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Assessment of hybrid Phillips Curve specifications

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HIGHLIGHTS

• The first-difference of inflation negatively depends on its own lag.
• The stylized fact rejects the forward-looking NKPC and its hybrid variant with a lag of inflation.
• We show that the stylized fact can be reconciled with the hybrid NKPC with lags of inflation.
• Firm’s forward-looking behavior is relatively more important than backward-looking behavior.

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ABSTRACT

Rudd and Whelan (2006) document evidence that the first-difference of inflation negatively depends on its own lag, and highlight that the puzzling negative dependence is inconsistent with the hybrid New Keynesian Phillips Curve (NKPC) with lags of inflation. Our theoretical results show that the negative dependence appears only when firms’ forward-looking pricing behavior is relatively more important than backward-looking behavior.

1. Introduction

The New Keynesian Phillips Curve (NKPC) is a centerpiece of dynamic stochastic general equilibrium (DSGE) models for the study of monetary policy and business cycle fluctuations. However, Rudd and Whelan (2006) document evidence that the first-difference of inflation negatively depends on its own lag, and highlight that this puzzling negative dependence is an important feature that is absent from the hybrid NKPC with a lag of inflation.\(^1\)

In addition, this feature is also not consistent with the purely forward-looking NKPC in which current inflation does not rely on lags of inflation. This article investigates whether the stylized fact can be reconciled with an alternative hybrid NKPC with lags of inflation, instead of a single lag of inflation. To this end, we derive the closed-form solution of the alternative hybrid model, and find that the first-difference of inflation is determined by its lagged value and expected values of future output. Interestingly, the coefficient governing the relationship between the first-difference of inflation and its own lag is negative when firms’ forward-looking pricing behavior is relatively more important than backward-looking behavior; while it is positive when the opposite is true.

Our empirical results show that the stylized fact is consistent with the prediction of the hybrid model with lags of inflation, while, as Rudd and Whelan (2006) point out, it is inconsistent with the prediction of the hybrid NPC with a lag of inflation. We also find

\(^{1}\) Rudd and Whelan (2006) find that the stylized fact is robustly observed even after commodity price and measurement error are controlled for in their regression analysis. See Rudd and Whelan (2006) for a detailed discussion about this issue.
that expected values of future output play an important role in determining inflation dynamics. This evidence is consistent with the prediction of the closed-form solution of the hybrid model with lags of inflation. Overall, we find that the hybrid model with lags of inflation is supported by data.

2. Inflation dynamics and hybrid NKPC

The aim of this article is to investigate whether the stylized fact documented by Rudd and Whelan (2006) can be reconciled with a hybrid NKPC emphasizing the role of firms’ forward-looking pricing behavior in accounting for inflation dynamics. To this end, we employ a hybrid model given by

\[ \pi_t - \kappa \bar{\pi}_t = \beta \pi_{t-1} - \beta \pi_{t-2} + \eta y_t + \theta \tau_t \]

(1)

where \( \pi_t \equiv t_1 \pi_{t-1} + t_2 \pi_{t-2} \) and \( \tau_t = 1 \). \( \pi_t \) and \( y_t \) denote the inflation rate and output, respectively. The term \( \theta \pi_t \) represents an exogenous innovation to inflation. The parameter \( \beta \) is the discount factor, and the parameter \( \kappa \in [0, 1] \) captures the degree of indexation to lags of inflation. The hybrid model nests the purely forward-looking NKPC \((\kappa = 0)\) and the hybrid NKPC with a lag of inflation \((\tau_2 = 0)\) employed in Fuhrer (2009), Bekaert et al. (2010), and many others.

Rearranging (1) yields

\[ \pi_t = \frac{\beta}{1 + \beta \kappa} \pi_{t-1} + \frac{\kappa}{1 + \beta \kappa} \pi_{t-2} + \frac{\eta}{1 + \beta \kappa} y_t + \frac{\theta}{1 + \beta \kappa} \tau_t \]

(2)

where \( \pi_t \equiv \pi_{t-1} + \theta \pi_{t-2} + \lambda y_t + \theta^2 \tau_t \).

Interestingly, (6) implies that the first-difference of inflation, \( \Delta \pi_t \), negatively depends on its own lag when firms’ forward-looking pricing behavior is more important than backward-looking behavior \((\theta > 1/2)\). On the other hand, when the role played by backward-looking behavior is dominant \((\theta < 1/2)\), the model predicts a positive relationship of \( \Delta \pi_{t-1} \) with \( \Delta \pi_{t-2} \).

When \( \theta > 1/2 \), the model yields a negative relationship of the first-difference of inflation with its own lag. By contrast, the model with \( \theta < 1/2 \) predicts positive autocorrelation of \( \Delta \pi_t \). Thus, we find that the model properties hold for both the partial and full indexation model.

The full indexation model, \( \bar{X}_t \), that appears in (6) and (9) makes it difficult to estimate the full and partial indexation models. To avoid this problem, we define the summation term as \( \bar{X}_t \equiv \sum_{k=0}^{\infty} E_t y_{t+k} \) and introduce an equation governing the dynamics of \( \bar{X}_t \) for estimation as follows

\[ \bar{X}_{t+1} = E_t X_{t+1} + y_t. \]

(10)

Notice that iterating \( X_t \) forward results in \( X_{t+1} = \sum_{k=0}^{\infty} E_t y_{t+k} \). The summation term \( \sum_{k=0}^{\infty} E_t y_{t+k} \) of (6) and (9) can be written as \( \eta^2 \) because the supply shock follows the AR(1) process. Thus, the indexation models, (6) and (9), can be expressed as

\[ \Delta \pi_t = \left( \frac{1 - 2 \theta}{\theta} \right) \Delta \pi_{t-1} + \frac{\eta}{\theta} \Delta X_{t-1} + \frac{\theta^2}{\theta} \tau_t \]

(11)

\[ \Delta \pi_t = \left( \frac{1 - 2 \theta}{\theta} \right) \Delta \pi_{t-1} + \frac{\eta}{\theta} \Delta X_{t-1} + \frac{2 - 3 \theta - \theta^2}{\theta} \pi_{t-2} \]

(12)

respectively.

\footnote{In contrast to (9), the parameter \( \theta_0 \) does not appear in (6). This is because there is a one-to-one relationship between \( \theta_0 \) and \( \theta_1 \) in the full indexation model due to the restriction of \( \kappa = 1 \). Notice that we hold \( \hat{t}_1 = 2 - 3 \theta \) when the parameter \( \kappa \) is restricted to be one.

Rearranging this equation results in \( \Delta \pi_t = \left( \frac{1 - \theta}{\theta} \right) \Delta \pi_{t-1} + \eta \sum_{k=0}^{\infty} E_t y_{t+k} + \frac{1}{\theta} \sum_{k=0}^{\infty} E_t \pi_{t+k} \). This model predicts that \( \Delta \pi_t \) is predicted by \( \pi_{t-1} \) rather than \( \Delta \pi_{t-1} \). The full indexation model with \( t_2 = 0 \) (equivalently, \( \theta = 1/2 \)) implies that the first-difference of inflation can be expressed as \( \Delta \pi_t = \eta \sum_{k=0}^{\infty} E_t y_{t+k} + \frac{1}{\theta} \sum_{k=0}^{\infty} E_t \pi_{t+k} \). This model also predicts that \( \Delta \pi_{t-1} \) does not have any contribution to \( \Delta \pi_t \).}
For estimation of the model parameters, we consider a simple DSGE model that includes a IS curve, a monetary policy rule, and the equations describing inflation dynamics such as (10) and either (11) or (12). The IS curve for output dynamics is given by

\[ y_t = \gamma E_t y_{t+1} + (1 - \gamma) y_{t-1} - \sigma \left( i_t - E_t \pi_{t+1} \right) + \epsilon_t. \]  

(13)

The term \( i_t \) denotes the nominal interest rate. (13) can be driven under the assumption of external habit formation in consumption. This equation links current output to expected future output, lagged output, and the real interest rate. The term \( \epsilon_t \) represents a preference shock, which follows an AR(1) process, \( \epsilon_t = \delta \epsilon_{t-1} + \nu_t \) with \( \nu_t \sim N(0, \sigma^2_\nu) \). The monetary policy rule has a form of

\[ i_t = \rho i_{t-1} + (1 - \rho) \left( \alpha_0 E_t \pi_{t+1} + \alpha_1 y_t + \alpha_2 \Delta y_t + \epsilon_t \right). \]  

(14)

The central bank sets the short term interest rate in response to expected future inflation, current output, and output growth with interest rate smoothing. The term \( \epsilon_t \) represents a monetary shock, which follows an AR(1) process, \( \epsilon_t = \delta \epsilon_{t-1} + \nu_t \) with \( \nu_t \sim N(0, \sigma^2_\nu) \).

We estimate the DSGE system of (10), (11), (13), and (14) which is referred to as the full indexation model in this article. We also estimate the alternative DSGE system of (10), (12), (13), (14), which is referred to as the partial indexation model.

3. Empirical and simulation results

We use quarterly US data to estimate the DSGE models. The dataset includes real GDP, the GDP deflator, and the effective Federal Funds rate from the FRED database of the Federal Reserve Bank of St. Louis. The sample period spans from 1960:1 to 2008:4. We choose the sample period that ends in 2008:4 to avoid issues related to the zero lower bound of the short-term interest rate. We adopt a Bayesian framework with the Metropolis–Hastings algorithm for estimation of the DSGE models. We compute the posterior mean estimates of the model parameters using a sequence of 50,000 draws after an initial burn-in of 50,000 draws.

The prior distributions of the model parameters are summarized in the second column of Table 1. The parameter \( \gamma \) determining the weight between expected future output and past output in the IS curve is assumed to follow a Beta distribution with mean 0.50 and standard deviation 0.20. The slope of the IS curve (13), \( \sigma \), has a Normal distribution with mean 1.00 and standard deviation 0.50. The parameter \( \theta \) governing the relative importance of inflation expectations to lags of inflation has a Beta distribution with mean 0.50 and standard deviation 0.25. The prior mean of \( \theta \) implies that the first-difference of inflation is not related to its own lag. The coefficient \( \eta \) on \( X_t \) in the close form solutions of the hybrid models follows a Normal distribution with mean 0.01 and standard deviation 0.05. The prior distribution is set to include zero. It is worth mentioning that the determinacy condition is not violated when the parameter \( \eta \) is zero. The parameter \( \theta_1 \) follows a Beta distribution with mean 0.25 and standard deviation 0.10. The interest rate smoothing parameter \( \rho \) has a Beta distribution with mean 0.70 and standard deviation 0.5. The parameter \( \alpha_0 \) governing the responsiveness of the interest rate to inflation follows a Normal distribution with mean 2.00 and standard deviation 0.50, while the parameters, \( \alpha_1 \) and \( \alpha_2 \), measuring the response of the interest rate to current output and output growth have a Normal distribution with mean 0.50 and standard deviation 0.10.

The third and fourth columns report the posterior mean estimates for the full indexation model parameters and their 95% confidence intervals, respectively. The estimates of the partial indexation model parameters are presented in fifth and sixth columns. Focusing on the key parameters of interest, we find that the parameter \( \theta \) is estimated at 0.61 and 0.60 for the full and partial indexation model, respectively. The 95% confidence intervals reveal that the posterior mean estimates are sufficiently different from 0.50. This evidence indicates that the negative dependence of the first-difference on its own lag can be reconciled by the US data. The parameter \( \theta_1 \) is estimated at 0.25 for the partial indexation model. The parameter \( \theta_2 \) computed to be 0.15 due to the restriction of \( \theta + \theta_1 + \theta_2 = 1 \). For the full indexation model, we do not directly estimate the parameter \( \theta_1 \) due to the restriction \( \theta_1 = 2 - 3\theta \) as discussed in the previous section. The restriction suggests that the parameter \( \theta_1 \) is 0.18. The parameter \( \theta_2 \) is computed to be 0.21. For the hybrid models, the sum of the coefficients for the lagged inflation terms is about 0.40. These results highlight that the lagged inflation terms play an important role in accounting for inflation dynamics along with inflation expectations. The parameter \( \eta \) is estimated at 0.01 for the hybrid models, and the estimate is statistically different from zero.

We find that the remaining model parameters are also estimated to be very similar across the models, indicating our results are not sensitive to the model specifications for inflation dynamics. The weight on expected future output (\( \gamma \)) in the IS curve is estimated to be 0.72 for both models, highlighting the importance of output expectations in determining output dynamics. The estimate of \( \sigma \) is 0.43 and 0.38 for the full and partial indexation model, respectively. The monetary policy rule parameters \( (\rho, \alpha_0, \alpha_1) \) are estimated at 0.83, 1.87, and 0.48, respectively. The estimates are the same across the model specifications. The estimate of \( \alpha_2 \), is 0.21 and 0.20 for the full and partial indexation model, respectively.

We study whether the estimated DSGE models are able to provide a good description of the observed relationship between \( \Delta \pi_t \) and \( \Delta \pi_{t-1} \). To this end, we first plot actual changes in inflation (x-axis) and its lagged values (y-axis) in Fig. 1(a). The line with stars indicates the predicted values from a simple OLS regression of \( \Delta \pi_t \) on \( \Delta \pi_{t-1} \). The coefficient on \( \Delta \pi_{t-1} \) in this regression is estimated at –0.31. This estimate is statistically different from zero. Fig. 1(a) clearly confirms that the first-difference in inflation is negatively correlated with its own lag which is consistent with the finding of Rudd and Whelan (2006). This OLS regression has an adjusted \( R^2 \) of 0.15 which indicates a need for an additional lag of inflation in the hybrid NKPC of Christiano et al. (2005) and Smets and Wouters (2007).

In order to determine the performance of the estimated DSGE models in replicating the observed scatter plot of \( \Delta \pi_t \) against \( \Delta \pi_{t-1} \), we conduct simulation exercises by feeding 2000 random draws from the posterior distributions of the exogenous shocks into the estimated DSGE models. Fig. 1(b) and (c) display the scatter plot of the artificial \( \Delta \pi_t \) series against \( \Delta \pi_{t-1} \) based on the full and partial indexation model, respectively. The figures show the negative correlation between \( \Delta \pi_t \) and \( \Delta \pi_{t-1} \). The regression of the artificial \( \Delta \pi_t \) series on \( \Delta \pi_{t-1} \) suggests that the coefficient on \( \Delta \pi_{t-1} \) is estimated to be –0.32 for the full indexation model and –0.31 for the partial indexation model. These estimates are exactly

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5 Imposing the restriction of \( \eta = 0 \), inflation dynamics are driven by the stationary supply shock. Therefore, inflation follows a stationary process. The stationarity of output is achieved by the Fed’s strong response to output and inflation. Once inflation and output are stationary, the short-term interest rate also becomes stationary. Thus, the hybrid model with \( \eta = 0 \) does not violate the determinacy condition.

6 The fact that forward-looking behavior is relatively more important than backward-looking behavior is consistent with the finding of Gall and Gertler (1999).

7 See footnote 3 for a detailed discussion.
consistent with the counterpart based on actual data.\(^8\) We find that the observed distribution of \(\Delta\pi_t\) (or \(\Delta\pi_{t-1}\)) is also matched with the artificial data. The change in inflation is mostly distributed between \(-0.8\) and \(0.8\) with mean zero across actual and artificial data. The standard deviation of \(\Delta\pi_t\) is 0.28 for actual data, while it is 0.29 (0.28) for the full (partial) indexation model. The adjusted \(R^2\) of the regression is 0.11 for the full indexation model and 0.09 for the partial indexation model. These values are slightly lower than an adjusted \(R^2\) of 0.15 for actual data.

4. Robustness check

We study whether our results are sensitive to an alternative inflation measure. We consider the inflation rate based on the personal consumption expenditure (PCE). The PCE series has been taken from the FRED dataset of the Federal Reserve Bank of St. Louis. We also investigate whether the parameters of interest are consistently estimated over the sample periods:1960:1–2008:4,

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\(^8\) The estimates appear to be slightly biased upward in that the estimate of \(\theta\) presented in Table 1 implies that the coefficient on the lagged change in inflation is \(-0.36\). The bias may result from the missing variable \(X_t\) in the simple OLS regression for \(\Delta\pi_t\).
Table 2
Robustness check: subsample analysis.

<table>
<thead>
<tr>
<th>Inflation</th>
<th>$\theta$</th>
<th>$\theta_1$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full indexation model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960:1–2008:4</td>
<td>GDP inflation</td>
<td>0.6083 (0.5603, 0.6556)</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>PCE inflation</td>
<td>0.6296 (0.5829, 0.6774)</td>
<td>–</td>
</tr>
<tr>
<td>1960:1–1979:4</td>
<td>GDP inflation</td>
<td>0.6257 (0.5676, 0.6948)</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>PCE inflation</td>
<td>0.6508 (0.5866, 0.7173)</td>
<td>–</td>
</tr>
<tr>
<td>1983:1–2008:4</td>
<td>GDP inflation</td>
<td>0.6479 (0.6040, 0.6952)</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>PCE inflation</td>
<td>0.6446 (0.6059, 0.6830)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Partial indexation model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960:1–2008:4</td>
<td>GDP inflation</td>
<td>0.5958 (0.5553, 0.6396)</td>
<td>0.2491 (0.0759, 0.4256)</td>
</tr>
<tr>
<td></td>
<td>PCE inflation</td>
<td>0.6043 (0.5638, 0.6454)</td>
<td>0.2501 (0.0855, 0.4304)</td>
</tr>
<tr>
<td>1960:1–1979:4</td>
<td>GDP inflation</td>
<td>0.5993 (0.5524, 0.6473)</td>
<td>0.2665 (0.0775, 0.4435)</td>
</tr>
<tr>
<td></td>
<td>PCE inflation</td>
<td>0.6126 (0.5675, 0.6611)</td>
<td>0.2875 (0.1095, 0.4713)</td>
</tr>
<tr>
<td>1983:1–2008:4</td>
<td>GDP inflation</td>
<td>0.6136 (0.5709, 0.6583)</td>
<td>0.2480 (0.0420, 0.4532)</td>
</tr>
<tr>
<td></td>
<td>PCE inflation</td>
<td>0.6173 (0.5779, 0.6568)</td>
<td>0.1968 (0.0047, 0.4024)</td>
</tr>
</tbody>
</table>

1960:1–1979:4 and 1983:1–2008:4. Table 2 reports the full sample and subsample estimates based on the PCE inflation rate. For comparison, we also report estimation results based on the GDP inflation rate. The top panel of the table involves the full indexation model, and the bottom panel concerns the partial indexation model.

The top panel finds the full sample estimate of $\theta$ to be 0.61 for the GDP inflation rate and 0.63 for the PCE inflation rate. In the pre-1980 period, the estimate of $\theta$ is 0.63 for the GDP inflation rate and 0.65 for the PCE inflation rate. In the post-1980 period, the estimate of $\theta$ is 0.65 for the GDP inflation rate and 0.64 for the PCE inflation rate. The fact that forward-looking behavior is relatively more important than backward-looking behavior is consistently observed across the sample periods and the inflation measures. We also find that the forward-looking term $X_t$ has a statistically significant contribution to inflation dynamics regardless of the measures of inflation and the sample periods. In contrast to the finding of Rudd and Whelan (2006), our empirical results show that expectations on future economic activity play a crucial role in determining inflation. This difference may result from methodology used to estimate the hybrid models. We estimate the parameters of the hybrid models using the DSGE system, while Rudd and Whelan (2006) find the values of the hybrid model parameters using both the GMM estimator and grid search. The bottom panel of Table 2 presents the estimates of the partial indexation model. We find that the estimates of $\theta$ and $\eta$ are similar to those reported in the top panel. Once again, the statistically significant estimates of the model parameters indicate that the hybrid models with lags of inflation are supported by the US data, and that forward-looking behavior plays a crucial role in accounting for inflation dynamics.

5. Conclusion
This article shows that the puzzling negative dependence in inflation on its own lag can be reconciled by the hybrid model with lags of inflation. In addition, expected values of future output have statistically significant contributions to inflation dynamics. Our results favor the hybrid NKPC with lags of inflation to the purely forward-looking NKPC and the hybrid models with a lag of inflation proposed by Christiano et al. (2005) and Smets and Wouters (2007).

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