crisis. This article studies amplifications of financial frictions on the equity premium using the medium-scale dynamic stochastic general equilibrium model in which the capitalist uses housing as collateral to finance investment. My findings from this study are two-fold. First, the accelerating mechanism of financial frictions has a very limited contribution to the equity premium even though it amplifies the impact of macroeconomic shocks on business cycle fluctuations. Second, a liquidity shock and a housing demand shock while a nonstationary technology shock is able to account for the
observed equity premium regardless of the presence of the financial accelerator channel.
Chapter 4

Housing and Relative Risk Aversion with Generalized Recursive Preferences

4.1 Introduction

The interaction of the housing market and the macroeconomy has received substantial attention since recent financial crises. Because disruptions in house prices affect the household’s consumption and labor, the housing market might be an important source of business cycle fluctuations (e.g., Iacoviello, 2005; Iacoviello and Neri, 2010; and Liu, Wang, and Zha, 2013). This provides a hint that housing might change the household’s attitudes toward risk. At the same time, a growing macro-finance literature uses generalized recursive preferences that allow standard macroeconomic models to match basic asset pricing facts (e.g.,
Generalized recursive preferences break the tight link between the intertemporal elasticity of substitution and the coefficient of relative risk aversion, which is the essential parameter for measuring risk.

In the present paper, I study the role of housing on the coefficient relative risk aversion with generalized recursive preferences, focusing on deriving the closed-form expressions for risk aversion. By deriving risk aversion with generalized recursive preferences, the closed-form expression of the coefficient of relative risk aversion can be utilized in the asset pricing literature immediately. My finding suggests that house prices lower risk aversion. This is because when housing is included in the household’s period utility function, it partially absorbs aggregate shocks to consumption and labor. This finding echoes Swanson (2012), who shows that the labor margin provides a buffer for shocks. The result is also analogous to Zanetti (2014), but I add house prices and generalized recursive preferences, and do not depend on the additively separable utility function.

The organization of the paper is as follows. Section 2 describes the model set up. Section 3 shows derivation of risk aversion with generalized recursive preferences in the presence of house price. Section 4 presents examples. Section 5 concludes and discusses future research.

4.2 The Model

Time is discrete and continues forever. Each period, the household chooses: consumption, $c_t$, beginning-of-period housing services, $h_{t+1}$, and labor, $l_t$, given two state variables, the
quantity of real risk-free bonds, \( b_t \), and its stock of housing, \( h_t \). The flow budget constraint of the household is given by:

\[
b_{t+1} = (1 + r_t)b_t + w_tl_t + d_t - c_t - q^h_t(h_{t+1} - h_t)
\] (4.1)

where \( r_t \) is the real risk-free return, \( w_t \) is the real wage, \( q^h_t \) is the real house price, and \( d_t \) is net transfer payments.

Each household has multiplier preferences following Hansen and Sargent (2001):\(^1\)

\[
\bar{V}(b_t, h_t; \Theta_t) = (1 - \beta)u(c_t, h_{t+1}, l_t) - \beta \frac{1}{\alpha} \log \left[ E_t \exp \left( -\alpha V(b_{t+1}, h_{t+1}; \Theta_{t+1}) \right) \right].
\] (4.2)

The household maximizes (4.2) subject to the budget constraint (4.1) and a standard no-Ponzi constraint:

\[
V(b_t, h_t; \Theta_t) = \max_{c_t, h_{t+1}, l_t \in \Gamma} \left( 1 - \beta \right)u(c^*_t, h^*_{t+1}, l^*_t) - \beta \frac{1}{\alpha} \log \left[ E_t \exp \left( -\alpha V(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1}) \right) \right]
\] (4.3)

where \( V(b_t, h_t; \Theta_t) \) is the household’s generalized value function, \( \Gamma \) is the choice set for \( c_t, h_{t+1}, \) and \( l_t \), \( u \) is the period utility function, \( \beta \in (0, 1) \) is the household’s time discount factor, \( \alpha \) measures the additional curvature of multiplier preferences, and the state of the aggregate economy, \( \Theta_t \), controls the processes for \( w_t, r_t, q^h_t \), and \( d_t \). I also assume that \( c^*_t = c^*(b_t, h_t; \Theta_t) \), \( h^*_{t+1} = h^*(b_t, h_t; \Theta_t) \), and \( l^*_t = l^*(b_t, h_t; \Theta_t) \) are the optimal interior solutions for consumption, housing, and labor, respectively. So, \( b^*_{t+1} \equiv (1 + r_t)b_t + w_t l^*_t + d_t - c^*_t - q^h_t(h^*_{t+1} - h_t) \).

\(^1\)Multiplier preferences is a version of generalized recursive preferences. The expectation operator is “twisted” by the factor \(-\alpha\) and exponential function and “untwisted” by the factor \(-\alpha^{-1}\) and the natural logarithm function.
Following Arrow (1965), Pratt (1964), and Swanson (2012), I consider the household’s aversion to a hypothetical one-shot gamble in period $t$, as follows:

$$b_{t+1}^* = (1 + r_t)b_t + w_t l_t^* + d_t - c_t^* - q_t^h (h_{t+1}^* - h_t) + \sigma \epsilon_{t+1}$$  \hspace{1cm} (4.4)

where $\epsilon_{t+1} \in [\underline{\epsilon}, \bar{\epsilon}]$ is a random variable which represents the gamble, $\sigma$ is size of the gamble, $E_t \epsilon_{t+1} = 0$, and $E_t \epsilon_{t+1}^2 = 1$.

As in Arrow (1965), Pratt (1964), and Swanson (2012), I consider how much the household would be willing to pay to avoid the gamble. If the household pays a one-time fee, $\mu$, to avoid the gamble, its budget constraint accordingly is given by:

$$b_{t+1}^* = (1 + r_t)b_t + w_t l_t^* + d_t - c_t^* - q_t^h (h_{t+1}^* - h_t) - \mu.$$  \hspace{1cm} (4.5)

### 4.3 The Coefficient of Risk Aversion with Housing

In this section, I derive the closed-form expression for risk aversion with housing and generalized recursive preferences. In the conventional wisdom, a house is considered a risky asset because house prices are volatile. However, owning a house can also reduce risk since the homeowner can avoid the uncertainty of housing costs (Sinai and Souleles, 2005). When housing is included in period utility, how does it change economic agent’s attitudes toward risk at the nonstochastic steady state? Does it increase risk avoidance or provide space for taking additional risk? This is an important issue in measuring the risk premium, as shown in Swanson (2012 and 2015).

**Proposition 1. Absolute Risk Aversion** Let $(b, h; \Theta)$ be an interior point of the domain of $(b, h; \Theta)$. Then, the coefficient of absolute risk aversion, $R^a(b, h; \Theta)$, exists and
\[ R^a(b, h; \Theta) = -\frac{V_{11}(b, h; \Theta)}{V_1(b, h; \Theta)} + \alpha V_1(b, h; \Theta) \]  

(4.6)

where \( V_1 \) and \( V_{11} \) are the first and second partial derivatives of the household’s value function with respect to the first element.\(^2\)

**Proof of Proposition 1** The household’s first-order conditions with respect to \( c^*_t, h^*_t, \) and \( l^*_t \) are:

\[ u_1(c^*_t, h^*_{t+1}, l^*_t) = \frac{1}{1 - \beta} E_t M_{t+1} V_1(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1}) \]

\[ u_2(c^*_t, h^*_{t+1}, l^*_t) = \frac{1}{1 - \beta} E_t M_{t+1} \left[ q^h V_1(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1}) - V_2(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1}) \right] \]  

(4.7)

\[ u_3(c^*_t, h^*_{t+1}, l^*_t) = -w_t \frac{1}{1 - \beta} E_t M_{t+1} V_1(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1}) \]

where \( u_1, u_2, \) and \( u_3 \) denote the corresponding partial derivatives of period utility, and \( M_{t+1} \equiv \beta \frac{e^{-aV(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1})}}{E_t e^{-aV(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1})}}. \) Note that the partial effect between choice variables, \( \partial h^*_{t+1} / \partial c^*_t \) or \( \partial h^*_{t+1} / \partial l^*_t, \) is zero. In (4.7), the first and third equations are the standard optimality conditions, while the second equation determines the house price \( q^h_t \) as the discounted infinite sum of the future marginal rate of substitution between housing and consumption.\(^3\)

Rearranging the budget constraint with the one-time fee (4.5) yields:

\[ b^*_{t+1} = (1 + r_t) \left( b_t - \frac{1}{(1 + r_t)} \mu \right) + w_t l^*_t + d_t - c^*_t - q^h_t (h^*_{t+1} - h_t), \]  

(4.8)

so the household can be thought of using the bond to pay the fee rather than housing. Using housing to pay \( \mu \) does not change the result, but the analysis below is much simpler if we use the bond.

\(^2\)I rigorously derive the closed-form expressions for risk aversion with housing in line with Swanson (2012 and 2015). To derive (4.6), more assumptions are needed such as \( u' > 0 \) and \( u'' < 0. \) For simplicity, in this paper, I assume that all the assumptions in Swanson (2015) hold.

\(^3\)This can be seen more clearly by equation (4.21), which is equivalent to the second equation in (4.7).
Then, for an infinitesimal fee $d\mu$, the first-order effect on household’s welfare is

$$
-\frac{1}{(1 + r_t)} V_1(b_t, h_t; \Theta_t)d\mu.
$$

(4.9)

As shown in Appendix A, the first order condition of household’s welfare with respect to state variable $b_t$ yields:

$$
V_1(b_t, h_t; \Theta_t) = E_t M_{t+1} V_1(b^*_t, h^*_{t+1}; \Theta_{t+1})(1 + r_t).
$$

(4.10)

Hence, (4.9) is the same as $-E_t M_{t+1} V_1(b^*_t, h^*_{t+1}; \Theta_{t+1})d\mu$.

Appendix A shows that the first-order effect on household’s welfare with respect to size of the gamble $\sigma$ in equation (4.3) is zero at $\sigma = 0$.

The second-order effect in equation (4.3) with respect to $\sigma$, evaluated at $\sigma = 0$ yields:

$$
\left[-\beta \alpha \frac{1}{E_t e^{-\alpha V_{t+1}} E_t e^{-\alpha V_{t+1}} V^2_{1,t+1}} + \beta \frac{1}{E_t e^{-\alpha V_{t+1}} E_t e^{-\alpha V_{t+1}} V_{11,t+1}} \right] \frac{d\sigma^2}{2}
$$

(4.11)

because of the optimality of $c^*_t$, $h^*_{t+1}$, and $l^*_t$ (derived in the appendix).

Following Arrow (1965), Pratt (1964), and Swanson (2012), absolute risk aversion is defined as

$$
R^a(b_t, h_t; \Theta_t) \equiv \lim_{\sigma \to 0} \frac{\mu(b_t, h_t; \Theta_t; \sigma)}{\sigma^2/2},
$$

(4.12)

where $\mu(b_t, h_t; \Theta_t; \sigma)$ denotes the household’s willingness to pay, which depends on the economic state. Finally, absolute risk aversion at the nonstochastic steady state is:

$$
R^a(b, h; \Theta) = -\frac{V_{11}(b, h; \Theta)}{V_1(b, h; \Theta)} + \alpha V_1(b, h; \Theta)
$$

(4.13)
which is exactly the same expression as in Swanson (2015) although there is one more state variable. Intuitively, this is because we can think of the household as using the bond to pay the one-time fee, $\mu$.

**Proposition 2. Closed-form of Absolute Risk Aversion** The household’s coefficient of absolute risk aversion at the nonstochastic steady state is given by

$$R^a(b, h; \Theta) = \frac{\lambda h u_{12} + \lambda^l u_{13} - u_{11}}{u_1} \frac{r(1 + r)}{1 + w \lambda l (1 + r) - q^h r \lambda h} + \alpha r u_1$$  \hspace{1cm} (4.14)

where $u_1$ and $u_{11}$ denote the corresponding partial derivatives of the household’s period utility at the nonstochastic steady state, and $\lambda h$ and $\lambda^l$ are parameters (defined below).

**Proof of Proposition 2** Given (4.13), we need to compute explicit functional forms of the first derivative, $V_1$, and the second derivative, $V_{11}$. Combining the first equation in (4.7) and (4.10) gives the standard envelope theorem

$$V_1(b_t, h_t; \Theta_t) = (1 - \beta)u_1(c^*_t, h^*_{t+1}, l^*_t)(1 + r_t).$$  \hspace{1cm} (4.15)

Differentiating (4.15) with respect to $b_t$ yields:

$$V_{11}(b_t, h_t; \Theta_t) = (1 - \beta) \left[ u_{11}(c^*_t, h^*_{t+1}, l^*_t) \frac{\partial c^*_t}{\partial b_t} 
+ u_{12}(c^*_t, h^*_{t+1}, l^*_t) \frac{\partial h^*_{t+1}}{\partial b_t} 
+ u_{13}(c^*_t, h^*_{t+1}, l^*_t) \frac{\partial l^*_t}{\partial b_t} \right] (1 + r_t).$$  \hspace{1cm} (4.16)

Before deriving the derivative $\partial c^*_t / \partial b_t$, it is convenient to find the relation between the derivatives of other choice variables. The first and the third equation of the household’s
first-order conditions (4.7) yield the intratemporal optimality condition:

\[-u_3(c_t^*, h_{t+1}^*, l_t^*) = w_t u_1(c_t^*, h_{t+1}^*, l_t^*). \tag{4.17}\]

Differentiating (4.17) with respect to $b_t$ yields:

\[
(u_3 u_{13} - u_{13} u_{33}) \frac{\partial l_t^*}{\partial b_t} = (u_1 u_{13} - u_{33} u_{11}) \frac{\partial c_t^*}{\partial b_t} + (u_1 u_{23} - u_{33} u_{12}) \frac{\partial h_{t+1}^*}{\partial b_t} \tag{4.18}\]

at the nonstochastic steady state.

Similarly, the first and the second equation of the household’s first-order conditions (4.7) yield the intertemporal optimality condition

\[u_2(c_t^*, h_{t+1}^*, l_t^*) = q_t^h u_1(c_t^*, h_{t+1}^*, l_t^*) - \frac{1}{1 - \beta} E_t \mathcal{M}_{t+1} V_2(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}). \tag{4.19}\]

Appendix A shows, $V_2(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1})$ can be computed by differentiating (4.3) with respect to $h_t$:

\[V_2(b_t, h_t; \Theta_t) = E_t \mathcal{M}_{t+1} V_1(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) q_t^h, \tag{4.20}\]

which gives $V_2(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) = (1 - \beta) u_1(c_{t+1}^*, h_{t+2}^*, l_{t+1}^*) q_{t+1}^h$ because of (4.10) and (4.15). Therefore, the intertemporal optimality condition (4.19) is equivalent to

\[u_2(c_t^*, h_{t+1}^*, l_t^*) = q_t^h u_1(c_t^*, h_{t+1}^*, l_t^*) - E_t \mathcal{M}_{t+1} u_1(c_{t+1}^*, h_{t+2}^*, l_{t+1}^*) q_{t+1}^h. \tag{4.21}\]

Differentiating (4.21) with respect to $b_{t-1}$ yields:

\[
u_{21} \frac{\partial c_t^*}{\partial b_t} + u_{22} \frac{\partial h_{t+1}^*}{\partial b_t} + u_{23} \frac{\partial l_t^*}{\partial b_t} = q_t^h \left( u_{11,t} \frac{\partial c_t^*}{\partial b_t} + u_{12,t} \frac{\partial h_{t+1}^*}{\partial b_t} + u_{13,t} \frac{\partial l_t^*}{\partial b_t} \right) - E_t \mathcal{M}_{t+1} q_{t+1}^h \left( u_{11,t+1} \frac{\partial c_{t+1}^*}{\partial b_t} + u_{12,t+1} \frac{\partial h_{t+2}^*}{\partial b_t} + u_{13,t+1} \frac{\partial l_{t+1}^*}{\partial b_t} \right) \tag{4.22}\]
Note that $\frac{\partial q_{t+1}}{\partial b_t} = 0$ because house price is given for a household.

We can guess and verify a solution for the derivatives $\frac{\partial h^*_{t+1}}{\partial b_t} = -\lambda^h \frac{\partial c^*_t}{\partial b_t}$ and $\frac{\partial l^*_{t+1}}{\partial b_t} = -\lambda^l \frac{\partial c^*_t}{\partial b_t}$ for some unknown coefficients $\lambda^h$ and $\lambda^l$. From (4.15), $V_1(b_{t+1}, h_{t+1}; \Theta_{t+1}) = (1 - \beta)(1 + r_t)u^*_1,t+1$ holds, and substituting this into the first equation in (4.7) yields:

$$u_1(c^*_t, h^*_{t+1}, l^*_{t+1}) = E_t M_{t+1}(1 + r_t)u^*_1,c^*_{t+1},h^*_{t+2},l^*_{t+1})$$ (4.23)

Differentiating (4.23) with respect to $b_t$ yields:

$$u_{11} \frac{\partial c^*_t}{\partial b_t} + u_{12} \frac{\partial h^*_{t+1}}{\partial b_t} + u_{13} \frac{\partial l^*_{t+1}}{\partial b_t} = E_t \left( u_{11} \frac{\partial c^*_{t+1}}{\partial b_t} + u_{12} \frac{\partial h^*_{t+2}}{\partial b_t} + u_{13} \frac{\partial l^*_{t+1}}{\partial b_t} \right)$$ (4.24)

at the nonstochastic steady state. Then,

$$(u_{11} - \lambda^h u_{12} - \lambda^l u_{13}) \left( \frac{\partial c^*_t}{\partial b_t} - E_t \frac{\partial c^*_{t+1}}{\partial b_t} \right) = 0$$ (4.25)

and this holds for each period. Hence, for each $j = 1, 2, 3, \ldots$,

$$\frac{\partial c^*_t}{\partial b_t} = E_t \frac{\partial c^*_{t+j}}{\partial b_t}.$$ (4.26)

Then, (4.22) is equivalent to

$$u_{21} \frac{\partial c^*_t}{\partial b_t} + u_{22} \frac{\partial h^*_{t+1}}{\partial b_t} + u_{23} \frac{\partial l^*_{t+1}}{\partial b_t} = (1 - \beta) q^h \left( u_{11} \frac{\partial c^*_t}{\partial b_t} + u_{12} \frac{\partial h^*_{t+1}}{\partial b_t} + u_{13} \frac{\partial l^*_{t+1}}{\partial b_t} \right)$$ (4.27)

in a neighborhood of the nonstochastic steady state. Combining (4.18) and (4.27) verifies:

$$\frac{\partial h^*_{t+1}}{\partial b_t} = -\lambda^h \frac{\partial c^*_t}{\partial b_t},$$ (4.28)

and

$$\frac{\partial l^*_t}{\partial b_t} = -\lambda^l \frac{\partial c^*_t}{\partial b_t}.$$ (4.29)
where
\[ \chi^h = \frac{u_2(u_{13}^2 - u_{11}u_{33}) + u_1(u_{12}u_{33} - u_{12}u_{23}) - u_3(u_{12}u_{13} - u_{11}u_{23})}{u_3(u_{12}u_{23} - u_{13}u_{22}) - u_2(u_{12}u_{33} - u_{13}u_{23}) - u_1(u_{23}^2 - u_{22}u_{33})}, \] (4.30)
and
\[ \chi^l = \frac{u_2(u_{12}u_{13} - u_{11}u_{23}) + u_1(u_{12}u_{23} - u_{13}u_{22}) - u_3(u_{12}^2 - u_{11}u_{22})}{u_3(u_{13}u_{22} - u_{12}u_{23}) - u_2(u_{13}u_{23} - u_{12}u_{33}) - u_1(u_{23}u_{33} - u_{33}^2)} \] (4.31)

Forward recursion of the intertemporal budget constraint (4.1) with respect to the quantity bonds yields:
\[
\begin{align*}
 b_{t+1}^* & = \frac{1}{1 + r_{t+1}} \left( \frac{1}{1 + r_{t+2}} \left( b_{t+3}^* - w_{t+2}l_{t+2}^* - d_{t+2} + c_{t+2}^* + q_{t+2}^h(h_{t+3}^* - h_{t+2}^*) \right) \right) \\
 & \quad - w_{t+1}l_{t+1}^* - d_{t+1} + c_{t+1}^* + q_{t+1}^h(h_{t+2}^* - h_{t+1}^*) \\
& \quad \quad \cdots
\end{align*}
\] (4.32)

The term \( b_{t+3}^* \) in (4.32) can be eliminated by substituting, then differentiating the left-hand side of (4.32) with respect to \( b_t \) yields:
\[
(1 + r_t) + w_t \frac{\partial l_t^*}{\partial b_t} - \frac{\partial c_t^*}{\partial b_t} - q_t^h \frac{\partial h_{t+1}^*}{\partial b_t}. \] (4.33)

Similarly, differentiating the right-hand side of (4.32) with respect to \( b_t \) yields:
\[
\begin{align*}
\frac{1}{1 + r_{t+1}} \left( -w_{t+1} \frac{\partial l_{t+1}^*}{\partial b_t} + \frac{\partial c_{t+1}^*}{\partial b_t} + q_{t+1}^h \frac{\partial h_{t+2}^*}{\partial b_t} - q_t^h \frac{\partial h_{t+1}^*}{\partial b_t} \right) \\
+ \frac{1}{(1 + r_{t+1})(1 + r_{t+2})} \left( -w_{t+2} \frac{\partial l_{t+2}^*}{\partial b_t} + \frac{\partial c_{t+2}^*}{\partial b_t} + q_{t+2}^h \frac{\partial h_{t+3}^*}{\partial b_t} - q_{t+1}^h \frac{\partial h_{t+2}^*}{\partial b_t} \right) \\
\end{align*}
\] (4.34)

At the nonstochastic steady state, (4.34) is equivalent to
\[
\begin{align*}
\frac{1}{1 + r} \left( (1 + w\lambda^l) \frac{\partial c_{t+1}^*}{\partial b_t} \right) \\
+ \frac{1}{(1 + r)^2} \left( (1 + w\lambda^l) \frac{\partial c_{t+2}^*}{\partial b_t} \right) + \cdots
\end{align*}
\]

Therefore,
\[
\frac{1}{r} \left( (1 + w\lambda^l) \frac{\partial c_t^*}{\partial b_t} \right) = (1 + r) - (1 + w\lambda^l - q^h\lambda^h) \frac{\partial c_t^*}{\partial b_t}. \] (4.35)
holds. Rearranging (4.35) gives the derivative $\partial c^*_t / \partial b_t$:

$$\frac{\partial c^*_t}{\partial b_t} = \frac{r(1 + r)}{(1 + w\lambda) (1 + r) - q^h r \lambda^h}.$$  

(4.36)

At the nonstochastic steady state, (4.16) is:

$$V_{11}(b, h; \Theta) = (1 + r)(1 - \beta) \left( u_{11} - \lambda^h u_{12} - \lambda^l u_{13} \right) \frac{r(1 + r)}{(1 + w\lambda) (1 + r) - q^h r \lambda^h}. \tag{4.37}$$

In addition, from (4.15)

$$V_1(b, h; \Theta) = (1 - \beta) u_1 (1 + r). \tag{4.38}$$

Therefore, the coefficient of absolute risk aversion is:

$$R^a = \frac{\lambda^h u_{12} + \lambda^l u_{13} - u_{11}}{u_1} \frac{r(1 + r)}{(1 + w\lambda) (1 + r) - q^h r \lambda^h} + \alpha r u_1. \tag{4.39}$$

Equation (4.39) shows that, in effect, housing decreases risk aversion since $u_1 > 0$, $u_{11} < 0$, and $\lambda^h < 0$ by assuming housing is a normal good. In the absence of housing, households adjust their consumption and supply of labor to absorb shocks. In contrast, when housing is in period utility function, shocks are split and absorbed by consumption, labor, and housing in the utility, so that risk aversion can be reduced with the additional component.

For asset pricing and risk premia, the coefficient of relative risk aversion is more standard to use instead of absolute risk aversion as in Rudebusch and Swanson (2012) and Swanson (2012). Definition 1 provides the coefficient of relative risk aversion.

**Definition 1. Relative Risk Aversion** Let $(b, h; \Theta)$ be an interior point of the domain of $(b, h; \Theta)$. The household’s coefficient of relative risk aversion, $R^c(b, h; \Theta)$, is given by
$A^c R^a(b, h; \Theta)$ where $A^c$ is the nonstochastic steady state of the discounted sum of present and future consumption, $A^c_t \equiv (1 + r_t)^{-1} E_t \sum_{j=0}^{\infty} m_{t+j} c^*_t$.

**Proposition 3. Closed-form of Relative Risk Aversion** The coefficient of relative risk aversion at the nonstochastic steady state is given by

$$R^c(b, h; \Theta) = \frac{\lambda h u_{12} + \lambda l u_{13} - u_{11}}{u_1} \frac{(1 + r) c}{(1 + w \lambda l)(1 + r) - q^h r \lambda h + \alpha u_1}. \quad (4.40)$$

**Proof of Proposition 3** Let budget constraint is now:

$$b^*_{t+1} = (1 + r_t)b_t + w_t l^*_t + d_t - c^*_t - q^h_t (h^*_{t+1} - h_t) + A^c_t \sigma_{t+1} \quad (4.41)$$

Then,

$$R^c(b, h; \Theta) = A^c R^a(b, h; \Theta) \quad (4.42)$$

where $A^c \equiv \frac{c}{r}$ is the present discounted value of consumption at the nonstochastic steady state.

### 4.4 Examples

Many authors in the macroeconomy literature use the additive separable utility function

$$u(c_t, l_t) \equiv \log c_t - \eta \frac{l^{1+\chi}}{1 + \chi} \quad (4.43)$$

where $\eta > 0$ is the relative weight on labor, and $\chi > 0$ denotes the inverse Frisch elasticity of the labor supply. Thus, the coefficient of relative risk aversion is

$$R^c(b; \Theta) = \frac{1}{1 + \frac{\eta}{\chi}} + \alpha \quad (4.44)$$
when the nonstochastic steady state labor is normalized at 1, \( l = 1 \), by setting the relative weight of labor \( \eta \) appropriately for simplicity. \( R^c(b; \Theta) \) decreases because of the labor margin compared to the fixed-labor measures of risk aversion as in Arrow (1965), Pratt (1964), Epstein and Zin (1989), and Weil (1989).

Following Iacoviello (2005), the household’s period utility is given as

\[
 u(c_t, h_{t+1}, l_t) \equiv \log c_t + j \log h_{t+1} - \eta \frac{l^{1+\chi}}{1+\chi}
\]  

(4.45)

where \( j > 0 \) denotes the relative weight on housing. Using (4.40), the coefficient of relative risk aversion is

\[
 R^c(b, h; \Theta) = \frac{1}{1 + \frac{\eta}{\chi} + j} + \alpha
\]  

(4.46)

Equation (4.46) is very simple due to the separability assumption in period utility (4.45). Intuitively, the closed-form expression of risk aversion (4.46) reaffirms that shocks are absorbed by consumption, labor, and housing. In particular, the \( \frac{\eta}{\chi} \) term is due to labor and the \( j \) term is due to housing. In addition, (4.46) is equivalent to relative risk aversion derived from Swanson (2015) when housing is absent in the model \((j = 0)\).

### 4.5 Conclusion

If housing is one of the components of the household’s utility function, house prices should enter the intertemporal Euler equation. This implies that housing affects the household’s optimal consumption, labor, and attitudes toward risk. I rigorously derive the coefficient of relative risk aversion with house prices and generalized recursive preferences. The result shows that house price lowers risk aversion, since the household alters consumption, labor, and housing more flexibly to absorb shocks.
In the present paper, I focus on the derivation of the household’s aversion and its properties when housing is present. Extending the analysis to investigate the relationship between risk aversion and risk premia is beyond the scope of the present paper and open for future research.
This appendix shows how to derive equation (4.10), (4.20), and (4.11). Moreover, it shows that the first-order effect on household welfare with respect to the size of the gamble is zero at $\sigma = 0$, which is necessary as an intermediate step in deriving equation (4.11).

Household marginal welfare with respect to state variables $b_t$ and $h_t$

$$V_1(b_t, h_t; \Theta_t) = (1 - \beta) \left[ u_1(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial c_t^*}{\partial b_t} + u_2(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial h_{t+1}^*}{\partial b_t} + u_3(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial l_t^*}{\partial b_t} \right]$$

$$+ E_t M_{t+1} V_1(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) \left( (1 + r_t) + w_t \frac{\partial l_t^*}{\partial b_t} - \frac{\partial c_t^*}{\partial b_t} - q_i^h \frac{\partial h_{t+1}^*}{\partial b_t} \right)$$

$$+ E_t M_{t+1} V_2(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) \frac{\partial h_{t+1}^*}{\partial b_t}$$

$$= E_t M_{t+1} V_1(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) \frac{\partial c_t^*}{\partial b_t}$$

$$+ E_t M_{t+1} \left[ q_i^b V_1(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) - V_2(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) \right] \frac{\partial h_{t+1}^*}{\partial b_t}$$

$$- w_t E_t M_{t+1} V_1(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) \frac{\partial l_t^*}{\partial b_t}$$

$$+ E_t M_{t+1} V_1(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) \left( (1 + r_t) + w_t \frac{\partial l_t^*}{\partial b_t} - \frac{\partial c_t^*}{\partial b_t} - q_i^h \frac{\partial h_{t+1}^*}{\partial b_t} \right)$$

$$+ E_t M_{t+1} V_2(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1}) \frac{\partial h_{t+1}^*}{\partial b_t}$$

$$= E_t M_{t+1} V_1(b_{t+1}^*, h_{t+1}^*; \Theta_{t+1})(1 + r_t)$$

(4.47)

The second equality holds because of (4.7). Notice that $\partial h_t / \partial b_t = 0$ because $h_t$ and $b_t$ are given already. Accordingly, $\partial b_t / \partial h_t = 0$ as well.
Similarly, we can calculate \( V_2(b_t, h_t; \Theta_t) \) by differentiating the household welfare with respect to another state variable, \( h_t \):

\[
V_2(b_t, h_t; \Theta_t) = (1 - \beta) \left[ u_1(c^*_t, h^*_{t+1}, l^*_t) \frac{\partial c^*_t}{\partial h_t} + u_2(c^*_t, h^*_{t+1}, l^*_t) \frac{\partial h^*_{t+1}}{\partial h_t} + u_3(c^*_t, h^*_{t+1}, l^*_t) \frac{\partial n^*_t}{\partial h_t} \right]
+ E_t M_{t+1} V_1(b^*_t, h^*_{t+1}; \Theta_{t+1}) \left( w_t \frac{\partial l^*_t}{\partial h_t} - \frac{\partial c^*_t}{\partial h_t} - q_t^h \frac{\partial h^*_{t+1}}{\partial h_t} + q^h_t \right)
+ E_t M_{t+1} V_2(b^*_t, h^*_{t+1}; \Theta_{t+1}) \frac{\partial h^*_{t+1}}{\partial h_t}

= \left[ E_t M_{t+1} V_1(b^*_t, h^*_{t+1}; \Theta_{t+1}) \frac{\partial c^*_t}{\partial h_t} \right]
+ E_t M_{t+1} \left[ q^h_t V_1(b^*_t, h^*_{t+1}; \Theta_{t+1}) - V_2(b^*_t, h^*_{t+1}; \Theta_{t+1}) \right] \frac{\partial h^*_{t+1}}{\partial h_t}
- w_t E_t M_{t+1} V_1(b^*_t, h^*_{t+1}; \Theta_{t+1}) \frac{\partial l^*_t}{\partial h_t}
+ E_t M_{t+1} V_1(b^*_t, h^*_{t+1}; \Theta_{t+1}) \left( w_t \frac{\partial l^*_t}{\partial h_t} - \frac{\partial c^*_t}{\partial h_t} - q_t^h \frac{\partial h^*_{t+1}}{\partial h_t} + q^h_t \right)
+ E_t M_{t+1} V_2(b^*_t, h^*_{t+1}; \Theta_{t+1}) \frac{\partial h^*_{t+1}}{\partial h_t}

= E_t M_{t+1} V_1(b^*_t, h^*_{t+1}; \Theta_{t+1}) q^h_t

(4.48)

Total derivative of household welfare with respect to size of the gamble \( \sigma \)  The first-order effect on household welfare with respect to size of the gamble \( \sigma \) in equation (4.3)
is:

\[
\left[ (1 - \beta)u_1(c^*_t, h^*_t, l^*_t) \frac{\partial c^*_t}{\partial \sigma} + (1 - \beta)u_2(c^*_t, h^*_{t+1}, l^*_t) \frac{\partial h^*_{t+1}}{\partial \sigma} + (1 - \beta)u_3(c^*_t, h^*_t, l^*_t) \frac{\partial l^*_t}{\partial \sigma} \\
+ \beta \frac{1}{E_t e^{-\alpha V(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1})}} E_t e^{-\alpha V(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1})} V_1(b^*_t, h^*_t; \Theta_{t+1}) \left( \frac{w_t}{\partial \sigma} - \frac{\partial c^*_t}{\partial \sigma} - q_t h^*_t \frac{\partial h^*_t}{\partial \sigma} + \epsilon_{t+1} \right) \\
+ \beta \frac{1}{E_t e^{-\alpha V(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1})}} E_t e^{-\alpha V(b^*_{t+1}, h^*_{t+1}; \Theta_{t+1})} V_2(b^*_t, h^*_t; \Theta_{t+1}) \frac{\partial h^*_t}{\partial \sigma} \right] d\sigma.
\]

(4.49)

Note that \( \frac{\partial c^*_t}{\partial \sigma}, \frac{\partial h^*_{t+1}}{\partial \sigma}, \) and \( \frac{\partial l^*_t}{\partial \sigma} \) vanish at \( \sigma = 0 \) because of the usual envelope theorem. Moreover, \( E_t \epsilon_{t+1} = 0 \), and \( \epsilon_{t+1} \) is orthogonal to \( b^*_t, h^*_t, \) and \( \Theta_{t+1} \).

Hence, the first-order effect on household welfare is zero.

The second-order effect on household welfare with respect to size of the gamble \( \sigma \)
in equation (4.3) is:

\[
\begin{align*}
&(1 - \beta) \left\{ u_{11}(c_t^*, h_{t+1}^*, l_t^*) \left( \frac{\partial c_t^*}{\partial \sigma} \right)^2 + u_{22}(c_t^*, h_{t+1}^*, l_t^*) \left( \frac{\partial h_{t+1}^*}{\partial \sigma} \right)^2 + u_{33}(c_t^*, h_{t+1}^*, l_t^*) \left( \frac{\partial l_t^*}{\partial \sigma} \right)^2 \\
&+ 2u_{12}(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial c_t^*}{\partial \sigma} \frac{\partial h_{t+1}^*}{\partial \sigma} + 2u_{13}(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial c_t^*}{\partial \sigma} \frac{\partial c_t^*}{\partial \sigma} + 2u_{23}(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial h_{t+1}^*}{\partial \sigma} \frac{\partial l_t^*}{\partial \sigma} \\
&+ u_1(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial^2 c_t^*}{\partial \sigma^2} + u_2(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial^2 h_{t+1}^*}{\partial \sigma^2} + u_3(c_t^*, h_{t+1}^*, l_t^*) \frac{\partial^2 l_t^*}{\partial \sigma^2} \right\} \\
&+ \beta \frac{1}{[E_t e^{-aV_{t+1}}]^2} E_t e^{-aV_{t+1}} \left\{ V_{1, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) + V_{2, t+1} \frac{\partial h_{t+1}^*}{\partial \sigma} \right\} \\
&\times E_t e^{-aV_{t+1}} V_{1, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) \\
&- \beta \frac{1}{E_t e^{-aV_{t+1}}} E_t e^{-aV_{t+1}} \left\{ V_{1, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) + V_{2, t+1} \frac{\partial h_{t+1}^*}{\partial \sigma} \right\} \\
&\times V_{1, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) \\
&+ \beta \frac{1}{E_t e^{-aV_{t+1}}} E_t e^{-aV_{t+1}} \left\{ V_{1, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) \right\}^2 \\
&+ V_{12, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) \left( \frac{\partial h_{t+1}^*}{\partial \sigma} \right) \\
&+ \beta \frac{1}{E_t e^{-aV_{t+1}}} E_t e^{-aV_{t+1}} V_{1, t+1} \left( w_t \frac{\partial^2 l_t^*}{\partial \sigma^2} - \frac{\partial^2 c_t^*}{\partial \sigma^2} - q_t \frac{\partial^2 h_{t+1}^*}{\partial \sigma^2} \right) \\
&+ \beta \frac{1}{[E_t e^{-aV_{t+1}}]^2} E_t e^{-aV_{t+1}} \left\{ V_{1, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) + V_{2, t+1} \frac{\partial h_{t+1}^*}{\partial \sigma} \right\} \\
&\times E_t e^{-aV_{t+1}} V_{2, t+1} \frac{\partial h_{t+1}^*}{\partial \sigma} \\
&- \beta \frac{1}{E_t e^{-aV_{t+1}}} E_t e^{-aV_{t+1}} \left\{ V_{1, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) + V_{2, t+1} \frac{\partial h_{t+1}^*}{\partial \sigma} \right\} V_{2, t+1} \frac{\partial h_{t+1}^*}{\partial \sigma} \\
&+ \beta \frac{1}{E_t e^{-aV_{t+1}}} E_t e^{-aV_{t+1}} \left\{ V_{21, t+1} \left( w_t \frac{\partial l_t^*}{\partial \sigma} - \frac{\partial c_t^*}{\partial \sigma} - q_t h_{t+1}^* + \epsilon_{t+1} \right) \frac{\partial h_{t+1}^*}{\partial \sigma} + V_{22, t+1} \left( \frac{\partial h_{t+1}^*}{\partial \sigma} \right)^2 \right\} \\
&+ \beta \frac{1}{E_t e^{-aV_{t+1}}} E_t e^{-aV_{t+1}} V_{2, t+1} \frac{\partial^2 h_{t+1}^*}{\partial \sigma^2} \right\} \frac{\sigma^2}{2}.
\end{align*}
\]

(4.50)

Notice that the terms involving \(\frac{\partial^2 c_t^*}{\partial \sigma^2}, \frac{\partial^2 h_{t+1}^*}{\partial \sigma^2}, \text{and } \frac{\partial^2 l_t^*}{\partial \sigma^2}\) in (4.11) cancel each other, and \(\frac{\partial c_t^*}{\partial \sigma}, \frac{\partial h_{t+1}^*}{\partial \sigma}, \text{and } \frac{\partial l_t^*}{\partial \sigma}\) vanish at \(\sigma = 0\) as before. Therefore,
the second-order effect in equation (4.3) with respect to $\sigma$, evaluated at $\sigma = 0$ is:

\[
-\beta \alpha \frac{1}{E_t e^{-\alpha V_{t+1}}} E_t e^{-\alpha V_{t+1}} V^2_{1,t+1} \\
+ \beta \frac{1}{E_t e^{-\alpha V_{t+1}}} E_t e^{-\alpha V_{t+1}} V_{11,t+1} d\sigma^2 \\
\] (4.51)
Chapter 5

Conclusions

This dissertation examines the impacts of financial frictions on the equity premium with two modifications on a medium-scale New Keynesian DSGE model: the adoption of generalized recursive preferences and financial frictions. Quantitative analyses of the models show that financial frictions substantially improve the model’s asset pricing implications in the second chapter, while they have a negligible impact on the equity premium in the third chapter. The result indicates that these two different kinds of financial frictions have similar impact on the macroeconomy, but not on asset pricing.

In a nutshell, I believe the present dissertation makes progress on the task of consolidating the analysis of asset prices and macroeconomics with financial frictions. Still, the models have many rooms to improve. Firstly, it may be useful to extend the role of financial intermediaries in future research. In real world, financial intermediaries have a much more diverse role and products like mortgage, mutual fund, capital raising, and project finance. Financial intermediaries in the model, however, consider only corporate finance and
household deposits. Secondly, I plan to explore the impacts of financial frictions on various financial indicators such as the bond premium, credit spread premium, or exchange rate risk premium. Finally, I will explore alternative ways to increase risk in the model with the framework I developed in the present dissertation.
Bibliography


