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EFFECT OF VIBRATION-ROTATION INTERACTION
ON E2 TRANSITION PROBABILITIES IN SPHEROIDAL NUCLEI

Lucy Wu Person and John O. Rasmussen

August, 1960
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INTRODUCTION

Rotational spectra have been well established as an important feature of the excited states in deformed nuclei. The deviation from the simple rotational spectrum has been found for the most part to be small and often to exhibit the character of rotation-vibration corrections, especially in even-even nuclei. An examination of rotational spectra in Sm$^{152}$, Er$^{166}$, Th$^{232}$, U$^{234}$, and Pu$^{238}$ reveal a deviation from a simple structure which can be partially accounted for by the rotation-vibration correction.$^1$ This rotation-vibration correction in molecules is well known.

In the following sections, the sequence of the rotation-vibration perturbation are developed using a simplified model in the beginning in order to keep the essential points foremost. A rather detailed application is then made to the accurately measured energies of Sm$^{152}$, Er$^{166}$, Th$^{232}$, U$^{234}$, and Pu$^{238}$. It is also shown that the experimental values agree reasonably well with the theoretical results.

CALCULATION OF ROTATION-VIBRATION INTERACTION

In order to make clear the origin and generality of the particular rotational perturbation in which we are most interested, it is useful to consider a simple theoretical model. Subsequently, we shall generalize this model in order to make it applicable to actual nuclei.

Therefore, we first restrict ourselves to nuclear systems strictly analogous to diatomic molecules.$^7$ The Hamiltonian for the system is simply

$$\hat{H} = T_{\text{rot}} + V_{\text{vib}} + T_{\text{vib}},$$

$$\hat{H} = \hat{H}_{\text{rot}} + \hat{H}_{\text{vib}} + 1/2 \left( \alpha - \alpha_0 \right)^2 + 1/2 B \left[ (d/dt)(\alpha - \alpha_0) \right]^2, \quad (1a, 1b)$$
where

\( \mathcal{Q} \) is the principal moment of inertia,

\( I_x \) is the x component of the total angular momentum,

\( I_y \) is the y component of the total angular momentum,

\( \alpha \) is the distance between two atoms along the nuclear symmetry axis, and

\( \alpha = \beta R_0/10^6 \) \( \text{10} \)

(2)

where we use \( R_0 = 1.2 \times 10^{-13} \, \text{A}^{1/3} \text{cm}, \) and

\( \alpha_0 \) is the equilibrium distance between two atoms along the nuclear axis

\( B \) the mass parameter

\( C \) the force constant parameter or the surface tension

And \( I = f \alpha^2 B \) in cgs units

\[ I = f \beta \alpha^2 B \text{ in Mev.} \]

(3a)

(3b)

(Where (3a) and (3b) are corrections to the irrotational flow moment-of-inertia expression; the constant \( f \) has the value of 3 in the irrotational flow model but here it will be taken as a parameter.)

\[ \alpha = \alpha_0 + \delta \]

(4)

\( \delta \) is the distance from the equilibrium internuclear distance.

If we put (3a) and (4) into the first term of (1b), we get

\[ \frac{\hbar^2 \tilde{I}^2}{2 f \beta \alpha_0^2} (1 + \frac{\delta}{\alpha_0})^2 \]

(5a)

where \( \tilde{I} \) is the total angular momentum of nuclei \( \tilde{I}^2 = I_x^2 + I_y^2 \).

Since \( \delta \ll \alpha \) we can expand \( (1 + \delta/\alpha_0)^{-2} \) in binomial form, the (5a) turns out to be

\[ (\hbar^2 \tilde{I}^2/2 f \beta \alpha_0^2) (1 - 2\delta/\alpha_0 + ...) \]

(5b)

The second term defined in (5b) is an effect of rotation-vibration interaction, and it constitutes the main part of the interact, in Hamiltonian

\[ H_{\text{int}} = (\hbar^2/2 f \beta \alpha_0^2)(-2 \delta/\alpha_0) \tilde{I}^2 \]

(6)

This term expresses the interaction between vibration bands with

\( \Delta n = 1 \) where \( n \) is the vibrational quantum number.

Whether or not the \( H_{\text{int}} \) will be important depends essentially on the spectrum of energies for the vibrational band. If the spacing between the bands is large compared with the rotational energies i.e., if the rotation is adiabatic with respect to the particle motion, then the \( H_{\text{int}} \) is a small perturbation and we can use second-order perturbation theory.
Since it is not certain just what value of $f$ in eq. (3b) is appropriate for nuclei, we shall estimate an upper limit on the interaction of the ground rotational band and first excited beta vibrational band by attributing all the deviation from $I(I+1)$ spacing in the ground band to this one interaction.

The rotational spectra corresponding to the close lying configurations will be mixed by the rotation-vibration interaction, and the resulting energy spectrum will be given by

$$E' = h\nu - \sqrt{h\nu - 4H_{\text{int}}^2}$$  \hspace{1cm} (7a)

or

$$E = \frac{A^2}{2J} I (I+1) \varepsilon_{II}'^2 (E_{I}' - E_{I}) + h\nu$$  \hspace{1cm} (7b)

$$\varepsilon_{II}' = \sqrt{\frac{H_{\text{int}}}{h\nu - E}}$$  \hspace{1cm} (8a)

or

$$\varepsilon_{II}' = \sqrt{\frac{F I^2 (I+1)^2}{(E_{I}' - E_{I})}}$$  \hspace{1cm} (8b)

where $\nu$ is the vibration frequency corresponding to $B$, (the mass parameter) and $C$, (the force constant parameter) $\varepsilon_{II}'$ is the mixing amplitude.

We are interested in the electric-quadrupole reduced transition probabilities between mixed states. The results are:

with mixing

$$B(E2 \ 0' \rightarrow 2+) = Z^2 e^2 \frac{5}{2l\pi} \frac{a_0^2}{\sqrt{2/5C}} + \varepsilon \alpha^2$$  \hspace{1cm} (9a)

without mixing

$$B(E2 \ 0' \rightarrow 2+) = Z^2 e^2 \frac{5}{2l\pi} \left[ 2 \alpha \sqrt{\frac{A}{2/5C}} \right]^2$$  \hspace{1cm} (9b)

and

$$B(E2 \ 2' \rightarrow 0 +)$$

$$= Z^2 e^2 \frac{1}{4\pi} \left[ 2 \alpha_0 \sqrt{\frac{A}{2/5C}} - \varepsilon \alpha^2 \right]^2$$  \hspace{1cm} (10)

We can estimate the lower limiting values of $B$ and $C$ from the correction formulae given by Birbran et al.9
By assuming \( \Delta E_Y = -\left(\frac{h^2}{2I}\right)^2 \frac{1}{6\beta^2 C_Y} \) as mentioned in Bohr et al., and the correction due to the interaction with the \( \beta \) oscillation can be determined by taking into account that the moment of inertia is a function of \( \beta \) as in (3), we then get

\[
\Delta E_B = -\frac{1}{2C_B} \left(\frac{\partial}{\partial \beta} \frac{h^2}{2I}\right)^2 I^2 (I + 1)^2
\]

\[
= -\frac{1}{2C_B} B^2 \left(\frac{h^2}{2I}\right)^2 I^2 (I + 1)^2
\]

\[
\Delta E_o = \Delta E_B + \Delta E_Y = \left(\frac{h^2}{I}\right)^2 \left[ \frac{1}{6\beta^2 C_Y} + \frac{1}{2\beta^2 C_B} \right] I^2 (I+1)^2
\]

\[
= -F I^2 (I+1)^2.
\]

Applying these calculations to \( \text{Sm}^{152} \), \( \text{Er}^{166} \), \( \text{Th}^{232} \), \( \text{U}^{234} \), and \( \text{Pu}^{238} \) we find that since \( C_Y \leq C_B \), we can get lower limiting values of \( C_B' \) and \( B_B' \) by the following formula:

\[
B_B' \geq 2/3 \left(\frac{h^2}{I}\right)^2 \frac{1/FB^2(ho_B)^2}{1/FB^2(ho_B)^2}
\]

\[
C_B' \geq 2/3 \left(\frac{h^2}{I}\right)^2 1/FB^2.
\]

Here we assume \( C_B' \) and \( B_B' \) are the same for the different vibrational-rotational bands in a nucleus. The experimental energies of the first and second excited states of the first rotational bands (\( E_2^+ \) and \( E_4^+ \)) are substituted into equation (11) to give values for \( \frac{\alpha^2}{2I} \) and \( F \). Substituting \( F \) and the experimental energies of the first and second excited states of the \( \beta \) vibrational band (\( 0^+, 2^+ \)) and \( E_2^+ \) into equation (8b) we get \( \epsilon_{22}' \), the mixing amplitude. The values of \( \epsilon_{22}' \) are given in Table I, column (a). Substituting into
The lower limiting values of $C_B'$, $B_B'$, and the experimental value of $\alpha$ are substituted into Eqs. (9a), (9b), and (10) to give the upper limiting value of $B(\text{E2} \, 0^+ \rightarrow 2^+)$ with mixing and without mixing and the values of $B(\text{E2} \, 0^+ \rightarrow 2^+)$ with mixing. These values are tabulated in columns (a), (b), and (c) of Table II both in cgs units and $\text{Bsp}$ units.

### Table I

Theoretical values of mass parameters and force constant parameters.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(a) $\epsilon_{22}'$ (mixing amplitude) in spin 2 states</th>
<th>(b) $B_B'$</th>
<th>(c) $C_B'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sm$^{152}$</td>
<td>0.087</td>
<td>228.4 $4.36 \times 10^{-22}$ Mev$^{-1}$</td>
<td>107.0 $4.69 \times 10^{20}$ Mev $\text{gm/sec}^2$</td>
</tr>
<tr>
<td>Er$^{166}$</td>
<td>0.018</td>
<td>153.2 $2.71 \times 10^{-22}$ Mev$^{-1}$</td>
<td>332.4 $1.31 \times 10^{20}$ Mev $\text{gm/sec}^2$</td>
</tr>
<tr>
<td>Th$^{232}$</td>
<td>0.023</td>
<td>533.0 $7.56 \times 10^{-22}$ Mev$^{-1}$</td>
<td>278.8 $9.21 \times 10^{20}$ Mev $\text{gm/sec}^2$</td>
</tr>
<tr>
<td>U$^{238}$</td>
<td>0.017</td>
<td>458.6 $6.53 \times 10^{-22}$ Mev$^{-1}$</td>
<td>302.4 $9.87 \times 10^{20}$ Mev $\text{gm/sec}^2$</td>
</tr>
<tr>
<td>Pu$^{238}$</td>
<td>0.012</td>
<td>536.0 $9.47 \times 10^{-22}$ Mev$^{-1}$</td>
<td>464.5 $15.14 \times 10^{20}$ Mev $\text{gm/sec}^2$</td>
</tr>
</tbody>
</table>

### Table II

Calculated values of electric quadrupole reduced transition probabilities.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(a) $B(\text{E2} , 0^+ \rightarrow 2^+)$ (with mixing)</th>
<th>(b) $B(\text{E2} , 0^+ \rightarrow 2^+)$ (without mixing)</th>
<th>(c) $B(\text{E2} , 0^+ \rightarrow 2^+)$ (with mixing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e$^2$ x 10$^{-48}$ cm$^4$ Bsp</td>
<td>e$^2$ x 10$^{-48}$ cm$^4$ Bsp</td>
<td>e$^2$ x 10$^{-48}$ cm$^4$ Bsp</td>
<td></td>
</tr>
<tr>
<td>Sm$^{152}$</td>
<td>0.30</td>
<td>12.5</td>
<td>0.21</td>
</tr>
<tr>
<td>Er$^{166}$</td>
<td>0.29</td>
<td>10.9</td>
<td>0.26</td>
</tr>
<tr>
<td>Th$^{232}$</td>
<td>0.28</td>
<td>6.7</td>
<td>0.24</td>
</tr>
<tr>
<td>U$^{234}$</td>
<td>0.35</td>
<td>8.2</td>
<td>0.31</td>
</tr>
<tr>
<td>Pu$^{238}$</td>
<td>0.30</td>
<td>6.8</td>
<td>0.27</td>
</tr>
</tbody>
</table>

\* $B(\text{E2} \, 0^+ \rightarrow 2^+) = B(\text{E2} \, 0^+ \rightarrow 2^+) \text{ (without mixing)}$. 

DISCUSSION

From the calculated $B\left(E2\right)$ values it is indicated that the $B\left(E2\ 0^+\rightarrow 2^{+}\right)$ value is depressed and $B\left(E2\ 0^'+\rightarrow 2^{+}\right)$ is enhanced. Since we have estimated the interaction by perturbation of rotational spacings, the depression and enhancement are very sensitive to the values of energies of the rotational bands; for example, a 5% error in the experimental energies will result in approximately a 20% error in the calculated $B\left(E2\right)$ values.

There are few experimental data for comparison. The only available coulombic excitation datum is $B\left(E2\ 0^+\rightarrow 2^{+}\right) = 3.1$ in Bsp for Sm$^{152}$. There is qualitative agreement between the calculated upper limiting values for Sm$^{152}$ (5.28 in Bsp) with the experimental values. Generally, the enhancement of the de-excitation transition caused by mixing is increased as the nucleus approaches to the borderline of the region of strongly deformed nuclei. At the same time the depression of the Coulombic excitation transition $0^+\rightarrow 2^{+}$ becomes more pronounced.

On the other hand, from equations (8a), (8b), (9a), (9b), (10), and (11) we get the expression for the branching ratio of the $B(E2)$ transition from the $2^+$ state of the $\gamma$ vibrational band and first rotational band

\[
\frac{B(E2; 2^+ \rightarrow 2^+)}{B(E2; 2^+ \rightarrow 0^+)} = \frac{\left(\frac{\epsilon_{22}^2}{\epsilon_{00}}\right)^2 \left(\frac{B(E2)}{\epsilon_{22}^2 \epsilon_{00}}\right)}{\left(\frac{B(E2)}{\epsilon_{00}}\right)}
\]

(A-10)

\[
\frac{B(E2; 2^+ \rightarrow 4^+)}{B(E2; 2^+ \rightarrow 0^+)} = \frac{\left(\frac{\epsilon_{42}^2}{\epsilon_{00}}\right)^2 \left[\frac{B(E2)}{\epsilon_{42} \epsilon_{00}}\right]}{\left(\frac{B(E2)}{\epsilon_{00}}\right)^2 \left[\frac{B(E2)}{\epsilon_{42} \epsilon_{00}}\right]}
\]

(A-1b)

where $B(E2)$ is the electric quadrupole transition between the $2^+$ state and first rotational band without considering the effect of mixing—these values can be easily calculated from experimental values$^{10}$ of $B(E2; 2^+ \rightarrow 0^+)$.
$B_{\text{vib}}^{(E2)}$ is the electric quadrupole transition between the $2^{'+}$ state of the $\beta$ vibration band and ground state without considering the effect of mixing — these values are assumed to be 5 times the values of $B_{\text{sp}}$, respectively, except "Sm" and Th which we could estimate easily from experimental data.

$\epsilon_{22}, \epsilon_{44}$ is the mixing amplitude which can be evaluated the same way as we mentioned before. The calculated results are tabulated in Table III:

**TABLE III**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$B(E2 \ 2'+ \rightarrow 0^+)$</th>
<th>$B(E2 \ 2'+ \rightarrow 2^+)$</th>
<th>$B(E2 \ 2'+ \rightarrow 4^+)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sm$^{152}$**</td>
<td>1</td>
<td>3.52</td>
<td>9.37</td>
</tr>
<tr>
<td>Er$^{166}$</td>
<td>1</td>
<td>1.79</td>
<td>5.06</td>
</tr>
<tr>
<td>Th$^{232}$</td>
<td>1</td>
<td>1.88</td>
<td>6.17</td>
</tr>
<tr>
<td>U$^{234}$**</td>
<td>1</td>
<td>1.84</td>
<td>5.44</td>
</tr>
<tr>
<td>Pu$^{238}$</td>
<td>1</td>
<td>1.66</td>
<td>4.10</td>
</tr>
<tr>
<td>without mixing</td>
<td>1</td>
<td>1.43</td>
<td>2.57</td>
</tr>
</tbody>
</table>

*In the Th$^{232}$ case the $\beta$ and $\gamma$ vibrational bands are so close together that the experimentally measured values of $B(E2)$ will be a mixture of two bands and then the estimated value is not accurate.

**Sm$^{152}$ Th$^{232}$ cases the third excited states of first rotational band are not known so we use $\epsilon_{44} = \frac{20}{6} \epsilon_{22}$ as an approximation.

From the Table III we see that the $B(E2 \ 2'+ \rightarrow 4^+)$ transitions are enhanced by mixing more than the $B(E2 \ 2'+ \rightarrow 2^+)$ transition, and the enhancement in general increases toward the edge of the apheroidal nuclei region. This is the same conclusion as we get from the $B(E2)$ absolute values.
REFERENCES

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