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K_{e4} DECAYS AND THE DETERMINATION OF LOW-ENERGY PION-PION PHASE SHIFTS

Nicola Cabibbo and Alexander Maksymowicz

June 16, 1964
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LOW-ENERGY PION-PION PHASE SHIFTS

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I. INTRODUCTION

In this paper we discuss the angular correlations in the decay
K⁺ → π⁺π⁻e⁺ν and their relevance to the determination of π-π phase shifts.

We also give the results of a model calculation of these correlations.

II. ANGULAR CORRELATIONS

We visualize the reaction K⁺ → π⁺π⁻e⁺ν as a two-body decay into a
dipion of mass M_{ππ} and a dilepton of mass M_{eν}. We then consider the sub-
sequent decay of each of these two "particles" in its own c.m. (center-of-mass
system).

The total decay is described by the following five variables: (Fig. 1):

M_{ππ} = (- R^2)^{1/2} = the effective mass of the two-pion system;
M_{eν} = the effective mass of the two-lepton system;
θ = the angle of the π⁺ in the c.m. of the two pions with respect to
the line of flight of the dipion in the K⁺ rest system;
ζ = the angle of the e⁺ in the c.m. of the leptons with respect to
the line of flight of the dilepton in the K⁺ rest system;

ϕ = the angle between the plane formed by the two pions in the K⁺
rest system and the corresponding plane formed by the leptons; it has the
same sign as (p⁺ × p⁻) · p_e, where p⁺, p⁻, and p_e are the three-momenta
of the π⁺, π⁻, and e⁺ in the K⁺ rest system.
We consider the following four correlations:

(a) The distribution in $\cos \theta$ or the forward-backward asymmetry of the $\pi^+$ in the pion c.m.

(b) The distribution in $\sin \phi$ or the up-down asymmetry of the positron with respect to the plane formed by the two pions in the $K^+$ rest system.

(c) The distribution in $\cos \phi$ or the right-left asymmetry of the positron with respect to the plane formed by the line of flight of the dipion and the normal to the plane of the pions.

(d) The distribution in $R^2$ or the $\pi^-\pi^+$ effective mass spectrum.

III. MATRIX ELEMENT

We set the positron mass equal to zero and write the matrix element as

$$\langle G/\sqrt{2} \rangle \left[ \nabla \gamma (1 + \gamma_5) e \right] \left[ f(R^2) (p_+ + p_-) + g(R^2) (p_+ - p_-) \right]
+ h(R^2) \epsilon_{\lambda \mu \nu \sigma} p_+^\mu p_+^\nu p_-^\sigma \right] .$$

In this expression, $G$ is the universal Fermi coupling constant for weak interactions; $p^+_{K^+}, p^+_\pi^+, p^-_{\pi^-}$ are the four-momenta of the $K^+, \pi^+, \pi^-$; and $f(R^2), g(R^2), h(R^2)$ are form factors. The rest of the notation is obvious.

The $f$ and $g$ terms come from the axial current of the strongly interacting particles, while the $h$ term comes from the vector current. In writing $f, g, h$ as functions of $R^2$ we have assumed that the pions are emitted only in $I = 0, \ell = 1$ relative angular momentum states. If we further assume that the $|(\Delta I)| = 1/2$ rule is valid, then from symmetry considerations the $f$ term is just the amplitude for the pions to be emitted in an $I = 0, \ell = 0$ state, while the $g$ and $h$ terms give the amplitude for emission in an $I = 1, \ell = 1$ state.
The final-state interaction of the pions has a twofold effect. First of all, by the final-state theorem of Fermi, it defines the phases of \( f, g, \) and \( h \) as follows:

\[
f = |f| \exp[i\delta_0(R^2)] , \quad g = |g| \exp[i\delta_1(R^2)] , \quad h = |h| \exp[i\delta_1(R^2)].
\] (2)

The quantities \( \delta_0 \) and \( \delta_1 \) are the \( I = 0, J = 0 \) and \( I = 1, J = 1 \) \( \pi-\pi \) scattering phase shifts; they are functions of the c.m. energy of the pions. In the second place, the \( \pi-\pi \) interaction gives rise to enhancement effects, which are assumed to determine the \( R^2 \) dependence of \(|f|, |g|, \) and \(|h|\).

IV. ANGULAR DISTRIBUTION

Integrating the decay probability obtained from (1) over \( M_{\pi^0} \) and \( \cos \xi \), we obtain the following expression for the differential decay rate:

\[
\frac{dI(K^+ \rightarrow \pi^+\pi^-e^+\nu)}{dR^2 \cos \theta d\phi} = R^2 \cos \theta \sin^2 \phi
\times \{ |f|^2 a(R^2) + |g|^2 [b(R^2) + c(R^2) \cos^2 \theta + d(R^2) \sin^2 \theta \sin^2 \phi ]
+ |h|^2 e(R^2) \sin^2 \theta (1 + 2 \cos^2 \phi) + |f| |g| \cos(\delta_0 - \delta_1)s(R^2) \cos \theta
+ |f| |g| \sin(\delta_0 - \delta_1) t(R^2) \sin \theta \sin \phi + |f| |h| \cos(\delta_0 - \delta_1) u(R^2) \sin \theta \cos \phi
+ |g| |h| \nu(R^2) \cos \theta \sin \theta \cos \phi \}.
\] (3)

The quantities \( a, b, c, d, e, s, t, u, \) and \( \nu \) are well-defined functions of \( R^2 \).

An examination of Eq. (3) shows that the \( \pi^+ \) forward-backward asymmetry arises from the \( s \) term; the positron up-down asymmetry from the \( t \) term; the positron right-left asymmetry from the \( u \) term; and the \( \pi-\pi \) effective mass spectrum from the \( a, b, c, d, \) and \( e \) terms.

By measuring angular distributions in \( \theta \) and \( \phi \) at a fixed value of the pion c.m. energy, one can obtain \( \tan(\delta_0 - \delta_1) \) at that energy from the ratio of the up-down to forward-backward asymmetries, without having to know the values of \(|f|, \) and \(|g|\). Furthermore, it is clear from Eq. (3) that the various
correlations can, in principle, also be used to determine $|f|$, $|g|$, and $|h|$ at any given pion c.m. energy.

In view of poor experimental statistics it will be more practical to look at the angular correlations averaged over $R^2$. Even though one lacks knowledge of the dependence of $|f|$ and $|g|$ on $R^2$, one should still be able in this manner to get an idea of the sign and magnitude of the average value of $(\delta_0-\delta_1)$ in the energy region under consideration.

V. MODEL CALCULATION

An alternative approach would be to attempt a fit of the experimental data to a given model for $f$ and $g$. For example, one can assume that the behavior of $f$ is dominated by a large scattering length and that $g$ has a constant value $g_0$. (One would expect $g$ to be dominated by the rather distant $\rho$ singularity.) The $R^2$ dependence of $f$ should then be well described by the relativistic Watson enhancement factor:

$$f \propto \frac{1}{\beta} \sin \delta_0 \exp(i\delta_0)$$

where $\beta$ is the velocity of either pion in the pion c.m. The energy dependence of the phase shift is assumed to be given by the Chew-Mandelstam effective range formula:

$$\cot \delta_0 = \frac{1}{\beta a_0} + \frac{2}{\pi} \log \left\{ (R^2)^{1/2}/2m_\pi \right\} (1+\beta),$$

where $a_0$ is the $\pi-\pi$ scattering length for the $I=0$, $J=0$ state in units of the pion Compton wavelength. One can then proceed to find that value of the parameter $a_0$ which best fits the experimental data.

With this model we obtain the angular distribution averaged over $R^2$ in the form
\[
\frac{d^2T}{d\cos \theta d\phi} = 10^{-5} \frac{G^2 \pi^2 M_K^5}{8(2\pi)^8} f_0^2 \{ A_1 x A_2 \cos \theta + x A_3 \sin \theta \sin \phi + x^2 A_4 \cos^2 \theta + x^2 A_5 \{ 1 + 2 \sin^2 \theta \sin^2 \phi \} \} ,
\]

where \( f_0 = f(R^2 = -4m_\pi^2) \) and \( x = g_0/f_0 \). The values of \( A_1 \), \( A_2 \), \( A_3 \), \( A_4 \), and \( A_5 \) are given in Table I as functions of \( a_0 \).

Table I. The coefficients in the angular distribution for the decay
\( K^+ \rightarrow \pi^+ \pi^- e^+ \nu \) as functions of the \( I = 0, j = 0 \) \( \pi-\pi \) scattering length, \( a_0 \).

<table>
<thead>
<tr>
<th>( a_0 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
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<td>4.67</td>
<td>0</td>
<td>1.26</td>
<td>0.23</td>
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<td>1</td>
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<td>0.23</td>
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<td>1.26</td>
<td>0.23</td>
</tr>
<tr>
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<td>1.26</td>
<td>0.23</td>
</tr>
<tr>
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<td>0.23</td>
</tr>
<tr>
<td>5</td>
<td>0.61</td>
<td>0.94</td>
<td>-0.62</td>
<td>1.26</td>
<td>0.23</td>
</tr>
</tbody>
</table>
FOOTNOTES AND REFERENCES

*Work performed under the auspices of the U. S. Atomic Energy Commission.


2. Considering only the states \( f = 0, 1 \) is equivalent to neglecting the dependence of \( f, g, \) and \( h \) on \( (p_K p_+ \) and \( (p_K p_-. \) This is reasonable, since the singularities in these variables are far from the physical region.


4. In this section we ignore the contribution of the vector current, which should be very small compared with that of the axial current.

5. A large \( I = 0, f = 0 \) \( \pi-\pi \) scattering length is suggested by the results of N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters 7, 35 (1961).


Fig. 1. Angular variables for the decay $K^+ \rightarrow \pi^+ \pi^- e^+ \nu$.

The range of the variables is: $0 \leq \theta, \xi \leq \pi$, $-\pi \leq \phi \leq \pi$. 
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