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Cyclic threshold strains in clays versus sands and the change of secant shear modulus and pore water pressure at small cyclic strains

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Cyclic threshold strains in clays versus sands and the change of secant shear modulus and pore water pressure at small cyclic strains

A dissertation submitted in partial satisfaction of the requirement for the degree Doctor of Philosophy in Civil Engineering

by

Ahmad Reza Mortezaie

2012
ABSTRACT OF THE DISSERTATION

Cyclic threshold strains in clays versus sands and the change of secant shear modulus and pore water pressure at small cyclic strains

by

Ahmad Reza Mortezaie

Doctor of Philosophy in Civil Engineering

University of California, Los Angeles, 2012

Professor Mladen Vucetic, Chair

When fully saturated soils are subjected to cyclic loading in undrained conditions involving moderate and large cyclic shear strain amplitudes, \( \gamma_c \), their stiffness and strength decrease and the pore water pressure changes permanently with the number of cycles, N. Such cyclic degradation of stiffness and pore water pressure change are among the most fundamental and important phenomena in soil dynamics. Fully saturated sands are most susceptible to cyclic degradation and significant pore pressure buildup. During cyclic loading, they can completely lose their stiffness and strength while large excess pore pressure develops and they can eventually liquefy. On the other end of the spectrum are fully saturated clays of high plasticity that under significant cyclic loading can lose only a fraction of their original stiffness and
strength while the pore water pressure change is relatively small. Moreover, in overconsolidated clays of high plasticity pore pressure can decrease instead of increase with N while cyclic degradation is even smaller.

As opposed to that, when fully saturated soils are subjected to very small cyclic shear strains, \( \gamma_c \), soil’s structure practically does not change and, consequently, neither cyclic degradation nor permanent cyclic pore pressure change occur. The amplitude \( \gamma_c \) below which there is no cyclic degradation and above which the degradation occurs is known as the threshold shear strain for cyclic degradation, \( \gamma_{td} \). The amplitude \( \gamma_c \) below which there is no permanent pore water pressure change with N and above which the permanent pore pressure is recorded after every cycle is known as the threshold shear strain for cyclic pore water pressure change, \( \gamma_{tp} \).

For sands, \( \gamma_{tp} \) has been extensively investigated but \( \gamma_{td} \) has been hardly investigated at all. For clays, both \( \gamma_{tp} \) and \( \gamma_{td} \) were investigated to quite a limited extent. For example, \( \gamma_{tp} \) in overconsolidated clays has practically not been investigated.

The thesis describes laboratory investigation focused on \( \gamma_{td} \) in a clean sand and \( \gamma_{tp} \) in two laboratory-made normally consolidated (NC) and overconsolidated (OC) clays, kaolinite clay having PI=28 and kaolinite-betonite clay having PI=55, and on the comparison and connection between different thresholds. The Norwegian Geotechnical Institute (NGI) type of direct simple shear device (DSS) was employed for the constant-volume equivalent-undrained cyclic testing. Two types of tests were conducted, single-stage cyclic strain-controlled test with constant \( \gamma_c \) throughout the test, and the multi-stage cyclic strain-controlled test in which \( \gamma_c \) was constant in
each stage but larger in every subsequent stage. The magnitude of $\gamma_c$ covered the range from 0.003% to 2%.

In the context of investigating $\gamma_{td}$ in sand and $\gamma_{tp}$ in clays, the following tasks were also performed: (1) the NGI-DSS device was adapted for small-strain cyclic testing and a procedure for eliminating false loads and deformations from test records was developed, (2) the effect of the vertical consolidation stress, $\sigma_{vc}'$, and frequency of cyclic straining, $f$, on cyclic degradation and pore water pressure change in clays was tested, and (3) cyclic stress-strain behavior and the change in secant shear modulus, $G_{SN}$, with $N$ in sands at very small cyclic strains was investigated.

The following conclusions are derived and results obtained: (1) small-strain cyclic testing at $\gamma_c$ as small as 0.003% can be conducted in the standard NGI-DSS device and the results can be used in soil dynamics analyses if the device is properly modified and the false loads are eliminated from the test records, (2) cyclic degradation in clays is affected moderately to significantly by $\sigma_{vc}'$ and frequency, $f$, (3) in sands modulus $G_{SN}$ increases with $N$ at very small $\gamma_c$ below $\gamma_{tp}$ (sand is stiffening) while the cyclic pore water pressure does not develop; (4) in sands at small to moderate $\gamma_c$ above $\gamma_{tp}$ modulus $G_{SN}$ first increases and then decreases with $N$ while the cyclic pore water pressure continuously increases, (5) because of such behavior in which soil stiffness starts degrading after certain number of cycles, $\gamma_{td}$ in sand cannot be defined like $\gamma_{td}$ in clays, (6) in both clays tested $\gamma_{td}$ is not visibly affected by overconsolidation ratio, OCR, (7) in kaolinite clay $\gamma_{tp}$ is not affected by OCR, (8) in kaolinite-bentonite clay with PI=55 and overconsolidated to OCR=4 and 7.8 the pore water pressures between $\gamma_c=0.003\%$ and 0.3% did not change in a
consistent manner and $\gamma_{tp}$ could not be evaluated, (9) the thresholds $\gamma_{tp}$ tested show increase with PI just like in the previous studies, but their values are at or below the lower bound of published $\gamma_{tp}$-PI trends, and (10) the tested $\gamma_{td}$ thresholds in clays do not follow the trend of increase with PI like in the previous studies and they are smaller than those published earlier for similar soils.
The dissertation of Ahmad Reza Mortezaie is approved.

Scott Brandenberg

Macan Doroudian

Harold Monbouguette

Jonathan Stewart

Mladen Vucetic, Committee Chair

University of California, Los Angeles

2012
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LIST OF SYMBOLS

\[ A \] = area of specimen

\[ d_c \] = horizontal cyclic displacement amplitude

\[ e \] = void ratio

\[ F_c \] = horizontal cyclic shear force

\[ f \] = frequency of cyclic loading

\[ G_{\text{max}} \] = \( G_s \) at \( \gamma_c = 0.0001 \% \) or maximum shear modulus

\[ G_s \] = secant shear modulus

\[ G_s / G_{\text{max}} \] = normalized secant shear modulus

\[ H \] = height of specimen

\[ \text{LL} \] = liquid limit

\[ N \] = number of cycles

\[ \text{PI} \] = plasticity index

\[ T \] = period of cyclic straining
\( t \) = degradation parameter

\( w \) = water content

\( S_r \) = degree of saturation

\( \gamma \) = shear strain

\( \gamma_c \) = cyclic shear strain amplitude

\( \gamma_{td} \) = threshold shear strain for cyclic degradation

\( \gamma_{sp} \) = threshold shear strain for cyclic pore water pressure change

\( \gamma_{ts} \) = threshold shear strain for cyclic stiffening

\( \gamma_{tv} \) = threshold shear strain for cyclic compression

\( \Delta u \) = excess pore water pressure

\( \Delta u_N \) = cyclic excess pore water pressure at the end of strain cycle \( N \)

\( \Delta W \) = area of cyclic loop

\( \delta_N \) = degradation index

\( \lambda \) = equivalent viscous damping ratio
$\sigma'_h$ = horizontal effective stress

$\sigma'_v$ = vertical effective stress

$\sigma'_{vc}$ = vertical effective consolidation stress

$\tau$ = shear stress

$\tau_c$ = cyclic shear stress amplitude
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VITA

March 25, 1977
Born, Tehran, Iran

2000
Bachelor of Engineering
Amir Kabir University (Tehran Polytechnic)
Tehran, Iran

2004
Master of Engineering with Thesis
Bahonar University
Kerman, Iran

2010
Master of Engineering
University of California, Los Angeles
Los Angeles, USA

PUBLICATIONS


1. Introduction

When fully saturated soils are subjected to cyclic loading in undrained conditions involving moderate and large cyclic shear strain amplitudes, $\gamma_c$, their stiffness and strength decrease and the cyclic pore water pressure, $\Delta u_N$, changes permanently with the number of cycles, $N$. Here, $\Delta u_N$ is the pore water pressure at the end of strain cycle $N$. It is the pore water pressure that remains when the cycling stops and is therefore sometimes called the residual pore water pressure. The change of stiffness with $N$ is typically expressed as the change of the secant shear modulus $G_{SN}$ with $N$. Such cyclic degradation of stiffness and pore water pressure change are among the most fundamental and important phenomena in soil dynamics. For example, they are key considerations in the analyses of the response of soil deposits to earthquake shaking (see Kramer, 1996; Ishihara, 1996) and foundations of offshore structures subjected to ocean wave loads (see Andersen et al., 1980). Fully saturated sands are most susceptible to cyclic degradation and significant pore pressure buildup. During cyclic loading they can completely lose their stiffness and strength while the large excess pore pressure develops, and they can eventually liquefy. On the other end of the spectrum are fully saturated clays of higher plasticity that under significant cyclic loading can lose only a fraction of their original stiffness and strength while the pore water pressure change is relatively small (see Dobry and Vucetic, 1987). Moreover, in overconsolidated (OC) clays of high plasticity cyclic pore water pressure, $\Delta u_N$, can decrease instead of increase with $N$ while the cyclic degradation is even smaller (Vucetic, 1988).
As opposed to that, when fully saturated soils are subjected to very small cyclic shear strains, $\gamma_c$, soil’s structure practically does not change and, consequently, neither the cyclic degradation nor the permanent cyclic pore pressure change occur. The amplitude $\gamma_c$ below which there is no cyclic degradation and above which the degradation occurs is known as the threshold shear strain for cyclic degradation, $\gamma_{td}$ (Tabata and Vucetic, 2010). The amplitude $\gamma_c$ below which there is no permanent pore water pressure change with N and above which the permanent pore pressure is recorded after every cycle is known as the threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$ (Dobry et al, 1982; Vucetic, 1994).

For sands $\gamma_{tp}$ has been extensively investigated (Dobry and Ladd, 1980; Dobry et al, 1982; Dyvik et al., 1984) while the threshold $\gamma_{td}$ for stiffness change has been hardly investigated at all. For clays, both $\gamma_{tp}$ and $\gamma_{td}$ were investigated to a limited extent (Tabata and Vucetic, 2010; Hsu and Vucetic, 2004). For example, the threshold strain $\gamma_{tp}$ for pore water pressure in OC clays has not been directly investigated to a satisfactory extent.

The focus of the investigation described in this thesis was on the existence and magnitude of $\gamma_{td}$ in sand and $\gamma_{tp}$ in normally consolidated (NC) and OC clays, and on the comparison and connection between different thresholds in sands and clays and difference in thresholds of NC and OC clays. With these goals in mind three sands and two laboratory-made clays are tested. They include two fine clean sands and one coarse clean sand, and the laboratory prepared pure kaolinite clay having PI=28 and a clay prepared in laboratory from a mix of kaolinite and bentonite powder which is named the kaolinite-bentonite clay and has PI=55.
These soils are tested in the Norwegian Geotechnical Institute (NGI) type of direct simple shear device (DSS) introduced by Bjerrum and Landva (1966). The NGI-DSS device was originally designed for monotonic loading testing and was modified and automated at UCLA for cyclic testing. In this investigation the UCLA NGI-DSS device was used for the constant-volume equivalent-undrained type of cyclic testing. The NGI-DSS specimen has the shape of a short cylinder which is consolidated vertically and sheared horizontally. The specimen is confined during consolidation and shear within a wire-reinforced rubber membrane. The reinforced membrane greatly restricts and almost prevents lateral deformation of the specimen during consolidation and shear. In such a testing, during the shear the volume of the specimen is kept constant, the water drains are kept open to atmospheric pressure, and consequently the pore water pressure is always zero. In such an arrangement, the pore water pressure that would have developed in a true undrained test is equivalent to the change in the vertical stress on the specimen that is necessary to maintain the volume of the specimen constant. This change in vertical stress is thus called the equivalent pore water pressure. Because of the negligible lateral deformations of the specimen due to the confinement in the wire-reinforced rubber membrane, the volume is maintained practically constant by maintaining the height of the specimen constant. For the validity of the NGI-DSS constant-volume equivalent-undrained testing concept see Dyvik, et al. (1987).

Two types of cyclic tests are conducted, single-stage cyclic strain-controlled test with constant $\gamma_c$ throughout the test, and the multi-stage cyclic strain-controlled test in which $\gamma_c$ was constant in each stage but larger in every subsequent stage. The amplitudes of $\gamma_c$ varied from 0.003% to 2%.
This range spans the magnitudes of $\gamma_c$ from well below the threshold shear strains obtained in previous studies to well above them.

In the context of investigating $\gamma_{td}$ in sand and $\gamma_{tp}$ in clays, the following tasks and more narrowly focused investigations are performed: (1) the NGI-DSS device is adapted for small-strain cyclic testing and a procedure for eliminating false loads and deformations from the test records is developed, (2) the effect of the vertical consolidation stress, $\sigma_{vc}'$, and frequency of cyclic straining, $f$, on cyclic degradation and pore water pressure change in clays is tested, and (3) cyclic stress-strain behavior and the change of secant shear modulus, $G_{SN}$, with $N$ in sands at very small cyclic strains is investigated.

Altogether, around 50 tests on clays and 20 tests on sands are conducted. The results of all tests are included in appendix, while the results of some representative tests and the results of the analyses of certain groups of related tests are presented throughout the thesis.

In the following chapter, the modifications of the NGI-DSS device and the development of the procedure and software for processing and analyzing the data, in particular the small-strain data, is described. In the next chapter, the investigation of the effects of the vertical consolidation stress, $\sigma_{vc}'$, and frequency of cyclic straining, $f$, on cyclic degradation and pore water pressure change in the kaolinite clay is described. This is followed with the chapter on the investigation of the cyclic behavior of sands at small cyclic strains. After that is the chapter that includes the analyses of the cyclic threshold shear strains for both sands and clays. This chapter also includes their comparison with the similar results and trends published earlier. The last chapter lists the
conclusions of the entire investigation, and is followed by the appendices with the list of references and the test results.

The results obtained can be summarized as follows: (1) small-strain cyclic testing with $\gamma_c$ as small as 0.003% can be conducted in standard NGI-DSS device if it is properly modified and equipped and false loads are eliminated before data analyses, (2) cyclic degradation in clays is affected moderately to significantly by $\sigma_{vc}'$ and frequency, $f$, (3) at very small $\gamma_c$ the modulus $G_{SN}$ increases with $N$, i.e., sand is stiffening, while at small and moderate $\gamma_c$ modulus $G_{SN}$ is first increasing and then decreasing while the cyclic pore water pressure is continuously increasing, (4) because of such behavior $\gamma_{td}$ in sand cannot be defined in the standard manner, (5) in both clays the threshold strain for degradation, $\gamma_{td}$, is not visibly affected by the overconsolidation ratio, (6) in kaolinite clay the threshold strain for pore water pressure, $\gamma_{tp}$, is not affected by the overconsolidation ratio, (7) in kaolinite-bentonite clay with PI=55 overconsolidated to OCR=4 and 7.8 the pore water pressures between $\gamma_c = 0.003\%$ and 0.3% did not change in a consistent manner and, consequently, $\gamma_{tp}$ could not be evaluated, (8) the threshold strain for pore water pressure, $\gamma_{tp}$, shows increase with PI just like in the previous studies, but its values are at or below the lower bound of published $\gamma_{tp}$ - PI trends, and (9) the threshold strains for degradation in clays, $\gamma_{td}$, do not follow the trend of increase with PI like in the previous studies and are smaller than those published earlier for similar soils.
2. Preparation of the standard NGI-DSS device for small-strain cyclic testing and development of software for filtering of false load records, and for the analyses, interpretation and presentation of the data

2.1. Introduction

Small cyclic shear strain amplitudes, \( \gamma_c \), are dominant in many soil dynamics problems, such as the vibrations of ground due to traffic and machines, seismic response of soil deposits, and cyclic loading of soil below the foundations of offshore structures subjected to ocean wave loads. For their solution the understanding of the small-strain cyclic soil behavior and the ability to conduct the small-strain cyclic soil testing are therefore essential (see e.g., Jamiolkowski et al., 1999; Shibuya et al., 1994). Amplitudes \( \gamma_c \) smaller than the cyclic threshold shear strain below which the soil structure practically does not change with the number of cycles are usually called small, and this strain is typically no larger than 0.1% (Vucetic, 1994). Amplitudes larger than 0.1% can be called medium to large.

The most popular devices for the small-strain cyclic testing are various versions and adaptations of regular cyclic triaxial, cyclic torsional shear and cyclic simple shear devices, and the resonant column device (see e.g., Woods, 1994). Some of these modifications to accommodate cyclic testing are quite sophisticated and innovative. The subject of this chapter is how the standard Norwegian Geotechnical Institute direct simple shear (NGI-DSS) device, originally introduced by Bjerrum and Landva (1966), has been modified for small-strain cyclic testing and how the test
records are processed to account for inherent testing errors to obtain the data that describe true cyclic soil behavior. Specific configuration of the NGI-DSS device at the University of California, Los Angeles (UCLA) is shown in Figure 2.1 along with the definitions of cyclic shear strain amplitude, $\gamma_c$, and cyclic shear stress amplitude, $\tau_c$. In the NGI-DSS device the specimen is a short cylinder of soil that is sitting on the porous stone and is covered by another porous stone. The porous stones are firmly fixed in the bottom and top caps. Around the vertical boundary the specimen is enclosed tightly in the wire-reinforced rubber membrane. The purpose of such a special membrane is to greatly restrict (and if properly selected almost prevent) the radial strains during consolidation and shear. In such a setup the specimen can be consolidated under conditions which are close to the $K_0$ conditions with no lateral strains.

During the cyclic shearing, the stress-strain curve describes cyclic loops. Sketch of such a loop is presented in Figure 2.2 along with the definition of two important cyclic loading parameters, the secant shear modulus, $G_S$, and the equivalent viscous damping ratio, $\lambda$. These two parameters and their variation with the number of cycles, $N$, are important components of many soil dynamics analyses.

Since the standard NGI-DSS device is originally designed for the classical soil mechanics problems involving medium and large shear strains, it is not suitable for the small-strain cyclic testing unless it is either modified or the recorded data are corrected. It is not suitable because of the inherent false displacements and false forces which are picked up by the displacement transducers and load cells but are not felt by the soil specimen.
The original NGI-DSS device has been modified and upgraded at UCLA by adding a computer-controlled closed-loop servo-hydraulic system for the application of either desired cyclic horizontal displacement or desired cyclic load on the specimen via a horizontal hydraulic piston.
Also, electronic load cells and displacement transducers are installed in configurations that are suitable for recording of small displacements and loads.

In the specimen setup in Figure 2.1 the false displacements are actually negligibly small, while the false loads are significant. Precise horizontal displacements between the bottom and top faces of the specimen can be applied and measured because the horizontal displacement transducer is bridged directly between the bottom and top specimen caps. In such a setup, the horizontal false deformations consist only of the shear deformations of the porous stones and a fraction of the shear deformations of the bottom and top caps which are made of brass. Since these deformations are truly negligible in comparison to those of the soils tested in this study, there is no need for their correction.

Figure 2.2: Stress-strain cyclic loop.
As opposed to that, the false loads recorded by the horizontal load cell are relatively large and need to be accounted for. Besides the shear resistance of the soil specimen the horizontal load cell detects the following: (i) shear resistance of the wire-reinforced rubber membrane, (ii) force applied on the top specimen cap by the probe of the displacement transducer (force of the spring in the transducer), (iii) friction of the horizontal linear roller bearing, and (iv) the friction of the horizontal plate roller bearing. These false forces are relatively small and can be neglected at large $\gamma_c$ when the soil resistance is very large, but not at small $\gamma_c$ when they can easily become as large as the soil resistance itself and even larger. It should be noted that at small $\gamma_c$ the false loads from the two roller bearings are the main problem. They are many times larger than the forces from the membrane and displacement transducer which can be often neglected. A procedure of how to quantify all four false forces and subtract them from the horizontal load cell record to obtain the accurate stress-strain behavior at small cyclic strains is the main topic of this chapter.

The other alternative for using the NGI-DSS testing concept for small strain testing is to substantially re-design the apparatus or come up with an entirely new design. One such entirely new design is the Dual Specimen Direct Simple Shear device (DS-DSS device) introduced at UCLA by Doroudian and Vucetic (1995, 1998). In this device the false deformations and loads essentially do not exist and the behavior of soil at $\gamma_c$ as small as 0.001%, and even smaller, has been accurately tested (see e.g. Matesic and Vucetic, 2003; Tabata and Vucetic, 2004; Vucetic et al., 1998). However, the DS-DSS device has two limitations. First is that the precise amplitude and pattern of sinusoidal or triangular displacement and loading cannot be easily applied, which means that the precise strain-controlled (constant $\gamma_c$) and stress-controlled (constant $\tau_c$) testing
cannot be conducted. Furthermore, the equivalent pore water pressure variation cannot be recorded. In fact, these limitations of the DS-DSS device disqualified it for the precise small-strain cyclic strain-controlled testing to obtain the threshold cyclic shear strains for cyclic degradation, $\gamma_{td}$, and cyclic pore water pressure change, $\gamma_{tp}$.

To better understand the significance and timing of the procedure for the processing and analysis of the raw test data proposed below, it is put here in historical perspective. The DSS devices, such as the Swedish (Kjellman, 1950), Cambridge (Roscoe, 1953) and Norwegian (Bjerrum and Landva, 1966), were introduced to geotechnical practice and research between 1950 and 1966 and subsequently modified for cyclic testing (Woods, 1978). From that time until approximately 1985, the analog signals from the displacement, load and pressure transducers generated during cyclic testing were typically recorded by scanners and analog strip chart recorders and X-Y plotters. With scanners the electrical signals were sampled at certain frequency and recorded digitally on paper, with strip chart recorders the continuous plots of time histories were recorded on paper rolls, and with X-Y plotters different combinations of measured parameters were cross-plotted on paper sheets. In other words, everything was recorded in real time on paper. The false loads and deformations could be easily recognized on these paper records, but it was extremely time consuming and therefore impractical to eliminate them and generate new accurate plots and data columns. Consequently, the DSS devices were essentially not used for small-strain cyclic testing where the false loads and displacements must be accounted for. They were used instead to successfully investigate many aspects of the cyclic soil behavior at moderate and large $\gamma_c$ above approximately 0.1% where the false loads and deformations are from the practical point of view negligible. For example, the test data obtained in the cyclic DSS tests by many investigators
from different laboratories that are reported in the state-of-the-art reports on cyclic soil behavior by Finn (1981) and Dobry and Vucetic (1987) do not cover very small cyclic strains, but \( \gamma_c \) from around 0.1\% to 10\%. Between approximately 1985 and 1990 the tools for the electronic acquisition of data and their digitization into computer files that can be manipulated in various ways became available to geotechnical laboratories. It then became possible to identify the false loads and deformations very precisely, correct the data for these false components, and generate new digitized records of correct data and various correct plots. However, such a procedure, although doable for the last 20 years, has not been very attractive until recently when the powerful personal computers with truly powerful graphics and sophisticated software for manipulation of large data files became available. That is why the seemingly simple procedure described below in this chapter has not been employed before in the cyclic DSS testing.

An interesting thing that needs to be mentioned at this point is that in the triaxial and torsional shear devices the false load due to the friction between the vertical piston and its ball bearing is small in comparison to the friction of the two roller bearings in the DSS device. Also, in the triaxial and torsional shear devices the ball bearing friction can be easily bypassed if the load cell is built into the bottom or top specimen cap in the specimen cell (Woods, 1994). To do something like this with the DSS device is very difficult because of the different load-transfer configuration. Consequently, the procedure described here is very attractive for the DSS small-strain cyclic testing, but it is not attractive for the triaxial and torsional shear small-strain cyclic testing.
2.2 Comparison between the correct stress-strain results and the results affected by false loads

The effects of the false loads on the test records are demonstrated here by comparing the records obtained on similar soils in testing devices without and with the false loads. In Figure 2.3 are the results of a small-strain cyclic test on low-plasticity clay with plasticity index PI=26, which was conducted in the DS-DSS device that has practically no false loads (Vucetic et al., 1999). Only 3.5 cycles are applied in this test. The shape of the cyclic strain, and consequently that of the cyclic stresses, was close to triangular with rather sharp peaks of the cyclic strain-time and stress-time histories. In Figure 2.4 are the results of a test on kaolin clay having PI=28 which was conducted in the present study in the NGI-DSS device with the configuration sketched in Figure 2.1. In this test the specimen was first subjected to 10 cycles of perfectly triangular and then 10 cycles of perfectly sinusoidal cyclic strains. Only cycles 1, 2, 11 and 12 of this test are presented. In both tests $\gamma_c$ was around 0.035%, which is quite small. The representative cyclic stress-strain loops recorded in the tests are also displayed in Figures 2.3 and 2.4.

In the record in Figure 2.3 representing a correct soil behavior, the stress is consistently increasing with increasing strain, and decreasing with decreasing strain, which resulted in the stress-strain loops with rather perfectly pointed tips. In the record in Figure 2.4 from the test segment with the triangular straining, however, immediately after the strain reversal the stress-time record exhibits a sudden drop of approximately 2.5 kPa which is denoted by $\Delta \tau_f$. Similar stress drop of around 2.0 kPa occurred in the case of the sinusoidal straining. These stress drops
are also visible at the tips of the stress-strain loops which are now not pointed but rather square-shaped. Such tips make the loops unrealistically large.

Figure 2.3: Records from test conducted in the DS-DSS device on low-plasticity clay having $PI=26$ consolidated to $\sigma_{vc}^c=390$ kPa (from Vucetic et. al., 1999).
Figure 2.4: Records from test conducted in the NGI-DSS device at UCLA (Figure 2.1) on clay having \( PI = 28 \) consolidated to \( \sigma_{vc}' = 148 \text{ kPa} \).

The drop \( \Delta \tau_f \) occurs due to the false friction forces of the horizontal plate roller bearing and linear roller bearing that act in the direction opposite to the movement of the loading piston. When the specimen top cap changes the direction at the strain-reversal, the direction of these friction forces changes too and this sudden change is recorded in a stress-time diagram as \( \Delta \tau_f \). These circumstances are illustrated in Figure 2.5 on a strain-time diagram. In this diagram \( (\Delta \tau_f/2) \) is the stress corresponding to the magnitude of these two friction forces when the piston is moving in one of the directions, either left or right. Consequently, the magnitude of \( \Delta \tau_f \) is twice
the stress corresponding to the sum of the two friction forces generated by two roller bearings. During the loading phase when the movement of the loading piston and top cap is to the left, both roller bearings are resisting the motion of the piston with the cumulative friction force which is detected by the horizontal load cell as compression. This compression corresponds approximately to half of $\Delta \tau_f$. During the unloading phase after the strain reversal, the movement of the piston and top cap is to the right and both roller bearings are resisting the motion with a friction force in opposite direction which is now recorded by the load cell as extension. This extension load corresponds approximately to the other half of $\Delta \tau_f$.

Figure 2.5: Directions of stresses applied to the soil and false stresses due to the friction of roller bearings with respect to the strain reversal and movement of the loading piston (example from test with $\gamma_c=0.035\%$, $f=0.01$ Hz, and $\sigma'_{vc}=148$ kPa).
The effects of the other two sources of false loads, the shear resistance of the wire-reinforced rubber membrane and the force applied by the probe of the displacement transducer, cannot be distinguished in the records in Figure 2.4 because of their different character and much smaller magnitude. This is explained and illustrated further below in detail.

The false stresses caused by the roller bearings have different effects on different cyclic loading parameters. For example, the true cyclic shear stress amplitude, \( \tau_c \), is enlarged by approximately \( \Delta\tau_f/2 \). In the case presented in Figure 2.4 the true \( \tau_c \approx 10 \) kPa, and \( \Delta\tau_f/2 \) which is between 1.0 and 1.25 kPa causes an error in \( \tau_c \) and associated modulus \( G_S \) between 10\% and 12.5\%. This is a significant error, but not an enormous one. However, if the damping ratio, \( \lambda \), is calculated from the area of the distorted cyclic loop, the error in \( \lambda \) is huge. Given the fact that in both tests (Figure 2.3 vs. Figure 2.4) the soil has similar plasticity index and was sheared under practically same \( \gamma_c \), the shapes of the cyclic loops due to the triangular mode of shearing should be pretty much the same. They shapes should differ just slightly due to the different frequency of cyclic straining and different vertical consolidation stress, \( \sigma'_{vc} \), applied in two tests. However, the shape of the loop in Figure 2.4 is much thicker than that of the loop in Figure 2.3, corresponding to the error in \( \lambda \) of about 300\%. In other words, \( \lambda \) obtained from the loops in Figure 2.4 is about 300\% larger than real \( \lambda \) of the soil.

Such errors in \( \lambda \) have been noticed earlier. An example are four damping curves, \( \lambda \) versus \( \gamma_c \), in Figure 2.6, which were obtained in the NGI-DSS device at UCLA for four different soils (Hsu and Vucetic, 2002). The average damping curves for different plasticity indices, PI, which have proper trends at small \( \gamma_c \) are included in the figure for comparison. It can be seen that below
certain $\gamma_c$ four damping curves plot much higher than the average curves, while at large $\gamma_c$ they are similar to the average curves. The deviation of the four curves at small $\gamma_c$ is due to the effect of $\Delta \tau_f$ which in relative terms increases as $\gamma_c$ decreases. In this particular case the level of $\gamma_c$ below which $\lambda$ values are unacceptable and require correction is approximately 0.1%.

Figure 2.6: Effect of friction of the horizontal plate roller bearing and linear roller bearing on damping ratio (Hsu and Vucetic, 2002) – average curves for different PI values are from Vucetic and Dobry (1991).
2.3 Magnitudes of false loads and corresponding false stresses

Many tests on different soils revealed that $\Delta \tau_f$ due to the combined false loads of the horizontal linear and plate roller bearings varies between 1 and 3 kPa and is largely independent of $\gamma_c$ (e.g., in the test in Figure 2.4 $\Delta \tau_f$ was between 2 and 2.5 kPa). Furthermore, in each individual test $\Delta \tau_f$ varies very little, i.e., it is pretty much constant. Also, a quantifiable influence of vertical stress and loading frequency on $\Delta \tau_f$ is not found.

Figure 2.7: Displacement-force diagram of the spring in the horizontal displacement transducer and the corresponding stress-strain relationship.
The load-displacement diagram of the spring in the horizontal displacement transducer currently mounted in the NGI-DSS device is presented in Figure 2.7a. Figure 2.7b displays the corresponding stress-strain relationship for typical specimen dimensions: height \( H = 18 \) mm, diameter \( D = 6.7 \) cm and the area \( A = 35 \) cm\(^2\). It can be seen that for very large shear strain of 10\% the equivalent stress is about 0.067 kPa, which corresponds to shear modulus of 0.67 kPa. These are very small stress and modulus indeed, and they are extremely small at small \( \gamma_c \). Consequently, for most practical purposes involving small \( \gamma_c \) this false load can be neglected. Other transducers with similar design have, of course, different but still negligible spring force.

To evaluate the shear resistance of the wire-reinforced rubber membrane, a special test was conducted. In this test, instead of the soil specimen a thin rubber balloon filled with water, with almost zero shear resistance, was sheared. The balloon tightly filled the space within the membrane and the bottom and top caps. For several membranes a practically linear load-displacement relationship was obtained in the domain of small strains, which for specimen having \( H = 18 \) mm, \( D = 6.7 \) cm and \( A = 35 \) cm\(^2\) corresponds on average to the shear modulus of around 90 kPa. The results of representative test are shown in Figure 2.8. In this test the modulus of around 87 kPa was obtained for three different levels of \( \gamma_c \), which is approximately 130 times larger than that of the spring of the horizontal displacement transducer. This means that the false load of the transducer is just 0.8\% of the false load of the membrane. The above values are also in a relatively good agreement with the membrane stiffness correction curves used internally at NGI (see e.g. Vucetic, 1981). It was found that the shear stiffness of the wire-reinforced rubber membrane is negligibly affected by lateral stresses.
Figure 2.8: Shear resistance of the wire-reinforced rubber membrane -- strain-time pattern and resulting stress-strain curve obtained on thin rubber balloon filled with water and surrounded by the NGI wire-reinforced rubber membrane.

2.4 Relative influence of different false loads on test results

The relative influence of different false loads on the shape of the soil’s stress-strain curve can be assessed in three steps. First is to construct the cyclic stress-time history and the stress-strain
cyclic loop of each false load corresponding to certain uniform $\gamma_c$ and specimen size. In the second step, in order to assess the effect of a particular false load, the above curves must be compared to the curve of the pure soil behavior corresponding to the same $\gamma_c$. Finally, in the third step, the stresses of individual false loads must be combined and added to the soil’s stresses to assess their combined effects.

Figure 2.9: Sketches of the strain-time history and the stress-time histories of the soil and false loads.
Figure 2.9 contains sketches of the strain-time and stress-time histories of typical soft clayey soil consolidated to approximately $\sigma'_{vc} = 80$ kPa and subjected to uniform $\gamma_c = 0.44\%$, and of the stress-time histories of three different false loads corresponding to the same strain-time history. All curves pertain to the specimen with $H=18$ mm and $A=35$ cm$^2$. The corresponding cyclic loops are presented in Figure 2.10. It should be noted that the vertical scales in different plots in Figures 2.9 and 2.10 are very different. A relatively large amplitude $\gamma_c = 0.44\%$ is selected to obtain large enough false stresses of the displacement transducer spring and the wire-reinforced rubber membrane. However, the cyclic stress amplitude of the displacement transducer force is still extremely small, around 0.003 kPa. The stress amplitude of the resistance of the membrane is much larger, but in absolute terms it is still small, around 0.39 kPa. In this example the amplitude of the shear resistance of the roller bearings is 1.3 kPa.

The cyclic loops in Figure 2.10 are consistent with the materials and load-deformation mechanisms they pertain to. While observing the loops it should be kept in mind that the area of a cyclic stress-strain loop represents specific energy (energy per unit volume) spent in one cycle of loading. The stress-strain loop of soil is elliptical, because the soil is nonlinear elastic-plastic material which dissipates energy during cyclic loading. The ideal cyclic loop of roller bearings is a rectangle, because they exhibit rigid-perfectly plastic behavior characteristic of the mechanism with pure friction which also has significant energy dissipation. The equivalent stress-strain behavior of displacement transducer and membrane is essentially linearly elastic with practically no energy dissipation, so the corresponding areas of the loops are in both cases practically zero. Consequently, the area of the cyclic loop derived from the uncorrected horizontal load cell
record is essentially not affected by the false loads of the displacement transducer and membrane, while it is affected significantly by the false load of the roller bearings.

Figure 2.10: Cyclic stress-strain loops of soil and the loops corresponding to the false loads of displacement transducer, wire-reinforced rubber membrane and the friction of two roller bearings.

Figures 2.11 and 2.12 show the individual and combined effects of the false loads on the stress-time history and the stress-strain loops of the soil. In Figure 2.11 the effect of the displacement transducer alone is barely noticeable, the effect of the membrane alone is very small and also hardly noticeable, while the effect of the friction of roller bearings is clearly visible. The same
can be concluded from Figure 2.12, which also shows that the effect of the friction of the roller bearings on the area of the loop is large.

![Graphs showing stress-time history due to stresses caused by false loads](image)

**Figure 2.11: Change of the stress-time history due to the stresses caused by false loads.**

The relative effects of the false loads on the cyclic loading parameters $G_S$ and $\lambda$ are summarized in Table 2.1 where they can be easily compared. The data in the table reveal that the false load due to the friction of the roller bearings is the only one that significantly affects the magnitude of
$G_S$ and $\lambda$. This is particularly true for $\lambda$ which is affected the most. It can be therefore concluded that, from a practical point of view, in many instances the test results need to be corrected only for the effect of the friction of the horizontal roller bearings and do not need to be corrected for the effects of the displacement transducer and membrane. In other words, if only the friction of the roller bearings is subtracted from the horizontal load cell records, for many practical purposes the soil behavior can be described accurately enough. This point is evident from the percentages of change in the last row of Table 2.1 and is illustrated further in Figure 2.13 where the loop corrected in this manner is compared to the loop of pure soil.

![Figure 2.12: Change of the stress-strain loop due to the stresses caused by false loads.](image-url)
Table 2.1: Relative errors due to the false loads for $\gamma_c = 0.44 \%$.

<table>
<thead>
<tr>
<th></th>
<th>Area of the loop $\Delta W$ (kPa/100)</th>
<th>Change in (%)</th>
<th>$W$ (kPa/100)</th>
<th>Change in (%)</th>
<th>Damping $\lambda$ (%)</th>
<th>Change in (%)</th>
<th>Modulus $G_s$ (kPa)</th>
<th>Change in (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>6.65</td>
<td>-</td>
<td>3.11</td>
<td>-</td>
<td>17.0</td>
<td>-</td>
<td>3150</td>
<td>-</td>
</tr>
<tr>
<td>Soil + Transducer</td>
<td>6.65</td>
<td>0.0</td>
<td>3.12</td>
<td>0.3</td>
<td>16.95</td>
<td>-0.3</td>
<td>3161</td>
<td>0.34</td>
</tr>
<tr>
<td>Soil + Membrane</td>
<td>6.65</td>
<td>0.0</td>
<td>3.19</td>
<td>2.6</td>
<td>16.57</td>
<td>-2.5</td>
<td>3232</td>
<td>2.60</td>
</tr>
<tr>
<td>Soil + Friction of roller bearings</td>
<td>8.95</td>
<td>34.6</td>
<td>3.40</td>
<td>9.3</td>
<td>20.94</td>
<td>23.2</td>
<td>3443</td>
<td>9.29</td>
</tr>
<tr>
<td>Soil + All</td>
<td>8.95</td>
<td>34.6</td>
<td>3.49</td>
<td>12.2</td>
<td>20.40</td>
<td>20.0</td>
<td>3535</td>
<td>12.22</td>
</tr>
<tr>
<td>Soil + Transducer + Membrane</td>
<td>6.65</td>
<td>0.0</td>
<td>3.20</td>
<td>2.9</td>
<td>16.52</td>
<td>-2.8</td>
<td>3243</td>
<td>2.94</td>
</tr>
</tbody>
</table>

Figure 2.13: Effect of the correction for the false loads caused only by the friction of the roller bearings.
2.5 Procedure for the correction of recorded stress-strain data

The above approach for correcting only for the friction of the roller bearings is applied here to the cyclic test results obtained on kaolinite clay classified as MH and having PI=28. The clay was consolidated under $\sigma_{vc}^\prime=148$ kPa and cycled in sinusoidal mode at $\gamma_c=0.035\%$ at the frequency of $f=0.01$ Hz. A section of the uncorrected test results have already been shown in Figure 2.4. One cycle of loading is shown again in Figures 2.14 and 2.15 in much larger scales such that the stress drops, $\Delta\tau_f$, can be better assessed and measured. They are in this particular case $(\Delta\tau_f)_b=2.1$ kPa and $(\Delta\tau_f)_c=2.2$ kPa. Knowing these values the stress record can be corrected for $(\Delta\tau_f/2)$ between each two consecutive strain reversals. Between the reversal points $a$ and $b$ the false stress $[(\Delta\tau_f)_b/2]$ must be added, and between the points $b$ and $c$ the stress $[(\Delta\tau_f)_c/2]$ must be subtracted.

The results of such a correction are displayed in Figures 2.16, 2.17 and 2.18. Figure 2.16 shows the correction in detail. Figure 2.17 presents full cycle of strain and stress and it shows that the correction yields almost perfectly smooth stress-time history. Figure 18 shows that the correction yields a realistic stress-strain loop. It should be noted that this procedure still includes a slight error, because $(\Delta\tau_f)_b$ and $(\Delta\tau_f)_c$ are not exactly the same. This can result in a discrepancy at the strain reversal, i.e., in the stress-time curve not being perfectly connected. However, many cyclic NGI DSS test revealed that the consecutive drops are always quite similar. They are similar to the point that the error involved is generally negligible.
Figure 2.14: Uncorrected strain-time and stress-time histories in one cycle of the sinusoidal straining on kaolinite clay.

Figure 2.15: Identification of the effect of the false stresses – sudden stress change, $\Delta \tau_b$, at strain reversals.
Figure 2.16: Correction of the stress record for the false stresses.

The above procedure looks simple and can be easily implemented manually to just one loading cycle. However, for tests with many cycles such a manual procedure is extremely time-consuming and is therefore not an option. Consequently, the procedure has been automated by coding it with the Matlab software. The code includes automatic and precise identification of the points on the stress-time history curve where the stress drop starts and ends, yielding a precise measurement of $\Delta \tau_f$. It should be noted on Figure 2.15 that the drop $\Delta \tau_f$ does not occur
instantaneously at the strain reversal, but gradually over a short period of time. This is because the reversal of the friction of real roller bearings is rather complex. However, the Matlab code can automatically and correctly capture the beginning and end of this process. The code also includes calculation of damping and some other features. For convenience, a sample of Matlab code applied to one multi-staged cyclic strain-controlled test on kaolinite clay is provided in Appendix 2. In this test there are 7 cyclic strain-controlled stages with $\gamma_c=0.007\%$, 0.021\%, 0.035\%, 0.055\%, 0.097\%, 0.172\%, 0.687\%, and $\sigma'_{vc}=229$ and $f=0.01$ Hz.

Figure 2.17: Strain-time history and the comparison between the corrected (solid line) and uncorrected (dashed line) stress-time histories.
2.6 Example of the application of the corrected test results

The results of a multi-stage cyclic strain-controlled test on the same kaolinite clay having PI=28 are corrected here with the help of the Matlab code and used to derive several practical cyclic soil parameters and relationships. The clay was overconsolidated to OCR=4. Prior to the cyclic shearing $\sigma'_{vc}=211$ kPa. The test had 6 consecutive cyclic strain-controlled stages, with larger but constant $\gamma_c$ in each subsequent stage. The time histories of the test are presented in Figure 2.19 and the stress-strain loops in Figure 2.20. Each stage had 10 cycles with the smallest $\gamma_c=0.026\%$. 

Figure 2.18: Corrected and uncorrected stress-strain loops for the sinusoidal straining of kaolinite clay.
in the first stage and the largest $\gamma_c = 0.84\%$ in the last stage. It can be observed that in the last four stages the cyclic shear stress amplitude, $\tau_c$, slightly decreased with the number of cycles, which indicates that in these stages the clay has experienced cyclic degradation. A more precise analysis actually revealed that small cyclic degradation also occurred in the first two stages. Such a degradation in the cyclic strain-controlled mode is traditionally described by the degradation index, $\delta$, and the degradation parameter, $t$ (Idriss et al., 1978):

$$\delta = \frac{G_{SN}}{G_{S1}} = \frac{\tau_{CN}}{\gamma_c} = \frac{\tau_{CN}}{\tau_{C1}}$$ , and

(Eq. 2.1)

$$t = -\frac{\log \delta}{\log N}.$$ (Eq. 2.2)

Here, $G_{SN}$ is the secant shear modulus $G_S$ defined in Figure 2.2, but in cycle $N$. Modulus $G_{S1}$ is thus $G_S$ in the first cycle. Stress $\tau_{CN}$ is the cyclic shear stress amplitude $\tau_c$ in cycle $N$ and $\tau_{C1}$ is $\tau_c$ in the first cycle. The degradation index, $\delta$, describes the reduction of $G_S$ with $N$. Many published test results on different clays show that if $\delta$ is plotted versus $N$ in a log-log scale, a more or less straight line is obtained. The slope of this line is the degradation parameter $t$ defined above. Parameter $t$ describes the rate of cyclic degradation at certain $\gamma_c$.

The following cyclic properties are derived from the corrected multi-stage cyclic test data and carefully examined: (1) variation of $\lambda$ with $\gamma_c$, (2) variation of $\lambda$ with $N$ in several stages, (3) cyclic degradation in every stage, (4) variation of $t$ with $\gamma_c$, and (5) the cyclic threshold shear strain for cyclic degradation, $\gamma_{td}$. 

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Figure 2.19: *Strain-time and corrected stress-time history of a multi-stage cyclic strain-controlled test on kaolinite clay.*

Figure 2.20: *Corrected stress-strain loops of a multi-stage cyclic strain-controlled test on kaolinite clay.*
Figure 2.21: *Comparison between the trends of damping ratio \( \lambda \), in the second cycle obtained from uncorrected and corrected data.*

The comparison between \( \lambda \) values obtained for the second cycle of every stage from the uncorrected and corrected data is presented in Figure 2.21. The increasing influence of the false loads on \( \lambda \) as \( \gamma_c \) is getting smaller is evident. The corrected damping curve is in a good agreement with the average damping curves for different levels of PI by Vucetic and Dobry (1991) presented in Figure 2.6. Accordingly, the damping data are successfully corrected all the way down to \( \gamma_c = 0.026\% \).

The variation of \( \lambda \) with the number of cycles, \( N \), in the last three cyclic stages is presented in Figure 2.22. It can be seen that in the cyclic strain-controlled test \( \lambda \) of this clay does not change significantly with \( N \) even if the soil exhibits a relatively significant cyclic degradation. Similar
results were obtained on clays before (see e.g., Dobry and Vucetic, 1987; Hsu and Vucetic, 2002).

![Graph](image)

Figure 2.22: Variation of damping ratio, $\lambda$, with the number of cycles, $N$, in three cyclic stages.

Cyclic degradation in all six stages is presented in Figure 2.23 in terms of the variation of $\delta$ with $N$ in a log-log scale. In all stages the relationship is pretty much a straight line the slope of which is the degradation parameter $t$. As $\gamma_c$ increases from stage to stage, $t$ also increases. The relationship between $\gamma_c$ and $t$ is presented in Figure 2.24. Each data cluster represents slight variation of $t$ with $N$ in a single stage. If the trend of $t$ with $\gamma_c$ is extrapolated to $t=0$ axis, the cyclic shear strain amplitude below which there is practically no degradation is obtained. This cyclic shear strain amplitude that divides the zones of degradation and no degradation is called the threshold shear strain for cyclic degradation, $\gamma_{td}$. For this particular clay $\gamma_{td} \approx 0.013\%$, which is in a reasonable agreement with the trend of $\gamma_{td}$ with PI obtained in the DS-DSS device by Tabata and Vucetic (2010).
Figure 2.23: Cyclic degradation in the six cyclic stages of the multi-stage cyclic strain-controlled test.

Figure 2.24: Relationship between the cyclic shear strain amplitude, $\gamma_c$, and the degradation parameter, $t$. 

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It should be noted that, although in the above example the smallest $\gamma_c$ is 0.026%, the procedure described in this paper has been successfully applied by the writers to $\gamma_c$ as small as 0.003%.

2.7 Repeatability of testing

The repeatability of testing should be examined in any experimental investigation involving new testing apparatus, modification of existing apparatus, or new type of testing or procedure. Repeatability of testing comprises of conducting at least two identical tests. If the results of the tests are practically identical, it is very unlikely that the testing procedure depends on the operator. It also indicates that the testing conditions, and with it the test results, cannot change from test to test due to some out of control, unseen environmental reasons. If the test results are considerably different, the testing is not reliable.

To check the repeatability of cyclic tests performed in the present study, two almost identical single-stage cyclic strain-controlled tests are conducted on kaolinite clay. The test results are presented in Figures 2.25 and 2.26. It can be seen that the only difference between the tests are somewhat different vertical consolidation stresses. In the first test $\sigma'_v=487$ kPa while in the second $\sigma'_v=446$ kPa, which is just 8% smaller. In Figure 2.27 the normalized stress-time histories and the normalized equivalent pore water pressure changes with time are plotted on top of each other for comparison. The comparison shows that the test results are almost the same, which means that the repeatability is good and the testing is reliable.
Figure 2.25: Results of the repeatability test No. 1.
Figure 2.26: Results of the repeatability test No. 2.
2.8 Summary and conclusions

In the standard NGI-DSS device the horizontal load cell record contains several false loads which do not represent the soil resistance. They include the shear resistance of the wire-reinforced rubber membrane, force applied by the probe of the displacement transducer (force of the spring in the transducer), friction of the horizontal linear roller bearing and the friction of the horizontal plate roller bearing. If such a load cell record is used straightforwardly without any
correction, the resulting stress-time history and thus the stress-strain curve will be incorrect. In the case of cyclic loading, the relative magnitude of the error due to the false loads increases as the cyclic strain amplitude, $\gamma_c$, decreases. Consequently, the NGI-DSS test results cannot be used to describe properly the cyclic soil behavior at small $\gamma_c$ unless they are corrected for the false loads. The largest false loads come from the friction of the roller bearings which definitely need to be corrected for, while the false loads of the membrane and displacement transducer may be often neglected. Among important cyclic soil properties, the equivalent viscous damping ratio, $\lambda$, is affected by the false loads the most. The secant shear modulus, $G_S$, is affected much less.

If the false loads are properly identified and accurately quantified and subtracted from the load cell record, a realistic soil behavior can be obtained at $\gamma_c$ as small as 0.003%. A procedure how to do that with the help of modern computer technologies is proposed and it is verified for $\gamma_c$ as small as 0.026%. On an example of the multi-stage cyclic strain-controlled test with $\gamma_c$ between 0.026% and 0.84% it is verified that with this procedure the correct stress-strain loops, correct damping ratios, $\lambda$, and the correct variations of $G_S$ with the number of cycles, $N$, can be obtained for the entire strain range.
3. Materials tested

The primary soils tested in this investigation are two clays and one sand. The two clays are laboratory made. First is a clay made of pure commercially available kaolinite powder, called here the kaolinite clay. The second is a clay made of a mixture of 85% kaolinite powder and 15% commercially available bentonite powder, called here the kaolinite-bentonite clay. The sand tested is, so called, Nevada sand that has been used in the past in a number of investigations (e.g., Arulanandan and Scott, 1993; Hsu and Vucetic, 2002). The sand was provided by Professor B. Kutter (1992). All three soils have been tested and classified according to the Unified Soil Classification System (USCS) and the results are tabulated in Table 3.1.

Table 3.1: Classification data for three primary soils tested in this investigation

<table>
<thead>
<tr>
<th>Soil</th>
<th>Nevada sand</th>
<th>Kaolinite</th>
<th>Kaolinite-Bentonite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient Of Uniformity (C_u)</td>
<td>1.93</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td>Coefficient Of Curvature (C_c)</td>
<td>1.13</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td>Liquid Limit</td>
<td>___</td>
<td>61</td>
<td>93</td>
</tr>
<tr>
<td>Plastic Limit</td>
<td>___</td>
<td>33</td>
<td>38</td>
</tr>
<tr>
<td>Plasticity Index</td>
<td>___</td>
<td>28</td>
<td>55</td>
</tr>
<tr>
<td>USCS classification</td>
<td>SP</td>
<td>MH</td>
<td>CH</td>
</tr>
</tbody>
</table>
Figure 3.1 shows how the two cohesive soils plot on the Casagrande’s plasticity chart. It can be seen that the point corresponding to kaolinite clay plots below A-Line and to the right of the 50% liquid limit line. According to the Unified Soil Classification System (USCS) it therefore classifies as the high plasticity silt (MH). The point corresponding to the kaolinite-bentonite clay plots just above the A-Line and far to the right of the 50% liquid limit line, and the clay therefore classifies as the high plasticity clay (CH).

![Casagrande’s plasticity chart](image)

**Figure 3.1: Casagrande’s plasticity chart**

A sieve analysis is performed on Nevada sand and the grain size distribution curve is shown in Figure 3.2. From the grain size distribution curve the following important grain diameters are derived: \(D_{10}=0.11\) mm, \(D_{30}=0.16\) mm and \(D_{60}=0.21\) mm. From these grain diameters the coefficient of uniformity, \(Cu=D_{60}/D_{10}=1.93\), and the Coefficient of curvature,
Cc=$D_{30}^2/(D_{10}*D_{60})=1.13$. Based on these grain sizes and the coefficients Nevada sand is classified as the poorly graded sand (SP).

![Soil: Nevada Sand (SP)](image)

Figure 3.2: *Grain size distribution curve of Nevada sand*

An additional aspect of the NGI-DSS testing is examined as part of the present investigation. It is how the consolidation in the NGI-DSS device wire-reinforced rubber membrane compares to the consolidation in a steel ring in a standard oedometer test, i.e., how close is the consolidation in the NGI-DSS device to $K_o$ conditions with no lateral strains. Figure 3.3 shows how two compression curves obtained in the NGI-DSS device on kaolinite-bentonite clay compare to a curve obtained in the odometer device. It can be seen the three curves are very similar, which is
quite encouraging. This means that the consolidation in the wire-reinforced rubber membrane in the NGI-DSS test is close to the consolidation under no lateral strains in a standard oedometer test.

Figure 3.3: **Comparison between compression curves obtained for kaolinite-bentonite clay in the wire-reinforced rubber membrane in the NGI-DSS test and the steel ring in a standard oedometer test.**
4. Effect of the frequency and vertical stress on the cyclic degradation and pore water pressure in clay

4.1 Introduction

When fully saturated soils are subjected to cyclic loading in undrained conditions involving moderate to large shear strains their stiffness and strength decreases with the number of cycles, N. Fully saturated sands are most susceptible to such cyclic degradation. Due to cyclic loading they can completely lose their stiffness and strength while the cyclic pore water pressure continuously increases and can eventually liquefy. On the other end of the spectrum are fully saturated clays of high plasticity that under significant cyclic loading can lose only a fraction of their original stiffness and strength. Cyclic pore water pressure also changes in clays during cyclic loading. It always increases in NC clays, but in OC clays it may first decrease and then increase, depending on the magnitude of OCR (Dobry and Vucetic, 1987; Vucetic, 1988). The cyclic degradation of stiffness and cyclic pore pressure changes are among the most important phenomena in soil dynamics and for a quality soil dynamics analyses all of their aspects should be known. However, there are still some important aspects that need to be investigated. The primary topic of the investigation described in this chapter is how the cyclic degradation of clay is affected by the frequency of cyclic loading and effective vertical consolidation stress. The secondary topic is how the cyclic pore water pressure change is affected by the frequency and vertical stress. Very little is known about these effects. Knowing the extent of these effects is important when the cyclic degradation and pore pressure data obtained at different vertical stresses and different
loading frequencies are compared, or when the frequencies and stresses in laboratory testing program are not the same as those in the real problem in the field.

Cyclic degradation and pore water pressure change can be investigated systematically based on the results of either strain-controlled or stress-controlled cyclic tests. In the cyclic stress-controlled test the cyclic stress amplitude, \( \tau_c \), is maintained constant, while in the cyclic strain-controlled test the cyclic strain amplitude, \( \gamma_c \), is maintained constant. Cyclic degradation and pore water pressure change are larger if these cyclic amplitudes are larger, and for a given soil the cyclic amplitude and the number of cycles, \( N \), are the most important factors influencing the degradation and cyclic pore water pressure change.

In the investigation described in this chapter the cyclic degradation was tested in the constant-volume equivalent-undrained single-stage cyclic strain-controlled NGI-DSS tests. In each test the constant cyclic shear strain amplitude, \( \gamma_c \), was applied in sinusoidal mode, while the variation of the cyclic shear stress, \( \tau_{cN} \), with \( N \) and the change in pore water pressures, \( \Delta u_N \), with \( N \) were recorded. Here, \( \Delta u_N \) is the pore water pressure at the end of strain cycle \( N \). It is the pore water pressure that remains when the cycling stops and is therefore sometimes called the residual pore water pressure. Only laboratory-made kaolinite clay with plasticity index \( \text{PI}=28 \) was tested. It was tested in a series of 16 tests under three different amplitudes, \( \gamma_c \), two vertical effective consolidation stresses, \( \sigma'_{vc} \), and three cycling frequencies, \( f \). In all of these tests the soil was normally consolidated (NC soil) prior to cyclic shearing. Accordingly, the findings presented here are applicable to NC clays of similar plasticity and for the ranges of \( \sigma'_{vc} \) and \( f \) applied.
Comprehensive studies on the fundamental aspects of the cyclic degradation of clays and associated pore water pressure changes started between 1975 and 1980 in connection with the design of foundations of offshore structures subjected to cyclic ocean wave loads. They include, among others, a survey of the cyclic triaxial stress-controlled loading behavior of a number of different soils by Lee and Focht (1976), detailed investigation of the cyclic triaxial and simple shear stress-controlled behavior of one clay by Andersen (1976) and Andersen et al. (1980), and the investigation and modeling of the cyclic triaxial strain-controlled behavior of a soft clay by Idriss et al. (1978). These studies were then followed by: (i) the investigations of certain specific factors influencing the magnitude and rate of cyclic degradation and pore water pressure, such as the type of soil and overconsolidation ratio, OCR (e.g., Vucetic and Dobry, 1988; Tan and Vucetic, 1989), (ii) the studies about the cyclic threshold shear strain for cyclic degradation and cyclic pore water pressure (Vucetic, 1994; Tabata and Vucetic 2010), and (iii) by the development of computer models for the cyclic response of foundation soil that incorporates cyclic degradation and pore water pressure change (e.g., Idriss et al. 1976; Andersen et al., 1988; Andersen and Lauritzsen, 1988; Matasovic and Vucetic, 1993). These studies revealed that, besides the cyclic amplitude and the number of cycles, the cyclic degradation of clays and their pore pressures are significantly affected by the type of clay and OCR. Clays with higher plasticity index, PI, for example, experience smaller degradation, i.e., degradation decreases with PI. If clay has larger OCR the degradation is smaller, i.e., cyclic degradation also decreases with OCR. These studies also revealed that below a certain small cyclic shear strain amplitude, $\gamma_c$, cyclic degradation and permanent cyclic pore water pressure change practically do not take place. These amplitudes are called the cyclic threshold shear strain for cyclic degradation, $\gamma_{td}$.
and the cyclic threshold shear strain for cyclic pore water pressure, \( \gamma_{tp} \). These threshold strain amplitudes range approximately between \( \gamma_c = 0.01\% \) and 0.1% and generally increase with PI. However, to the writers’ knowledge, in all of the above studies and similar studies the effects of the vertical consolidation stress and frequency of cyclic shearing on cyclic degradation were not explicitly and systematically investigated.

Figure 4.1: Variation of strain, stress and pore water pressure with time in a cyclic strain-controlled test on kaolinite clay with \( \sigma'_{vc} = 691 \text{ kPa} \) and \( f=0.001 \text{ Hz} \).
These effects on the cyclic degradation and cyclic pore water pressure change are investigated here not only to fill the associated knowledge gap, but also to throw more light on how they can influence the cyclic threshold strains, $\gamma_{td}$ and $\gamma_{tp}$. The general idea is that if $\sigma'_vc$ and $f$ affect the cyclic degradation and pore pressures, they should affect $\gamma_{td}$ and $\gamma_{tp}$. These effects on $\gamma_{td}$ and $\gamma_{tp}$ can also be tested directly by conducting series of cyclic threshold tests at different $\sigma'_vc$ and $f$, but this is not so easy. As shown below in Chapter 6, the cyclic threshold tests are quite complex small-strain tests. They are very sensitive and time consuming, and therefore expensive, and to conduct a large series of such tests with different combinations of $\sigma'_vc$ and $f$ would be an entirely new full-scale investigation. Still, in the cyclic threshold tests elaborated in Chapter 6 a couple of different $\sigma'_vc$ and $f$ are applied just to get a preliminary idea on their influence on $\gamma_{td}$ and $\gamma_{tp}$.

In Figures 4.1 through 4.4 the results of two cyclic tests conducted at the same $\gamma_c=0.5\%$ are presented. In Figures 4.1 and 4.2 are the results of test with $\sigma'_vc=691$ kPa and $f=0.001$ Hz, while in Figures 4.3 and 4.4 are the results of test with $\sigma'_vc=216$ kPa and $f=0.1$ Hz. As shown in Figures 4.1 and 4.3, in both tests the cyclic stress amplitude, $\tau_{cN}$, decreased with the number of cycles, $N$, indicating the cyclic degradation, while the equivalent pore water pressure, $\Delta u$, cyclically increased. The cyclic degradation is also clearly evident from the stress-strain loops in Figures 4.2 and 4.4 showing the reduction of the secant shear modulus, $G_{SN}=\tau_{cN}/\gamma_c$. For convenience, in Figure 4.4 the modulus $G_{SN}$ is indicated for cycles 1 and 5.
Figure 4.2: Stress-strain loops of a cyclic strain-controlled test on kaolinite clay with $\sigma'_{vc} = 691$ kPa and $f=0.001$ Hz.

The cyclic degradation with N can be expressed with the degradation index, $\delta$, and the degradation parameter, $t$, already defined in Chapter 2 in Equations 2.1 and 2.2. The degradation index, $\delta$, describes the degradation of $G_{SN}$ with N at constant $\gamma_c$, which means that for a given $\gamma_c$ index $\delta$ is a function of N. The degradation parameter, $t$, describes the rate of degradation at given $\gamma_c$, which means that for a given $\gamma_c$ parameter $t$ is a single value, a constant.

The index, $\delta$, and parameter, $t$, were introduced by Idriss et al. (1978) in the context of the evaluation of the cyclic degradation of marine clay deposits underlying offshore structures for oil
explorations. Idriss et al. (1978) found that for many clays $\delta$ versus $N$ relationship in a log-log format is approximately a straight line the slope of which is the degradation parameter, $t$. For overconsolidated clays $\delta$ versus $N$ in log-log format is also approximately a straight line, while for sands it is usually curved.

![Image of test results]

**Figure 4.3:** Variation of strain, stress and pore water pressure with time in a cyclic strain-controlled test on kaolinite clay with $\sigma'_{vc} = 216$ kPa and $f = 0.1$ Hz.
The difference in the change of cyclic pore water pressure, $\Delta u_N$, and the cyclic degradation in the two tests presented in Figures 4.1 through 4.4 is illustrated in Figure 4.5. While the difference in $\Delta u_N$ trend is relatively small, the difference in cyclic degradation is significant. In the first test with $\sigma'_{vc} = 691$ kPa and $f=0.001$ Hz the degradation parameter $t=0.065$, while in the second test with smaller $\sigma'_{vc} = 216$ kPa and higher $f=0.1$ Hz parameter $t=0.157$. This indicates that the degradation may be considerably larger if $\sigma'_{vc}$ is smaller and $f$ is higher. These two trends are, in fact, confirmed below. It should be noted that in Figure 4.5 the variation of $\Delta u_N$ is presented in the normalized form with respect to $\sigma'_{vc}$, $\Delta u_N^* = \Delta u_N / \sigma'_{vc}$.

Figure 4.4: Stress-strain loops of a cyclic strain-controlled test on kaolinite clay with $\sigma'_{vc} = 216$ kPa and $f=0.1$ Hz.
Figure 4.5: The difference in cyclic degradation and pore water pressure change in two tests with different $\sigma'_{vc}$ and $f$

The program of 16 tests is summarized in Table 4.1. It encompasses three levels of $\gamma_c$, 0.1%, 0.25% and 0.5%, two levels of $\sigma'_{vc}$ of approximately 220 kPa and 680 kPa, and three relatively low frequencies, 0.001 Hz, 0.01 Hz and 0.1 Hz. The above range of $\sigma'_{vc}$ corresponds approximately to the depth of fully saturated clay deposit between 20 and 70 meters. The seemingly low frequencies generally cover the range of frequencies of the cyclic loads that are applied by the large design ocean wave storms to the foundations of offshore structures. For
example, the highest energy waves in the oil fields of North Sea, including the 100-year design wave, have periods of 15-20 seconds, which corresponds to \( f = 0.05 \text{ Hz} \) to 0.07 Hz (Schjetne, 1976; see also Ellers, 1982). Cyclic soil degradation is also important phenomena in geotechnical earthquake engineering where the frequencies of seismic shaking are larger. The frequency of ground accelerations during earthquakes varies widely from earthquake to earthquake and in a given earthquake over the different geologic and soil formations, and it ranges generally between 0.5 Hz and 15 Hz. However, the characteristic frequencies of ground velocities and displacements are much smaller. In any case, the trends presented in this investigation for the frequencies between 0.001 Hz and 0.1 Hz can be extrapolated to higher frequencies applicable to earthquake shaking. The values of degradation parameter, \( t \), in Table 4.1 are the slopes of straight lines fitted through the log\( N \) versus log \( \delta \) data points for 20 cycles of loading.

Table 4.1: Testing program of 16 cyclic strain-controlled tests on normally-consolidated laboratory-made kaolinite clay with PI=28

<table>
<thead>
<tr>
<th>( \gamma_c ) (%)</th>
<th>( \sigma_{vc}^{'} )</th>
<th>Fastest</th>
<th>Medium</th>
<th>Slowest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f = 0.1 \text{ Hz} )</td>
<td>Void ratio, ( e )</td>
<td>( \sigma_{vc}^{'} ) (kPa)</td>
<td>( t )</td>
</tr>
<tr>
<td>0.1</td>
<td>Low 0.1%-Lo-Fa</td>
<td>1.32</td>
<td>219</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>High 0.1%-Hi-Fa</td>
<td>1.07</td>
<td>677</td>
<td>0.042</td>
</tr>
<tr>
<td>0.25</td>
<td>Low 0.25%-Lo-Fa</td>
<td>1.34</td>
<td>217</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>High 0.25%-Hi-Fa</td>
<td>1.11</td>
<td>684</td>
<td>0.083</td>
</tr>
<tr>
<td>0.5</td>
<td>Low 0.5%-Lo-Fa</td>
<td>1.35</td>
<td>216</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>High 0.5%-Hi-Fa</td>
<td>1.07</td>
<td>688</td>
<td>0.098</td>
</tr>
</tbody>
</table>
4.2 Effect of the frequency on cyclic straining

It has been known for a long time that soils as viscous materials are susceptible to the effect of the rate of loading or deformation. For example, decades ago Taylor (1948) and Bjerrum (1972) investigated and quantified the rate effect on the shear strength of clay. The basically concluded that in clays the shear strength increases by about 10% if the rate of shearing is increased 10 times. This also means that the stress-strain curve from the test in which the rate of shearing is higher must plot higher, which has been shown by many in many different tests conducted in different geotechnical testing devices.

The same rate of loading phenomena has been obtained in this study, although the monotonic loading tests (static tests) are not conducted. However, the first quarter of the first cycle of a cyclic test is an initial part of a monotonic loading test. The average rate of monotonic shearing, \( \dot{\gamma} = d\gamma/dt \), in the NGI-DSS device during the first quarter of the first cycle is:

\[
\dot{\gamma} = \frac{4\gamma_c}{T} = 4f\gamma_c, \tag{Eq. 4.1}
\]

where \( T \) is the period of cyclic straining. This equation not only shows that \( \dot{\gamma} = d\gamma/dt \) and frequency, \( f \), are related, but that \( f \) must have quite a significant effect on the cyclic soil behavior. Consequently, various effects of \( f \) on cyclic soil behavior have also been investigate by many (e.g., Isenhower and Stokoe, 1981; Matesic and Vucetic, 2003; Tabata and Vucetic, 2004, 2010).
The first quarters of the first cycles of cyclic tests on kaolinite clay conducted at different $\gamma_c$, $\sigma'_vc$ and $f$ are shown in four plots in Figure 4.6. The tests presented in each plot are conducted at the same $\gamma_c$ and $\sigma'_vc$ but different $f$. The effect of $\dot{\gamma}$ is clear in all four cases. This also means that the soil started degrading in tests with the same $\gamma_c$ and $\sigma'_vc$ but different $f$ at different levels of the secant shear modulus $G_{S1}$.

Figure 4.6: *The effect of the rate of loading on the initial loading stress-strain curves obtained on kaolinite clay having PI=28.*
The effect of frequency, $f$, on the equivalent normalized cyclic pore water pressure, $\Delta u_N^*$, is presented in Figure 4.7. The results show that at the same $\gamma_c$ and $\sigma'_{vc}$ the pore water pressure $\Delta u_N^*$ is higher if the frequency is lower. The reason for such a trend is not apparent and should be a subject of future investigation. However, a contributing factor to the trend could be the tendency towards secondary compression due to the creep that is always present in soil subjected to vertical stresses. This tendency towards compression translates in undrained conditions to pore water pressure increase and is larger if the soil is exposed to the creep for a longer period of time. It should be noted that the tendency towards compression is amplified by the soil disturbance due to cyclic shearing.

Figure 4.7: *The effect of frequency on the equivalent cyclic pore water pressure, $\Delta u_N^*$.*
The effect of $f$ on the cyclic degradation is presented in Figures 4.8, 4.9 and 4.10. The degradation is consistently larger if the frequency is higher. The data in Table 4.1 reveal that for the tenfold increase of $f$ (increase by one order of magnitude) parameter $t$ increases by 20 to 50%. This is a substantial increase and should be taken into consideration in cyclic degradation.
analyses. The reason for such a trend is also not apparent and requires further investigation. As discussed above, a larger initial secant shear modulus $G_{S1}$ was obtained in tests with higher $f$ due to the effect of the rate of shearing, $\dot{\gamma} = d\gamma/dt$. According to Eq. (1) larger $G_{S1}$ would result in smaller degradation if $G_{SN}$ decrements at a given $N$ are the same in tests with different frequencies. However, a careful examination of the test data revealed that $G_{SN}$ decrements are larger in tests with higher $f$, in fact they are so much larger that at higher $f$ the degradation is larger in spite of the larger $G_{S1}$.

Figure 4.10 reveals that for given $\gamma_c$ and $\sigma'_{vc}$ parameter $t$ increases practically linearly with the logarithm of $f$. Such a consistent trend is convenient because it allows the extrapolation of $t$ to higher frequencies with a relatively high degree of confidence. Furthermore, Figure 4.9 and the data in Table 4.1 show that the trends of $t$ with $f$ at two different $\sigma'_{vc}$ are not very different, which means that the frequency effect does not depend significantly on $\sigma'_{vc}$. For example, if at $\gamma_c = 0.1\%$ and $\sigma'_{vc} \approx 219$ kPa $f$ increases by two orders of magnitude from 0.001Hz to 0.1Hz, $t$ increases by 103% from 0.029 to 0.060. At the same $\gamma_c = 0.1\%$ but larger $\sigma'_{vc} \approx 678$ kPa $t$ increases for the same increase of $f$ by 96% from 0.022 to 0.042. Similarly, if at $\gamma_c = 0.5\%$ and $\sigma'_{vc} \approx 218$ kPa $f$ increases from 0.001Hz to 0.1Hz, $t$ increases by 72% from 0.092 to 0.157, while at the same $\gamma_c = 0.5\%$ and larger $\sigma'_{vc} \approx 689$ kPa $t$ increases by 52% from 0.065 to 0.098.
At the end it should be noted that higher frequency causes simultaneously larger degradation and smaller pore water pressures. Considering that smaller pore water pressure means larger effective stress and potentially stiffer soil, this trend seems counterintuitive. This trend therefore suggests that the cyclic pore water pressure increase in normally consolidated clays may not be a dominant contributor to the cyclic degradation. In this regard it is interesting to note that in overconsolidated clays the cyclic pore water pressure change may be in negative direction (the
pressure decreases with N) while the significant cyclic degradation still takes place (see Andersen, 1976; Andersen et al., 1980; Vucetic, 1988).

Figure 4.10: The effect of frequency, f, on cyclic degradation.
4.3 Effect of the vertical consolidation stress on cyclic straining

It is well known that in soils, which are frictional particulate materials, at larger vertical consolidation stress, $\sigma'_{vc}$, shear strength is higher and the stress-strain curve plots higher. Because soils are frictional materials, it was found that for many fully saturated clays sheared in undrained conditions the shear strength increases proportionally with, $\sigma'_{vc}$. This also means that if the normalized stress strain curves, the $(\tau/\sigma'_{vc})-\gamma$ curves, of the same soil consolidated at different $\sigma'_{vc}$ are plotted together they should more or less overlap. This is the basis of the so called SHANSEP method, which stands for the Soil History And Normalized Soil Engineering Properties method for the evaluation of shear strength in clays. The normalized soil behavior of clays was studied and the SHANSEP method was introduced by Ladd and Foot (1974) and Ladd et al. (1977). Consequently, if for the kaolinite clay the stress-strain curves of the first quarters of the first cycles of tests with the same $\gamma_c$ and $f$ but different $\sigma'_{vc}$ are plotted on the same plot in normalized form they should more or less overlap. This exercise is presented in Figure 4.11, but it reveals that the stress-strain curves do not exactly overlap. The curves pertaining to larger $\sigma'_{vc}$ consistently plot lower than those for smaller $\sigma'_{vc}$. It can be therefore concluded that the kaolinite clay does not exhibit normalized behavior, just like some other clays.

The effect of the vertical consolidation stress, $\sigma'_{vc}$, on the cyclic pore water pressure, $\Delta u_N^*$, is presented in Figure 4.12. The results clearly indicate that at the lower $\sigma'_{vc}$ the pressure $\Delta u_N^*$ is higher. This is most likely because at the lower $\sigma'_{vc}$ the void ratio is larger and the soil structure has larger capacity to volume reduction which in undrained conditions translates to the pore water pressure increase.
Figure 4.11: The effect of vertical consolidation stress, $\sigma'_{vc}$, on the initial loading stress-strain curve.
The effect of $\sigma'_{vc}$ on cyclic degradation is presented in Figures 4.13 and 4.14. It can be seen that the degradation is consistently larger at lower $\sigma'_{vc}$. This may be in some part due to the higher cyclic pore water pressures at lower $\sigma'_{vc}$ which cause larger reduction of effective stresses and thus higher rate of softening. Since the increase in pore water pressure may not be a dominant cause of degradation in clays, as already mentioned, the above trend must be also caused by some other factors which may actually be more important. Consequently, this trend also deserves to be investigated further.

Figure 4.12: The effect of vertical consolidation stress $\sigma'_{vc}$, on the equivalent normalized cyclic pore water pressure, $\Delta u_N^*$. 

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The data in Table 4.1 show that for the approximately threefold increase of $\sigma'_{vc}$ from 220 to 680 kPa the degradation parameter $t$ will decrease by 20% to 38%. Furthermore, the same data reveal that at a given $\gamma_c$ the trends of $t$ with $\sigma'_{vc}$ for three different frequencies are not very different. The trends are as follows. At $\gamma_c=0.1\%$ parameter $t$ decreases by 20% to 30% if $\sigma'_{vc}$ increases from approximately 218 to 683 kPa. At $\gamma_c=0.25\%$ the decrease is between 20% and 23%, while at $\gamma_c=0.5\%$ the decrease ranges between 29% and 38%. This means that the effect of $\sigma'_{vc}$ on $t$ depends relatively little on frequency, $f$.

Figure 4.13: The effect of vertical consolidation stress, $\sigma'_{vc}$, on cyclic degradation.
4.4 Discussion and conclusions

The test results presented above show clearly that the cyclic degradation and pore water pressure change in normally-consolidated clays can be affected substantially by the vertical consolidation stress, $\sigma_{vc}'$, and the frequency of cyclic straining, $f$. The degradation parameter, $t$, that measures the rate of cyclic degradation with the number of cycles, $N$, increases with $f$ and decreases with $\sigma_{vc}'$. More specifically, this study shows that for the kaolinte clay having PI=28 and for the range of $\gamma_c$ between 0.1% and 0.5%, parameter $t$ may increase 20% to 50% if $f$ is ten times
higher. This is a substantial increase and should be taken into consideration in cyclic degradation analyses. If $\sigma'_{vc}$ is increased more than 3 times, from approximately 220 to 680 kPa, $t$ decreases between 20% and 38%. The magnitude of the combined effect of $\sigma'_{vc}$ and $f$ on $t$ is illustrated further in Figure 4.15 where the data points from two graphs in Figure 4.9 are plotted together. The significant width of the data band indicates that for a proper evaluation of cyclic degradation the cyclic tests must be conducted at the frequency and under vertical stress corresponding quite closely to that in the problem under consideration. The other alternative is to conduct test series at several different frequencies and vertical stresses and develop trends similar to those presented here, and then use the trends to project the data to desired frequency and vertical stress.

Figure 4.15: Combined effect of the vertical stress and frequency on cyclic degradation.
In comparison to other factors affecting the cyclic degradation, the above effects of $f$ and $\sigma'_{vc}$ are relatively significant. They seem as important as the effects of OCR and PI investigated earlier. Previous studies have shown, for example, that if OCR is increased from 1 to 4 for clays having PI between 25 and 55 parameter $t$ may decrease 35% to 40% (Vucetic and Dobry, 1988). Furthermore, parameter $t$ corresponding to $\gamma_c \approx 0.25\%$ may decrease in normally-consolidated clays by roughly 70% if PI increases from 15 to 50 (Vucetic, 1994).

The effects of $\sigma'_{vc}$ and $f$ on the normalized cyclic pore water pressure, $\Delta u_N^*$, are also considerable. However, from the practical point of view the effects on the pore water pressure are less important than the effects on the degradation. What matters the most in soil dynamics analyses is how much of soil stiffness and strength may be lost due to cyclic loading regardless of the pore water pressure development. The magnitude of the pore water pressure change at the end of cyclic event is still important in certain practical problems, however, because it will influence the post-cyclic settlement or expansion of the soil. For example, if at the end of cyclic event the cyclic pore water pressure increase is high a significant consolidation will ensue, and with it a considerable settlement.

In the end, it must be noted that from this study it is not clear how much the consolidation stress and frequency may affect the cyclic degradation and cyclic pore water pressure of high plasticity and very low plasticity clays that significantly differ from the kaolinite clay. This can be determined with the help of cyclic testing programs similar to the program described in this chapter.
5. Stiffness degradation and pore water pressure in sands

at small cyclic strains

5.1 Introduction

It has already been stated that fully saturated sands are most susceptible to cyclic degradation. They can completely lose their stiffness and strength due to cyclic loading and eventually liquefy. On the other end of the spectrum are fully saturated clays of high plasticity that under significant cyclic loading can lose only a fraction of their original stiffness and strength. Such a difference between the cyclic behavior of normally consolidated clay and sand is illustrated in Figures 5.1 and 5.2 where the results of two cyclic simple shear strain-controlled tests with cyclic shear strain amplitude, \( \gamma_c \), of around 0.5% are presented. The clay is kaolinite clay with plasticity index, PI=28, and the sand is uniform Nevada sand. Figure 5.1 shows the cyclic stress-strain loops and how the secant shear modulus, \( G_{SN} = \tau_N / \gamma_c \), decreases with N. Here, \( \tau_{CN} \) is the cyclic shear stress amplitude at cycle N. For convenience, the moduli \( G_{S1} \) and \( G_{S5} \) for cycles 1 and 5 are indicated on the loops obtained from test on clay. The reduction of \( G_{SN} \) with N is typically quantified by the degradation index, \( \delta_N \):

\[
\delta_N = \frac{G_{SN}}{G_{S1}} = \frac{\tau_{CN}}{\tau_{C1}} = \frac{\tau_{CN}}{\gamma_c}. \tag{5.1}
\]

This is the same degradation index defined in Equation 2.1, except that here it was given subscript N to indicate that it changes with N. The variation of \( \delta_N \) with N is presented in a log-
log scale in Figure 5.2. It can be observed that for the clay tested this relationship is practically a straight line while for the sand it is somewhat curved downwards. As already stated, Idriss et al. (1978) found that for several different clays $\delta_N$ versus $N$ relationship in a log-log format is typically a straight line the slope of which they defined as the degradation parameter, $t$:

$$
  t = -\frac{\log \delta_N}{\log N}.
$$

(5.2)

This is the same degradation parameter defined in Equation 2.2. It was subsequently found that for overconsolidated clays $\delta_N$ versus $N$ in log-log format is also approximately a straight line (Vucetic and Dobry, 1989). The parameter $t$ can be therefore used to describe very well the rate of cyclic degradation of clays with $N$ in a cyclic strain-controlled mode of loading. For sands, however, such characterization of degradation with $t$ can only be done approximately, because $\delta_N$ versus $N$ relationship in a log-log format has shown to be always more or less curved, just as that in Figure 5.2.

In Figure 5.2 the increase of cyclic pore water pressure, $\Delta u_N$, with $N$, measured at the end of each strain cycle $N$, is also presented. It is presented in the normalized form with respect to the effective vertical consolidation stress, $\sigma'_{vc}$. It can be seen that this normalized cyclic pore water pressure, $\Delta u^*_N = \Delta u_N / \sigma'_{vc}$, continuously increases in both soils, much more in sand than in the normally consolidated clay. It is evident that much larger degradation in sand is accompanied by much larger increase in pore water pressures and associated decrease in effective stresses. In other words, the behavior of sand and normally consolidate clay just described is consistent with the notion of the effective stress principal.

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Figure 5.1: Typical stress-strain behavior of fully saturated normally consolidated clay and sand in constant-volume equivalent-undrained cyclic strain-controlled simple shear test.
However, soil is a very complex material the behavior of which can be complicated and sometimes quite surprising. One such aspect of the cyclic behavior of clay is the cyclic pore water pressure change in overconsolidated clays while they are cyclically degrading. In spite of the fact that in the cyclic strain-controlled mode the overconsolidated clays are consistently degrading with N just like normally consolidated clays, the cyclic pore water pressure may decrease with N, or first decrease and then increase with N (Vucetic et al. 1985; Vucetic 1988).
The changes of clay structure and the forces between clay particles that are believed to be responsible for such a behavior are discussed in Dobry and Vucetic (1987). This investigation deals with another complexity of cyclic soil behavior that is equally intriguing. This is the complexity of the behavior of fully saturated sand in undrained conditions at small cyclic strains.

A typical example of the cyclic strain-controlled behavior of a uniform sand in simple shear test at \( \gamma_c = 0.08\% \) is presented in Figures 5.3 and 5.4. As shown in Figure 5.3 the stress-strain behavior is described by a series of uniform cyclic loops and the pore water pressure consistently increases with the number of cycles, \( N \). However, a close examination of the stress-time history reveals that the cyclic shear stress, \( \tau_{cN} \), actually increases in the first three cycles and then decreases only after the forth cycle. This is further illustrated in Figure 5.4 where the variations of the normalized cyclic pore water pressure, \( \Delta u_N^* \), and the cyclic degradation index, \( \delta_N \), with \( N \) are plotted along with the relationship directly between \( \delta_N \) and \( \Delta u_N^* \). This means that in the first 3 cycles the sand is becoming stiffer in spite of the relative increase of the cyclic pore water pressure \( \Delta u^* = 0.19 \) and the associated 19% decrease of the vertical effective stress. It also means that in this case, that is at such relatively small \( \gamma_c \), the cyclic pore water pressure cannot be used in the context of the effective stress principal as the parameter of soil softening due to cyclic loading. Furthermore, because in such a case the secant shear modulus, \( G_{SN} \), does not consistently decreases with \( N \), the index \( \delta_N \) which can still be used to describe the change of \( G_{SN} \) with \( N \) cannot be called the degradation index. Instead, in this chapter and from now on the index \( \delta_N \) is called the stiffness index, i.e., the index describing the relative change of \( G_{SN} \) with \( N \) irrespective of the trend.
Figure 5.3. Cyclic strain-controlled behavior of sand in undrained conditions at relatively small cyclic strains.
Figure 5.4. Change of cyclic pore water pressure, $\Delta u_N^*$, and cyclic stiffness index, $\delta_N$, with the number of cycles, $N$, at a relatively small cyclic strain, $\gamma_c = 0.08\%$, and their relationship.
5.2 Small-strain cyclic behavior of sand under different conditions

Because of the potential importance of the above described behavior in modeling of the response of fully saturated sand deposits to cyclic loads, including the seismic loads leading to liquefaction, it is important to explore whether the same behavior has been recorded elsewhere for different sands tested under different conditions. With this in mind, the results of the cyclic testing investigations on several different sands conducted in different laboratories under triaxial and simple shear conditions and at different cyclic shear strains are carefully studied and elaborated below. As shown below, the same complex relationship between the stiffness index, $\delta_N$, and the cyclic pore water pressure, $\Delta u_N^*$, can be derived from the results of these investigations.

![Diagram showing cyclic strain-controlled behavior of sand at very small cyclic strains, $\gamma_c$, below the cyclic threshold shear strain for pore water pressure buildup $\gamma_{th} \approx 0.01\%$.](image)

Figure 5.5. *Cyclic strain-controlled behavior of sand at very small cyclic strains, $\gamma_c$, below the cyclic threshold shear strain for pore water pressure buildup $\gamma_{th} \approx 0.01\%$.**
To examine whether the same behavior applies to very small cyclic shear strain amplitudes below the threshold shear strain for cyclic pore water pressure, $\gamma_{tp}$, which for sands is around 0.01% (Dobry et al, 1982), three cyclic strain-controlled tests were conducted first at UCLA on three different sands at $\gamma_c$ ranging between 0.0037% and 0.0045%. The test results are presented in Figure 5.5. The grain size distributions of these three sands, along with the distribution for one more sand treated in this investigation, are presented in Figure 5.6. Figure 5.5 reveals that the stiffness index, $\delta_N$, also increases with $N$ at very small $\gamma_c$ below $\gamma_{tp}$, and that after a relatively large number of cycles $\delta_N$ may start to decrease.

![Graph showing grain size distributions](image)

Figure 5.6. *Grain size distribution curves of sands treated in this investigation.*
Figure 5.7. Cyclic stress-controlled behavior of Nevada sand at small cyclic strains.
To examine whether the same phenomenon occurs in the cyclic stress-controlled mode, a cyclic stress-controlled simple shear test was conducted at UCLA on Nevada sand at $\tau_c/\sigma'_vc=0.068$. The results of the test are presented in Figures 5.7 and 5.8. Figure 5.7 reveals that in this test the cyclic pore water pressure continuously increased with N while $\gamma_c$ varied with N around 0.02%, mainly decreasing with N, especially in the first several cycles. Accordingly, and this is very clear from the relationship between $\delta_N$ and N in Figure 5.8, it is evident that the sand stiffens with N, in particular in the first 5 cycles. Apparently, the phenomenon of stiffening while the cyclic pore water pressure increases occurs also in the cyclic stress-controlled mode.

![Graph showing the change of stiffness index with the number of cycles.](Figure 5.8. Change of stiffness index, $\delta_N$, with the number of cycles, N, in stress-controlled cyclic test on Nevada sand presented in Figure 5.7.)
Figure 5.9. Relationship between the cyclic pore water pressure, $\Delta u_N^*$, and the secant shear modulus index, $\delta_N$, obtained in triaxial cyclic strain-controlled tests on reconstituted specimens of Monterey No. 0 sand (Dobry et al., 1982)
Dobry et al. (1982) conducted extensive cyclic triaxial strain-controlled testing program on clean sand in the context of developing a new approach for the prediction of pore water pressure buildup and liquefaction of sands during earthquakes. The tests were conducted on, so called, Monterey No. 0 sand in the laboratory of the geotechnical firm Woodward-Clyde Consultants in Clifton, New Jersey. A chart comparing the relationship between $\delta_N = G_{SN}/G_{S1}$ and the change of the cyclic pore water pressure expressed in terms of $(1-\Delta u^* = 1-\Delta u_N/\sigma'_3)$ is presented in Figure 5.9. Here, $\sigma'_3$ is the initial effective isotropic confining pressure in triaxial test. The cyclic shear strain amplitudes, $\gamma_c$, in Figure 5.5 are calculated from the applied one-way cyclic axial strain amplitudes, $\varepsilon_c$, using the equation $\gamma_c = 1.5 \varepsilon_c$. The range of $\gamma_c$ in 11 tests was 0.03% to 0.3%. It should be noted that the original chart presented in Dobry et al. (1982) contains only the data points corresponding to certain cycles and the average trend line. In Figure 5.9, however, the $(G_{SN}/G_{S1})$ versus $(1-\Delta u_N/\sigma'_3)$ relationships for three tests with small $\gamma_c$ (Tests 2 and 3 with $\gamma_c = 0.1\%$ and Test 5 with $\gamma_c = 0.03\%$) are constructed. Although the cycle numbers are not specified on the plot, it is clear that for these three tests conducted at relatively small $\gamma_c$ the relationship between the stiffness change and cyclic pore pressure is equivalent to the relationship between $\delta_N$ and $\Delta u_N^*$ for Nevada sand presented in Figure 5.4.

A similarly extensive cyclic strain-controlled testing program was conducted at the soil dynamics laboratory of the Rensselaer Polytechnic Institute (RPI) in Troy, New York by Vucetic and Dobry (1986; 1988). In this investigation, both cyclic triaxial and cyclic simple shear tests were conducted on liquefiable silty sand from the Wildlife Site in California that liquefied in the past. Both intact and reconstituted specimen of the natural sand from Wildlife Site were tested at $\gamma_c$ in
the range of 0.03% to 1.0%. The results of the tests derived from the original RPI report (Vucetic and Dobry, 1986) are presented in Figure 5.10. It is evident from the Figure 5.5 that in the range of small $\gamma_c$ from 0.03% to 0.1%, the stiffness index $\delta_N$ first increases and then decreases while the pore water pressure continuously increases. At larger $\gamma_c = 0.3\%$ and 1.0% stiffness index , $\delta_N$, only decreases with N just like it is presented in Figure 5.2 for Nevada sand.

![Figure 5.10. Variation of stiffness index, $\delta_N$, with the number of cycles, N, and its relationship with cyclic pore water pressure, $\Delta u_N^*$, obtained in triaxial and simple shear cyclic strain-controlled tests on reconstituted and intact specimens of liquefiable silty sand from Wildlife Site (derived from data in Vucetic and Dobry, 1986).](image-url)
All of the above results in Figures 5.1 to 5.10 show clearly, and beyond any doubt, that during the cyclic straining of fully saturated sand in undrained conditions at small cyclic shear strains, $\gamma_c$, the cyclic secant shear modulus $G_{SN}$ first increases and then decreases with the number of cycles, $N$, while the cyclic pore water pressure consistently and continuously increases with $N$. At larger cyclic strains, the secant shear modulus only decreases with $N$ while the large cyclic pore pressure buildup takes place. Such a complex cyclic behavior occurs in clean sands and silty sands under simple shear and triaxial loading conditions. It also occurs at very small cyclic shear strains below the threshold shear strain for pore water pressure buildup, $\gamma_{tp}$, and in naturally structured sand specimens as well as the specimens reconstituted from fully disturbed sand. It also occurs in the cyclic stress-controlled mode of cyclic loading. It should be also noted that this kind of behavior has been obtained independently in several geotechnical laboratories by different research teams.

5.3 Cyclic strain-controlled testing of Nevada sand

To investigate in more depth the above complexities of the cyclic behavior of sand, a series of cyclic strain-controlled constant-volume equivalent-undrained simple shear tests were conducted at UCLA in the NGI-DSS device. In each test the constant cyclic shear strain amplitude, $\gamma_c$, was applied in sinusoidal mode while the variation of the cyclic shear stress, $\tau_{cN}$, with $N$ and the change in pore water pressures, $\Delta u$, were recorded. The testing program is summarized in Table 5.1 and it includes 7 tests on Nevada sand. The results of three of these tests have already been
presented in Figures 5.1 through 5.5. Prior to cyclic shearing the sand specimens height was approximately 19 mm and diameter 67 mm.

Table 5.1. Program of cyclic simple shear strain-controlled tests on Nevada sand

<table>
<thead>
<tr>
<th>t</th>
<th>Void ratio, e</th>
<th>Vertical consolidation stress, $\sigma_{vc}'$ (Kpa)</th>
<th>Cyclic shear strain amplitude, $\gamma_c$ (%)</th>
<th>Frequency, $f$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.60</td>
<td>154.9</td>
<td>0.0045</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.66</td>
<td>153.4</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>0.63</td>
<td>145.1</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>0.66</td>
<td>155.4</td>
<td>0.12</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>0.65</td>
<td>154.1</td>
<td>0.15</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>0.64</td>
<td>158.1</td>
<td>0.24</td>
<td>0.01</td>
</tr>
<tr>
<td>7</td>
<td>0.66</td>
<td>157.1</td>
<td>0.47</td>
<td>0.01</td>
</tr>
</tbody>
</table>

It can be seen in Table 5.1 that the void ratio, $e$, and the vertical effective consolidation stress, $\sigma_{vc}'$, varied relatively little from test to test. Cyclic straining frequency was $f=0.01$ Hz in all tests. Such a relatively low frequency was applied in order to have a better control of the small-strain tests and also because the sand behavior is affected very little by the rate of loading and frequency. The cyclic shearing at lower frequency is particularly helpful for maintaining the height of the specimen constant and with it its volume, and in this way for facilitating a more precise recording of the variation of the equivalent pore water pressures.
As shown in Table 5.1, the range of $\gamma_c$ was between 0.0045% and 0.47%, i.e., it included very small and relatively large cyclic strains. While testing at small $\gamma_c$ was done to reconfirm the complex cyclic behavior at small cyclic strains, the testing at larger $\gamma_c$ was aimed at obtaining a gradual transition between the small and medium cyclic strain behavior. The processed test results are presented in Figure 5.11, while their details are available in Appendix 1.

The results in Figure 5.11 confirm that at small $\gamma_c$ below 0.15% the stiffness index $\delta_N$ first increases and then decreases with N while the pore water pressure continuously increases. At $\gamma_c = 0.15\%$ the index $\delta_N=1$ in the 2$^{nd}$ cycle and then it decreases with N while the cyclic pore water pressure increases. At larger $\gamma_c$ above 0.15% index $\delta_N$ consistently decreases with N while the significant pore water pressure buildup takes place. In this particular case of Nevada sand, it seems that $\gamma_c = 0.15\%$ divides two trends of the cyclic stiffness change. Below $\gamma_c = 0.15\%$, index $\delta_N$ first increases and then decreases with N, while above $\gamma_c=0.15\%$ index $\delta_N$ only decreases with N. To examine further the magnitude of this transitional $\gamma_c$ the results obtained on Wildlife sand and Nevada sand from Figures 5.10 and 5.11 are presented together in Figure 5.12. In Figure 5.12a are the results of tests that exhibit increase and then decrease of $\delta_N$ with N. In Figure 5.12b are the results of tests that exhibit no change of $\delta_N$ in the second cycle. In Figure 5.12c are the results of tests that exhibit only decrease of $\delta_N$ with N. Consequently, Figure 5.12 reveals that the transitional $\gamma_c$ is approximately between 0.1% and 0.15%, which means that it can still be slightly lower than 0.1% or slightly above 0.15%. The exact magnitude of the transitional $\gamma_c$ apparently depends on the soil type and particular testing conditions, such as void ratio, fabric and confining stress.
Another thing to observe in Figure 5.12 is that $\delta_N$ does not increase beyond 1.15, which means that in general one should not expect that at small cyclic strains $G_{SN}$ will increase for more than approximately 15%. Furthermore, the largest $\delta_N$ increment is always recorded in the 2nd cycle, apparently as a consequence of the changes of the soil structure in the last three quarters of the first cycle and the first quarter of the second cycle.

Figure 5.11. Variation of stiffness index, $\delta_N$, with the number of cycles, $N$, and its relationship with cyclic pore water pressure, $\Delta u_N^*$, obtained in simple shear cyclic strain-controlled tests on Nevada sand.
Figure 5.12. *Relationship between stiffness index, $\delta_N$, and cyclic pore water pressure, $\Delta u_N^*$, for Wildlife silty sand and Nevada clean sand.*

It should be also noticed that at small $\gamma_c$, when $\delta_N$ first increase and then decreases, the normalized cyclic pore water pressure may reach approximately 0.3 before $\delta_N$ starts to decrease.
below 1.0, i.e., before the sand starts to degrade with N with respect to its original stiffness in the first cycle. This means that the residual cyclic pore water pressure may reach 30% of its initial effective vertical stress before the soil stiffness starts to drop below the initial stiffness in the first cycle. This is quite a surprising finding, because the pore water pressure that is 30% of the initial effective vertical stress is a large pore water pressure indeed. Such behavior indicates that the cyclic pore water pressure cannot be used to measure the soil softening, especially the softening at smaller cyclic strains.

5.4 Possible mechanisms responsible for the observed stiffness changes

From the above data and behavioral patterns it can be concluded that two parallel processes are responsible for the changes of $G_{SN}$ and $\delta_N$ with N. One is the continuous increase in the cyclic pore water pressure which causes the reduction of the effective stresses and associated softening of the sand structure. This process, which is in agreement with the effective stress principle, is responsible for the decrease of $G_{SN}$ and $\delta_N$ with N. This process is, of course, present only at $\gamma_c$ that is larger than the cyclic threshold shear strain for pore water pressure buildup, $\gamma_{tp}$, which is around 0.01%. For example, it is not present in the case of tests included in Figure 5.5 when $\gamma_c$ is between 0.0037% and 0.0045%. It can be seen that in these cases $\delta_N$ monotonically increases with N over a number of cycles and then may start to slightly decrease. It should be noted that below $\gamma_{tp}=0.01\%$ the sand particles are not permanently displaced and the deformation of the sand structure is caused almost exclusively by the deformation of the particles themselves and the deformation due to alteration of their contacts. When cycling at $\gamma_c$ smaller than $\gamma_{tp}$ is stopped, the sand structure is practically unchanged and it is more or less the same as before the cyclic
loading. Similarly, in the stress controlled test presented in Figures 5.7 and 5.8 amplitude $\gamma_c$ varied around 0.02% which is very close to $\gamma_{tp} \approx 0.01\%$, so in this case too $\delta_N$ just monotonically increased with N and beyond $N \approx 10$ it just leveled off.

Such a behavior below $\gamma_{tp} \approx 0.01\%$ points to the existence of the second process that causes the increase of $G_{SN}$ and $\delta_N$ with N. This process of cyclic stiffening is believed to be caused by the changes at the contacts of the sand particles. During cyclic loading the material at particle surfaces in contact is deformed and probably damaged as the particles tend to move with respect to each other, i.e., tend to rub against each other without permanent particle displacements. The fact that $G_{SN}$ and $\delta_N$ increase with N means that each cycle causes the change at the particle contacts such that either the contacts become stronger or more contacts are generated.

More than 40 years ago a cyclic behavior phenomenon somewhat similar to the present was studied and explained along similar lines, and it is therefore of interest to look into the conclusions of that study. Drnevich and Richart (1970) studied the effect of the cyclic prestraining on the increase of $G_{SN}$ and damping in dry sand that was allowed to change in volume (densify) during cyclic loading. In their resonant column-torsional shear cyclic tests on hollow cylinder specimens they observed very large increase of $G_{SN}$ and damping due to the cyclic prestraining, so large that it cannot be attributed to the observed very small decrease in void ratio but to some other cause. They first measured $G_{SN}$ and damping of sand specimen at a very small $\gamma_c$. Then, they applied a large number of cycles of larger $\gamma_c = 0.06\%$. Following such prestraining, they then again measured $G_{SN}$ and damping at the same previous very small $\gamma_c$. They provided the following explanation for the significant increase of $G_{SN}$ and damping due to the
cyclic prestraining: “What then is the cause of the observed changes? It is postulated that the wear process on particle contacts is responsible. The large amplitude shearing strains occurring within the specimen were large enough that some relative particle motion existed. However, the motions were not large enough to cause gross particle reorientation (densification or dilation) to occur. Hence, the points on the surface of a particle that were in contact with neighboring particles essentially remained in contact throughout the prestraining. The prestraining applied abrasive action and caused the nature of these points of contact to wear. Original contacts were composed of the minute asperities of each particle touching. The actual contact areas were quite small. Relative particle motion probably flattened these asperities, increased contact areas, and allowed additional contacts to form. Because these changes could occur without significantly changing the pore volume, the void ratio and density did not change perceptibly. The wearing process here is analogous to the "wearing in" of a shaft on a bearing or piston rings in a cylinder of an engine or a compressor. An increase of the contact areas and the number of contacts would account for the observed modulus and damping increases.”

The variable-volume cyclic testing by Drnevich and Richart (1970) and the constant-volume equivalent-undrained cyclic testing described in the present study have important similarities and important differences. They are similar in that in both testing at the sand particle contacts there is abrasive action which changes the surface. These changes are believed to be quite similar in the first, or perhaps the first two cycles, but not after that as the cyclic loading continues. During the continuous cyclic loading in the variable-volume cyclic testing it is easy to envision how particles are getting closer to each other as their contacts are getting thinner and wider due to the abrasive action and are thus becoming increasingly stronger. At the same time, at stronger
contacts with more frictional resistance along the larger areas of abrasion more energy is needed for the same cyclic deformation (cyclic straining), which explains that the damping increases with the number of cycles.

Figure 5.13. *Cyclic loops and the variation of the area of the loop, $\Delta W$, with the number of cycles, $N$, in cyclic strain-controlled test on Nevada Sand presented in Figures 5.3 and 5.4 ($\Delta W$ represents specific energy spent in a cycle)*

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During the continuous cyclic loading in the constant-volume cyclic testing, which is equivalent to the undrained cyclic testing of fully saturated soil, adjacent particles cannot get closer to each other following the abrasive action at their contacts. Consequently, the abrasive action produces the largest change in the first few cycles, with by far the largest change and associated increase in $G_{SN}$ and $\delta_N$ in the very first cycle. After that the changes with $N$ diminish as the surface gets “polished”. This is in agreement with the changes of $\delta_N$ with $N$ presented throughout this investigation (e.g., see Figures 5.4, 5.5 and 5.8).

Such a mechanism of the increasingly smaller changes at particle contacts and diminishing of the rise in frictional resistance at the contacts should also result in the largest energy loss in the first cycle, somewhat smaller in the second cycle, and then smaller and smaller as the cycling continues.

Considering that the area of a cyclic loop, $\Delta W$, is specific energy (energy per volume) spent to generate the loop, such mechanism should yield smaller and smaller $\Delta W$ with $N$. In Figure 5.13 are the cyclic loops and the variation of $\Delta W$ with $N$ obtained in the test on Nevada Sand with $\gamma_c=0.08\%$ which is presented in Figures 5.3 and 5.4. It can be seen that in this example $\Delta W$ is the largest in the first cycle, it decreased the most from the first to second cycle, and after that $\Delta W$ gradually decreased with $N$ in smaller and smaller decrements. In conclusion, the change of the specific energy $\Delta W$ in this test is in agreement with the proposed mechanism. The same trend of decreasing $\Delta W$ with $N$ has been more or less observed in the results of a couple of other tests with very small amplitude $\gamma_c$ below $\gamma_{tp}$. These tests are conducted in other investigations in DSDSS device and their results are not reported here because the cyclic straining was not
perfectly uniform. They basically just indicate that most likely the same trend would have been clearly obtained had the cyclic straining been perfectly uniform.

To get a better insight into the changes at particle contacts that are responsible for observed stiffness changes and fully confirm the above hypothesis more tests at small $\gamma_c$ need to be conducted. Very precise tests at $\gamma_c$ smaller than $\gamma_{tp}$ can be particularly reveling. Such a fine and complex testing can easily constitute a separate research investigation.

The above two processes, the cyclic pore water pressure buildup that causes the cyclic degradation and the alteration of the particle contacts that causes the cyclic stiffening, occur during the cyclic straining simultaneously. The conceptual model showing how these two processes yield the observed variation of stiffness index, $\delta_N$, with $N$ at relatively small cyclic shear strain amplitude, $\gamma_c$, is presented in Figure 5.14. At the beginning of cyclic straining with small $\gamma_c$, when the cyclic pore water pressures are small, the cyclic stiffening prevails and $G_{SN}$ increases with $N$. After a certain number of cycles, when the contact areas are more or less completely worn off, the cyclic pore water pressure which still keeps increasing begins to dominate the change of $G_{SN}$, and $G_{SN}$ consequently starts to decrease. This behavior is illustrated in Figure 5.12a. During the cyclic straining with large $\gamma_c$ the cyclic pore water pressures can increase and the effective stress decrease in the first cycle so much to completely override the effect of the stiffening. In such a case, which is presented in Figure 5.12c, sand stiffness just degrades with $N$. In the intermediate case presented in Figure 5.12b, $G_{SN}$ and $\delta_N$ are the same in the first two cycles and afterwards they just decrease.
Figure 5.14: Conceptual model of the variation of stiffness index, $\delta_N$, in fully saturated sands during undrained cyclic strain-controlled loading at relatively small cyclic shear strain amplitude, $\gamma_c$. 
5.6 What is the threshold shear strain for cyclic degradation of sand?

The cyclic threshold shear strain for cyclic degradation is the cyclic shear strain amplitude, $\gamma_c$, below which there is no cyclic degradation with N and above which the degradation with N takes place. In clays, this threshold strain is denoted by $\gamma_{td}$ and it is easily defined (Tabata and Vucetic, 2010; Vucetic 1994). It is $\gamma_c$ below which the degradation parameter, $t$, defined by Eqs. 2.2 and 5.2 is zero, and above which $t$ is greater than zero. This definition assumes that for a given clay $t$ parameter is pretty much constant for $\gamma_c$ values around $\gamma_{td}$, i.e., that $\delta_N$ versus N relationship in a log-log format is a straight line. This is true for most clays while for sands it is not, because in sands at small $\gamma_c$ index $\delta_N$ increases and then decreases with N. So the definition based on the degradation parameter, $t$, is not suitable.

However, the threshold shear strain for cyclic degradation can be defined as the cyclic shear strain amplitude, $\gamma_c$, below which the cyclic softening with respect to the original soil modulus, $G_{S1}$, does not take place at all, and above which it eventually takes place irrespective if some stiffening occurs prior to that. According to such a definition, $\gamma_{td}$ is actually $\gamma_{tp}$, which is around 0.01%.

Another definition can be that the threshold shear strain for cyclic degradation is the cyclic shear strain amplitude, $\gamma_c$, below which the cyclic stiffening does not take place at all and above which only the degradation with respect to the original stiffness takes place. According to such a definition, the threshold shear strain for cyclic degradation is the transitional $\gamma_c$ defined in Figure 5.12 b. Below this transitional $\gamma_c$ sand first stiffens and then softens with N, and above it only
softens. In other words, below this transitional $\gamma_c$ index $\delta_N$ can be larger than 1 and parameter $t$ negative, while above it $\delta_N$ is always smaller than 1 and $t$ positive, irrespective of the number of cycles, $N$. According to this definition and Figure 5.12b, $\gamma_{td}$ for sand is approximately between 0.1% and 0.15%. Of course, this definition is not perfect, because at $\gamma_c$ smaller than such $\gamma_{td}$ sand may still soften after many cycles of loading. According to Figures 5.10 to 5.12, for the sands treated in this investigation this softening can be after 20 cycles of loading ($N=20$) as large as 20 to 30 %, i.e., $\delta_N$ can be 0.8 to 0.7. In this respect, the first definition of $\gamma_{td} \approx \gamma_{tp}$ is better. It is not only defined more rigorously, but it is also more attractive for soil dynamics practice. It is attractive for the practice because it defines whether any cyclic degradation can or cannot take place in a soil dynamics event.
6. Threshold shear strain for cyclic degradation, $\gamma_{td}$, and cyclic pore water pressure, $\gamma_{tp}$, in clay and sand

6.1. Introduction

During the cyclic loading of soils with moderate and large cyclic shear strains (moderate and large cyclic shear strain amplitudes, $\gamma_c$) under different conditions, four basic phenomena of soil change may occur. In fully saturated soils cyclically sheared in undrained conditions, (1) soil may degrade in stiffness and strength and (2) the pore water pressure may change. In dry and partially saturated soils cyclically sheared in drained conditions, (3) soil may densify (experience cyclic compression) and (4) it may increase in stiffness. During the cyclic loading at very small cyclic shear strains (very small $\gamma_c$) none of these phenomena practically occurs. Consequently, in each of these four phenomena of soil change due to the cyclic loading there is the cyclic threshold shear strain above which the phenomenon occurs and below which it does not. All corresponding four thresholds are fundamental cyclic soil properties and have therefore been investigated in the past to greater or lesser extent. As already stated, the threshold shear strains for cyclic degradation in sands and clays and for the pore water pressure change in clays sheared cyclically in undrained conditions have not been investigated to a satisfactory extent. This is the reason why they are investigated here.

6.2. Threshold shear strain for cyclic degradation, $\gamma_{td}$, in clays

Figure 6.1 shows the variation of strain, stress and pore water pressure with time in a multi-stage cyclic strain-controlled test on kaolinite clay. The test includes nine stages of cyclic shearing
with 10 cycles in each stage. The cyclic shear strain amplitude, $\gamma_c$, is maintained constant throughout each stage, but is larger in each subsequent stage. The stress-strain loops of the test are shown in Figure 6.2.

![Graph showing variation of strain, stress, and pore water pressure with time in a multi-stage cyclic strain-controlled test on kaolinite clay.](image)

**Figure 6.1**: Variation of strain, stress and pore water pressure with time in a multi-stage cyclic strain-controlled test on kaolinite clay.

In Figures 6.1 and 6.2 one can see that in the last few stages the magnitude of the cyclic shear stress amplitude, $\tau_c$, decreases and the magnitude of the cyclic pore water pressure, $\Delta u_N$,
increases with N. This means that the cyclic shear strain amplitudes, $\gamma_c$, in these stages are clearly above the threshold shear strains for pore water pressure, $\gamma_{tp}$, and degradation, $\gamma_{td}$. On the other hand, it can be seen that in the first few stages of this test there are neither signs of the cyclic degradation nor the signs of the cyclic pore water pressure change. This indicates that the two threshold strains are somewhere in the middle between these stages. The procedure for a precise determination of the magnitude of the threshold strain for cyclic degradation, $\gamma_{td}$, is described below. Also, the magnitudes of $\gamma_{td}$ for two clays tested under different vertical consolidation stresses and frequencies are provided and discussed.

Figure 6.2: Stress-strain loops of a multi-stage cyclic strain-controlled test on kaolinite clay.
(a) Cyclic degradation in all 9 stages of a multi-stage cyclic strain-controlled test.

(a) Cyclic degradation in first 3 stages of a multi-stage cyclic strain-controlled test.

Figure 6.3: Cyclic degradation in 9 stages of a multi-stage cyclic strain-controlled test on kaolinite clay.
Figure 6.3a shows the variation of the degradation index, $\delta_N$, defined and discussed in detail in chapter 4, versus the number of cycles, $N$, from all 9 stages of the test shown above in Figures 6.1 and 6.2. The first three curves for the first three stages with very small cyclic strains are plotted in Figure 6.3b. It can be seen that these three curves are jagged and fluctuate widely around $\delta_N=1$. The fluctuation is a consequence of the difficulties of recording strains and stresses at such small cyclic strain magnitudes. However, since these three degradation curves are fluctuating around $\delta_N=1$ their cyclic strains, $\gamma_c$, are smaller than the threshold for cyclic degradation, $\gamma_{td}$. Consequently, these three curves are eliminated from further analysis and the rest of the curves, six of them, are plotted separately in Figure 6.4.

Figure 6.4: Cyclic degradation in 6 stages of a multi-stage cyclic strain-controlled test on kaolinite clay.
The slopes of the $\delta_N$-log$N$ lines in Figure 6.4, defined as the degradation parameter, $t$, are calculated for all 10 cycles of every stage and plotted versus the shear strain amplitude, $\gamma_c$, in Figure 6.5. Above each $\gamma_c$ tested there are 10 data points of parameter $t$ corresponding to 10 cycles of the corresponding cyclic strain-controlled stage. From this plot in linear scales the exact value of $\gamma_{td}$ cannot be determined precisely enough by the extrapolation of the trend to $t=0$. Consequently, the same data are shown in a semi-log format in Figure 6.6. If the data trend is now extrapolated to $t=0$ level, the extrapolation will yield certain range of threshold strain $\gamma_{td}$, which is apparently somewhere between 0.010% and 0.017%. However, because of the data scatter at small $\gamma_c$ the final extrapolation is done in Figure 6.7 where the picture of the whole trend is simplified. In Figure 6.7 four cyclic stages just above the anticipated $\gamma_{td}$ are represented by single data points which correspond to the average degradation parameters for all 10 cycles of every stage. The curve interpolated through these data points and then extrapolated to $t=0$ can yield at $t=0$ more meaningful estimation of the threshold strain for cyclic degradation, $\gamma_{td}$. The data points corresponding to the first three stages with $\gamma_c$ below anticipated $\gamma_{td}$ are also included as the $t=0$ points. In this particular case such a procedure yields the cyclic threshold shear strain for cyclic degradation, $\gamma_{td}=0.012\%$. This procedure is used to evaluate $\gamma_{td}$ from all multi-stage cyclic strain-controlled tests on clays. Nine such multi-stage cyclic tests are conducted on clays, therefore yielding 9 values of $\gamma_{td}$. 
Figure 6.5: Variation of the degradation parameter, $t$, with the cyclic shear strain amplitude, $\gamma_c$, in 6 stages of the multi-stage cyclic test on kaolinite clay.

Figure 6.6: Variation of the degradation parameter, $t$, with the cyclic shear strain amplitude, $\gamma_c$, in 6 stages of the multi-stage cyclic test on kaolinite clay presented in the semi-log format.
6.2.1. Threshold shear strain for cyclic degradation, $\gamma_{td}$, of kaolinite clay (MH; PI=28)

The trends of the average degradation parameters, $t$, with the cyclic shear strain amplitudes, $\gamma_c$, for five different multi-stage cyclic tests on kaolinite clay constructed according to the above procedure are presented in Figure 6.8. In these tests on the same clay having PI=28 different vertical effective consolidation stresses, $\sigma'_{vc}$, different OCR-s, and different frequencies of cyclic loading, $f$, were applied. Their values are specified on top of each plot. Vertical effective consolidation stress, $\sigma'_{vc}$, varied between 210 kPa and 680 kPa, OCR between 1 and 4, and the frequency of cyclic loading between 0.1 Hz and 0.01 Hz.
Figure 6.8: Variation of the average degradation parameter, $t$, with the cyclic shear strain amplitude, $\gamma_c$, in 5 multi-stage cyclic tests on kaolinite clay.
To compare the values of $\gamma_{td}$ obtained under different conditions, the five trends of $t$ with $\gamma_c$ from Figure 6.8 are plotted together in Figure 6.9. This plot reveals that for this kaolinite clay and the different testing conditions applied the threshold shear strain for cyclic degradation, $\gamma_{td}$, is between 0.011% to 0.014%. This is a relatively narrow range, and moreover, no particular trend with $\sigma_{vc}'$, OCR and $f$ can be noticed. In conclusion, the threshold shear strain for cyclic
degradation, $\gamma_{td}$, is not affected noticeably by either the vertical effective consolidation stress, $\sigma'_{vc}$, OCR or frequency of cyclic loading, $f$. This conclusion, of course, relates only to the ranges of $\sigma'_{vc}$, OCR and $f$ applied. For much larger ranges and extremely precise testing some effects will most likely emerge.

6.2.2. Threshold shear strain for cyclic degradation, $\gamma_{td}$, of kaolinite-bentonite clay (CH; PI=55)

The trends of the average degradation parameters, $t$, with the cyclic shear strain amplitudes, $\gamma_c$, for four multi-stage cyclic tests on kaolinite-bentonite clay are presented in Figure 6.10. In these tests on the same clay having PI=55, just like in the tests on the kaolinite clay above, different vertical effective consolidation stresses, $\sigma'_{vc}$, different OCRs, and different frequencies of cyclic loading, $f$, were applied. Their values are again specified on top of each plot. Vertical effective consolidation stress, $\sigma'_{vc}$, varied between 113 kPa and 668 kPa, OCR between 1 and 7.8, and the frequency of cyclic loading between 0.1 Hz and 0.01 Hz.

In Figure 6.11 the trend lines from Figure 6.10 are plotted together. It can be seen that for different testing conditions the threshold shear strain for cyclic degradation, $\gamma_{td}$, ranges between 0.010\% to 0.016\%. Similar to kaolinite clay, this range of $\gamma_{td}$ is relatively narrow. Accordingly, the threshold shear strain for cyclic degradation, $\gamma_{td}$, of kaolinite-bentonite clay does not seem to be much affected by the variation of $\sigma'_{vc}$, OCR and the frequency of cyclic loading, $f$. 109
Figure 6.10: Variation of the average degradation parameter, \( t \), with the cyclic shear strain amplitude, \( \gamma_c \), in 4 multi-stage cyclic tests on kaolinite-bentonite clay.

The most surprising aspect of the results on kaolinite clay and the kaolinite-bentonite clay is that the ranges of \( \gamma_{td} \) for these two different clays having significantly different values of plasticity index, PI, are practically the same. According to previous investigations (Tabata and Vucetic, 2004; 2010) larger \( \gamma_{td} \) should be expected for markedly larger PI.
Figure 6.11 *Comparison of the variation of the average degradation parameter, t, with the cyclic shear strain amplitude, $\gamma_c$, in 4 multi-stage cyclic tests on kaolinite-bentonite clay.*
6.3 Threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$, in clays

6.3.1 Threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$, in kaolinite clay (MH; PI=28)

In the present study the approach to find the threshold shear strain for pore water pressure change, $\gamma_{tp}$, by Hsu and Vucetic (2002) is used. The approach is based on the results of multi-stage cyclic strain-controlled tests in which in each subsequent stage a larger amplitude $\gamma_c$ is applied. It consists of the measuring of the magnitude of cyclic pore water pressure, $\Delta u_N$, at the end of each cycle in all cyclic stages and then plotting the variation of $\Delta u_N$ versus the cyclic shear strain amplitudes, $\gamma_c$, in a semi-log scale. In this procedure each cyclic stage is treated as a separate cyclic strain-controlled test. In this context, the conditions of the specimen at the end of previous stage, the end of previous “cyclic test”, correspond to the conditions at the beginning of the next stage, i.e., the next “cyclic test”. The small difference in the effective vertical stress at the beginning of each subsequent stage caused by the small change of $\Delta u_N$ by the end of the previous stage is determined and taken into account.

The processed results of a multi-stage cyclic strain-controlled test with seven cyclic stages is plotted in Figure 3.12. Cyclic shear strain amplitudes, $\gamma_c$, applied in different stages are plotted on horizontal axis, while the equivalent cyclic pore water pressure normalized to the effective vertical stress determined at the beginning of each $i$ stage, $\Delta u_{Ni}^*$, are plotted on the vertical axis. Each vertical set of data points in this chart corresponds to a single cyclic stage, i.e., to a
separate ”cyclic test”. As indicated on top of the plot this multi-stage cyclic strain-controlled test was conducted on normally consolidated kaolinite clay.

![Image of Figure 6.12](image)

**Figure 6.12:** Change of the cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on normally consolidated kaolinite clay.

It can be seen that in the first two stages conducted at $\gamma_c=0.007\%$ and $0.021\%$ there was practically no cyclic pore water pressure change. In the third stage with $\gamma_c=0.035\%$ a rather small but clearly noticeable increase in cyclic pore water pressure, $\Delta u_{N3}$, was recorded. In the fourth stage conducted at $\gamma_c=0.055\%$, the cyclic pore water pressure, $\Delta u_{N4}$, was larger, as expected. In
the subsequent stages with ever larger $\gamma_c$ it was, of course, larger and larger. From this chart it is evident that the threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$, is somewhere between $\gamma_c=0.02\%$ in the second cyclic stage and $\gamma_c=0.034\%$ in the third cyclic stage. In other words, $\gamma_{tp}$ is slightly larger than 0.02% and smaller than 0.034%. Eventually, it is estimated that it is between 0.02% and 0.025%.

Figure 6.13: Change of the cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on normally consolidated kaolinite clay.
Figures 6.13 shows similar plot for another multi-stage cyclic test on the same normally consolidated kaolinite clay, but with larger loading frequency. From this test \( \gamma_{tp} \) is estimated to be between 0.02% and 0.03%.

Figure 6.14 shows yet another plot for the third multi-stage cyclic tests on normally consolidated kaolinite clay with different vertical effective consolidation stress, \( \sigma'_{vc} \). From this test \( \gamma_{tp} \) is estimated to be between 0.01% and 0.013%.

Figure 6.14: *Change of the cyclic pore water pressure, \( \Delta u_N \), with the cyclic strain amplitude, \( \gamma_c \), and the number of cycles, \( N \), in multi-stage cyclic strain-controlled test on normally consolidated kaolinite clay.*
To assess the threshold shear strain for cyclic pore water pressure, $\gamma_{tp}$, in the over-consolidated kaolinite clay, two multi-stage cyclic strain-controlled tests have been conducted. The results of these two tests are shown in Figures 6.15 and 6.16. As revealed in past investigations on undrained cyclic behavior of clay (Andersen et al., 1980; Dobry and Vucetic, 1986; Vucetic 1988, and Ohara and Matsuda, 1988), in OC clays the cyclic pore water pressure may decrease with N or decrease and then increase, depending on the levels of OCR and $\gamma_c$. At large OCR the negative cyclic pore water pressure usually develops. At small OCR the negative cyclic pore water pressure may actually not develop, but instead very small positive pressures. Furthermore, the negative cyclic pore water pressures recorded in the past studies were often very small. All of this makes the estimation of $\gamma_{tp}$ in overconsolidated clay rather difficult. In the present tests on the overconsolidated kaolinite and kaolinite-bentonite clays this was pretty much the case too, because the negative cyclic pore water pressures at small levels of $\gamma_c$ were often very small indeed. Nonetheless, for the kaolinite clay, in Figure 6.15 $\gamma_{tp}$ is estimated to be between 0.011% and 0.017%, while in Figure 6.16 it is estimated to range between 0.013% and 0.016%.

To summarize the results of all multi-stage tests aimed at finding $\gamma_{tp}$ in the kaolinite clay, the cyclic pore water pressures lines corresponding to N=10 from the above charts (Figures 6.12 to 6.16) are plotted together in Figure 6.17. This chart with cumulative results reveals rather clearly that the threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$, for kaolinite clay (MH; PI=28) is between 0.01% and 0.025%. This is a pretty narrow range given the different conditions of the specimens tested. The chart therefore reconfirms that the threshold shear strain
for cyclic pore water pressure change, $\gamma_{tp}$ of kaolinite clay is not affected noticeably by the vertical effective consolidation stress, $\sigma'_vc$, OCR and the frequency of cyclic loading, $f$.

Figure 6.15: Change of the cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on overconsolidated kaolinite clay.
Figure 6.16: Change of the cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on overconsolidated kaolinite clay.
Figure 6.17: Change of the cyclic pore water pressure, $\Delta u_N$, in 10 cycles ($N=10$) in five different multi-stage cyclic strain-controlled tests on normally consolidated and overconsolidated kaolinite clay.

6.3.2. Threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$, in kaolinite-bentonite clay (CH; PI=55)

The results of two multi-stage cyclic strain-controlled tests on the normally consolidated kaolinite-bentonite clay (CH; PI=55) are presented Figures 6.18 and 6.19. The vertical effective
consolidation stress, $\sigma'_{vc}$, and the frequency of cyclic loading, were different in these two tests. The threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$, is estimated to be between 0.035% and 0.040% from Figure 6.18, and between 0.075% and 0.080% from Figure 6.19.

Figure 6.18: Change of the cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on normally consolidated kaolinite-bentonite clay.
Figure 6.19: Change of the cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on normally consolidated kaolinite-bentonite clay.

In Figure 6.20 the pore water pressures lines corresponding to $N=10$ from the above two tests (Figures 6.18 and to 6.19) are plotted together. This comparison of $\gamma_{tp}$ from two tests on normally consolidated kaolinite-bentonite clay reveals that the cyclic threshold shear strain for pore water pressure change, $\gamma_{tp}$, ranges roughly between 0.03% and 0.08%, which is a significant range.
Therefore, no conclusions can be derived from the comparison between these two tests in regards to the effects of $\sigma'_{vc}$ and the frequency of cyclic loading, $f$.

![Figure 6.20](Image)

Figure 6.20: Change of the cyclic pore water pressure, $\Delta u_n$, in 10 cycles ($N=10$) in two different multi-stage cyclic strain-controlled tests on normally consolidated kaolinite-bentonite clay.

To estimate the cyclic threshold shear strain for pore water pressure change, $\gamma_{tp}$, in the over-consolidated kaolinite-bentonite clay, two strain-controlled multi-stage tests have been conducted and the results are shown in Figures 6.21 and 6.22. In these two tests, OCR was 4 and 7.8. It can be seen in the figures that the cyclic pore water pressures are quite small and rather inconsistent in most cyclic stages, all the way up to stages with $\gamma_c$ of approximately 0.3%. The
estimation of $\gamma_p$ from these two charts with a reasonable degree of confidence is therefore practically impossible. In Figure 6.21 it may be concluded that $\gamma_p$ is somewhere below 1.00%, while from Figure 6.22 perhaps it can be concluded that it is below 0.3%. It is evident that more research with more precise testing needs to be done to evaluate what $\gamma_p$ is in the highly plastic overconsolidated kaolinite–bentonite clay, or similar clays.

Figure 6.21: Change of cyclic pore water pressure, $\Delta u_n$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on overconsolidated kaolinite-bentonite clay.
Figure 6.22: Change of cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on overconsolidated kaolinite-bentonite clay.

6.4. Threshold shear strain for cyclic pore water pressure, $\gamma_{tp}$, in sands

The threshold shear strain for cyclic pore water pressure in sands is evaluated here using the same approach based on the multi-stage cyclic strain-controlled test. Three such tests are conducted on normally consolidated Nevada sand at the vertical effective consolidation stresses,
σ′_vc, of 154, 220 and 216 kPa, and at the frequencies of cyclic loading, f, of 0.01 and 0.1 Hz. The results are presented Figures 6.23, 6.24 and 6.25. The threshold shear strains for the cyclic pore water pressure, γ_tp, are estimated as follows. From Figures 6.23, γTp ranges from 0.007% to 0.008%, from Figure 6.24 from 0.008% to 0.0095%, and from Figure 6.25 from 0.01% to 0.013%. These three ranges are pretty close to each other, pointing to the fact that γTp is not affected very much by σ′_vc and frequency, f. This is not surprising because it has been established in the past by several extensive studies (e.g., Dobry et al., 1982; Dyvik et al., 1984). These ranges are also in excellent agreement with many past studies, which is discussed below in connection with the next Figure 6.26.

Figure 6.23: Change of cyclic pore water pressure, Δu_N, with the cyclic strain amplitude, γ_c, and the number of cycles, N, in multi-stage cyclic strain-controlled test on Nevada sand.
Figure 6.24: Change of cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on Nevada sand.

Figure 6.25: Change of cyclic pore water pressure, $\Delta u_N$, with the cyclic strain amplitude, $\gamma_c$, and the number of cycles, $N$, in multi-stage cyclic strain-controlled test on Nevada sand.
Figure 6.26: \textit{Change of the cyclic pore water pressure, }\Delta u_N^*, \textit{in 10 cycles } (N=10) \textit{in three different multi-stage cyclic strain-controlled tests on Nevada sand and comparison with the data band of equivalent cyclic triaxial tests presented by Dobry (1985)}

To summarized and compare the results of the above three multi-stage cyclic tests with the results obtained in the past by others, the pore water pressures lines corresponding to N=10 are plotted together in Figure 6.26. On the same figure is plotted the band of the data points of the pore water pressure buildup in 10 cycles obtained in the cyclic triaxial tests on many different sands tested under various conditions. This trend is provided by Dobry (1985). It can be seen
that the cyclic simple shear tests data obtained in the present study are in an excellent agreement with these data from the cyclic triaxial tests on many different sands tested under different conditions. This not only reconfirms that the cyclic threshold shear strain for pore water pressure change, $\gamma_{tp}$, in sands is between 0.007\% and 0.013\%, and that the cyclic simple shear test results, if properly interpreted, are comparable to the cyclic triaxial test results (see also Vucetic and Dobry, 1988), but also that the present testing is of high quality.

6.5. Threshold shear strain for cyclic degradation, $\gamma_{td}$, in sands

It is evident that due to the complex behavior of sands at small cyclic strains involving the phenomenon of cyclic stiffening, which is treated in detail in Chapter 5, even the definition of the threshold shear strain for cyclic degradation, $\gamma_{td}$, in sands is difficult to come up with. As explained in Chapter 5, from a practical point of view, a plausible definition of the cyclic shear strain amplitude, $\gamma_c$, below which the cyclic degradation with N will not occur at all is the amplitude of the cyclic threshold shear strain for cyclic pore water pressure, $\gamma_{tp}$.

6.6. Summary and comparison to previously published data

The results of the threshold shear strain for cyclic degradation, $\gamma_{td}$, and the threshold shear strain for cyclic pore water pressure change, $\gamma_{tp}$, of clays and sands tested in this investigation under different conditions are tabulated for comparison in Table 6.1. They are also compared below to the trends of the cyclic threshold shear strains with PI published earlier.
Table 6.1: Summary of the cyclic threshold shear strain results.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Nevada sand</th>
<th>Kaolinite</th>
<th>Kaolinite-Bentonite</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>USCS classification</strong></td>
<td>SP</td>
<td>MH</td>
<td>CH</td>
</tr>
<tr>
<td><strong>Plasticity</strong></td>
<td>NP</td>
<td>PI=28</td>
<td>PI=55</td>
</tr>
<tr>
<td><strong>OCR</strong></td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td><strong>(\gamma_{tp}) (%)</strong></td>
<td>0.0070-0.0080</td>
<td>0.010-0.013</td>
<td>0.013-0.016</td>
</tr>
<tr>
<td></td>
<td>0.0080-0.0095</td>
<td>0.020-0.026</td>
<td>0.011-0.017</td>
</tr>
<tr>
<td></td>
<td>0.010-0.013</td>
<td>0.020-0.025</td>
<td>-</td>
</tr>
<tr>
<td><strong>(\gamma_{td}) (%)</strong></td>
<td>?</td>
<td>0.011</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>?</td>
<td>0.012</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>?</td>
<td>0.012</td>
<td>-</td>
</tr>
</tbody>
</table>

Previously published data of \(\gamma_{tp}\) and the threshold shear strain for cyclic compression, \(\gamma_{tv}\), are presented in Figure 6.27. Threshold \(\gamma_{tv}\), which is not treated in the present study, is the cyclic shear strain amplitude dividing the domains of the volume-change and no-volume-change during the cyclic loading of dry and partially saturated soils in drained conditions. The thresholds \(\gamma_{tp}\) obtained in this study are compared in Figure 6.28 with the trend of these previously published data. It can be seen that the new \(\gamma_{tp}\) values generally increase with PI just like the data in the previous studies, but the present values are at or below the lower bound published previously. It is believed that the more precise testing in this narrowly focused investigation enabled detection of \(\gamma_{tp}\) at smaller \(\gamma_c\) than before.
Figure 6.27: Trend of threshold shear strain for cyclic pore water pressure, \( \gamma_{tp} \), and threshold shear strain for cyclic compression, \( \gamma_{tv} \), suggested by Hsu and Vucetic (2007)

Figure 6.28: The thresholds \( \gamma_{tp} \) obtained in this study compared to the trend of \( \gamma_{tp} \) and threshold \( \gamma_{tv} \), suggested by Hsu and Vucetic (2007)
Previously published data of $\gamma_{td}$ and the threshold shear strain for cyclic stiffening, $\gamma_{ts}$, are presented in Figure 6.29. Threshold $\gamma_{ts}$, which is not treated in the present study, is the cyclic shear strain amplitude dividing the domains of soil stiffening (increase of small–strain shear modulus $G_{\text{max}}$) due to cyclic pre-shearing and no such stiffening. The thresholds $\gamma_{td}$ obtained in this study are compared in Figure 6.30 with the trend of these previously published data. It can be seen that the new $\gamma_{td}$ values are practically the same for PI=28 and PI=55, and that they are lower than those previously obtained on three soils. In these respects, the newly obtained results are not in a good agreement with the data obtained earlier. More research on $\gamma_{td}$ is needed to explain this discrepancy and to evaluate further if and how $\gamma_{td}$ is changing with PI.

Figure 6.29: Trend of threshold shear strain for cyclic degradation, $\gamma_{td}$, and threshold shear strain for cyclic stiffening, $\gamma_{ts}$, suggested by Tabata and Vucetic (2010)
Figure 6.30: The thresholds $\gamma_{td}$ obtained in this study compared to the trends of $\gamma_{td}$ and, $\gamma_{ts}$, suggested by Tabata and Vucetic (2010)
7. Conclusions

The focus of the investigation are the threshold shear strain for cyclic degradation, $\gamma_{td}$, in sand and threshold shear strain for cyclic pore water pressure, $\gamma_{tp}$, in normally consolidated (NC) and overconsolidated (OC) clays, including their existence, magnitude and mutual comparison. These cyclic threshold shear strains have not been investigated in the past to a satisfactory extent. Three sands and two laboratory-made clays are tested. They include two fine clean sands and one coarse clean sand all classified as SP, and the laboratory prepared pure kaolinite clay and a clay prepared in laboratory from a mix of kaolinite and bentonite powder which is called the kaolinite-bentonite clay. The kaolinite clay is classified as MH and has PI=28, while kaolinite-bentonite clay is classified as CH and has PI=55. These soils are tested in the Norwegian Geotechnical Institute (NGI) type of direct simple shear device (DSS) modified and automated at UCLA for small-strain cyclic testing. Two types of cyclic tests are conducted, single-stage cyclic strain-controlled test with constant cyclic shear strain amplitude, $\gamma_{c}$, throughout the test, and the multi-stage cyclic strain-controlled test in which $\gamma_{c}$ was constant in each stage but larger in every subsequent stage. The amplitudes of $\gamma_{c}$ ranged from 0.003% to 2.0%.

In the context of investigating $\gamma_{td}$ in sand and $\gamma_{tp}$ in clays, the following tasks and investigations are performed before investigating directly the cyclic thresholds: (1) the NGI-DSS device is adapted for small-strain cyclic testing and a procedure for eliminating false loads and deformations from test records is developed, (2) the effect of the vertical consolidation stress, $\sigma_{vc}$, and the frequency of cyclic straining, $f$, on the cyclic degradation and pore water pressure change in clays is tested, and (3) the cyclic stress-strain behavior and the change of secant shear
modulus, $G_{SN}$, with $N$ at very small cyclic strains is investigated. Altogether, around 50 tests on clays and 20 tests on sands are conducted.

From the test results, their analyses and their discussion the following conclusions are derived.

(1) Small-strain cyclic testing with $\gamma_c$ as small as 0.003% can be conducted in the standard NGI-DSS device and obtain reliable results if the device is properly modified and equipped and if the false loads are eliminated from the test records before the data are analyzed. The false loads can be identified, quantified and removed from the test records with the help of modern computer graphics software and modern computer software for data analysis and management.

(2) Cyclic degradation in clays is affected moderately to significantly by $\sigma_{vc}'$ and frequency, $f$. More specifically, for the kaolinite clay having PI=28 and for the range of $\gamma_c$ between 0.1% and 0.5%, degradation parameter, $t$, that measures the rate of decrease of $G_{SN}$ with $N$ may increase 20% to 50% if $f$ is ten times higher. This is a substantial increase and should be taken into consideration in cyclic degradation analyses. If $\sigma_{vc}'$ is increased more than 3 times, from approximately 220 to 680 kPa, $t$ decreases between 20% and 38%.

(3) When fully saturated sand is subjected in undrained conditions to very small $\gamma_c$, smaller than threshold $\gamma_{tp}$, the modulus $G_{SN}$ increases with $N$, i.e., sand is stiffening while the cyclic pore water pressure is not developing. At small to moderate $\gamma_c$ somewhat larger than $\gamma_{tp}$, between $\gamma_{tp}$ and approximately 0.1%, the modulus $G_{SN}$ is first increasing and then decreasing while the cyclic pore water pressure is continuously increasing. These aspects of small-strain cyclic soil behavior of sand are truly fundamental and are therefore very important. Some data published in the past
also reveal this kind of behavior, but interestingly enough this behavior has never been studied before. When fully saturated sand is subjected in undrained conditions to moderate and large $\gamma_c$, approximately larger than 0.15%, $G_{SN}$ is consistently decreasing with N while the large permanent cyclic pore water pressure develops. This large-strain behavior is very well documented and has been known for decades

(4) Because of such a behavior of sand at small cyclic strains, i.e., the behavior when sand is stiffening instead of softening with N, $\gamma_{td}$ in sand cannot be defined in the standard manner like for clays. For a given sand there is no single amplitude $\gamma_c$ that can be called the threshold shear strain for cyclic degradation, $\gamma_{td}$. This is because at small cyclic strains $G_{SN}$ is first increasing and then decreasing with N, which means that the onset of degradation depends not only on $\gamma_c$ but also on the number of cycles, N.

(5) In both clays tested the threshold shear strain for cyclic degradation, $\gamma_{td}$, is not visibly affected by the overconsolidation ratio, OCR, and vertical effective consolidation stress, $\sigma_{vc}'$. For kaolinite clay, approximately the same $\gamma_{td}$ is obtained for OCR=1 and OCR=4. For kaolinite-bentonite clay, approximately the same $\gamma_{td}$ is obtained for OCR=1, OCR=4 and OCR=7.8. For kaolinite clay, approximately the same $\gamma_{td}$ is obtained for $\sigma_{vc}'=210$ kPa and 680 kPa. For kaolinite-bentonite clay, approximately the same $\gamma_{td}$ is obtained for $\sigma_{vc}'=220$ kPa and 670 kPa.

(6) In kaolinite clay the threshold strain for cyclic pore water pressure, $\gamma_{tp}$, is not visibly affected by the overconsolidation ratio, OCR, and the vertical effective consolidation stress, $\sigma_{vc}'$. Approximately the same $\gamma_{tp}$ is obtained for OCR=1 and OCR=4, and approximately the same $\gamma_{tp}$
is obtained for $\sigma_{vc}' = 210$ kPa and 680 kPa. For NC kaolinite-bentonite clay, approximately the same $\gamma_{tp}$ is obtained for $\sigma_{vc}' = 220$ kPa and 670 kPa.

(7) In kaolinite-bentonite clay with PI=55 overconsolidated to OCR=4 and 7.8 the cyclic pore water pressures between $\gamma_c = 0.003\%$ and 0.3% did not change in a consistent manner and, consequently, $\gamma_{tp}$ could not be evaluated. It seems that the equipment setup is not responsive and precise enough to capture sensitive pore pressure changes in this high plasticity clay.

(8) The threshold strain for cyclic pore water pressure, $\gamma_{tp}$, shows increase with PI just like in the previous studies, but its values are at or below the lower bound of published $\gamma_{tp}$-PI trends. It is believed that the more precise testing in this narrowly focused investigation enabled detection of $\gamma_{tp}$ at smaller $\gamma_c$ than before. The threshold strain values are listed in Table 6.1.

(9) The threshold strains for cyclic degradation in clays, $\gamma_{td}$, do not follow the trend of increase with PI like in the previous studies and are smaller than those published earlier for similar soils. This discrepancy calls for further studies. The threshold strain values are listed in Table 6.1.
Appendix 1: Test results

Below is the table that summarizes all of the tests conducted in this investigation. After the table the results of each test are presented. For each test the following plots are provided:

1. Strain-time history
2. Stress-time history
3. Equivalent pore water pressure e change with time
4. Stress-strain relationship

It has to be noted that some of the tests have not been corrected for false loads. These uncorrected tests are used for analyses of the cyclic pore water pressure change that are not affected by the correction. Some of them were also used for the analyses of the stiffness change that in some cases is negligibly affected by the correction.
<table>
<thead>
<tr>
<th>Test #</th>
<th>Test Name</th>
<th>Stage #</th>
<th>Cyclic Shear Strain Amplitude, $\gamma_c$ (%)</th>
<th>Vertical effective consolidation stress, $\sigma'_v$ (kPa)</th>
<th>Frequency of cyclic loading, $f$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Nev1</td>
<td>0.60</td>
<td>1 0.00 0.01 0.02 0.03 0.08 0.16</td>
<td>155 154 143 126 83</td>
<td>0.01 0.01 0.01 0.01 0.01 0.01 0.01</td>
</tr>
<tr>
<td>10</td>
<td>Nev2</td>
<td>0.66</td>
<td>1 0.08</td>
<td>153</td>
<td>0.01</td>
</tr>
<tr>
<td>11A</td>
<td>Nev0</td>
<td>0.63</td>
<td>1 0.08 0.02</td>
<td>0.03</td>
<td>145 74 74</td>
</tr>
<tr>
<td>11</td>
<td>Nev6</td>
<td>0.63</td>
<td>1 0.46</td>
<td>158</td>
<td>0.1</td>
</tr>
<tr>
<td>16</td>
<td>Nev5</td>
<td>0.64</td>
<td>1 0.24</td>
<td>158</td>
<td>0.01</td>
</tr>
<tr>
<td>17</td>
<td>Nev6</td>
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<td>1 0.47</td>
<td>157</td>
<td>0.01</td>
</tr>
<tr>
<td>18</td>
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<td>0.62</td>
<td>1 0.80</td>
<td>159</td>
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</tr>
<tr>
<td>19</td>
<td>Nev8</td>
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<td>1 0.10</td>
<td>162</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>Nev9</td>
<td>0.65</td>
<td>1 0.12</td>
<td>155</td>
<td>0.01</td>
</tr>
<tr>
<td>21</td>
<td>Nev20</td>
<td>0.65</td>
<td>1 0.15 0.10 0.13 0.13</td>
<td>0.13 154</td>
<td>0.01 0.01 0.01 0.01</td>
</tr>
<tr>
<td>22</td>
<td>Nev21</td>
<td>0.64</td>
<td>1 0.10 0.12 0.15 0.10</td>
<td>207 121 94 75</td>
<td>0.01 0.01 0.01 0.01</td>
</tr>
<tr>
<td>23</td>
<td>Nev22</td>
<td>0.65</td>
<td>1 0.12 0.10 0.08 0.06 0.08 0.08 0.10 0.12 0.15</td>
<td>156 157 156 159 157 156 157 157</td>
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132
Test Nev1, Soil: Nevada Sand; e=0.6, w=0%, \( \sigma'_{vc}=155 \text{ (kPa)} \); OCR=1; f=0.01 Hz

Shear strain, \( \gamma \) vs. Time (s)

Normalized horizontal shear stress, \( \tau = \tau / \tau_{uc} \) vs. Time (s)

Normalized pore water pressure, \( \Delta u = \Delta u / \tau_{uc} \) vs. Time (s)

Normalized horizontal shear stress, \( \tau = \tau / \tau_{uc} \) vs. Shear strain, \( \gamma \) (%)
Test Nev12: Soil: Nevada Sand;  e=0.65;  w=0%

\(\sigma_{v0}=156\) (kPa);  OCR=1;  \(f=0.01\) Hz

Shear strain, \(\gamma\) (%)

Normalized horizontal shear stress, \(\tau = \tau / \sigma_{v0}\)

Normalized equivalent pore water pressure, \(\Delta p = \Delta p / \sigma_{v0}\)

Normalized horizontal shear stress, \(\tau = \tau / \sigma_{v0}\)

Shear strain, \(\gamma\) (%)

144
Test: Nev21; Soil: Nevada Sand; \( e = 0.63; w = 0\% \)
\( \sigma_{vc} = 162 \text{ (kPa)} \); OCR = 1; \( f = 0.01 \text{ Hz} \)

Shear strain, \( \gamma \) (%)

Time (s)

Normalized horizontal shear stress, \( \tau / \tau_{vc} \)

Time (s)

Normalized equivalent pore water pressure, \( \Delta u / \tau_{vc} \)

Time (s)

Normalized horizontal shear stress, \( \tau / \tau_{vc} \)

Shear strain, \( \gamma \) (%)

148
Test: Toy1, Soil: Toyoura Sand, $\phi=0.71$, $w=0\%$
$\sigma_{v0}=146$ (kPa), OCR=1

Shear strain, $\gamma$ (%)

Time (s)

Normalized horizontal shear stress, $t_e = t / \sigma_{v0}$

Time (s)

Normalized equivalent pore water pressure, $\Delta t = \Delta t / \sigma_{v0}$

Time (s)

Test: Toy1, Soil: Toyoura Sand, $\phi=0.71$, $w=0\%$
$\sigma_{v0}=146$ (kPa), OCR=1

Normalized horizontal shear stress, $t_e = t / \sigma_{v0}$

Shear strain, $\gamma$ (%)

0.15

0.1

0.05

0

-0.05

-0.1

-0.08 -0.06 -0.04 -0.02 0 0.02 0.04 0.06 0.08

0.15

0.1

0.05

0

-0.05

-0.1

-0.08 -0.06 -0.04 -0.02 0 0.02 0.04 0.06 0.08
Test Kaol: Soil Kaolinite Clay, e=1.03; w=0.39\%
σ’_{vC}=131 (kPa); OCR=6; f=0.01 Hz

Normalized horizontal shear stress, \( t = t / t_{cC} \)

Normalized equivalent pore water pressure, \( \Delta u = \Delta u / \gamma_{vC} \)

Normalized shear stress, \( t^* = t / \gamma_{vC} \)

Shear strain, \( \gamma \ (%) \)
Test: Kaolinite Clay; PI=26 (MH); e=1.26; w=48% 
$\sigma_{ve} = 219$ (kPa); OCR=1; $f = 0.01$ Hz

- Shear strain, $\gamma$ (%)
- Normalized horizontal shear stress, $\tau^* = \frac{\tau}{\sigma_{ve}}$
- Normalized equivalent pore water pressure, $\Delta u = \frac{\Delta u}{\sigma_{ve}}$

Shear strain, $\gamma$ (%)
- Normalized horizontal shear stress, $\tau^* = \frac{\tau}{\sigma_{ve}}$

Normalized horizontal shear stress, $\tau^* = \frac{\tau}{\sigma_{ve}}$ vs. Shear strain, $\gamma$ (%)

Page 160
Test: Kao11; Soil: Kaolinite Clay. Pi=28 (Mpa); e=1.01; w=0.38% 
\(\sigma_{vd}=211\) (kPa); OCR=4; \(f=0.01\) Hz
Test: Kao12, Soil: Kaolinite Clay, P_I=28 (MH); e=0.97; w=36% 
$\sigma'_v=223$ (kPa); OCR=1; f=0.01 Hz
Test Kao22A: Soil: Kaolinite Clay; PI=28 (MH); e=1.4; w=58%
\( \sigma_{vc} = 216 \) kPa; OCR=1; \( f = 0.001 \) Hz

Shear strain, \( \gamma \) (%)

Time (s)

Normalized horizontal shear strain, \( \tau = \tau / \tau_{vc} \)
Test: Kao22B; Soil: Kaolinite Clay; P=28 (MH); e=1.13; w=43% 
\( \sigma_{vc}=691 \text{ (kPa)} \); OCR=1; f=0.001 Hz

Normalized horizontal shear stress, \( \tau \), vs. time (s)

Normalized equivalent pore water pressure, \( \Delta u \), vs. time (s)

Normalized horizontal shear stress vs. shear strain, \( \gamma \), (%)

172
Test: Kac22B, Soil: Kaolinite Clay, PI=28 (MH), e=1.07, w=40% 
$\sigma_{\gamma_c}=688$ (kPa), OCR=1, f=0.1 Hz

Normalized horizontal shear stress, $\tau = \tau_{\gamma_c}$

Normalized equivalent pore water pressure, $\Delta u = \Delta u_{\gamma_c}$

Normalized horizontal shear stress, $\tau = \tau_{\gamma_c}$

Shear strain, $\gamma$ (%)
Test: Kao25B; Soil: Kaolinite Clay; PL=28 (MH); e=1.13; w=43%
$\sigma_{vc}=681$ (kPa); OCR=1; $f=0.01$ Hz

Shear strain, $\gamma$ (%)

Time (s)

Normalized horizontal shear stress, $\sigma^* / \sigma_{vc}$

Time (s)

Normalized equivalent pore water pressure, $\Delta u / \sigma_{vc}$

Time (s)
Test: Kao26A; Soil: Kaolinite Clay; \( P_l=28 \) (MH); \( e=1.34\); \( w=58\% \)
\( \sigma_{vc}=217 \) (kPa); \( OCR=1\); \( f=0.1 \) Hz

![Shear strain vs Time](image1)

![Normalized horizontal shear stress vs Time](image2)

![Normalized deviation pore water pressure vs Time](image3)

![Normalized shear stress vs Shear strain](image4)
Test: Kac26B; Soil: Kaolinite Clay; PI=23 (MH); e=1.11; w=42% 
$\sigma'_{vc}=684$ (kPa); OCR=1; $f=0.1$ Hz
Test: Kao27A; Soil: Kaolinite Clay; PI=28 (MH); e=1.33; w=57% 
$\sigma_{vc}=215$ (kPa); OCR=1; f=0.01 Hz
Test: Kac27B; Soil: Kaolinite Clay; F1=28 (MH); e=1.09; w=41%  
\(\sigma_{uv}=691\) (kPa); OCR=1; \(f=0.01\) Hz
Test: Kao28A; Soil: Kaolinite Clay; PI=28 (MH); e=1.13; w=57%
c_vc=219 (kPa); OCR=1; f=0.1 Hz
Test: Kao28B; Soil: Kaolinite Clay; PI=28 (MH); e=1.07; w=41%
$\sigma_{vc}=677$ (kPa); OCR=1; f=0.1 Hz

Normalized horizontal shear stress $\tau_c = \tau / \sigma_{vc}$

Shear strain, $\gamma$ (%) vs. Normalized horizontal shear stress $\tau_c$
Test: Kao29B; Soil: Kaolinite Clay; PI=28 (MH); e=1.05; w=40%
$\sigma_{vc}=678$ (kPa); OCR=1; f=0.001 Hz
Test: Kao30A. Soil: Kaolinite Clay. PI=28 (MH): e=1.32; w=57% \( \sigma_{vc} = 220 \text{ (kPa)} \). OCR=1; f=0.001 Hz

187
Test: Kao31A; Soil: Kaolinite Clay; P_l=28 (MH); e=0; w=0%
$\sigma_{vc}=216$ (kPa); OCR=1; f=0.1 Hz
Test: Kao32; Soil: Kaolinite Clay; PI=28 (MH); e=1.23; w=45%
σ'\(_v\)=218 (kPa); OCR=1; f=0.1 Hz

Normalized horizontal stress, \(\tau^* = \frac{\tau}{\sigma'_v}\)

Normalized equivalent pore water pressure, \(\Delta \psi = \frac{\Delta \psi}{\sigma'_v}\)

Normalized horizontal shear stress, \(\tau^* = \frac{\tau}{\sigma'_v}\)

Normalized shear strain, \(\gamma^* = \frac{\gamma}{\gamma_{\text{ref}}}\)
Test KaoBen01; Soil: Kaolinite-Bentonite Clay; PL=55 (CH); e=1.42, w=54%
$\sigma_{vc}=220$ (kPa); OCR=1; f=0.1 Hz
Test: Kaol03, Soil: Kaolinite-Bentonite Clay; $P_l = 55$ (kPa); $\epsilon = 1.14$; $\omega = 43\%$

$\sigma_{vc} = 212$ (kPa); $OCR = 4$; $f = 0.01$ Hz

Normalized horizontal shear stress, $\tau / \sigma_{vc}$

Normalized horizontal water pressure, $\Delta \phi / \sigma_{vc}$

Normalized horizontal shear stress vs. shear strain, $\tau / \sigma_{vc}$ vs. $\gamma$ (%)
Test KaoBen04: Soil: Kaolinite-Bentonite Clay. Pl=55 (CH). e=1.17; w=44% 
\(\sigma'_{vc}=113\) (kPa). OCR=7.8; f=0.01 Hz

- Shear strain, \(\gamma\) (%)
- Normalized horizontal shear stress, \(\tau^* = \tau / \sigma'_{vc}\)
- Normalized equivalent pore water pressure, \(\Delta \sigma' = \Delta \sigma / \sigma'_{vc}\)

Graphs showing the relationships over time.
Test on rubber balloon filled with water: $\sigma_{vc} = 70.4 \text{ (kPa)}$; $f = 0.01 \text{ Hz}$
Appendix 2: Sample of Matlab code

This sample of Matlab code applies to the multi-staged cyclic strain-controlled test on kaolinite clay, number Kao12 in Appendix 1. In this test there are 7 cyclic strain-controlled stages with $\gamma_c=0.0071\%, \ 0.0205\%, \ 0.0347\%, \ 0.0549\%, \ 0.0966\%, \ 0.1722\%$, and $0.6868\%$. Furthermore $\sigma'_v c = 229\ kPa$ and $f=0.01\ Hz$.

```matlab
clc
clear all
fileName='TE_2010_11_30_142112.txt';
fid=fopen(fileName,'r');

Area=fscanf(fid,'%f',1);
H=fscanf(fid,'%f',1);
GainHD=fscanf(fid,'%f',1);
GainHL=fscanf(fid,'%f',1);
GainVD=fscanf(fid,'%f',1);
GainVL=fscanf(fid,'%f',1);
Num_Test=fscanf(fid,'%f',1);
OCR=fscanf(fid,'%f',1);
Initial_Void_Ratio=fscanf(fid,'%f',1);
Initial_Water_Content=fscanf(fid,'%f',1);

for kk=1:Num_Test;
    Freq(kk)=fscanf(fid,'%f',1);
    % To read frequencies for each multi-stage test
end

range(kk)=fscanf(fid,'%f',1);
% To read range for each multi-stage test

for kk=1:Num_Test;
    Freq_Rec(kk)=fscanf(fid,'%f',1);
    % To read frequencies for each multi-stage test
end

for kk=1:Num_Test;
    NumbCyc(kk)=fscanf(fid,'%f',1);
    % To read Number of cycles for each stage
end

for ii=1:4;
    fgetl(fid);
end
fgetl(fid);

ii=1;
while feof(fid)~=1
    time(ii)=fscanf(fid,'%f',1);
end
```
chan1(ii)=fscanf(fid,'%f',1);
chan2(ii)=fscanf(fid,'%f',1);
chan3(ii)=fscanf(fid,'%f',1);
chan4(ii)=fscanf(fid,'%f',1);
chan5(ii)=fscanf(fid,'%f',1);
ii=ii+1;
end
close(fid);

%% Setting values to be extracted from data file
Xmin=1; Xmax=length(chan5);  % Range of Interests of X
if GainHD == 1                 % Horizontal Displacement Calibration Values
    Horz_Dspl_Cal=-.38;         % Damages Transducer
else
    Horz_Dspl_Cal=-.038;       % Updated on 06/14/2010
end
if GainHL == 1                  % Horizontal Load Calibration Values, Updated on 06/14/2010
    Horz_Load_Cal=-11.99;
else
    Horz_Load_Cal=-1.199;
end
if GainVD == 1                   % Vertical Displacement Calibration Values
    Vert_Dspl_Cal=-.4044;
else
    Vert_Dspl_Cal=-.04;
end
Vert_Load_Cal=103.33;          % Vertical Load Calibration Values Updated on 06/14/2010
Horz_Dspl_Zero= mean(chan1(1:N_mean)); Horz_Load_Zero=mean(chan2(range(1)-N_mean:range(1)));
Vert_Dspl_Zero= mean(chan3(1:N_mean)); Vert_Load_Zero=.019; % Updated on 06/14/2010
for jj=1:ii
    Horz_Strain(jj)=(chan1(jj)-Horz_Dspl_Zero)*Horz_Dspl_Cal/H*100;
    Horz_Stress(jj)=(chan2(jj)-Horz_Load_Zero)*Horz_Load_Cal/Area*98.0665*(+1); % Multiplied by 98.0665 to convert to kPa
    Vert_Strain(jj)=(chan3(jj)-Vert_Dspl_Zero)*Vert_Dspl_Cal/H*100;
    Vert_Stress(jj)=(((chan4(jj)-Vert_Load_Zero)*Vert_Load_Cal)+.574)/Area*98.0665*(-1); % Multiplied by 98.0665 to convert to kPa; Wtop-cap=.574 kg
end
Vert_Stress_Initial=mean(Vert_Stress(range(1)-N_mean:range(1))); % Initial Vertical Stress for Normalization
for jj=1:ii
    Equi_PWP(jj)=(Vert_Stress(jj)-Vert_Stress_Initial)*(-1); % Equivalent Pore Water Pressure
    Norm_Equi_PWP(jj)=Equi_PWP(jj)/Vert_Stress_Initial;    % Normalized Equivalent Pore Water Pressure
    Norm_Horz_Stress(jj)=Horz_Stress(jj)/Vert_Stress_Initial;
end
Out=[time' Horz_Strain' Horz_Stress' Vert_Stress']; % output results Equi_Vert_Stress

%% Constant Height, just for myself
Xmax=length(chan5); Xmin=1;
% Xmin=range(3);
% Xmax=range(4);
figure('Name','Constant Height, check','NumberTitle','off'),clf,
subplot(3,1,1)
plot(time (Xmin:Xmax),Horz_Strain (Xmin:Xmax),'linewidth',2),xlabel('Time (s)'),ylabel('Shear strain,
\gamma (%)'),grid
%title([documentation here])
%xlim([Xmin Xmax])

subplot(3,1,2)
plot(time(Xmin:Xmax),Horz_Stress(Xmin:Xmax),'linewidth',2),xlabel('Time (s)'),ylabel({"Horizontal shear; stress, \tau (kPa)"})
%ylim([-10 10])

subplot(3,1,3)
plot(time(Xmin:Xmax),Norm_Equi_PWP(Xmin:Xmax),'linewidth',2),xlabel('Time (s)'),ylabel({"Normalized equivalent pore; water pressure, \Delta u^* = \Delta u / \sigma_{v_c}"})
%ylim([-10 10])

%%  To plot the Initial loops and Stress Path
% Xmin=range(3)
% Xmax=range(4)
% figure('Name','Strain-Stress Loop','NumberTitle','off'),clf,% To plot loops
% plot(Horz_Strain(Xmin:Xmax),Horz_Stress(Xmin:Xmax)),xlabel('Shear strain, \gamma (%)'),ylabel({"Horizontal shear; stress, \tau (kPa)"}),grid minor,%ylim([-40 40])
% %ylim([-10 10])
% figure('Name','Stess Path','NumberTitle','off'),clf, % To plot stress path
% plot(Vert_Stress(Xmin:Xmax),Horz_Stress(Xmin:Xmax)),xlabel({"Vertical normal; stress, \sigma (kPa)"}),ylabel({"Horizontal shear; stress, \tau (kPa)"}),grid minor,%ylim([-18 18])

% OuT=[time(Xmin:Xmax)';Horz_Strain(Xmin:Xmax)';Horz_Stress(Xmin:Xmax)'];

%% To Find and correct the erro based on Stress Time history
%% Step 1, Finding and confirming the Index matrix
%% Num_Test=7; %Here is set to 3, but later changed to 1 to Only correct the 1st stage
%% NumbCyc(4)=10; %Ignoring the 11th cycle in the 4th stage for Kao12
%% for ii=2:length(Horz_Stress)
% Diff(ii)=abs(Horz_Stress(ii)-Horz_Stress(ii-1));
%% end

for kk=1:Num_Test
  L(kk)=(1/Freq(kk))*Freq_Rec(kk);
  L2=round(L/2);
  L4=round(L/4);
  L16=round(L/16);
  % for jj=1:NumbCyc(kk)*2 %number of Half cycles
  % Diff_cut=Diff(range(kk)+(jj-1)*L2(kk)+L4(kk):range(kk)+(jj*L2(kk)-00); % -10 is for the 40th half cycle
  % Index(kk,jj)=find(Diff_cut==max(Diff_cut)) + range(kk)+(jj-1)*L2(kk)+ round(L(kk)/4);
  %
  % end
  % if NumbCyc(kk)<max(NumbCyc) %This part is for when NumbCyc is the same for all the stages
  % Index(kk,2*NumbCyc(kk)+1:max(NumbCyc))=Index(kk,jj);
  % end
end
%% To check the Index
%% NumStage=7;
%% HalfCycNum=20;
%% xmin=range(NumStage)+(HalfCycNum-1)*L2(NumStage)-00;
%% xmax=range(NumStage)+(HalfCycNum)*L2(NumStage)+00;

% figure(1), clf, % To plot and check if Index is between the friction drop for each half cycle
% plot(time(xmin:xmax),Horz_Stress(xmin:xmax),'.',time(Index(NumStage,HalfCycNum)),Horz_Stress(Index
% (NumStage,HalfCycNum)),'.',time(xmin:xmax),20*Diff(xmin:xmax)),xlabel('time'),ylabel({'H
% orizontal shear'; 'stress, \tau (kPa)'},grid minor;%ylim([-40 40])
%% plot(time(xmin:xmax),Horz_Stress(xmin:xmax),'.',time(Index(NumStage,HalfCycNum)),Horz_Stress(Index
% (NumStage,HalfCycNum)),'.',time(xmin:xmax),20*Diff(xmin:xmax)),xlabel('time'),ylabel({'H
% orizontal shear'; 'stress, \tau (kPa)'},grid minor;%ylim([-40 40])
% OOUt=[time(xmin:xmax)' Horz_Strain(xmin:xmax)'  Horz_Stress(xmin:xmax)'
% plot(time,Horz_Stress,time(Index(NumStage,HalfCycNum)),Horz_Stress(Index(NumStage,HalfCycNum)
% ),'.',time,20*Diff),xlabel('time'),ylabel({'Horizontal shear'; 'stress, \tau (kPa)'},grid minor;ylim([-40 40])
% OOUt=[time(xmin:xmax)' Horz_Strain(xmin:xmax)'  Horz_Stress(xmin:xmax)'
% plot(time,Horz_Stress,time(Index(NumStage,HalfCycNum)),Horz_Stress(Index(NumStage,HalfCycNum)
% ),'.',time,20*Diff),xlabel('time'),ylabel({'Horizontal shear'; 'stress, \tau (kPa)'}),grid minor;ylim([-40 40])
% % Step 2, Printing graphs to Eye-ball Friction, & Number of pionts to drop before and after the Index
% % Only to get started, commented once finalized
% % NumCycBefore=[1 2 3 4 5 6 7 8 9 10 1 2 3 4 5 6 7 8 9 10 ];
% % NumCycAfter=[1 2 3 4 5 6 7 8 9 10 1 2 3 4 5 6 7 8 9 10 ];
% Index(1,1:20)=[481 981 1487 1973 2483 2979 3484 3987 4473 4981 5486 5978 6481 6984 7485
% 7985 8483 8981 9481 9982];
% Index(2,1:20)=[10819 11320 11819 12319 12819 13319 13819 14319 14819 15319 15819 16319
% 16819 17319 17819 18319 18819 19319 19819 20319];
% Index(3,1:20)=[21060 21508 21958 22407 22860 23309 23760 24210 24659 25112 25562 25998
% 26435 26854 27267 27670 28073 28477 28872 29268];
% Index(4,1:20)=[30240 30590 30940 31288 31640 31989 32338 32688 33039 33387
% 33738 34084 34438 34785 35135 35484 35835 36185 36535 36885];
% Index(5,1:20)=[36121 36371 36622 36870 39121 39371 39621 39871 40121 40371
% 40621 40871 41121 41371 41621 41871 42121 42371 42621 42871];
% Index(6,1:20)=[43578 44075 44577 45076 45578 46077 46578 47078 47578 48078
% 48578 49076 49578 50078 50579 51076 51578 52078 52578 53078];
% Index(7,1:20)=[54108 54608 55110 55610 56110 56609 57110 57610 58110 58608
% 59110 59610 60110 60610 61110 61610 62110 62610 63110 63610];
% Index(11,1:20)=[10334 10836 11337 11838 12339 12835 13338 13838 14340 14837
% 15339 15837 16339 16838 17340 17840 18338 18838 19338 19819];
% Index(41,1:20)=[20320 20819 21319 21819 22319 22819 23321 23816 24314 24798
% 25273 25740 26200 26649 27087 27517 27940 28347 28751 29148];
% NumCycBefore(1,1:max(NumCyc)*2)=10; NumCycAfter(1,1:max(NumCyc)*2)=5;
% NumCycBefore(2,1:max(NumCyc)*2)=8; NumCycAfter(2,1:max(NumCyc)*2)=4;
% NumCycBefore(3,1:max(NumCyc)*2)=7; NumCycAfter(3,1:max(NumCyc)*2)=2;
% NumCycBefore(4,1:max(NumCyc)*2)=6; NumCycAfter(4,1:max(NumCyc)*2)=1;
% NumCycBefore(5,1:max(NumCyc)*2)=4; NumCycAfter(5,1:max(NumCyc)*2)=1;
% NumCycBefore(6,1:max(NumCyc)*2)=6; NumCycAfter(6,1:max(NumCyc)*2)=1;
% NumCycBefore(7,1:max(NumCyc)*2)=5; NumCycAfter(7,1:max(NumCyc)*2)=1;
% NumCycBefore(:,:)=0; NumCycAfter(:,)=0; %Before correcting the data
% NumStage=7;
% HalfCycNum=19;
% KK=NumStage;JJ=HalfCycNum; % Only to make the equations shorter
% JJ2=HalfCycNum+1; % For 2 consequive half cycles
% Domain=+35;
xmin1=Index(KK,JJ)-L16(KK)+Domain;
xmax1=Index(KK,JJ)+L16(KK)-Domain;
xmin2=Index(KK,JJ2)-L16(KK)+Domain;
xmax2=Index(KK,JJ2)+L16(KK)-Domain;
% NumCycBefore(KK,JJ)=4;
% NumCycAfter(KK,JJ)=2;
figure(2), clf, % To plot and obtain NumCycBefore and NumCycAfter
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```matlab
subplot(2,1,1)
plot(time(xmin1:xmax1),Horz_Stress(xmin1:xmax1),'.',time(Index(KK,JJ)-NumCycBefore(KK,JJ)+NumCycAfter(KK,JJ)),Horz_Stress(Index(KK,JJ)-NumCycBefore(KK,JJ)+NumCycAfter(KK,JJ)),'*',time(Index(KK,JJ)),Horz_Stress(Index(KK,JJ)),'+'),
xlabel('time'),ylabel({'Horizontal shear'; 'stress, \tau (kPa)'},'interpreter','latex'),grid, grid minor,
title({['Stage: ',num2str(NumStage),';   Half Cycle Number = ',num2str(HalfCycNum,4)]})
subplot(2,1,2)
plot(time(xmin2:xmax2),Horz_Stress(xmin2:xmax2),'.',time(Index(KK,JJ2)-NumCycBefore(KK,JJ2)+NumCycAfter(KK,JJ2)),Horz_Stress(Index(KK,JJ2)-NumCycBefore(KK,JJ2)+NumCycAfter(KK,JJ2)),'*',time(Index(KK,JJ2)),Horz_Stress(Index(KK,JJ2)),'+'),
xlabel('time'),ylabel({'Horizontal shear'; 'stress, \tau (kPa)'},'interpreter','latex'),grid, grid minor,
title({['Stage: ',num2str(NumStage),';   Half Cycle Number = ',num2str(HalfCycNum+1,4)]})
```

%% To check if we have all the numbers for NumCycBefore, NumCycAfter
%% Input Friction, NumCycBefore and NumCycAfter by hand from the printed pages
%Friction(1,1:2:19)=(1.20);Friction(1,2:2:20)=(-1.40);
%Friction(2,1:2:19)=(1.70);Friction(2,2:2:20)=(-1.60);
%Friction(3,1:2:19)=(1.70);Friction(3,2:2:20)=(-1.30);
%Friction(4,1:2:19)=(1.90);Friction(4,2:2:20)=(-1.80);
%Friction(5,1:2:19)=(1.95);Friction(5,2:2:20)=(-1.90);
%Friction(6,1:2:19)=(1.95);Friction(6,2:2:20)=(-1.90);
%Friction(7,1:2:19)=(2.10);Friction(7,2:2:20)=(-1.04);
%Friction(22:2:40)=(-1.15);
Friction(1,1:20)=[1.19 -1.18 1.14 -1.12 1.09 -1.08 1.06 -1.05 1.04 -1.03 1.02 -1.01 1.0 1.0 0.99 -0.98 0.98 -0.97 0.97 -0.96];
Friction(2,1:20)=[1.11 -0.97 1.10 -0.97 1.08 -0.97 1.07 -0.97 1.06 -0.97 1.05 -0.97 1.04 -0.97 1.04 -0.97 0.97 -0.96];
Friction(3,1:20)=[1.11 -1.50 1.06 -1.37 1.03 -1.30 1.02 -1.30 1.00 -1.21 0.99 -1.25 1.05 -1.15 0.98 -1.20 0.97 -1.11 0.97 -1.09];
Friction(4,1:20)=[1.40 -1.45 1.22 -1.35 1.14 -1.29 1.08 -1.24 1.04 -1.21 1.01 -1.18 0.98 -1.16 0.96 -1.14 0.94 -1.12 0.92 -1.11];
Friction(5,1:20)=[1.26 -1.43 1.22 -1.37 1.20 -1.33 1.19 -1.31 1.18 -1.29 1.18 -1.27 1.17 -1.26 1.16 -1.25 1.16 -1.24 1.16 -1.23];
Friction(6,1:20)=[1.36 -1.59 1.32 -1.57 1.30 -1.56 1.29 -1.55 1.28 -1.54 1.27 -1.53 1.27 -1.53 1.26 -1.52 1.25 -1.52];
Friction(7,1:20)=[1.42 -1.21 1.24 -1.20 1.16 -1.20 1.11 -1.19 1.07 -1.19 1.03 -1.19 1.01 -1.18 0.99 -1.18 0.97 -1.18 0.95 -1.18];
Friction(21:40)=[1.51 -1.19 1.49 -1.17 1.47 -1.16 1.45 -1.14 1.43 -1.13 1.41 -1.12 1.40 -1.11 1.38 -1.10 1.37 -1.09 1.35 -1.08];
% NumCycBefore=[6 7 8 8 13 6 7 9 7 5 0 0 0 0 0 0 0 0 0 0 0 0];
% NumCycAfter=[2 3 8 4 6 10 5 8 5 3 0 0 0 0 0 0 0 0 0 0 0];
% NumIndexBefore=[0 1 2 3 4 5 6 7 8 9 0 0 0 0 0 0 0 0 0 0];
% NumIndexAfter=[10 11 12 13 14 15 16 17 18 19 0 0 0 0 0 0 0 0 0];
% HalfCycNum=[20 21 22 23 24 25 26 27 28 29 30 31 0 0 0 0 0 0 0];
% NumStage=[32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51];
```
%% Generating NumIndexBefore
 NumIndexBefore=ones(Num_Test,max(NumbCyc)*2);
for kk=1:Num_Test
 for jj=1:max(NumbCyc)*2
   if  NumCycAfter(kk,jj)== 0
     NumIndexBefore(kk,jj)=0;
   end
 end
end
InvNumIndexBef=NumIndexBefore';
%% New IndexNew method
InvNumbef=NumCycBefore';
InvNumAft=NumCycAfter';
SizeIndex=size(Index);
InvInd=Index';
%Num_Test=1; %Only correcting the 1st stage
for kk=1:Num_Test
 for jj=1:NumbCyc(kk)*2
   Rank(kk,jj)=(kk-1)*SizeIndex(2)+jj;
   IndexNew(kk,jj)=Index(kk,jj)-sum(InvNumbef(1:Rank(kk,jj)-1))-sum(InvNumAft(1:Rank(kk,jj)-1))-sum(InvNumIndexBef(1:Rank(kk,jj)-1));
 end
end
%%%%%%%%%%%%%
%% Step 3, Magnifying the points corresponding to Friction drop
Horz_StressEdit1=Horz_Stress;
TooBig(1:Num_Test)=(95);  % Pick a big enough point
for kk=1:Num_Test
 for jj=1:NumbCyc(kk)*2 %number of Half cycles
   Horz_StressEdit1(Index(kk,jj)-NumCycBefore(kk,jj):Index(kk,jj)+NumCycAfter(kk,jj))=TooBig(kk); %TooBig is an arbitrary number to distinguish the points
 end
end
Horz_StressEdit2=Horz_StressEdit1;
figure(3),clf, % To plot and see if the dropping is correct
plot(time, Horz_StressEdit1,'-*'),xlabel('time'),ylabel({'Horizontal shear'; 'stress, \tau (kPa)'})
grid minor;ylim([min(Horz_StressEdit1)-2 TooBig(kk)+2])
%%%%%%%%%%%%%
%% Step 4, Subtract or add friction from the data
for kk=1:Num_Test
 for jj=2:NumbCyc(kk)*2 %number of Half cycles
   for pp=Index(kk,jj-1)+NumCycAfter(kk,jj-1):Index(kk,jj)-NumCycBefore(kk,jj);
     Horz_StressEdit2(pp)=Horz_StressEdit1(pp)-Friction(kk,jj)/2;
   end
 end
 Horz_StressEdit2(range(kk):Index(kk,1)-NumCycBefore(kk,1)=Horz_StressEdit1(range(kk):Index(kk,1)-NumCycBefore(kk,1))-Friction(kk,1)/2; %Only for the start of the curve
end
figure(4),clf,
plot(time, Horz_Stress,time, Horz_StressEdit1,'-*',time, Horz_StressEdit2,'.'),xlabel('time'),ylabel({'Horizontal shear'; 'stress, \tau (kPa)'})
grid minor;ylim([min(Horz_StressEdit1)-2 TooBig(kk)+2])
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%% Step 5, To zero the negative points at the start of the stage and drop the points
%% Horz_StressEdit1 is when the Friction points are distinguished
%% Horz_StressEdit2 is when Friction has been deducted, but outlaw points exist
%% Horz_StressEdit3 is Horz_StressEdit2, but with zero values for outlaw Pts
%% Horz_StressEdit is when outlaw points are taken out
for kk=1:Num_Test
    for jj=range(kk)-110:range(kk)+5;
        if Horz_StressEdit2(jj)<0
            Horz_StressEdit2(jj)=0;
        end
    end
end
% To check the zero padding at the start of each stage
plot(time, Horz_StressEdit1,time, Horz_StressEdit2,'*','time(range(1)),Horz_Stress(range(1))','*r')
%% To drop the points
Horz_StressEdit=Horz_StressEdit2;
Horz_StrainEdit=Horz_Strain;
Vert_StrainEdit=Vert_Strain;
Vert_StressEdit=Vert_Stress;
Equi_PWPEdit=Equi_PWP;
Norm_Equi_PWPEdit=Norm_Equi_PWP;
Norm_Horz_StressEdit=Horz_StressEdit2/Vert_Stress_Initial;
timeEdit=time;
%%
for kk=1:Num_Test
    for jj=1:NumbCyc(kk)*2;
        timeEdit(IndexNew(kk,jj)-NumCycBefore(kk,jj):IndexNew(kk,jj)+NumCycAfter(kk,jj))=[];
        Horz_StressEdit(IndexNew(kk,jj)-NumCycBefore(kk,jj):IndexNew(kk,jj)+NumCycAfter(kk,jj))=[];
        Horz_StressEdit(IndexNew(kk,jj)-NumCycBefore(kk,jj):IndexNew(kk,jj)+NumCycAfter(kk,jj))=[];
        Vert_StressEdit(IndexNew(kk,jj)-NumCycBefore(kk,jj):IndexNew(kk,jj)+NumCycAfter(kk,jj))=[];
        Equi_PWPEdit(IndexNew(kk,jj)-NumCycBefore(kk,jj):IndexNew(kk,jj)+NumCycAfter(kk,jj))=[];
        Num_Horz_StressEdit(IndexNew(kk,jj)-NumCycBefore(kk,jj):IndexNew(kk,jj)+NumCycAfter(kk,jj))=[];
        Norm_Horz_StressEdit(IndexNew(kk,jj)-NumCycBefore(kk,jj):IndexNew(kk,jj)+NumCycAfter(kk,jj))=[];
    end
end
% To plot and see if the dropping is correct
%%plot(time, Horz_Stress,timeEdit, Horz_StressEdit2,'*','time(range(1)),Horz_Stress(range(1))','*r')
%% plot(Horz_Stress,Horz_StressEdit,Horz_StressEdit2,Horz_Strain,Horz_Stress)
%% Step 6, Plotting the results for the final review
NumStage=7;
HalfCycNumStart=1;
HalfCycNumFinish=20;
KK=NumStage;JJ=HalfCycNumStart; % Only to make the equations shorter
JJ2=HalfCycNumFinish; % For 2 consecutive half cycles
xmin1=Index(KK, JJ)-L4(KK)-50;
xmax1=Index(KK, JJ2)+L4(KK);
Xmin1=IndexNew(KK, JJ)-L4(KK)-50;
Xmax1=IndexNew(KK, JJ2)+L4(KK);
figure('Name', 'Selected Strain-Stress time histories','NumberTitle', 'off'),clf,
%subplot(2,1,1)
%plot(time(xmin1:xmax1),Horz_Stress(xmin1:xmax1),'.',time(Index(KK,JJ)),Horz_Stress(Index(KK,JJ)),'*r',xlabel('time'),ylabel('Horizontal shear; stress, \(\tau\) (kPa)'),grid minor;%ylim([-18 18])
%subplot(2,1,2)
plot(timeEdit(Xmin1:Xmax1),Horz_StressEdit(Xmin1:Xmax1),'.',timeEdit(IndexNew(KK,JJ)),Horz_StressEdit(IndexNew(KK,JJ)),'*r','linewidth',2)
xlabel('time'),ylabel('Horizontal shear; stress, \(\tau\) (kPa)'),grid minor;%ylim([-18 18])

% figure('Name','Corrected and Recorded Strain-Stress time histories for that Stage','NumberTitle','off'),clf,
plot(time(xmin1:xmax1),Horz_Stress(xmin1:xmax1),timeEdit(Xmin1:Xmax1),Horz_StressEdit(Xmin1:Xmax1),xlabel('time'),ylabel('Horizontal shear; stress, \(\tau\) (kPa)'),grid minor;%ylim([-18 18])

%% Finding the Degradation
Horz_StressEdit3=Horz_StressEdit2;
for kk=1:Num_Test % To zero the outlaw points
    for jj=1:length(Horz_StressEdit2)
        if Horz_StressEdit2(jj) >= TooBig(kk)-10
            Horz_StressEdit3(jj)=0;
        end
    end
end

%range(2)=10700; range(3)=21250; % To account for the dropped points
%NumbCyc(3)=30;
% checking the data to see if Rec_Freq fits the data properly
figure('Name','Check data','NumberTitle','off'),clf,
kk=7;DD=range(kk);  %kk is the stage we want to check
%DD=150; %To check cycles 20-30
%L=980; %To make the last 3 cycles work
%range(kk)=259+500;  %To check what happened if we ignore the 1st half cycle
subplot(2,1,1) % for 10 cycles
plot(time(DD:DD+(1)*L(kk)),Horz_Strain(DD:DD+(1)*L(kk)),time(DD+(1)*L(kk):DD+(2)*L(kk)),Horz_Strain(DD+(2)*L(kk):DD+(3)*L(kk)),time(DD+(3)*L(kk):DD+(4)*L(kk)),Horz_Strain(DD+(4)*L(kk):DD+(5)*L(kk)),time(DD+(5)*L(kk):DD+(6)*L(kk)),Horz_Strain(DD+(6)*L(kk):DD+(7)*L(kk)),Horz_Strain(DD+(7)*L(kk):DD+(8)*L(kk)),time(DD+(8)*L(kk):DD+(9)*L(kk)),Horz_Strain(DD+(9)*L(kk):DD+(10)*L(kk)),Horz_Strain(DD+(10)*L(kk):DD+(11)*L(kk)),grid
subplot(2,1,2)
plot(time(DD:DD+(1)*L(kk)),Horz_StressEdit3(DD:DD+(1)*L(kk)),time(DD+(1)*L(kk):DD+(2)*L(kk)),Horz_StressEdit3(DD+(2)*L(kk):DD+(3)*L(kk)),time(DD+(3)*L(kk):DD+(4)*L(kk)),Horz_StressEdit3(DD+(4)*L(kk):DD+(5)*L(kk)),time(DD+(5)*L(kk):DD+(6)*L(kk)),Horz_StressEdit3(DD+(6)*L(kk):DD+(7)*L(kk)),Horz_StressEdit3(DD+(7)*L(kk):DD+(8)*L(kk)),time(DD+(8)*L(kk):DD+(9)*L(kk)),Horz_StressEdit3(DD+(9)*L(kk):DD+(10)*L(kk)),grid
subplot(4,1,1) % for 20 cycles

% subplot(4,1,1) % for 20 cycles
for kk=1:Num_Test;
    for pp=1:NumbCyc(kk);
        lower_bound = range(kk)+(pp-1)*L(kk);
        upper_bound = range(kk)+pp*L(kk);

        Max_Horz_Strain(kk,pp)=max(Horz_Strain(lower_bound:upper_bound));
        Min_Horz_Strain(kk,pp)=min(Horz_Strain(lower_bound:upper_bound));
        Med_Horz_Strain(kk,pp)=(Max_Horz_Strain(kk,pp)-Min_Horz_Strain(kk,pp))/2;

        Max_Horz_Stress(kk,pp)=max(Horz_StressEdit3(lower_bound:upper_bound));
        Min_Horz_Stress(kk,pp)=min(Horz_StressEdit3(lower_bound:upper_bound));
        Med_Horz_Stress(kk,pp)=(Max_Horz_Stress(kk,pp)-Min_Horz_Stress(kk,pp))/2;

        Gs(kk,pp)=Med_Horz_Stress(kk,pp)/Med_Horz_Strain(kk,pp)*100;
        Gs_Gmax(kk,pp)=Gs(kk,pp)/Gs(kk,1);

        %           Gs_Gmax(kk,pp)=Gs(kk,pp)/Gs(kk,1);
        %           Max_Vert_Strain(kk,pp)=max(Vert_Strain(lower_bound:upper_bound));
    end
end

% for kk=1:Num_Test;
%     for pp=1:NumbCyc(kk);
%         lower_bound = range(kk)+(pp-1)*L(kk);
%         upper_bound = range(kk)+pp*L(kk);
%         Max_Horz_Strain(kk,pp)=max(Horz_Strain(lower_bound:upper_bound));
%         Min_Horz_Strain(kk,pp)=min(Horz_Strain(lower_bound:upper_bound));
%         Med_Horz_Strain(kk,pp)=(Max_Horz_Strain(kk,pp)-Min_Horz_Strain(kk,pp))/2;
%         Max_Horz_Stress(kk,pp)=max(Horz_StressEdit3(lower_bound:upper_bound));
%         Min_Horz_Stress(kk,pp)=min(Horz_StressEdit3(lower_bound:upper_bound));
%         Med_Horz_Stress(kk,pp)=(Max_Horz_Stress(kk,pp)-Min_Horz_Stress(kk,pp))/2;
%         Gs(kk,pp)=Med_Horz_Stress(kk,pp)/Med_Horz_Strain(kk,pp)*100;
%         Gs_Gmax(kk,pp)=Gs(kk,pp)/Gs(kk,1);
%         %           Gs_Gmax(kk,pp)=Gs(kk,pp)/Gs(kk,1);
%         %           Max_Vert_Strain(kk,pp)=max(Vert_Strain(lower_bound:upper_bound));
%     end
% end
% Min_Vert_StrainAVE(kk,pp)=min(Vert_Strain(lower_bound:upper_bound));
% Max_Vert_Stress(kk,pp)=max(Vert_Stress(lower_bound:upper_bound));
% Min_Vert_Stress(kk,pp)=min(Vert_Stress(lower_bound:upper_bound));
% Med_Vert_Stress(kk,pp)=(Max_Vert_Stress(kk,pp)+Min_Vert_Stress(kk,pp))/2;
end
end

%Gs_Gmax(1,:)=1; %Since fluctuation is due to DAQ digitization
%% calculating the legends
%%
for ii=1:Num_Test;
    Strains(ii)=mean(Max_Horz_Strain(ii,1:max(NumbCyc(ii))));
end
Leg1 = num2str(Strains(1),'%%%4.3f');
Leg2 = num2str(Strains(2),'%%%4.3f');
Leg3 = num2str(Strains(3),'%%%4.3f');
Leg4 = num2str(Strains(4),'%%%4.2f');
Leg5 = num2str(Strains(5),'%%%4.2f');
Leg6 = num2str(Strains(6),'%%%4.2f');
Leg7 = num2str(Strains(7),'%%%4.2f');
%In case we have more than 5 stages
%ShowLegend6 = num2str(Strains(6)*100,'%%%4.1f');
%ShowLegend7 = num2str(Strains(7)*100,'%%%4.1f');
%% Calculating the best fits
%Cycle=[1:20];
%Cycle=[1:round(max(NumbCyc))]; % or delete 1 if necessary
%% Plotting the degradation. The graph is saved and manually Corrected
%%
figure('Name','Degradation','NumberTitle','off'),clf,
loglog(Cycle,Gs_Gmax(:,:),'-','linewidth',2),legend(Leg1,Leg2,Leg3,Leg4,Leg5,Leg6,Leg7,'location','SE'),grid
grid minor,xlim([1 round(max(NumbCyc))]),xlabel('Number of cycles , N'),ylabel('Degradation index, \delta'),
title({['Test: Kao12;  Soil: Kaolinite Clay;', '   e=',num2str(Initial_Void_Ratio,3),';',
          'w=',num2str(Initial_Water_Content,4),'%';','
          '\sigma^,\_v\_c\=',num2str(Vert_Stress_Initial,4),' (kPa);','
          OCR=',num2str(OCR,4),'; f =',num2str(Freq(1),2),' Hz']);
%ylim([min(Gs_Gmax) 1]), %text(11,1.01,'Only the 1st stage is corrected.','FontSize',12)
%Out2=[Max_Horz_Strain(:,pp) Gs_Gmax];
%%%%%%%%%%%%%%%%
%% Preparing final graphs
%%%%%%%%%%%%%%%%
%% General Both Recorded and Corrected
% figure('Name','General Strain, Recorded and Corrected Stress Time histories','NumberTitle','off'),clf,
% To plot averaged time history and loops
% subplot(3,1,1)
% plot(time, Horz_Strain,'linewidth',2),xlabel('Time (s)'),ylabel('Shear strain, \gamma (%)'),grid
% subplot(3,1,2)
% title({['Test: Kao12;  Soil: Kaolinite Clay;', '   e=',num2str(Initial_Void_Ratio,3),';',
          'w=',num2str(Initial_Water_Content,4),'%';','
          '\sigma^,\_v\_c\=',num2str(Vert_Stress_Initial,4),' (kPa);','
          OCR=',num2str(OCR,4),'; f =',num2str(Freq(1),2),' Hz']);
% subplot(3,1,3)
% plot(time, Horz_Strength,Horz_StrengthEdit,'linewidth',2),xlabel('Time (s)'),ylabel('Horizontal shear; stress, \tau (kPa)'),grid
% subplot(3,1,3)
% plot(time,Equi_PWP,'linewidth',2),xlabel('Time (s)'),ylabel('Equivalent pore water pressure, \Delta u (kPa)'),grid
% figure('Name','General Strain-Stress Loops','NumberTitle','off'),clf, % To plot general loops
% plot(Horz_Strain,Horz_Stress,Horz_StrainEdit,Horz_StressEdit,'linewidth',2),xlabel('Shear strain, \gamma (%)'),ylabel('Horizontal shear stress, \tau (kPa)'),grid ,grid minor,legend('Recorded','Corrected','location','SE')
% title([('Test: Kao12;  Soil: Kaolinite Clay; ', ' e=',num2str(Initial_Void_Ratio,3),'; w=',num2str(Initial_Water_Content,4),'%'],[\'\sigma^\_v_c=',num2str(Vert_Stress_Initial,4),' (kPa); OCR=',num2str(OCR,4),'; f =',num2str(Freq(1),2),' Hz'])
% figure('Name','General Normalized Time histories','NumberTitle','off'),clf, % To plot averaged time history and loops
% subplot(3,1,1)
% plot(time, Horz_Stress,'linewidth',2),xlabel('Time (s)'),ylabel('Horizontal shear stress, \tau (kPa)'),grid minor,legend('Recorded','Corrected','location','SW')
% title([('Test: Kao12;  Soil: Kaolinite Clay; ', ' e=',num2str(Initial_Void_Ratio,3),'; w=',num2str(Initial_Water_Content,4),'%'],[\'\sigma^\_v_c=',num2str(Vert_Stress_Initial,4),' (kPa); OCR=',num2str(OCR,4),'; f =',num2str(Freq(1),2),' Hz'])
% subplot(3,1,2)
% plot(time, Equi_PWP,'linewidth',2),xlabel('Time (s)'),ylabel('Equivalent pore water pressure, \Delta u (kPa)'),grid minor,legend('Recorded','Corrected','location','SE')
% title([('Test: Kao12;  Soil: Kaolinite Clay; PI=28 (MH); ', ' e=',num2str(Initial_Void_Ratio,3),'; w=',num2str(Initial_Water_Content,2),'%'],[\'\sigma^\_v_c=',num2str(Vert_Stress_Initial,3),' (kPa); OCR=',num2str(OCR,2),'; f =',num2str(Freq(1),2),' Hz'])
% subplot(3,1,1)
% plot(time(1:20000), Norm_Horz_Stress(1:20000),timeAVEZerod, Norm_Horz_StressAVEZerod,'linewidth',2),grid minor To see the effect of correction in large strains
% Preparing final graphs New Only Corrected
% General Only Corrected
figure('Name','General Strain, Recorded and Corrected Stress Time histories','NumberTitle','off'),clf, % To plot averaged time history and loops
% subplot(3,1,1)
% plot(time, Horz_Strain,'linewidth',1),xlabel('Time (s)'),ylabel('Shear strain, \gamma (%)'),grid minor ,grid ,legend('Recorded','Corrected','location','SE')
% title([('Test: Kao12;  Soil: Kaolinite Clay; ', ' e=',num2str(Initial_Void_Ratio,3),'; w=',num2str(Initial_Water_Content,4),'%'],[\'\sigma^\_v_c=',num2str(Vert_Stress_Initial,4),' (kPa); OCR=',num2str(OCR,4),'; f =',num2str(Freq(1),2),' Hz'])
% subplot(3,1,2)
% plot(timeEdit, Horz_StressEdit,'linewidth',1),xlabel('Horizontal shear stress, \tau (kPa)'),grid minor,legend('Recorded','Corrected','location','SE')
% subplot(3,1,3)
% plot(time, Equi_PWP,'linewidth',1),xlabel('Equivalent pore water pressure, \Delta u (kPa)'),grid minor,legend('Recorded','Corrected','location','SE')
figure('Name','General Strain-Stress Loops','NumberTitle','off'), clf, % To plot general loops
plot(Horz_StrainEdit,Horz_StressEdit,'linewidth',1),xlabel('Shear strain, \(^\gamma\) (%), grid %,grid minor,%legend('Recorded','Corrected','location','SE')
title({['Test: Kao12;  Soil: Kaolinite Clay;  PI=28 (MH); e=',num2str(Initial_Void_Ratio,3),'; OCR=',num2str(OCR,2),'; f=',num2str(Freq(1),2),'; Hz']});
figure('Name','General Normalized Time histories','NumberTitle','off'),clf, % To plot averaged time history and loops
subplot(3,1,1)
plot(time, Horz_Strain,'linewidth',1),xlabel('Time (s)'),ylabel('Shear strain, \(^\gamma\) (%), grid %,grid minor,%legend('Recorded','Corrected','location','SE')
title({['Test: Kao12;  Soil: Kaolinite Clay;  PI=28 (MH); e=',num2str(Initial_Void_Ratio,3),'; OCR=',num2str(OCR,2),'; f=',num2str(Freq(1),2),'; Hz']});

text(250,.1,[\(\gamma_c=\),num2str(Strains(1),2),'%']),'FontSize',7),
text(1350,.12,[num2str(Strains(2),2),'%']),'FontSize',7),
text(2400,.12,[num2str(Strains(3),2),'%']),'FontSize',7),
text(3600,.15,[num2str(Strains(4),2),'%']),'FontSize',7),
text(4600,.2,[num2str(Strains(5),2),'%']),'FontSize',7),
text(5700,.25,[num2str(Strains(6),2),'%']),'FontSize',7),
text(6800,.75,[num2str(Strains(7),2),'%']),'FontSize',7)
subplot(3,1,2)
plot(timeEdit, Norm_Horz_StressEdit,'linewidth',1),xlabel('Time (s)'),ylabel('Normalized horizontal shear stress, \(\tau^* = \tau / \sigma_{v_c}\)');
grid %,grid minor,%legend('Recorded','Corrected','location','SE')
subplot(3,1,3)
plot(time, Norm_Equi_PWP,'linewidth',1),xlabel('Time (s)'),ylabel('Normalized equivalent pore water pressure, \(\Delta u^* = \Delta u / \sigma_{v_c}\)');
grid %,grid minor,%legend('Recorded','Corrected','location','SE')
figure('Name','General Normalized Strain-Stress Loops','NumberTitle','off'),clf, % To plot General Normalized loops
plot(Horz_StrainEdit,Norm_Horz_StressEdit,'linewidth',1),xlabel('Shear strain, \(^\gamma\) (%), grid %,grid minor,%legend('Recorded','Corrected','location','SE')
title({['Test: Kao12;  Soil: Kaolinite Clay;  PI=28 (MH); e=',num2str(Initial_Void_Ratio,3),'; OCR=',num2str(OCR,2),'; f=',num2str(Freq(1),2),'; Hz']});

clear timeZerod;
clear Horz_StrainZerod;
clear Horz_StressZerod;
clear Equi_PWPEnd;
clear Norm_Equi_PWPEnd;
clear Norm_Horz_StressZerod;
Xmin=IndexNew(NumStage,1)-L4(NumStage)-50;Xmax=IndexNew(NumStage,NumCyc(NumStage)*2)+L4(NumStage); %50 is just to get zero before and after the cycles

clear timeZerod;
for kk=Xmin:Xmax;
timeZerod(kk-Xmin+1)=timeEdit(kk)-timeEdit(Xmin);
Horz_StrainZerod(kk-Xmin+1)=Horz_StrainEdit(kk)-Horz_StrainEdit_Zero;

NumStage=6; %The stage we are interested to plot its data
...
Horz_StressZerod(kk-Xmin+1)=Horz_StressEdit(kk)-Horz_StressEdit_Zero;
Norm_Horz_StressZerod(kk-Xmin+1)=Norm_Horz_StressEdit(kk)-Norm_Horz_StressEdit_Zero;
Equi_PWPZerod(kk-Xmin+1)=Equi_PWPEdit(kk)-Equi_PWPEdit_Zero;
Norm_Equi_PWPZerod(kk-Xmin+1)=Norm_Equi_PWPEdit(kk)-Norm_Equi_PWPEdit_Zero;
end
Out1=[timeZerod' Horz_StressZerod'];
figure('Name','Each Stage Corrected','NumberTitle','off'),clf,
subplot(3,1,1)
plot(timeZerod, Horz_StrainZerod,'linewidth',2),xlabel('Time (s)'),ylabel('Shear strain, \gamma (%)'),grid ,grid minor,
title({['Test: Kao12;  Soil: Kaolinite Clay;', ' e=',num2str(Initial_Void_Ratio,3),';
  w=',num2str(Initial_Water_Content,4),'%],[\sigma^v, \nu, c', '=', num2str(Vert_Stress_Initial,4),';
  OCR=',num2str(OCR,4),'; f=',num2str(Freq(1),2),' Hz']});
subplot(3,2)
plot(timeZerod, Horz_StressZerod,'linewidth',2),xlabel('Time (s)'),ylabel('Horizontal shear stress, \tau (kPa)'),grid minor,
subplot(3,3)
plot(timeZerod, Equi_PWPZerod,'linewidth',2),xlabel('Time (s)'),ylabel('Equivalent pore water pressure, \Delta u (kPa)'),grid minor,
figure('Name','Strain-Stress Loops for Each Stage','NumberTitle','off'),clf,
subplot(3,1,1)
plot(Horz_StrainZerod,Horz_StressZerod,'linewidth',2),xlabel('Shear strain, \gamma (%),'),ylabel('Horizontal shear stress, \tau (kPa)'),grid ,grid minor,
title({['Test: Kao12;  Soil: Kaolinite Clay;', ' e=',num2str(Initial_Void_Ratio,3),';
  w=',num2str(Initial_Water_Content,4),'%],[\sigma^v, \nu, c', '=', num2str(Vert_Stress_Initial,4),';
  OCR=',num2str(OCR,4),'; f=',num2str(Freq(1),2),' Hz']});
subplot(3,3)
plot(Horz_StrainZerod,Norm_Horz_StressZerod,'linewidth',2),xlabel('Shear strain, \gamma (%),'),ylabel('Normalized horizontal shear stress, \tau^* = \tau / \sigma^v'),grid minor,
figure('Name','Normalized Strain-Stress Loops for Each Stage','NumberTitle','off'),clf,
subplot(3,1,1)
plot(Horz_StrainZerod,Norm_Horz_StressZerod,'linewidth',2),xlabel('Shear strain, \gamma (%),'),ylabel('Normalized horizontal shear stress, \tau^* = \tau / \sigma^v'),grid minor,
figure('Name','Normalized Strain-Stress Loops for Each Stage','NumberTitle','off'),clf,
Vert_Stress_Init1=Vert_Stress(1);
Vert_Stress_Init2=Vert_Stress(2);
for kk=2:Num_Test

Vert_Stress_Init1=Vert_Stress_Init1+Vert_Stress(range(kk-50:range(kk)-10));
end

Vert_Stress_Init2=Vert_Stress_Init2+Vert_Stress(range(kk-50:range(kk)-10));
end
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