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Implications of the eccentric Kozai–Lidov mechanism for stars surrounding supermassive black hole binaries

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ABSTRACT
An enhanced rate of stellar tidal disruption events (TDEs) may be an important characteristic of supermassive black hole (SMBH) binaries at close separations. Here, we study the evolution of the distribution of stars around an SMBH binary due to the eccentric Kozai–Lidov (EKL) mechanism, including octupole effects and apsidal precession caused by the stellar mass distribution and general relativity. We identify a region around one of the SMBHs in the binary where the EKL mechanism drives stars to high eccentricities, which ultimately causes the stars to either scatter off the second SMBH or get disrupted. For SMBH masses \( 10^7 \) and \( 10^9 \, M_\odot \), the TDE rate can reach \( \sim 10^{-2} \, \text{yr}^{-1} \) and deplete a region of the stellar cusp around the secondary SMBH in \( \sim 0.5 \, \text{Myr} \). As a result, the final geometry of the stellar distribution between 0.01 and 0.1 pc around the secondary SMBH is a torus. These effects may be even more prominent in nuclear stellar clusters hosting a supermassive and an intermediate mass black hole.

Key words: black hole physics – galaxies: kinematics and dynamics – galaxies: nuclei.

1 INTRODUCTION
Supermassive black holes (SMBHs) are ubiquitous at the centres of galaxies (Kormendy & Ho 2013). Stars passing close to the SMBH can be tidally disrupted, and the fall back of the stellar debris produces a strong electromagnetic tidal disruption flare (e.g. Gezari 2012). More than a dozen tidal disruption event (TDE) candidates have been observed until present (e.g. Bade, Komossa & Dahlem 1996; Gezari et al. 2003, 2006, 2008, 2009; van Velzen et al. 2011; Gezari et al. 2012; Holoien et al. 2014), including two candidates with relativistic jets (Bloom et al. 2011; Levan et al. 2011; Zauderer et al. 2011; Cenko et al. 2012). TDEs can provide valuable information on dormant SMBHs, which are otherwise difficult to detect.

The rate of the TDEs provides information about the SMBH and the stellar distribution in the centre of galaxies (Stone & Metzger 2014). The rate of TDEs is highly uncertain observationally due to the small sample size. It is estimated to be in the range of \( 10^{-5} \)–\( 10^{-4} \) per galaxy per year by Donley et al. (2002), Gezari et al. (2008), Maksym (2012) and van Velzen & Farrar (2014). This roughly agrees with the theoretical estimates, discussed by Frank & Rees (1976), Lightman & Shapiro (1977), Cohn & Kulsrud (1978), Magorrian & Tremaine (1999), Wang & Merritt (2004), Brook-
through hierarchical three-body interactions. We examine the effect of these hierarchical three-body interactions. Specifically, the outer SMBH perturbs the stellar population around the inner SMBH, and leads to long-term variations in the eccentricities and inclinations of the stellar orbits while keeping the semimajor axes of their orbits fixed. In particular, when the orbit of the SMBH secondary is circular and if the mutual inclination between the orbits of the SMBHB and the stellar orbits is low, the stellar eccentricity and inclination undergo periodic oscillations, known as the quadrupole Kozai–Lidov mechanism (Kozai 1962; Lidov 1962). This is caused by the long-term (orbit-averaged) Newtonian (NT) gravitational effect expanded in multipole to the quadrupole order, i.e. second order in the semimajor axial ratio of the stellar and the outer SMBH’s orbit. More generally, it has been found that when the outer orbit is eccentric, the analogous octupole eccentric Kozai–Lidov (EKL) mechanism (third order in semimajor axial ratio) causes the eccentricity to be excited very close to the inner orbit to flip (Chang 2009). In light of recent developments in the understanding of hierarchical three-body interactions, we revisit this problem. Since the stellar eccentricity can be increased to a value much closer to unity by eccentric perturbers, we expect the EKL mechanism to enhance the TDE rates with respect to the circular case. We therefore seek to re-evaluate the total number of stars vulnerable to TDE due to EKL.

It is well known that apsidal precession quenches the EKL mechanism (e.g. Ford et al. 2000; Blaes, Lee & Socrates 2002; Naoz et al. 2013). In galactic nuclei, this may be due to the NT gravitational effect of the spherical stellar cusp or general relativistic (GR) precession, provided that the corresponding precession time-scale is much shorter than the Kozai time-scale (Chang 2009). Furthermore, the EKL mechanism may be quenched if the eccentricity of the star is changed by the stellar cluster due to scalar resonant relaxation, or if the orbital plane is reoriented by the stellar cluster due to vector resonant relaxation (Rauh & Tremaine 1996; Kocsis & Tremaine 2011, 2015) or Lense-Thirring precession (Merritt et al. 2010; Merritt & Vasileiou 2012). We find that NT precession and GR precession may have a large effect on the EKL mechanism, but tidal effects, scalar and vector resonant relaxation, and Lense-Thirring precession are typically less important. The time-scale on which the EKL mechanism operates increases if the outer SMBH mass is reduced. Thus, GR precession may dominate over and quench the EKL mechanism most efficiently if the outer SMBH is less massive than the inner SMBH (see fig. 2 in Naoz & Silk 2014). Similarly, we find that NT precession also suppresses the EKL mechanism most efficiently when the outer SMBH is less massive. Tidal disruption is expected in the opposite regime when the EKL mechanism is very prominent, i.e. when the outer SMBH is more massive than

\[ \epsilon = \frac{a_1}{a_2} \frac{e_2}{1 - e_2^2} < 0.1 , \]

where \( a \) and \( e \) are, respectively, the semimajor axis and eccentricity.

2.1 Comparison of time-scales

We examine the range of orbital parameters in order to identify the regions in which the EKL mechanism may operate. The relevant processes’ time-scales can be expressed as

\[ t_K = \frac{2\pi a_1^3}{\sqrt{G} a_1^3} \left(1 - e_2^2\right)^{3/2} \left(m_0 + m_1\right) \left(1 - e_1^2\right) \]

\[ t_{\text{GR1}} = \frac{2\pi a_1^{5/2} c_2 \left(1 - e_2^2\right)}{3G^{3/2} \left(m_0 + m_1\right)^{3/2}} \]

\[ t_{\text{GR2}} = \frac{2\pi a_2^{5/2} c_2 \left(1 - e_2^2\right)}{3G^{3/2} \left(m_0 + m_1 + m_2\right)^{3/2}} \]

\[ t_{\text{GR, int}} = \frac{16}{9} \frac{a_1^3 c_2 \left(1 - e_2^2\right)^2 \left(m_0\right)^{5/2}}{\sqrt{\pi} \left(1 - e_1^2\right) G^{3/2} m_2^2 m_3} \]

\[ t_{\text{GR, int}} = \frac{16}{9} \frac{a_1^3 c_2 \left(1 - e_2^2\right)^2 \left(m_0\right)^{5/2}}{\sqrt{\pi} \left(1 - e_1^2\right) G^{3/2} m_2^2 m_3} \]

\[ t_{\text{GR, int}} = \frac{16}{9} \frac{a_1^3 c_2 \left(1 - e_2^2\right)^2 \left(m_0\right)^{5/2}}{\sqrt{\pi} \left(1 - e_1^2\right) G^{3/2} m_2^2 m_3} \]
\[ t_{NT} = 2\pi \left( \frac{\sqrt{Gm_0/a_1^2}}{\tau m_0} \int_0^\infty f \sin \phi \frac{M_*(r) \cos \psi}{r} \right)^{-1} \]  

(7)

\[ t_{GR} = \frac{4\pi\omega}{\beta^2 \Omega^2} m_0 \frac{m_0}{M_*(r)m_1} \]  

(8)

\[ t_{GR,int} = \frac{2\pi f_{int}}{\Omega} m_0 \frac{1}{\sqrt{M_*(r)m_1}} \]  

(9)

\[ t_{RR,s} = 0.34 \frac{\sigma^3}{G^3 \rho m_1 \ln \Lambda} \]  

(10)

\[ t_{RR,v} = \frac{a_1^2 c^3 (1 - e^2)^{3/2}}{2G^2 m_0^2 \sigma} \]  

(11)

\[ t_{GW} = \frac{a_1^2}{4} \frac{5}{64} \frac{G^3 m_0 m_2 (m_0 + m_2)}{c^5} \]  

(12)

Here \( t_k \) is the quadrupole \((O(a_1/a_2)^2)\) Kozai time-scale. Following Naoz et al. (2013b), \( t_{oct} \) is the octupole \((O(a_1/a_2)^3)\) Kozai time-scale. \( t_{GR} \) and \( t_{GR,int} \) are the time-scales of the first-order post NT GR precession at the quadrupole order \((O(a_1/a_2)^2)\) on the inner and outer orbits, and \( t_{GR,int} \) is the time-scale associated with the first post-NT (1PN) order GR interaction between the inner and outer orbits. Following Kocsis & Tremaine (2011), \( t_{NT} \) is the time-scale of the NT precession caused by the stellar potential, and \( t_{RR,s} \) and \( t_{RR,v} \) are the time-scales of the scalar and vector resonant relaxation. \( t_{RR,v} \) is the two-body relaxation time-scale. \( t_{GR} \) is the Lense–Thirring precession time-scale and \( t_{GW} \) is the time-scale of the orbital decay of the binary SMBHB due to gravitational wave radiation. For the resonant relaxation time-scales, \( M_*(r) \) is the mass of the stars interior to \( r, \omega \) is the net rate of precession due to GR and NT, \( \beta \) is estimated to be \( 1.05 \pm 0.02 \) by Eilon, Kupi & Alexander (2013b). \( \Omega \) is the orbital frequency of the star and \( f_{int} \) is estimated to be 1.2 by Kocsis & Tremaine (2015). For the Lense–Thirring time-scale, \( sGm_0^2/c \) is the spin angular momentum of the inner SMBH (see references in e.g. Peters 1964; Kocsis & Tremaine 2011; Naoz et al. 2013b). We define some of these effects in more detail in Section 2.2 below.

The EKL mechanism operates if the following criteria are satisfied:

(i) The three-body configuration satisfies the hierarchical condition \((\epsilon < 0.1, \text{see equation 1})\).

(ii) The stars stay in the Hill sphere of the inner SMBH in order for them to remain bound to it, i.e. \( a_1/(1+\epsilon_1) < a_2/(1-\epsilon_2)(m_0/3m_2)^{1/3} \).

(iii) The quadrupole \((O(a_1/a_2)^2)\) Kozai time-scale, \( t_k \), needs to be shorter than the time-scales of the other mechanisms that modify the orbital elements, otherwise the EKL mechanism is suppressed. The competing mechanisms include NT precession, GR precession, scalar resonant relaxation, vector resonant relaxation, two-body relaxation, Lense–Thirring precession and the gravitational radiation.

Note that the secular approximation fails when the perturbation from the outer SMBH is too strong or when the eccentricity reaches values very close to unity (e.g. Antonini & Perets 2012; Katz & Dong 2012; Antognini et al. 2014; Antonini, Murray & Mikkola 2014; Bode & Wegg 2014). This means that there are some systems that are poorly described by our approximation. However, we expect that those systems reach even higher eccentricities than the one predicted by the octupole approximation (e.g. Antognini et al. 2014), and thus our overall qualitative conclusions may hold even for those systems, but the quantitative rate values possibly underestimate the true rates.

\[ \rho_0(r) = \frac{3 - \alpha}{2\pi r^2} \left( \frac{GM_0/m_0}{\sigma_0^2 r} \right)^{-3+\alpha} \]  

(13)

where \( k = 4, M_0 = 1.3 \times 10^6 \ M_\odot \) and \( \sigma_0 = 200 \cdot \text{km} \cdot \text{s}^{-1} \) (Tremaine et al. 2002), and we set \( \alpha = 1.75 \).

Fig. 2 shows the time-scales for the case of a 1 \( M_\odot \) star orbiting a 10\(^7 \) \( M_\odot \) SMBBH. The separation of the SMBHB is set to 0.3 pc. The upper panel corresponds to \( m_2 = 10^9 \ M_\odot \), and the lower panel corresponds to \( m_2 = 10^7 \ M_\odot \). For the Lense–Thirring time-scale, \( s \) is set to unity. The eccentricity of the star–SMBH system, \( e_s \), is assumed to be 2/3 and \( e_2 \) is assumed to be 0.7. The EKL-dominated region is larger for higher \( e_2 \) with fixed \( a_1 \) and \( a_2 \). Fig. 2 shows that the EKL mechanism is suppressed for a 10\(^7 \)–10\(^9 \) \( M_\odot \) binary at all radii, but it may operate at least in a restricted range for a 10\(^7 \)–10\(^9 \) \( M_\odot \) binary. Note that although the octupole time-scale \( t_{oct} \) is longer than some of the other secular time-scales, our simulations show that the eccentricity can nevertheless reach high values provided that \( t_k \) is the shortest time-scale and \( t_\text{sec} \) is at most moderately larger than the other time-scales. Since \( t_{t_{sec}} = t_k/\epsilon \) and \( 1/\epsilon \sim 10^{-30} \), \( t_{t_{oct}} \) is only moderately larger than the other time-scales in most of the relevant phase space when \( t_k \) is the shortest time-scale. Thus, in the following, we identify the regions where the eccentricity may be excited using conditions (i)–(iii) above irrespective of \( t_{t_{oct}} \). Typically, the conditions on the quadrupole Kozai time-scale \((t_k < t_{GR} \text{ and } t_k < t_{NT})\) set the lower limit for \( a_1 \) for a fixed \( a_2 \), and the
The $a_1$--$a_2$ parameter space, $m_0 = 10^7 M_\odot$, $m_1 = 1 M_\odot$, $e_2 = 0.7$. In the upper panel, $m_2 = 10^5 M_\odot$, and in the lower panel, $m_2 = 10^8 M_\odot$. The solid blue and red lines represent $e_1 = 0.001$ and the dashed blue and red lines represent $e_1 = 2/3$. Above the red or blue lines, the EKL mechanism is suppressed by the GR or the Newtonian precession. Below the black line or the grey lines, the hierarchical configuration or the Hill sphere limit is violated. The EKL mechanism is suppressed everywhere in the upper panel, and the EKL mechanism dominates in the shaded regions in the lower panel.

hierarchical configuration $\epsilon < 0.1$ and the Hill sphere limit set the upper limit on $a_1$.

Next, we examine the $a_1$--$a_2$ parameter space to identify the parameters where EKL dominates. We plot two examples in Fig. 3: $m_0 = 10^7 M_\odot$, $m_1 = 1 M_\odot$, $m_2 = 10^5 M_\odot$, $e_2 = 0.7$ in the upper panel, and $m_0 = 10^7 M_\odot$, $m_1 = 1 M_\odot$, $m_2 = 10^5 M_\odot$, $e_2 = 0.7$ in the lower panel. The EKL-dominated region is bigger for larger $e_2$. To test the dependence on $e_1$, we include two $e_1$ values: $e_1 = 0.001$ (solid lines) and $e_1 = 2/3$ (dashed lines), where $e_1 = 2/3$ corresponds to the mean value of $e_1$ in a thermal distribution. The parameter space is independent of the mass of the star as long as $m_1 \ll m_0$. The EKL-dominated region is bounded by $t_k = t_{GR}$ (blue line) and $t_k = NT$ (red line) from above and by the Hill sphere limit (grey line) and the hierarchical condition (black line) from below. In the upper panel, there is no region where the EKL mechanism dominates. In the lower panel, the region where EKL dominates is shaded with horizontal dashed lines for $e_1 = 2/3$ and it is shaded with vertical solid lines for $e_1 = 0.001$.

We calculate the number of stars affected by the EKL mechanism for the particular stellar density distribution around the inner SMBH (equation 13). In Fig. 4, we consider the parameter space of different $m_0$, $m_2$, $a_2$, $e_2$ and show the number of stars in the range of $a_1$ where all criteria are satisfied for the EKL mechanism to operate. Each panel shows the parameter plane of $m_0$ and $m_2$ (assuming $m_1 \ll m_0$), $a_2$ is varied in different columns of panels from 0.1 to 10 pc, and $e_2$ is varied in the different rows from 0.1 to 0.7. We set the stellar eccentricity to $e_1 = 2/3$ in all panels, the mean eccentricity for an isotropic thermal distribution. There is no systematic change in the number of stars affected by the EKL versus $e_1$. When $e_1 = 0.001$, the numbers typically increase to roughly twice the numbers of $e_1 = 2/3$, since the maximum $a_1$ allowed due to the Hill sphere criterion becomes larger. When $e_1 = 0.999$, the parameter region where stars can be affected in the $m_0$--$m_2$ plane increases, since the NT precession time-scale increases, while the changes in the numbers depend on the specific $m_0$--$m_2$ configurations. In regions where the EKL mechanism is important, approximately $10^{5-6}$ stars are affected. Thus, the EKL mechanism may significantly contribute to the TDEs. Note that the EKL mechanism is more likely to be suppressed for stars orbiting around the more massive SMBH. However, for parameters where the EKL mechanism is not suppressed everywhere around the more massive inner SMBH, the total number of stars affected by EKL may be higher for stars orbiting the more massive SMBH than for those orbiting the less massive SMBH.

### 2.2 Equations of motion

As shown in the previous section, GR and NT precessions represent important limitations for the EKL mechanism. In this section, we review the equations of motion which govern the long-term evolution of stars due to the EKL mechanism, GR and NT precessions, and tidal effects adopted from Naoz et al. (2013a,b) and Tremaine (2005). We use the Delaunay’s elements, which provide a convenient dynamical description of hierarchical three-body systems. The coordinates are the mean anomalies, $l_1$ and $l_2$, the arguments of periastron, $g_1$ and $g_2$ and the longitude of nodes, $h_1$ and $h_2$. Their conjugate momenta are

\begin{align}
L_1 &= \frac{m_0 m_1}{m_0 + m_1} \sqrt{G(m_0 + m_1) a_1} \\
L_2 &= \frac{m_2 (m_0 + m_1)}{m_0 + m_1 + m_2} \sqrt{G(m_0 + m_1 + m_2) a_2} \\
G_1 &= L_1 \sqrt{1 - e_1^2}, G_2 = L_2 \sqrt{1 - e_2^2} \\
H_1 &= G_1 \cos i_1, H_2 = G_2 \cos i_2,
\end{align}

where $i$ denotes the inclination relative to the total angular momentum of the three-body system and $G$ without subscript is the gravitational constant. To leading order, the two binaries follow independent Keplerian orbits where $l_j$ are rapidly varying and $L_j$, $G_j$, $H_j$, $g_j$ and $h_j$ are conserved for $j \in \{1, 2\}$. These quantities are slowly varying over longer time-scales due to the superposition of the perturbations: the EKL mechanism, GR and NT precessions, and tidal effects, discussed next.

#### 2.2.1 Eccentric Kozai–Lidov mechanism

The equations for the EKL mechanism may be derived using the double-averaged Hamiltonian (i.e. averaged over the rapidly varying $l_j$ and $l_i$ elements). We go beyond the analyses of Chen et al. (2011) and Wegg & Bode (2011), who considered only the quadrupole ($O(a_1/a_2^2)$) Kozai–Lidov mechanism, where the $z$-component of angular momentum is constant. This assumption does not hold when the orbit of the SMBHB is eccentric, and one needs to include the octupole order terms ($O(a_1/a_2^3)$) (e.g. Naoz et al. 2013a). The Hamiltonian can be decomposed as
2.2.2 GR effects

Next, we consider the leading order (1PN) effects of GR. We follow Naoz et al. (2013b), who derived the double-averaged 1PN Hamiltonian to the octupole ($O(a_1^3/a_2^3)$) order. The Hamiltonian consists of four terms: $H_{\text{Kozai,quad}}, H_{\text{Kozai,oct}}, H_{\text{int}}, H_{\text{NT}}$ (Naoz et al. 2013b). Here, $H_{\text{int}}$ does not contribute to the dynamical evolution, and the long-term effect of $H_{\text{int}}$ is typically negligible, as its time-scale is longer than that of the Kozai time-scale and the GR precession of the inner and outer orbits as long as the star stays within the Hill sphere of the inner SMBH. Thus, we only consider the effects of $H_{\text{Kozai}}$ and $H_{\text{NT}}$ which cause the GR precession of the arguments of periastrides,

\[
\frac{\mathrm{d}g_1}{\mathrm{d}r}_{\text{1PN,a1}} = -\frac{3G^{3/2}(m_0 + m_1)^{3/2}}{a_1^{5/2}c^2(1 - e_1)} ,
\]

\[
\frac{\mathrm{d}g_2}{\mathrm{d}r}_{\text{1PN,a2}} = -\frac{3G^{3/2}(m_0 + m_1 + m_2)^{3/2}}{a_2^{5/2}c^2(1 - e_2)} .
\]

Given that we neglect $H_{\text{int}}$, and higher order post-NT corrections such as Lense–Thirring precession and gravitational radiation, the other conserved quantities, $L_j, G_j, H_j, h_j$, are not affected for $j \in \{1, 2\}$.

2.2.3 NT precession

The NT potential of a spherical stellar cusp causes apsidal precession at the rate (Tremaine 2005):

\[
\dot{g}_{\text{1,NT}} = \frac{(1 - e_1^2)^{1/2}}{(Gm_0/a_1)^{1/2} a_1 e_1} \frac{\mathrm{d}\Phi_1}{\mathrm{d}r} \cos \psi ,
\]

where $\Phi_1$ is the stellar potential, $r$ is the distance to the central SMBH and $\psi$ is the true anomaly of the inner orbit. The averaged precession rate of $g_1$ due to NT precession is expressed below:

\[
\dot{g}_{\text{1,NT}} = \frac{(Gm_0/a_1)^{1/2}}{\pi a_1 e_1} \int_0^{\pi} \mathrm{d}\psi \, M_*(r) \cos \psi ,
\]
where $M(r)$ is the mass of the stellar system interior to $r$ and $r \equiv r(\psi) = a_1(1 - e_1^2)/(1 + e \cos \psi)$ from Kepler’s equation. Explicit analytic expressions for the apsidal precession rate are given in appendix A of Kocsis & Tremaine (2015).

### 2.2.4 Tidal dissipation

To investigate if tides can suppress eccentricity excitation, we consider the ‘equilibrium tide’ with constant time lag to calculate the inner binary’s orbital evolution when the pericentre distance is larger than $2R_\ast$. Similarly to Naoz, Farr & Rasio (2012) and Naoz & Fabrycky (2014), we include the differential equation governing the orbital evolution following Eggleton, Kiseleva & Hut (1998), Eggleton & Kiseleva-Eggleton (2001) and Fabrycky & Tremaine (2007). For the star, we assume the viscous time-scale is 10 yr, which corresponds to the quality factor (Goldreich & Soter 1966) $Q \sim 10^5$ for a 10 d orbit (or $Q \sim 4 \times 10^8$ for a 100 yr orbit).

In Fig. 5, we show a representative example of the evolution with and without tides. The effect of tides is negligible mainly because the orbital period is long and $Q$ is low.

### 3 SMBH-BINARY SYSTEM

Requiring the criteria listed in Section 2.1, the minimum and the maximum distance of the star affected by the EKL mechanism from the inner SMBH can be calculated. However, not all stars in this region will be disrupted, since the excitation of the eccentricity depends sensitively on the orbital orientation, and the parameter region where the eccentricity can be excited is complicated (Li et al. 2014b). In addition, when the Kozai time-scale is only slightly smaller than the GR or the NT time-scale (with $f_K$ still being the smallest), the evolution of the inner orbit is complex. For instance, the eccentricity of the inner orbit can be excited in configurations where the eccentricity cannot be excited due to the Kozai–Lidov mechanism alone. This excitation may be caused by the resonances between the NT, GR or Kozai–Lidov precessions (Naoz et al. 2013b).

We consider the following illustrative example: $m_1 = 10^4 M_\odot$, $m_2 = 10^6 M_\odot$, $a_2 = 0.5$ pc, $e_2 = 0.5$. We adopt the isotropic stellar distribution function of equation (13), assuming the stars have a solar mass, and that the eccentricity distribution is thermal $dN/de = 2e$. We run large Monte Carlo simulations, integrating the equations presented in Section 2, where the equations of motion for the EKL mechanism are given by Hamilton’s equations (equations A26–A35 in Naoz et al. 2013a), and $g_1 = g_{1,\text{EKL}} + g_{1,\text{GR}} + g_{1,\text{INT}}$, $g_2 = g_{2,\text{EKL}} + g_{2,\text{GR}}$. We distinguish three outcomes for the EKL evolution: ‘TDE’, ‘scattered by the SMBH companion’ and ‘surviving’ as explained now.

The eccentricity of the star needs to reach very close to unity to cause tidal disruption. The tidal radius is $R_t = 5 \times 10^{-6}$ pc around a $10^7 M_\odot$ SMBH. We identify the TDE with $a_1(1 - e_1) < 3R_t$ since the stars may still be disrupted due to accumulated heating under the strong tide outside the tidal radius (Li & Loeb 2013). Since the size of the Hill sphere of the less massive SMBH is small (i.e. 0.08 pc in our example), the star may reach the apocentre outside the Hill sphere before disruption as the eccentricity increases. Namely, the gravitational pull of the companion SMBH ($m_2$) will be larger than $m_1$. We refer to this as a ‘scattering event’ $(a_1(1 + e_1) > a_2(1 - e_2)(m_0/(3m_2))^{1/3}$). Note that the secular approximation is no longer valid for the scattering events. Three-body integrations of the dynamical evolution of scattering events show that they may either lead to an exchange interaction, where the star is captured by the outer SMBH, or they may be tidally disrupted. The scattering events resulting in a capture may systematically increase the eccentricity distribution of stars orbiting the companion SMBH. For the third category, we label the stars neither disrupted nor scattered by the companion after 1 Gyr as ‘survivors’.

Fig. 6 shows the results of the numerical simulation in the final $a_1-a_2$ and $a_1-e_1$ planes. We use open circles to mark stars that underwent TDEs, crosses for stars that were scattered by the companion and full circles for stars that survived. The disruption/scattering time is colour coded, and it indicates that most of the disruption events occur within ~0.5 Myr. This corresponds to the octupole Kozai time-scale, which is roughly 0.2–2 Myr for these systems at $a_1 = 0.03–0.08$ pc. Out of all 1000 stars between $a_1 = 0.0275$ and 0.075 pc, 57 are disrupted, and 726 are scattered by the outer black hole. According to the stellar density distribution in equation (13), there are $10^5$ stars in this semimajor axis range. Normalized by the total number of stars in this semimajor axis range, it indicates that the tidal disruption rate is ~$10^{-2}$ yr$^{-1}$ in the first ~0.5 Myr.
for the less massive black hole due to EKL, while \( \sim 7 \times 10^3 \) stars undergo scattering events by the outer SMBH.

Since the eccentricity of the stars with high inclinations are more likely to be excited, the stars with high inclinations are more vulnerable to tidal disruption, the final inclination distribution is no longer isotropic (the lower panels in Fig. 7) and the stars around the SMBH form a torus-like configuration (see Naoz & Silk 2014, for similar results). The stars with larger semimajor axis have higher probability to be scattered when their eccentricity become excited due to the EKL mechanism, and thus the final distribution of stars surrounding the less massive black hole will be truncated at a larger semimajor axis. In addition, the distribution of the eccentricity of the surviving stars shows deviations from thermal distribution with a suppression of very eccentric stars (as expected since they get scattered by \( m_2 \) more easily, and their eccentricity can be excited more easily at a lower inclination; Li et al. 2014a). Furthermore, as shown in Fig. 8, the stars that are closer to \( m_1 \) have an eccentricity distribution closer to thermal. The stars that are closer to \( m_2 \) have systematically smaller eccentricities. The thermal distribution for closely separated stars is similar to the observed S stars in the centre of the Milky Way galaxy (Genzel, Eisenhauer & Gillessen 2010), which shows a steeper slope.

4 SMBH–IMBH SYSTEM

Let us consider next the perturbations of an SMBH on stars orbiting an IMBH. IMBHs may form through runaway mergers during core collapse in globular clusters (Portegies Zwart & McMillan 2002). Since globular clusters sink to the galactic centre through dynamical friction, and the disrupted globular cluster could contribute to most of the mass in nuclei stellar cluster for galaxies with total mass below \( 10^{11} \text{M}_\odot \), this setup may be common in the Universe (Portegies Zwart et al. 2006; Antonini 2013; Gnedin, Ostriker & Tremaine 2014). Alternatively, IMBH may form at cosmologically early times from Population III stars in galactic nuclei (Madau & Rees 2001), or in accretion discs around SMBHs (Goodman & Tan 2004; McKernan et al. 2012, 2014). In the Milky Way centre, the orbits of the S stars are consistent with those caused by the dynamical interactions of IMBHs (Merritt, Gualandris & Mikkola 2009). In addition, IRS 13E may potentially host an IMBH, though its existence is controversial (Maillard et al. 2004; Schödel et al. 2005; Fritz et al. 2010). The TDE rate has been discussed by Chen & Liu (2013) and Mastrobuono-Battisti, Perets & Loeb (2014). Here, we consider the interactions of stars surrounding IMBHs in the centre of galaxies with the central SMBH due to the hierarchical three-body interactions, and consider the re-distribution of the stars as a result of the interaction.

We set the IMBH mass to \( 10^4 \text{M}_\odot \) at a distance of 0.1 pc from Sgr A* (\( a_2 = 0.1 \text{ pc, } e_2 = 0.7, m_2 = 10^4 \text{M}_\odot \) and \( m_2 = 4 \times 10^5 \text{M}_\odot \)). These parameters for the IMBH are allowed according to limits on the astrometric wobble of the radio image of Sgr A* (Hansen & Milosavljević 2003; Reid & Brunthaler 2004), the study of hyper-velocity stars (Yu & Tremaine 2003) and the study of the orbits of S stars (Gualandris & Merritt 2009). We set the distance of stars to be uniformly distributed between 0.00045 and 0.0028 pc. The tidal disruption radius for 1 M\(_\odot\) stars is 4.89 \( \times \) \( 10^{-7} \) pc. The minimum distance is set by requiring the GR precession time-scale to be longer than the Kozai time-scale, and the maximum distance is set by requiring the stars to stay in the Hill sphere of the IMBH. Note that in this case the hierarchical criterion (i) in Section 2.1, \( \epsilon < 0.1 \), is satisfied as long as the stars are within the IMBH’s Hill sphere. We assume the distribution of the stellar eccentricity to be uniform. We take into account GR precession, NT precession and EKL at octupole order in the integration.

In 1000 runs, we find that \( \sim 40 \) end up in tidal disruption and \( \sim 500 \) are scattered as shown in Fig. 9. The tidal disruption/scattering time...
served hyper-velocity stars. The TDE rate may reach
produces scattering events which may be responsible for the ob-
ticles). The predicted distribution may be resolved if the angular
resolution of the instrument is better than that corresponding to the
parameter space where the EKL mechanism may operate and
lead to TDEs. We also demonstrated that tidal effects are typically
negligible for the stellar orbital evolution (see Fig. 5).

To illustrate the EKL effects on stars surrounding the less massive
black hole, we ran 1000 numerical experiments with different initial
conditions for a star cluster surrounding a $10^4 M_\odot$ black hole, which
is being perturbed by a $10^6 M_\odot$ outer black hole. We have found
over ~50 out of the 1000 runs stars are disrupted in ~0.5 Myr.
Scaled with the total number of stars according to equation (13),
this corresponds to a TDE rate of $10^{-2} \text{ yr}^{-1}$ for the first ~0.5 Myr.
In contrast, Chen et al. (2011) considered tidal disruption rates for
stars surrounding the more massive SMBH, using numerical three-
body scattering experiments. They estimated the tidal disruption
rate to be as high as 0.2 yr$^{-1}$ mainly due to three-body scattering
effects, in the first $3 \times 10^3$ yr for stars surrounding a $10^7 M_\odot$
SMBH perturbed by an 81 times less massive outer SMBH. For
the same SMBHB configuration, EKL only affects at most ~$10^3$
stars surrounding the less massive SMBH as shown in Fig. 4, and
affects at most ~$10^4$ stars surrounding the more massive SMBH.
Thus, EKL contributes negligibly to the total tidal disruption rate in
this case, but EKL contributes significantly to the TDE rate of stars
around the secondary SMBH.

The EKL mechanism also affects the stellar distribution for stars
surrounding the less massive SMBH. As shown in Fig. 7, the sur-
vived stars within a particular range of radii are distributed in the
shape of a torus (Naoz & Silk 2014). In addition, a large number
of stars orbiting the less massive black hole will be scattered by the
outer black hole following the EKL-induced eccentricity increase.
In our illustrative example, ~670 out of 1000 stars are eventually
transferred to an orbit around the outer, more massive SMBH. This
may produce hyper-velocity stars (Guillochon & Loeb 2014).

Finally, we studied the tidal disruption of stars by an IMBH dur-
ing mergers of globular clusters with galactic nuclei. For an IMBH
of mass $10^4 M_\odot$ at a distance of 0.1 pc from Sgr A* 4 per cent
of stars get disrupted within the relevant distance range around the
IMBH, and ~50 per cent get scattered within $10^3$ yr. This yields
a temporary tidal disruption rate of $10^{-4} \text{ yr}^{-1}$. Some of the
scattering events may produce hyper-velocity stars or additional
TDEs. The EKL mechanism produces a torus-like stellar distribu-
tion for the surviving stars, which may be resolved by the Gemini,
VLT and Keck telescopes in near-infrared. Further investigations
of this process using numerical scattering experiments would be a
worthwhile in the future.

We first compared the Kozai time-scale with the secular time-
scales of other mechanisms that may suppress EKL in galactic
nuclei. These include NT precession, GR precession, resonant
relaxation, two-body relaxation, Lense–Thirring precession and
orbital decay due to gravitational wave emission. We have found that
for the SMBHB cases we considered, NT precession and GR pre-
cession may suppress EKL, especially when the inner SMBH is
more massive than the outer SMBH (as shown in Fig. 4). This is
consistent with the results by Naoz & Silk (2014) for dark matter
particles around SMBHBs, as well as the three-body scattering ex-
periments done by Chen et al. (2009), Wegg & Bode (2011) and
Chen et al. (2011), who observed that the TDEs were dominated
by the three-body chaotic interactions rather than EKL mechanism
for stars surrounding the more massive black hole. However, we
found that a massive outer binary allows a non-negligible region of
parameter space where the EKL mechanism may operate and
lead to TDEs. We also demonstrated that tidal effects are typically
negligible for the stellar orbital evolution (see Fig. 5).

Figure 9. The final distribution of stars surrounding a $10^4 M_\odot$ IMBH at a
distance of 0.1 pc from Sgr A* after 100 Myr. The open circles represent stars
that get tidally disrupted, and the crosses represent stars that get scattered.
Both are coloured according to the time of tidal disruption/scattering. We
find that ~50 per cent of the stars survived tidal disruption and scattering.
The final distribution of the star has a deficiency at high inclination relative
to the orbital plane of IMBH.

Figure 10. The initial distribution and the final distribution of the stars after
100Myr in our illustrative example for the IMBH, as shown in Fig. 9.
(colour coded) is around $10^5$ yr. As shown in Fig. 10, we predict
that the surviving stars form a torus-like configuration (similarly
to the result achieved by Naoz & Silk (2014) for dark matter
particles). The predicted distribution may be resolved if the angular
resolution of the instrument is better than that corresponding to the
Hill sphere around the IMBH, in this case 0.07 arcsec. This can be
achieved in near-infrared by the Gemini, Very Large Telescope
(VLT) and Keck telescopes. In addition, the EKL mechanism also
produces scattering events which may be responsible for the ob-
served hyper-velocity stars. The TDE rate may reach ~$10^{-4}$ \text{ yr}^{-1}
for a short ~$10^3$ yr duration episode after the globular cluster first
approaches the galactic nucleus at a distance of 0.1 pc, assuming
there are ~200 stars in a globular cluster around an $10^4 M_\odot$-IMBH
in the EKL-dominated region according to the density distribution
in equation (13).

5 CONCLUSIONS

SMBHBs are natural outcomes of galaxy mergers. An SMBHB
may show an enhanced TDE rates due to the EKL mechanism
and chaotic three-body interactions (Ivanov et al. 2005; Chen et al.
2009, 2011; Wegg & Bode 2011). The higher tidal disruption rates
may in turn serve as a flag to identify closely separated black hole
binaries on subparsec scale, which are difficult to detect otherwise.
We focused on the effect of the EKL mechanism (see Naoz et al.
2011, 2013a) on the surrounding stars in SMBHB. This mechanism
can excite the stars’ eccentricity to values very close to unity (e.g.
Naoz et al. 2013a,b; Li et al. 2014a,b). We identified the range of
physical parameters where EKL is important.

\footnote{Since, as we showed, the EKL is suppressed in this case.}
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