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(M.S. Thesis)

May 1986
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THREE-DIMENSIONAL STUDY OF DISLOCATION SUBSTRUCTURES
IN PUNCH-STRETCHED, AK, DQ, LOW-CARBON STEEL SHEETS

by

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Dislocation substructures developed in Aluminum Killed (AK), Drawing Quality (DQ), Low Carbon steel sheets during the Limiting Dome Height (LDH) test are investigated. Thin foils parallel to the sheet plane, longitudinal and transverse sections for different strain ratios have been observed by Transmission Electron Microscopy (TEM). Analysis of the cellular structure in three-dimensions allows the determination of the orientations of the cell wall planes inside individual ferrite grains. The observed cell wall planes for different strain ratios and grain orientations are compared with active slip planes calculated by using the Sach's model for polycrystal deformation. Cell walls are found to be roughly parallel to calculated slip planes for the range of strain ratios considered. Discrepancies observed in negative strain ratio samples are explained in terms of the validity of the Sach's model free-grain assumption.
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1. INTRODUCTION

Sheet metal forming has been for many decades one of the most important manufacturing techniques, particularly in the automobile industry. Today's automatic stamping lines make possible the achievement of high production rates of different automobile-body parts. The material most widely used for this application is low carbon steel, which offers among other advantages uniform thickness, good finish and low cost. Aluminum killed (AK), drawing quality (DQ), low carbon steel is characterized by a favorable plastic anisotropy as well as respectable strain hardening and strain-rate hardening.

Material failure during stamping operations is a common problem, appearing in the form of tears, cracks and/or necks in the finished part. Even with careful die design, good material selection, and thousands of successful stampings, failure can still occur. This results in an increase of the sheet metal scrap rate which leads to higher production costs. Moreover, the current demands for energy saving require the use of thinner and therefore more formable sheets to reduce the vehicle weight. For these reasons, many engineers and scientists have turned their attention to finding ways of predicting and controlling sheet metal formability.

The failure criterion in sheet metal forming is the onset of localized necking. It is an instability process \(^{(1,2)}\) determined by the competition between geometric softening and strain hardening during plastic deformation. Therefore, attempts to improve the formability of AK low carbon steel should be directed towards finding ways of either (1) delaying the geometric softening or, (2) increasing the strain
hardening of this material.

Regarding the first point, inclusions are known to have an important effect on the geometric softening of the material since they serve as void nucleation sites during plastic deformation. However, according to the study carried out by Shaffer\(^{(3)}\), a decrease of the inclusion content from the levels currently present in commercial AK low carbon steel would not result in a significant improvement of its formability in punch-stretching.

With regard to the second point, the possibility of controlling the strain hardening of this material is still uncertain. Since low carbon steel is basically a single phase material, its strain hardening behavior is determined by the evolution of the dislocation substructure inside the ferrite grains during plastic deformation. This substructure consists of misoriented cells separated by boundaries with a high density of dislocation tangles called cell walls. These are believed to be low energy configurations: dislocations tend to cluster into cell walls in order to decrease the elastic energy of the system\(^{(4-6)}\). However, whether this cellular structure can be controlled in some way to favor a better performance of the sheet metal during forming operations is still unknown. Therefore, as a first step, it is essential to have a complete and precise understanding of the evolution of dislocation substructures under different deformation conditions.

Some amount of research has been carried out on the characterization of cellular substructures in different materials under different loading conditions\(^{(7)}\). Most of these investigations have dealt with the variation of parameters such as cell size, dislocation density, and misorientation between cells with amount of plastic deformation.
These studies show that as the strain increases, the cell size decreases \((8-11)\) whereas the misorientation between cells increases \((12)\).

An important characteristic of the cell structures in single crystals and polycrystalline materials is the tendency of cell walls to be aligned along particular orientations. The majority of the investigations concerning cell wall orientation have been carried out in single crystals. In iron single crystals with different orientations deformed under tension, cell walls have been found to be roughly parallel to active slip planes \((13-16)\). Deviations of a few degrees are usual. In single crystals oriented for multiple slip, more than one cell wall family is usually present. The same finding has been observed in thin foils extended in an electron microscope where the cell formation process can be followed continuously \((17,18)\). Jackson \((19)\) modeled the formation of cell walls parallel to slip planes for the case of single glide deformation assuming cross-slip as the main cell-forming mechanism. In this model, prismatic loops serve as obstacles for dislocation motion and produce cross-slip of dislocations that transfer slip to a set of closely spaced glide planes. Thus, the associated shear is distributed over a band of neighboring planes to form well defined cell walls. Nevertheless, whatever the cell formation mechanism, it is clear that in single crystal deformation cell walls are associated with active slip planes such that the orientation of cell walls and the general cell configuration depend on the single crystal orientation with respect to the macroscopic stress axes.

In polycrystalline materials dislocation cells have in general the same characteristics as those observed in single crystals. Transmission Electron Microscopy (TEM) studies of dislocation substructures in low
carbon steel sheets show that the cell configuration depends on the loading conditions: elongated cells in uniaxial tension and plane strain deformation and equiaxed cells after equibiaxial stretching \(^{(20,21)}\). It seems likely that, as in single crystals, the orientation of the cell walls inside single grains is determined by the orientation of the grains with respect to the macroscopic stress axes. However, the correlation of cell walls with active slip planes is complicated by the fact that the local stress and strain states inside each grain are unknown due to the constraints imposed by the surrounding grains during plastic deformation.

One of the few detailed analyses of cell wall orientations in low-carbon steel sheets was carried out by Fernades and Schmitt \(^{(22)}\). Three different strain paths were analysed: uniaxial tension, plane strain and equibiaxial stretching. Good agreement was found between the cell wall orientations and the active slip plane orientations inside single grains only for the case of uniaxial tension. However, their TEM observations were limited to samples parallel to the sheet plane. The active slip systems were determined, under the assumption of pencil glide, by using two different models for polycrystal deformation: the Sach's model \(^{(23)}\), which assumes that the stress in the grains is equal to the macroscopic stress (free grains), and the Taylor model \(^{(24)}\), which assumes that the strain tensor in the grains is equal to the macroscopic strain (fully constrained grains). For uniaxial tension and equibiaxial stretching both models predicted the same principal slip systems. Their work establishes new links between the studies on single crystals and the more complicated deformation situation characteristic of polycrystalline materials.
A better understanding of dislocation cell structures related to formability may be achieved by extending the analysis of Fernandes and Schmitt to the case of punch-stretching deformation which induces a non-uniform strain distribution in the sheet metal characteristic of actual forming operations. Particularly, the Limiting Dome Height (LDH) test (25,26) (see Appendix A), which is used in the automotive industry for formability evaluation, can provide a wide range of major and minor principal strains combinations (different minor to major strain ratios) and allows the study of the dislocation cell geometry that results from different macroscopic strain conditions.

However, an important point to recognize is that the study of the cell structure in only one dimension of the sheet does not provide all the information necessary for the complete determination of the geometry of cells and cell walls inside the grains. Coincidence between the traces of cell walls and primary slip planes in sheet planes samples may not be sufficient to establish a correlation between cell wall planes and active slip planes. Similar analyses in longitudinal and transverse sections of the sheet are required.

The present investigation is a three dimensional analysis of the orientation of cell walls in punch-stretched, AK, low carbon steel sheet. The cell wall orientations observed in sheet plane, longitudinal section and transverse section samples are compared with the orientations of the active slip planes in individual grains determined through the Sach's model for polycrystal deformation. The analysis is carried out for different positive and negative strain ratios. From this study, a correlation is found between cell wall orientation, grain crystallography and macroscopic strain conditions.
2. EXPERIMENTAL PROCEDURE

2.1 Material

The material used for this study was a 0.035 in. (0.89 mm) thick sheet of a commercial AK low carbon steel. Its chemical composition and tensile properties are given in Tables 1 and 2, respectively. This material was provided by General Motors Corporation in the punch-stretched condition as a set of 7 limiting dome height test samples (see Appendix A, LDH test). For convenience, these samples (domes) were numbered from #1 to #7 according to increasing blank width from 1 in. to 7 in. (see figure 1). They exhibited both fracture and necked regions on opposite sides of the pole. Before deformation, the blanks were electrochemically-etched with a grid of 0.1 in. solid circles to allow for post-testing strain measurements.

2.2 Strain Measurement

The circle grid analysis technique\(^{(27)}\) was used for measuring strains on the different domes. After punch-stretching, the original 0.1 in. circles on the sheet became ellipses. Measurements of the major and minor axes of these ellipses at particular locations on the domes permitted the determination of the major and minor engineering strains respectively (see figure 2). For this purpose, pictures at 7x were taken from the necked regions of the domes. No more than 10 ellipses were included in each picture. The major and minor axes of each ellipse were carefully marked and their lengths were digitized using a CALCOMP 9000 digitizer. The data were then converted into major and minor engineering
strains through a computer program. This procedure was repeated at least 3 times for each ellipse to account for errors during digitizing.

2.3 Sample Selection for TEM study

Analysis of the major and minor strain data showed that close to the necked region of each dome the minor strain remained fairly constant (5% variation) while the major strain increased very rapidly with proximity to the localized neck. Samples for TEM analysis were selected from domes #7 and #3 since they offered positive and negative minor strains of approximately the same absolute values, respectively (from 10% to 15%), for a variation of major strain from 18% to about 60%. The symmetrical distribution of strain states characteristic of these domes permitted the preparation of TEM samples parallel to the sheet plane, to the longitudinal section, and to the transverse section with similar major and minor strains. In this way, the geometry of the dislocation cell structure developed during punch stretching could be studied in three dimensions for different positive and negative strain ratios \( \varepsilon_2/\varepsilon_1 \). The different strain states analyzed are included in the two bands shown in the figure 3 which is a forming limit curve typical of low carbon steel (see Appendix A, FLC). The values of engineering and true major and minor strains, equivalent strains and strain ratios studied in the three sections of the sheet are included in Tables 3 and 4 for positive and negative strain ratios, respectively.
2.4 TEM Sample Preparation

The steps involved in the preparation of sheet plane and cross-section samples for transmission electron microscopy are sketched in figure 4. Knowledge of the rolling direction through all the sample preparation procedure was maintained to allow for the determination of the major strain axis, which was perpendicular to the rolling direction, on the TEM micrographs. The general sample preparation procedure was as follows. A piece of the dome, which included the strain states (ellipses) of interest, was cut with a fine jeweler saw. The rolling direction was marked by two notches at opposite edges of the cut piece. The locations of the ellipses, corresponding to different strain states, were mapped with respect to the edges of the piece for later identification. At this stage, the procedures for preparation of sheet plane and cross-samples started differing from each other and are explained separately.

Sheet plane samples. The piece was chemically thinned in a solution of 5% HF in H₂O₂(30%) to a thickness of about 8 mils (0.20 mm). The positions of the ellipses’ centers were remarked on the thinned sheet using the map mentioned above. The piece was then scored with lines parallel to the rolling direction passing along the ellipses’ centers. At this stage, discs 3 mm in diameter were cut by spark cutting to avoid the introduction of dislocations characteristic of mechanical cutting. Under an optical microscope, the rolling direction was again indicated on the circumference of each disc by two small notches, which were made with a sharp blade (see figure 5a). The 3 mm discs were subsequently
hand ground carefully on 600 grit SiC paper down to about 2.5 mils (0.06 mm); material was removed equally from both sides. Thin foils were obtained by jet polishing these discs at room temperature in a solution of 400 ml CH₃COOH + 75 g Cr₂O₃ + 21 ml H₂O. The polishing current and voltage varied between 20 to 25 mA and 20 to 25 volts, respectively.

Cross-section samples.—The main inconvenience in the preparation of discs 3 mm in diameter from cross-section samples was that the thickness of the sheet was only 35 mils (0.89 mm) or less. Therefore, nickel plating was used to increase the thickness of the cut piece to at least 3 mm. After nickel plating, the locations of the ellipses were remarked on the piece which was then mounted in Koldmount. Thin slices of about 15 mils (0.38 mm), parallel to the rolling direction for transverse section samples and perpendicular to the rolling direction for longitudinal section samples, were cut through the ellipses of interest using an Isomet diamond wafering blade under flood cooling. These thin slices were ground on 600 grit SiC paper down to about 6 mils (0.15 mm). TEM discs were obtained by spark cutting followed by jet electropolishing under the same conditions used for the sheet plane samples. In longitudinal section samples, the sheet plane trace, clearly indicated by the interface between the nickel and the steel strip, was parallel to the major strain axis while in transverse section samples this trace was parallel to the minor strain direction (see figure 5b).
2.5 Transmission Electron Microscopy

Sheet plane and cross-section specimens at different positive and negative strain ratios were observed in a Phillips EM 301 electron microscope at an accelerating voltage of 100kV. Examination of the general dislocation cell structure configuration and orientation of the cell walls required low magnification bright field micrographs (between 7.5 and 13XX). In order to include as much information as possible for the individual grains, composite pictures were preferred. Three or more grains were analysed for each strain state. In sheet plane samples, most of the grains observed were in \{111\} orientation, which corresponds to the initial texture characteristic of AK low carbon steel sheet (\{111\} fiber axis normal to the sheet plane). Likewise, the grain orientations observed in cross-section samples were approximately perpendicular to the \{111\} direction. The amount of tilting necessary for optimum contrast conditions was always less than 5 degrees. Before taking any specimen out of the microscope, a low magnification picture (2,200x) of a portion of the hole was taken. This was necessary for later identification of the principal strain axes on the TEM micrographs.

The procedure for the determination of the major and minor strain axes on the TEM micrographs of sheet plane samples was as follows. The rolling direction, which was perpendicular to the major strain axis, was used as a reference line. The rolling direction was correlated to the shape of the holes by taking pictures of the 3 mm discs at 40x (see figure 5a) and 2,000x in an optical microscope. These optical pictures were then compared with the TEM micrographs of the same holes. By taking into account the image rotation at the magnification of the TEM
micrographs the major strain direction was determined with an accuracy of ±5 degrees. In cross-section samples the same procedure was followed. However, in this case, the reference line was given by the sheet plane trace on the 3 mm discs (see figure 5b).

2.6 Orientation of dislocation cell walls

Examination of the cellular substructure in the three-dimensions of the sheet was necessary for a complete determination of the cell wall plane orientation with respect to the principal strain axes. In sheet plane samples, the angle that cell walls formed with the major strain axis (α_sp, see figure 6) was measured on each micrographs for different strain states. Similarly, in cross-section samples, the angle between the cell wall direction and the sheet plane trace (α_L for longitudinal samples, α_T for transverse samples) was determined. The orientation of the (111) grains on the plane of the sheet was given in terms of the angle (β) that the closest <110> direction, on the respective <111> diffraction pattern, formed with the major strain axis (see figure 7). Due to crystal symmetry, this angle could only vary between 0 and 30 degrees.

The cell wall plane orientation was examined in three dimensions as a function of grain orientation on the sheet plane and as a function of strain state (minor to major strain ratio). The cell wall planes obtained from this analysis were then compared to the active slip planes calculated by using the maximum Schmid factor criterion for specific strain ratios and grain orientations.
2.7 Determination of active slip planes

In this investigation, the Sachs's model\(^{(7,23)}\) for polycrystal deformation was followed in the determination of the active slip system(s) inside individual grains of the sheet. In this model, the grains are treated as an array of single crystals that can deform independently. No accommodation between grains is considered; the stress inside each grain is equal to the macroscopic stress. Among the reasons for using this model are: a) the observation of dislocation cell walls roughly parallel to active slip systems in deformed \(\alpha\)-Fe monocrystals with different orientations\(^{(13-18)}\); b) the similarity between the dislocation cell structures developed in single crystals and polycrystalline materials; c) the existence of a strong texture in AK steel may favor deformation conditions in the individual grains that are close to those in single crystals; d) the simplicity of this model compared to the Taylor model\(^{(7,24)}\), which assumes the strain in the grains is prescribed by the macroscopic strain (fully constrained grains).

The active slip systems were calculated from the Schmid's law\(^{(28)}\). This law was originally stated in the following way: "A single crystal yields on any particular slip system if the shear stress resolved on that slip plane and slip direction reaches a critical value: the yield strength on that slip system"

For any stress state, the resolved shear stress on a slip system \(s\) is given by:

\[
\tau^s = m_{ij}^s \sigma_{ij} \quad (i,j = 1,2,3)
\]  
(1)
where $\sigma_{ij}$ is the applied stress tensor and $m_{ij}^s$ is the tensor transformation matrix. By defining $\bar{n}^s$ and $\bar{r}^s$ as the unit vectors denoting the slip plane normal and the slip direction respectively, $m_{ij}^s$ can be expressed as:

$$m_{ij}^s = r_i^s n_j^s$$  \hspace{1cm} (2)

$r_i^s$ and $n_j^s$ being the components of $\bar{n}^s$ and $\bar{r}^s$ in the coordinate system in which the stress tensor in given (see figure 8).

During punch-stretching deformation, the major and minor stress components are on the plane of the sheet, so that

$$\sigma_{11} = \sigma_1 \quad \sigma_{22} = \sigma_2 \quad \sigma_{33} = 0$$

and $\sigma_{ij} = 0 \ (i \neq j)$

Therefore, the resolved shear stress, eq. (1), is reduced to:

$$\tau^s = m_{11}^s \sigma_1 + m_{22}^s \sigma_2$$ \hspace{1cm} (3)

where $m_{11}^s = r_1^s n_1^s = \cos \psi_1 \cos \gamma_1$

and $m_{22}^s = r_2^s n_2^s = \cos \psi_2 \cos \gamma_2$

$\psi_1$ and $\psi_2$ are the angles that the normal to the slip plane $\bar{n}^s$ forms with the major and minor stress axes $\sigma_1$ and $\sigma_2$, respectively (see figure 8). Similarly, $\gamma_1$ and $\gamma_2$ are the angles that the slip direction $\bar{r}^s$ forms with the major and minor stress axes.

Substituting $m_{11}^s$ and $m_{22}^s$ and rearranging terms, eq. (3) has the final form:
\[ \tau^s = [ \cos\psi_1 \cos\gamma_1 + \alpha \cos\psi_2 \cos\gamma_2 ] \sigma_1 \]  

(4)

where \( \alpha \) is the ratio between the minor and major stresses,

\[ \alpha = \frac{\sigma_2}{\sigma_1} \]

Therefore, the slip system with maximum resolved shear stress will be that one for which the term in brackets in eq. (4) is maximum. This term is a form of the Schmid factor for deformation under plane stress conditions. Note that when \( \alpha \) is equal to zero, this term reduces to the Schmid factor for the case of pure tension.

The Schmid factors were then calculated, assuming crystallographic glide, for all possible slip systems in BCC crystals: \langle111\rangle, \langle110\rangle, \langle112\rangle, \langle123\rangle. This calculation required two steps: a) determination of the cosines of the angles \( \psi_1, \gamma_1, \psi_2, \gamma_2 \) (figure 8), b) determination of the stress ratio \( \alpha \). The analysis was carried out for crystals with the \langle111\rangle axis normal to the sheet plane which was the most common grain orientation observed in sheet plane TEM samples.

For the determination of the angles \( \psi_1, \gamma_1, \psi_2, \gamma_2 \) the major and minor stress directions were expressed in terms of the crystal cubic lattice coordinates (see Appendix B). By changing the angle \( \beta \) from 0 to 30 degrees (see figure 7), the whole range of major and minor stress axes directions could be determined.

Due to the characteristic normal and planar anisotropy of this sheet (see Table 2) the Hill anisotropic plasticity equations(29,30) were used for the determination of the ratio between the minor and major principal stresses \( \alpha \). For the special case of loading where \( \sigma_2 \) and \( \sigma_1 \)
are parallel and perpendicular to the rolling direction respectively, the ratio between the minor and major strain increments is given by (30):

\[
d s_2 / d s_1 = [ r_0( \sigma_2 - \sigma_1 ) + \sigma_2 ] / [ ( r_0/r_{90} )\sigma_1 + r_0( \sigma_1 - \sigma_2 ) ]
\]

(5)

where \( r_0 \) and \( r_{90} \) are the \( r \)-values (ratio of width-to-thickness strains) of this sheet obtained from tension tests parallel and perpendicular to the rolling direction respectively. For simplicity, the minor to major strain ratio \( \rho \) associated with each ellipse on the sheet was assumed to be constant during punch-stretching deformation. Therefore, replacing the strain increment ratio in eq. (5) by \( \rho \) and rearranging terms, the stress ratio \( \alpha \) can be expressed as:

\[
\alpha = \sigma_2 / \sigma_1 = [ r_0( \rho / r_{90} + \rho + 1 ) ] / [ r_0( \rho + 1 ) + 1 ]
\]

(6)

where \( \rho = \varepsilon_2 / \varepsilon_1 \). In this manner, the stress ratio can be determined from the major and minor strains measured on the original domes (see table 3 and 4). The \( r \)-values for this steel are \( r_0 = 1.5 \) and \( r_{90} = 1.85 \).

A computer program was developed for this analysis (only for the case of (111) grains parallel to the sheet plane). Introducing the values of the major and minor engineering strains and the angle \( \beta \), a list of all possible slip systems with their respective Schmid factors could be obtained. In this way, the slip system(s) with maximum Schmid factor for each specific strain ratio and grain orientation was determined. The active slip plane was then projected on the sheet plane, longitudinal section and transverse section planes, and the angles between the slip plane traces and the major and minor strain axes were
calculated (see figure 6 and Appendix B). These angles were identified as \( \alpha_{SP} \) on the sheet plane, \( \alpha_{L} \) on the longitudinal section and \( \alpha_{T} \) on the transverse section of the sheet. They were then compared with the angles between the cell wall traces and the strain axes measured on the TEM micrographs for the 3-sections of the sheet (\( \alpha_{SP}, \alpha_{L}, \alpha_{T} \)). Both calculated and measured angles were plotted with respect to the grain orientation, \( \beta \), and with respect to the strain ratio \( \rho \).

3. RESULTS AND DISCUSSION

The microstructure of commercial 1008 low carbon steel consists primarily of flat ferrite grains of medium size (~40 \( \mu \)m in diameter, ~11 \( \mu \)m in thickness) with a fine distribution of inter- and intragranular carbides. Before punch stretching, thin foils of this material parallel to the sheet plane showed individual dislocations uniformly distributed inside the ferrite grains (see figure 9). The origin of these dislocations could be attributed to the temper rolling characteristic of the processing of this sheet.

After punch-stretching, the microstructure changed drastically (see figure 10). As expected, there was a large increase in the dislocation density inside each grain. Dislocations tended to arrange into areas of high dislocation density (cell walls) enclosing areas of low dislocation density, forming what is known as a cellular substructure. The cell walls are low angle boundaries; the misorientation between adjacent cells is usually very small (2–3 degrees). This is evident in the diffraction pattern in figure 10 (taken from an area including more
than one cell) which shows almost no splitting of the diffraction spots.

Planar features, similar to the "microbands" found in cold rolled copper\(^{31,32}\) and in cold rolled low carbon steel\(^{33,34}\), were occasionally observed in longitudinal and transverse section samples. Figure 11 is a representative observation showing the displacements that these microbands can create when they intersect a grain boundary. However, in many cases, the distinction between microbands and elongated cells was not completely evident. Therefore both elongated cells and microband-like features were taken as dislocation cells for the orientation analysis.

In the following, the results of the dislocation cell geometry study for positive and negative strain ratio \(\varepsilon_2/\varepsilon_1\) samples are presented. These results are compared to the slip geometry predicted by the Sach's model for these two different deformation conditions.

### 3.1 Positive strain ratio

#### 3.1.1 Dislocation substructure

**Sheet plane samples:**

The dislocation substructures in samples parallel to the sheet plane with different positive strain ratios (positive minor strain, \(\varepsilon_2\)) were studied by TEM. Due to the characteristic texture of this material, most of the grains observed were in \(\langle 111 \rangle\) orientation (\(\langle 111 \rangle\) zone-axis parallel to the normal of the sheet). The orientation of these \(\langle 111 \rangle\) grains on the sheet plane was defined by the angle \(\beta\) between the closest \(\langle 110 \rangle\) direction and the major strain axis \(\varepsilon_1\) (see figure 6).
In general, for the range of positive strain ratios investigated, an elongated cell substructure was present. The traces of the long cell walls, which correspond to the intersection of the cell wall planes with the plane of the foil, were somewhat irregular and in some cases better defined than in others. However, the orientation of these traces with respect to the major strain axis could always be determined visually. This orientation is given as the angle \( \alpha_{SP} \): the angle between the cell wall trace and the major strain axis measured on each micrograph (see figures 6 and 8). Figures 12 and 13 are examples of cellular substructures for two different positive strain ratios. In these figures, the cell wall trace (CW) and the major strain axis (\( e_1 \)) are indicated, along with the angle \( \alpha_{SP} \) and the grain orientation \( \beta \). The angle \( \alpha_{SP} \) was always between 70 and 90 degrees for different \{111\} grain orientations (\( \beta \)).

**Cross-section samples:**

The grains observed in cross-section foils were usually perpendicular to a \( \langle 111 \rangle \) direction; the \( \langle 110 \rangle \) grain orientation being the most common. This observation indicated that the orientation of these grains in the sheet plane was close to \( \langle 111 \rangle \). Differences of up to 10 degrees from this perpendicularity were attributed to errors during sample preparation, i.e. cross-section samples not exactly at right angle from the sheet plane.

In general, elongated cells with well defined cell walls were observed. These cell walls were sharper and straighter than those in sheet plane samples. Figures 14 and 15 show the dislocation substructures in two longitudinal section samples with different strain...
ratios. Similarly, figure 16 is an example of dislocation cells in a transverse section foil.

In the longitudinal section samples, \( \alpha_L \), which is the angle that the cell wall trace forms with the major strain axis (see figure 8), lay between 15 and 30 degrees for all the strain ratios analyzed. In the transverse section samples, however, the cell walls were almost parallel to the sheet plane trace (parallel to \( e_2 \) axis); the angle \( \alpha_T \) was generally between 0 and 10 degrees.

The irregularity of the cell walls observed in the sheet plane samples compared to those in the cross-section samples could be simply a result of the different inclination of the cell wall plane with respect to the plane of the foil for each case. In sheet plane samples, cell walls are almost parallel to the plane of the foil \((\alpha_L \approx 20^\circ, \alpha_T \approx 0^\circ)\) such that the observed (apparent) cell wall thickness is greater than in cross-section samples. In addition, irregularities in the dislocation distribution on the cell wall plane through the thickness of the foil may be appreciable. In cross-section samples, however, cells are steeply inclined from the foil plane so that they have a sharper appearance on TEM micrographs.

In summary, for samples with positive strain ratio, elongated cells are observed in the three dimensions of the sheet. This indicates that cells formed inside each grain during punch-stretching are sheet-like features or parallelopipeds stacked somehow next to each other as proposed by Jackson\(^{19} \). These parallelopipeds are enclosed by fairly planar cell walls. The orientation of the cell wall habit plane in each grain is such that cell walls are: 1) roughly perpendicular to the major strain axis in sheet plane samples, 2) inclined about 20 to 30 degrees
with respect to the sheet plane in longitudinal section samples, and 3) almost parallel to the sheet plane in transverse section samples.

3.1.2 Comparison with orientation of active slip planes

As explained in section 2.7, the slip system(s) with the maximum resolved shear stress was determined for all the strain ratios studied and for the range of possible orientations of (111) grains on the sheet plane ($\beta = 0$ to 30 degrees). The orientations of the cell wall planes, given by the measured angles $\alpha_{SP}$, $\alpha_{L}$ and $\alpha_{T}$, were then compared with the calculated orientations of the active slip plane $\alpha_{SP}^{C}$, $\alpha_{L}^{C}$ and $\alpha_{T}^{C}$ respectively. Stereographic projections were extensively used to develop a clear understanding of the orientation of cell walls and active slip planes with respect to the crystal lattice coordinates. The data was plotted in two different ways: 1) calculated and measured angles $\alpha_{SP}$ versus grain orientation $\beta$ and 2) calculated and measured angles $\alpha_{SP}$, $\alpha_{L}$ and $\alpha_{T}$ versus strain ratio $\rho$.

Figures 17 and 18 are examples of the first type of plot for two different positive strain ratios. In these figures, the angle $\alpha_{SP}^{C}$ between the trace of the primary on the sheet plane and the major strain axis is plotted as a function of the grain orientation $\beta$, along with the measured angles $\alpha_{SP}$. The lines show the variation of the calculated angle $\alpha_{SP}^{C}$ with grain orientation for the active slip systems indicated and the solid circles represent the experimental data. In both cases, the values of the calculated and measured angles are between 70 and 90 degrees and their variation with grain orientation is similar. The plots for the other strain ratios investigated show approximately the same
behavior (see figures 19 to 21).

The second kind of plot is represented in figure 22. The calculated and measured angles in the three dimensions of the sheet are plotted as a function of strain ratio \( \rho \). The rectangles correspond to the possible ranges of the calculated angles \( \alpha_{SP}^C, \alpha_L^C \) and \( \alpha_T^C \) for the different (111) grain orientations (\( \beta \) from 0 to 30 degrees). This graph shows two important results. First, the correspondence between cell walls and active slip planes observed in sheet-plane samples also holds for longitudinal and transverse section samples. Both measured and calculated angles in the three sections of the sheet tend to be confined to well defined bands. Therefore, the orientation of cell walls developed under positive strain ratio conditions seems to be determined by the orientation of the slip plane with the maximum resolved shear stress inside each grain.

Second, the magnitude of the strain ratio has only a slight effect on the orientations of both the cell walls and the primary slip planes, at least within the range of positive strain ratios considered. In the sheet plane, the calculated angles tend to be closer to 90 degrees when the strain ratio increases. Although not indicated in the graph, this tendency continues up to \( \rho \) equal to 1.23, which corresponds to the case of equibiaxial stretching (\( \sigma_2/\sigma_1=1 \), see equation 6, section 2.7). Under this deformation condition, there are three different slip system with the same maximum Schmid factor for all values of \( \beta \), which is in agreement with the results of Fernandes and Schmitt\textsuperscript{(22)}. Their TEM observations for this case (only in foils parallel to the sheet plane) show an equiaxed cell structure that could result from the interaction of these three active slip systems.
The deviations observed in figure 17 to 22 between the orientations of cell walls and the orientation of primary slip planes are generally very small. Two explanations are proposed for these deviations: (1) The calculated Schmid factors of primary and secondary slip systems are very similar. Therefore, only a slight variation of the local stress inside the grains may be enough to favor slip on secondary slip planes and formation of cell walls roughly parallel to them. The orientations of primary and secondary slip systems with respect to the macroscopic strain axes are generally very similar and can account for the small deviations observed between cell walls and calculated primary slip planes. Another possibility is that cell walls originally on primary slip planes are deviated by the interaction with dislocations on secondary slip systems. This last situation is in agreement with the study of copper single crystals with different orientations where deviations of cell walls from the glide plane increase with the number of active slip systems; (2) the experimental error involved in the orientation analysis which arise mainly from the difficulty of keeping track of the major and minor principal strain axes through the TEM sample preparation up to the TEM micrographs (see section 2.5). This error, estimated within a the range of ±5 degrees, is carried over the measured angles $\alpha_{SP}$, $\alpha_L$, and $\alpha_T$ as well as over the grain orientation $\beta$, and can account for the difference between calculated and measured angles in the three sections of the sheet.

An important point to notice is that the active slip systems were determined by using the Sach's model for polycrystal deformation, where grains are treated as free single crystal and compatibility stresses between grains are neglected. In practice these compatibility stresses...
do exist due to the necessity of each grain to adjust its shape to that of the neighborhood. However, for the particular situation studied in this research, namely, flat ferrite grains with a preferred \(\langle 111\rangle\) direction perpendicular to the sheet plane under positive strain ratio deformation, the coincidence between cell wall plane orientations and calculated active slip planes indicates that the compatibility stresses may not be very significant and the behavior of individual \(\langle 111\rangle\) grains in the sheet can be adequately predicted by the behavior of single crystals under the same loading conditions.

A contributing factor for this single crystal-like behavior of \(\langle 111\rangle\) grains could be the similar orientations of the primary slip planes for different grain orientations and also for different strain ratios. This can be observed in figure 22 where, for \(\beta\) between 0 and 30 degrees, the calculated angles in the three dimensions of the sheet vary by: \(\Delta \alpha_p^C \approx 20\) degrees, \(\Delta \alpha_L^C \approx 3\) degrees, and \(\Delta \alpha_T^C \approx 10\) degrees. This similarity in the active slip plane orientation may somehow facilitate the propagation of slip from one grain to another as well as the accommodation of the shape change of each grain together with their surroundings even under local variations of the strain ratio.

In summary, the orientation of cell walls formed during punch-stretching under positive strain ratio conditions is coincident with the orientation of the slip plane with the maximum resolved shear stress in each grain determined by using the Sach's model for polycrystal deformation.
3.2 Negative strain ratio

3.2.1 Dislocation substructure

Sheet plane samples:

The observation of dislocation cells in negative strain ratio samples was also limited to (111) grains parallel to the sheet plane. However, deviation of the (111) zone axis from the microscope axis was common and the (111) diffraction patterns were generally not symmetrical. Moreover, the diffraction conditions necessary for the observation of dislocation walls were usually present only in localized areas of the grains. By tilting the sample a few degrees, dislocation cells would go out of contrast in the one area and would appear in another area of the same grain. The misorientation between different regions of individual grains built up during plastic deformation seems to be more pronounced in negative strain ratio samples (negative $\frac{a_2}{e_1}$).

Examples of the cellular substructures for two different strain ratios are shown in figures 23 and 24. Long cell walls are fairly sharp and well aligned. The angle $\alpha_{SP}$ the cell walls form with the major strain axis varied in the range of 40 and 70 degrees for different (111) grain orientations (different $\beta$) and different negative strain ratios.

Cross-section samples:

A larger variety of grain orientations were observed in cross-section samples with negative strain ratios compared to those with positive strain ratio. In most cases, these grain orientations were not
perpendicular to a \(\langle111\rangle\) direction; deviations of up to 12 degrees were usual. This variety in grain orientations and the misorientations inside individual grains characteristic of negative strain ratio samples was an indication of a more irregular arrangement of the grains inside the sheet that resulted from this deformation condition.

An elongated cell structure was also observed in cross-section samples (see figures 25 and 26). The angles between the cell wall traces and the principal strain axes varied significantly for different grains. In the longitudinal section samples, \(\alpha_L\) was between 25 to 60 degrees. In the transverse section samples, \(\alpha_T\) was either between 0 and 10 degrees or between 40 and 60 degrees (as stated earlier, the corresponding angular ranges for the positive strain ratio case were \(\alpha_L\) 15-30 degrees and \(\alpha_T\) 0-10 degrees). Figure 26 shows a case for which the cell walls are almost parallel to the sheet plane.

Taken together, the observations of the dislocation substructures in the three dimensions of the sheet for negative strain ratio samples indicate that cells are also sheet-like features. The orientations of the cell wall planes associated with negative strain ratios are clearly different from those associated with positive strain ratios. Determination of specific orientations of the cell wall planes offers some difficulty due to the large variation of the measured angles \(\alpha_{SP}, \alpha_L, \alpha_T\).

3.2.2 Comparison with orientation of active slip planes

The slip planes with the maximum resolved shear stress for negative strain ratio samples were also of the \(\{112\}\) and \(\{123\}\) types, again for
the case of (111) grains parallel to the sheet plane. However, the orientations of these planes inside each grain differ from those in positive strain ratio deformation. In figures 27 to 31 the measured and calculated angles $\alpha_{sp}$ are plotted versus the grain orientation $\beta$ for different negative strain ratios. Both calculated and measured angles vary between 45 and 70 degrees with grain orientation except in the lowest strain ratio case ($\rho = -0.33$) (see figure 27), where the trace of the active slip plane is almost perpendicular to the major strain axis for $\beta$ close to 30 degrees.

The orientation of cell walls and primary slip planes in sheet plane, longitudinal and transverse section samples are plotted versus strain ratio in figures 32, 33, 34, respectively. For comparison these plots are grouped together in figure 35. As in the positive strain ratio case, the rectangles in these figures correspond to the calculated angles for the primary slip systems whereas the solid symbols represent the angles measured on TEM micrographs. The main result shown in these three plots is that there is a reasonable agreement between the measured and calculated angles, being more evident in the sheet plane case (figure 32). The spread of the cell wall orientations observed in the three dimensions of the sheet seems to correspond to the spread of the primary slip plane orientations for different grain orientations $\beta$. Comparison of these figures with the same kind of plot for positive strain ratio samples (figure 22) indicates that the clearly different orientation of cell walls that results from positive and negative strain ratio deformation is a consequence of the different slip systems activated inside individual grains under these two deformation conditions.
Nevertheless, whether dislocations walls are parallel to calculated active slip planes in negative strain ratio cases is not completely clear. Significant discrepancies between the cell wall orientation and the primary slip plane orientation are observed in both longitudinal and transverse sections as shown in figures 33 and 34 respectively. In the longitudinal section samples (figure 33) the measured angles are in general smaller than those corresponding to calculated active slip planes. In the transverse section samples (figure 34), cell walls almost parallel to the sheet plane are usually present for the different strain ratios studied, which is not predicted by the theoretical calculations above (more negative than) \( \rho = -0.34 \). These deviations are too large be attributed to experimental errors and seem to indicate that the Sach's model fails in predicting the slip geometry inside individual grains.

There is some evidence that constraints around grains may be more significant during negative strain ratio deformation. Figure 36 shows optical micrographs of transverse sections of the sheet after punch-stretching for both positive and negative strain ratio cases. The different grain configuration after these two deformation conditions can be clearly observed. For positive minor strain (figure 36a) grains of more regular shape are present, with fairly straight boundaries parallel to the sheet plane. For negative minor strain the curvature and irregularity of the grains is remarkable (figure 36b). This irregular grain configuration is in correspondence with: (1) the apparent deviations of (111) grains from the sheet plane, (2) the variation of grain orientations in cross-section samples and, (3) the pronounced misorientation between different areas of single grains generally observed in negative strain ratio TEM samples (see section 3.2.1).
Apparently, under negative strain ratio conditions, the grains inside the sheet seem to encounter more difficulty in accommodating the imposed deformation together with their surroundings.

Local differences of the strain ratio inside the grains due to constraints may result in significant variations of the orientation of the slip planes activated. This situation is especially important in the negative strain ratio case where the orientations of the primary slip planes vary significantly with both grain orientation $\beta$ and strain ratio $\rho$ (see figures 32 to 35). Therefore, the observed discrepancies between calculated and measured angles may be explained through two different situations: (1) The local strain ratio inside the grains may be smaller than the macroscopic strain ratio due to the influence of constraints. This can explain the presence of cell walls parallel to the sheet plane ($\alpha_T \approx 0$ degrees) in transverse section samples for high strain ratios (see figure 34); (2) The microscopic stress and strain states due to the constraints around grains may enhance the activity of secondary slip systems. This situation may explain the tendency of the measured angles $\alpha_L$ and $\alpha_T$ of being in between the two calculated angle ranges (which correspond to different slip systems) for low strain ratios as shown in figures 33 and 34. The cell walls seem to be deviated by the influence of a slip plane with $\alpha_L \approx 20$ and $\alpha_T \approx 0$ degrees which is predicted to be active only for grains with $\beta$ close to 30 degrees (see figure 27 (211) slip plane). The same deviations are also observed for higher strain ratios in both longitudinal and transverse section samples, indicating that the local stress and strain states inside the grains may still favor shear on such a slip system.

In summary, the orientation of the cell walls in negative strain
ratio samples are not always predicted by the Sach's model for polycrystal deformation. Deviations are mainly observed in longitudinal and transverse section samples. If the TEM analysis had been limited to only sheet plane samples, then this deviations would not have been exposed. The three-dimensional analysis provided important information on the actual cell structure geometry.

Nevertheless, comparison of positive and negative strain ratio results shows that cell walls developed during punch-stretching are related to active slip planes. In the positive strain ratio case, this relationship is clearly shown by cell walls being roughly parallel to the active slip planes inside individual grains. This connection between cell walls and active slip planes could allow for the prediction of the dislocation cell geometry from the initial texture of the sheet. The strain hardening behavior of the material may then be controlled through the development of an adequate texture during the processing of the sheet for specific deformation conditions. The extension of the present three-dimensional analysis to different initial textures and different deformation conditions is required to develop a precise understanding of the relationship between the dislocation substructure geometry and the strain hardening of the material.
4. CONCLUSIONS

A three-dimensional observation of the dislocation cell structure developed in AK, low carbon steel sheet during punch-stretching shows that cells inside individual grains are sheet-like features enclosed by planar cell walls. The orientations of the cell walls are related to the orientations of the active slip planes for different grain orientations and strain conditions.

For positive strain ratio deformation and (111) grains parallel to the sheet plane, cell walls are roughly parallel to the slip planes with the maximum resolved shear stress, determined by using the Sach's model for polycrystal deformation. The constraints around grains do not seem to have a major effect on the primary slip systems. The dislocation cell geometry inside each grain can be predicted by the behavior of single crystals under the same loading conditions.

For negative strain ratio samples, however, significant deviations are observed between cell wall orientations and calculated active slip planes. Constraints around grains seem to play an important role under negative minor strain deformation. These constraints may alter the local stress tensor so that other slip systems different from those predicted by the Sach's model may be activated inside individual grains.
REFERENCES


APPENDIX A. LDH test and Forming Limit Curve (FLC).

The Limiting Dome Height (LDH) test was designed to evaluate sheet metal for forming operations where failure occurs from through thickness necks or splits during stretching. This test permits the generation of failure over a wide range of strain ratios and friction conditions\(^{(1,2)}\). Sheet metal samples are previously photogridded with 0.1in. circles to allow strain measurement after testing. The metal blanks are clamped at the periphery between two die-plates, and are stretched to failure over a 4in. dia hemispherical punch at a punch-stroke rate of 254 mm/min. The limiting dome height is taken at maximum load which correspond to the development of a split. The minor strain at failure is varied by testing progressively narrower blanks which permits greater amounts of lateral drawing-in (see figure 1). The major and minor strains are measured from the distorted circles by using the circle grid technique\(^{(3)}\) (see figure 2).

The Forming Limit Curve (FLC) marks the limit between failure and safe conditions of the sheet metal\(^{(4,5)}\)(see figure 3). The FLC is constructed by plotting the major and minor strains from areas close to and within necked and fracture areas. The major and minor strain combinations at a neck or fracture give the failure condition (strains above the top curve) while strains in areas not affected by localized thinning are considered safe (below the bottom curve). The area between the two curves, which is called the marginal zone, is used in the press shop for detecting potential trouble spots in sheet-metal forming operations. The lowest value of the major strain \(e_1\) occurs at plane strain conditions \((e_2=0)\). The whole level of the FLC for low-carbon steel rises with strain hardening exponent, \(n\), and the sheet
thickness \((3,5)\).

References


APPENDIX B

1. Schmid factor calculation

As mentioned in section 2.7, determination of the Schmid factors and the active slip plane orientation with respect to the macroscopic strain axes was limited to the case of [111] grains parallel to the sheet plane. The procedure followed for this calculation is explained below.

1.1 Determination of the major and minor strain directions

The components of the major and minor strain vectors in the crystal cubic lattice coordinates are expressed in terms of $\beta$, the angle between the [110] direction and the major strain direction (see figure 7). For this purpose, the scalar product of two vectors is used, provided the following conditions:

1) $e_1$ is perpendicular to [111] ($e_1$ is on the sheet plane):

$$ e_1 \cdot [111] = a_1 + b_1 + c_1 = 0 \quad e_1 = [a_1, b_1, c_1] \quad (B.1) $$

2) the angle between $e_1$ and [110] is equal to $\beta$:

$$ \cos \beta = (-a_1 + b_1) / \sqrt{2} \sqrt{(a_1^2 + b_1^2 + c_1^2)} \quad (B.2) $$

3) the angle between $e_1$ and [211] is $(30^\circ - \beta)$:

$$ \cos(30^\circ - \beta) = (-2a_1 + b_1 + c_1) / \sqrt{6} \sqrt{(a_1^2 + b_1^2 + c_1^2)} \quad (B.3) $$
Solving for \( a_1, b_1, c_1 \), the components of the major strain axis direction \( e_1 \) are

\[
\begin{align*}
a_1 &= -1, \\
b_1 &= x - 1, \\
c_1 &= 1 - b_1
\end{align*}
\] (B.4)

where

\[ x = \sqrt{3} \cos \beta / \cos(30^\circ - \beta) \]

Similarly, the components of the minor strain axis direction \( e_2 \), which is perpendicular to the \( e_1 \), are given by:

\[
\begin{align*}
a_2 &= -1, \\
b_2 &= (3 - x) / (3 - 2x) \\
c_2 &= 1 - b_2
\end{align*}
\] (B.5)

1.2 Determination of the cosines of the angles \( \psi_1, \gamma_1, \psi_2, \) and \( \gamma_2 \)

Once the directions of the major and minor strain (or stress) axes are known, the cosines of the angles \( \psi_1, \gamma_1, \psi_2, \) and \( \gamma_2 \) (see figure 8) can be calculated by using the scalar product of two vectors. If \([h,k,l]\) is the slip plane normal and \([u,v,w]\) is one of the particular \(\langle 111\rangle\) slip directions, the cosines of these angles are given by:

\[
\begin{align*}
cos \psi_1 &= (h a_1 + k b_1 + l c_1) / \| [h,k,l] \| \| [a_1,b_1,c_1] \| \\
cos \gamma_1 &= (u a_1 + v b_1 + w c_1) / \sqrt{3} \| [a_1,b_1,c_1] \| \\
cos \psi_2 &= (h a_2 + k b_2 + l c_2) / \| [h,k,l] \| \| [a_2,b_2,c_2] \| \\
cos \gamma_2 &= (u a_2 + v b_2 + w c_2) / \sqrt{3} \| [a_2,b_2,c_2] \|
\end{align*}
\] (B.6)
The Schmid factor (SF) for all possible slip systems in BCC crystals, \(\{111\}, \{110\}, \{112\}, \{123\}\), are determined by using equation (4) (see section 2.7):

\[
SF = \cos \psi_1 \cos \gamma_1 + \alpha \cos \psi_2 \cos \gamma_2
\]  

(B.7)

The stress ratio \(\alpha (\sigma_2 / \sigma_1)\) is calculated from the measured strain ratio \(\rho (\varepsilon_2 / \varepsilon_1)\) by using equation (6) of section 2.7. In this way, the slip system with the maximum Schmid factor can be determined, for different grain orientations \(\beta\) and strain ratios \(\rho\).

2. Determination of the active slip plane orientation \((\alpha_{\text{SP}}^c, \alpha_L^c, \alpha_T^c)\)

2.1 Determination of the angle \(\alpha_{\text{SP}}^c\) (sheet plane)

The projection (trace) of the slip plane on the sheet plane, \([s_1, s_2, s_3]\) (see figure 6), can be determined by using the vector product of the normal to the slip plane \([h,k,1]\) and the normal to the sheet plane \([111]\)

\[
[h,k,1] \times [1,1,1] = [s_1, s_2, s_3]
\]

(B.8)

where

\[
s_1 = k - 1
\]

\[
s_2 = 1 - h
\]

\[
s_3 = h - k
\]

Then, \(\alpha_{\text{SP}}^c\), which is the angle between this slip plane trace and the major strain axis \(e_1\), is calculated from the scalar product
2.2 Determination of the angles \( a_L^c \) and \( a_T^c \)

The trace of the slip plane on the longitudinal and transverse section planes (see figure 6) are determined by equations involving the vector product of the slip plane normal and the normal to the longitudinal or to the transverse section planes.

The normal to the longitudinal section plane \([u_1, u_2, u_3]\) is given by the vector product of the sheet plane normal \([111]\) and the major strain axis direction \([a_1, b_1, c_1]\)

\[
[1,1,1] \times [a_1, b_1, c_1] = [u_1, u_2, u_3]
\]  

(B.10)

where

\[
\begin{align*}
u_1 &= c_1 - b_1 \\
u_2 &= a_1 - c_1 \\
u_3 &= b_1 - a_1
\end{align*}
\]

Similarly, the normal of the transverse section plane \([n_1, n_2, n_3]\) is given by the vector product of the \([111]\) direction and the minor strain axes \([a_2, b_2, c_2]\)

\[
[1,1,1] \times [a_2, b_2, c_2] = [n_1, n_2, n_3]
\]  

(B.11)

where

\[
\begin{align*}
n_1 &= c_2 - b_2 \\
n_2 &= a_2 - c_2 \\
n_3 &= b_2 - a_2
\end{align*}
\]
Therefore, the trace of the slip plane on the longitudinal section plane \([p_1, p_2, p_3]\) is given by:

\[
[u_1, u_2, u_3] \times [h, k, l] = [p_1, p_2, p_3]
\]  
(B.12)

where

\[
p_1 = u_2 l - u_3 k
\]

\[
p_2 = u_3 h - u_1 l
\]

\[
p_3 = u_1 k - u_2 h
\]

In the same way, the trace of the slip plane on the transverse section plane \([t_1, t_2, t_3]\) is

\[
[n_1, n_2, n_3] \times [h, k, l] = [t_1, t_2, t_3]
\]  
(B.13)

where

\[
t_1 = n_2 l - n_3 k
\]

\[
t_2 = n_3 h - n_1 l
\]

\[
t_3 = n_1 k - n_2 h
\]

The angles \(\alpha_L\) and \(\alpha_T\) are determined from the following equations:

\[
\cos \alpha_L = [p_1, p_2, p_3] \cdot [a_1, b_1, c_1] / ||[p_1, p_2, p_3]|| ||[a_1, b_1, c_1]||
\]

and

\[
\cos \alpha_T = [t_1, t_2, t_3] \cdot [a_2, b_2, c_2] / ||[t_1, t_2, t_3]|| ||[a_2, b_2, c_2]||
\]  
(B.14)
**TABLE 1.** Chemical composition of AK steel (wt. %)

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<tr>
<th>Fe</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
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**TABLE 2.** Tensile properties of AK steel

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<tr>
<th>YIELD STRENGTH (MN/m²)</th>
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<th>NORMAL ANISOTROPY COEFFIC.</th>
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<td>300</td>
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<td>1.5</td>
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\[
\bar{r} = \frac{r_0 + 2r_{45} + r_{90}}{4}
\]

\[
\Delta r = \frac{r_0 - 2r_{45} + r_{90}}{2}
\]
### TABLE 3. Strain states of the TEM samples analyzed (positive strain ratio)

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<th>Sheet Plane</th>
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<th>Minor ε₂</th>
<th>Major ε₁</th>
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<td>0.31</td>
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<th>Transversal Section</th>
<th>Major ε₁</th>
<th>Minor ε₂</th>
<th>Major ε₁</th>
<th>Minor ε₂</th>
<th>ε₁</th>
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<th>ε₁/ε₂</th>
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<td>0.67</td>
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TABLE 4. Strain states of the TEM samples analyzed (negative strain ratio)

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<tr>
<th>ENGINEERING STRAIN (%)</th>
<th>TRUE STRAIN</th>
<th>EQUIVALENT STRAIN</th>
<th>STRAIN RATIO (p)</th>
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<tr>
<td>e_1</td>
<td>e_2</td>
<td>Major</td>
<td>Minor</td>
</tr>
<tr>
<td>Major</td>
<td>Minor</td>
<td>e_1</td>
<td>e_2</td>
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<tr>
<td>Sheet Plane</td>
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<tr>
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<td>0.22</td>
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</tr>
<tr>
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<td>0.26</td>
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</tr>
<tr>
<td>33</td>
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<td>0.29</td>
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<tr>
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<td>Longitudinal Section</td>
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<td>0.46</td>
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<tr>
<td>Transverse Section</td>
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<tr>
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<tr>
<td>49</td>
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<td>0.40</td>
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Fig. 1. Set of limiting dome height (LDH) samples (the numbers indicate the blank width in inches).
Fig. 2. Measurement of the major and minor engineering strains, $e_1$ and $e_2$, by using the circle grid analysis technique. The original circles of diameter $l_0$ (0.1 in.) become ellipses after deformation.

Major Strain: $e_1 = \frac{l_1 - l_0}{l_0} \times 100$

Minor Strain: $e_2 = \frac{l_2 - l_0}{l_0} \times 100$

Positive Strain Ratio: $\left( \frac{e_2}{e_1} \right)$

Negative Strain Ratio: $\left( \frac{e_2}{e_1} \right)$
Fig. 3. Forming limit curve (FLC) for AK steel. The different major and minor strain combinations analyzed are in the two shady bands (see tables 3 and 4).
Fig. 4. Schematic representation of the sample preparation procedure. Sheet plane sample: chemical thinning (II), spark cutting and marking of the rolling direction RD on the TEM disc (III), electropolishing (IV). Cross-section sample: nickel plating (II), isomet slicing (III), spark-cutting and electropolishing (IV).
Fig. 5. Optical pictures of TEM samples. Sheet plane sample (top): the rolling direction (RD) is indicated by two notches. Cross-section sample (bottom): the interface between the steel and the nickel gives the sheet plane trace direction (SP).
Fig. 6. Schematic representation of the measured and calculated angles $\alpha_{SP}$, $\alpha_L$, $\alpha_T$, which are used to define the cell wall orientation and the slip plane orientation. Only grains with $\langle111\rangle$ parallel to the sheet plane are analyzed.
Fig. 7. (111) diffraction pattern. The angle $\beta$ represents the grain orientation with respect to the major strain axis $e_1$, and $\alpha_{SP}$ gives the cell wall CW orientation on sheet plane samples.
Fig. 8. Schematic representation of the angles $\Psi_1$, $\gamma_1$, $\Psi_2$, and $\gamma_2$. The normal to the slip plane and the slip direction are represented by $\hat{n}$ and $\hat{r}$, respectively. $\sigma_1$ and $\sigma_2$ are the major and minor stress axes.
Fig. 9. TEM bright field image of AK steel sample before punch-stretching. Sample parallel to the sheet plane.
Fig. 10. TEM bright field image and SAD pattern of a sheet plane sample after punch-stretching. A typical dislocation cell substructure is observed.
Fig. 11. TEM bright field image and SAD pattern of a longitudinal section sample \((p=0.40)\). Note the displacement created by microbands when they intersect the grain boundary (shear deformation). The cell wall trace and the major strain axis are indicated by \(CW\) and \(e_1\), respectively.
Fig. 12. TEM bright field image and SAD pattern of a sheet plane sample with positive strain ratio ($\rho=0.52$). The grain orientation and the cell wall (CW) orientation are indicated by the angles $\beta$ and $\alpha_{SP}$, respectively.
Fig. 13. TEM bright field image and SAD pattern of a sheet plane sample ($\varphi=0.28$). The grain orientation and cell wall (CW) orientation are represented by $\beta$ and $\alpha_{SP}$, respectively.
Fig. 14. TEM bright field image and SAD pattern of a longitudinal section sample with positive strain ratio ($\rho=0.49$). The cell wall orientation (CW) with respect to the major strain axis $e_1$ is given by the angle $\alpha_L$. 
Fig. 15. TEM bright field image and SAD pattern of a longitudinal section sample with strain ratio $\rho=0.31$. 
Fig. 16. TEM bright field image and SAD pattern of a transverse section sample with positive strain ratio \((p=0.33)\). The cell wall orientation is given by \(\alpha_T\), the angle between the cell wall trace \(\text{CW}\) and the minor strain axis \(e_2\). Note that the cell walls are almost parallel to the sheet plane (parallel to \(e_2\)).
Fig. 17. Variation of the slip plane orientation $\alpha_{SP}$ (lines) and the cell wall orientation $\alpha_{SP}$ (points) with grain orientation $\beta$ for a sheet plane sample with strain ratio $\rho=0.34$. The active slip planes corresponding to each line are indicated.
$\alpha_{SP}$ Vs. GRAIN ORIENTATION ($\beta$)

Fig. 18. Calculated (lines) and measured (points) $\alpha_{SP}$ versus grain orientation $\beta$. Sheet plane sample with strain ratio $\rho=0.28$. 
Fig. 19. Calculated (lines) and measured (points) $\alpha_{SP}$ versus grain orientation $\beta$. Sheet plane sample with strain ratio $\rho=0.22$. 

$\alpha_{SP}$ Vs. GRAIN ORIENTATION ( $\beta$ )

$\rho = 0.22$

$\alpha_{SP}$ (degrees)

$\beta$ (degrees)
Fig. 20. Calculated (lines) and measured (points) \( \alpha_{SP} \) versus grain orientation \( \beta \). Sheet plane sample with strain ratio \( \rho = 0.52 \).
Fig. 21. Calculated (lines) and measured (points) $\alpha_{SP}$ versus grain orientation $\beta$. Sheet plane sample with strain ratio $\rho=0.57$. 
Fig. 22. Slip plane orientations (rectangles) and cell wall orientations (points) versus strain ratio $\rho$. Data from the three-sections of the sheet are included: $a_{SP}$ (sheet plane), $a_{L}$ (longitudinal section), $a_{T}$ (transverse section). The rectangles correspond to the range of calculated angles $\alpha$ for grain orientations $\beta$ from $0^\circ$ to $30^\circ$. 
Fig. 23. TEM bright field image and SAD pattern of a sample parallel to the sheet plane with negative strain ratio ($\rho = -0.47$). The grain orientation and cell wall (CW) orientation are represented by the angles $\beta$ and $\alpha_{SP}$, respectively.
Fig. 24. TEM bright field image and SAD pattern of a sheet plane sample with $\rho = -0.33$. 
Fig. 25. TEM bright field image and SAD pattern of a longitudinal section sample with negative strain ratio ($\rho=-0.39$). The orientation of the cell wall (CW) with respect to the major strain axis $e_1$ is given by the angle $\alpha_L$. 
Fig. 26. TEM bright field image and SAD pattern of a transverse section sample with $p = -0.34$. The orientation of the cell wall with respect to the minor strain axis $e_2$ is given by the angle $\alpha_T$. 
Fig. 27. Variation of the slip plane orientation $\alpha_{SP}^c$ (lines) and the cell wall orientation $\alpha_{SP}$ (points) with grain orientation $\beta$ for a sheet plane sample with $\rho = -0.33$. The active slip planes corresponding to each line are indicated.
Fig. 28. Measured and calculated angles \( \alpha_{SP} \) versus grain orientation \( \beta \). Sheet plane sample with strain ratio \( \rho = -0.38 \).
Fig. 29. Measured and calculated angles $\alpha_{sp}$ versus grain orientation $\beta$. Sheet plane sample with strain ratio $\rho = -0.42$. 
Fig. 30. Measured and calculated angles \( \alpha_{SP} \) versus grain orientation \( \beta \). Sheet plane sample with \( \rho = -0.47 \).
Fig. 31. Measured and calculated angles $\alpha_{SP}$ versus grain orientation $\beta$.
Sheet plane sample with $\rho=-0.52$. 
Fig. 32. Measured and calculated angles $\alpha_{SP}$ versus strain ratio $\rho$ for sheet plane samples. The rectangles correspond to the range of calculated angles $\alpha_{SP}$ for $\beta=0^\circ$ to $30^\circ$. 
Fig. 33. Measured and calculated angles $\alpha_L$ versus strain ratio for longitudinal section samples.
Fig. 34. Measured and calculated angles $\alpha_T$ versus strain ratio $\rho$ for transverse section samples.
Fig. 35. Slip plane orientation and cell wall orientation versus strain ratio $p$ on the three-sections of the sheet: (left) sheet plane ($\alpha_{SP}$), (center) longitudinal section ($\alpha_{L}$), and (right) transverse section ($\alpha_{T}$).
Fig. 36. Optical micrographs of the transverse section samples: a) positive strain ratio (positive minor strain $e_2$), b) negative strain ratio (negative minor strain $e_2$). Note the different grain configurations.
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