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Theoretical Estimates of the Rates of Radioactive Decay of Radium Isotopes by $^{14}$C Emission

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Abstract: The measured branching ratios for the decays of $^{222,223,224}$Ra by alpha or $^{14}$C emissions can be accounted for within a factor of 10 in terms of the ratios of Gamow penetrabilities through potential-energy barriers consisting of a Coulomb repulsion, the nuclear proximity attraction and an interpolation between the configuration of tangent fragments and the configuration of the parent nucleus.

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In this note we would like to point out that, in the spirit of ref. 7, the branching ratios between α-particle and \(^{14}\text{C}\) radioactivity, reported in ref. 1 (see also refs 2,3), can be interpreted with reasonable accuracy (within a power of 10, or so) by a quantal tunneling calculation, provided a realistic estimate of the potential-energy barrier is used.

To construct the deformation-energy barrier of a Ra nucleus disintegrating into a pair of fragments (Rn + α or Pb + \(^{14}\text{C}\)), we added to the Coulomb repulsion between the fragments the nuclear proximity attraction from refs. 2,3. After contact of the fragments, when the approximation of two spherical fragments ceases to be applicable, we used for the deformation energy a smooth, power-law interpolation between the contact configuration and the configuration of the parent nucleus (where the deformation energy is zero by definition). The explicit formula for the deformation energy \(V(L)\) is thus as follows:

\[
V(L) = -Q + \frac{Z_1 Z_2 e^2}{r} + V_p(z) \quad \text{for } L > L_c \quad (1)
\]

\[
V(L) = a (L - L_0)^\nu \quad \text{for } L_0 < L < L_c \quad . \quad (2)
\]

In the above, \(L\) is the major axis (i.e. the overall length) of the configuration in question, \(L_c\) refers to its value at contact (equal to the sum of the fragment diameters), \(L_0\) is the diameter of the parent nucleus, \(Q\) is the energy released in the disintegration, \(Z_1\) and \(Z_2\) are the atomic numbers of the fragments, \(r\) is the distance between fragment centers, \(z\) is the distance between the near surfaces of the fragments and \(V_p\) is the proximity potential, given by
\[ V_p(z) = K \phi(z/b) \quad , \quad (3) \]

where

\[ K = 4 \pi R_Y b \quad (4) \]

and \( \phi \) is the universal nuclear proximity function of ref. 2, to which an approximation, given in ref. 3, reads as follows:

\[ \phi(\zeta) \approx -4.41e^{-\zeta/0.7176} \quad \text{for } \zeta \geq 1.9475 \quad (5) \]

\[ \phi(\zeta) \approx -1.7817 + 0.9270\zeta + 0.01696\zeta^2 - 0.05148\zeta^3 \]

\[ \quad \text{for } 0 \leq \zeta \leq 1.9475 \quad (6) \]

In the above, \( \zeta = z/b \), \( b \) is the width (diffuseness) of the nuclear surface \((b \approx 1 \text{ fm})\) and \( \gamma \) is the specific nuclear surface tension, for which we used the expression given in ref. 2:

\[ \gamma = 0.9517 [1 - 1.7826 \left( \frac{N-Z}{A} \right)^2] \text{MeV/fm}^2 \quad , \quad (7) \]

where \( N, Z, A \) are the neutron, proton and mass numbers of the parent nucleus. The reduced radius \( \overline{R} \) is given by

\[ \overline{R} = C_1 C_2 / (C_1 + C_2) \quad , \quad (8) \]

where \( C_i \) are the central radii of the fragments, related to the effective sharp radii \( R_i \) by
Reference 2 gives the following semi-empirical formula for $R$ in terms of the mass number $A$

$$R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3}$$

Disregarding ground-state deformations, the value of $L_0$ in eq. 2 is given by $2C$, where $C$ is the central radius of the parent nucleus, calculated also according to eqs. 9,10.

The requirement of a smooth fit for $V(L)$ at $L = L_c$ defines the coefficients $a$ and $v$ in eq. 2 as

$$v = V'_c/(L_c - L_0)V_c$$

$$a = V'_c/(L_c - L_0)^v$$

where the subscript "c" refers to contact, and

$$V'_c = \frac{dV}{dL} \bigg|_c = -\frac{Z_1Z_2e^2}{r^2_c} + K \frac{d}{dz} \phi(z/b) \bigg|_c$$

$$= -\frac{Z_1Z_2e^2}{r^2_c} + 0.9270 \frac{K}{b}$$

Here, $r_c$ is the center separation at contact and 0.9270 is the dimensionless derivative of $\phi$ at $\zeta = 0$. 

$$C \approx R - \frac{b^2}{R}$$
The standard WKB expression for the Gamow penetrability factors was used both for the $\alpha$ particle and $^{14}$C, the effective mass in the disintegration degree of freedom being taken simply as the reduced mass $M_r$ of the fragments, since most of the barrier (also in the case of $^{14}$C) corresponds to separating fragments beyond scission.

The above formulae, including all relevant nuclear parameters, were taken from refs. 2,3, without any adjustments. The resulting ratios of the penetrability factors for $\alpha$ and $^{14}$C emissions from $^{222}$Ra, $^{223}$Ra and $^{224}$Ra were found to be $1.678 \times 10^{-9}$, $6.903 \times 10^{-9}$ and $6.147 \times 10^{-11}$, respectively. The ratios of these numbers to the measured branching ratios are 4.5, 11.3 and 0.83. Since the Gamow penetrability factors (for $^{14}$C) are in the range $10^{32} - 10^{38}$, agreement within about a factor of 10 implies an accuracy in the estimated deformation-energy barriers of a few percent.

The reasons why, in the present calculations, the penetrability ratios are several orders of magnitude smaller than for a pure Coulomb barrier cut off at a contact distance parameterized as $r_0(A_1^{1/3} + A_2^{1/3})$, as in refs. 1,4, are the inclusion of the nuclear proximity interaction and the use of more realistic expressions for the nuclear radii (eqs. 9,10).

A fuller account of these calculations, (ref. 6), including estimates of branching ratios for other exotic decays, will be published separately.

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