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ISBN
978-0-7918-5713-7

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Publication Date
2016

DOI
10.1115/DETC201547723

Peer reviewed

Proceedings of the ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2015
August 2-5, 2015, Boston, Massachusetts, USA

DETC2015-47723

SYNTHESIS OF A 10-BAR LINKAGE TO GUIDE THE GAIT CYCLE OF THE HUMAN LEG

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ABSTRACT

This paper uses path synthesis techniques to design four-bar linkage modules to constrain the movement of a 3R chain. The result is a 10-bar linkage. The goal is to develop a design procedure for a robotic system that guides the human leg during the walking gait cycle. A 3R chain is designed to match the dimensions of a human leg and the two four-bar linkages are synthesized using 9 point path synthesis to constrain the trajectory of the ankle and the toe. Precision points are derived from a basis spline equation. A numerical example is given using data collected from a motion capture system.

INTRODUCTION

In this paper, we present a synthesis procedure for a 10-bar linkage that is intended to guide and support the human leg during the walking gait cycle. The goal is to use linkage synthesis techniques to design a mechanism to support the cyclic motions required in robotic rehabilitation and exoskeleton applications. The design of this particular 10-bar linkage begins with a serial 3R chain that is then constrained by two four-bar linkages. The four-bar linkages are designed using a combination of path synthesis and basis spline techniques. The desired path of the linkages is gathered from joint locations of the leg during the gait cycle.

Many of the recent robotic rehabilitation devices and exoskeletons have many degrees of freedom and require a large collection of actuators. The Pelvic Assist Manipulator (PAM) and the Pneumatically Operated Gait Orthosis (POGO) utilize linear actuators that are pneumatically powered to move a user's pelvis and legs during treadmill training. This work was done by Aoyagi et al. and Ichinose et. al [1] - [4]. The Active Leg Exoskeleton (ALEX), by Banala et al. [5] - [7], is another powered orthosis that has multi-DOF and attaches to the trunk, thigh, and foot. Other devices that are relatively similar in function are the Ambulation-assisting Robotic Tool for Human Rehabilitation (ARTHuR), by Emken et al. [8], the Lokomat by Jezernik et al. and Klobucka et al. [9] - [11], and the Lower Extremity Powered Exoskeleton (LOPES), by Reinger et al. [12]. All of which require elaborate control theory.

Another area of existing research is in single-DOF walking mechanisms and can provide a basis for the design of similar linkages for robotic rehabilitation. The Theo Jansen linkage is a walking linkage that was implemented on a wind powered kinetic sculpture. This linkage has been studied by Aan and Heinloo, Giesbrech et al., Komoda and Wagatsuma [13]- [16]. The Klann linkage is another patented walking mechanism that is often used on multi-legged robots. Lockhande and Emche [17] use this linkage for the legs of a mechanical spider. Both of these linkages create locomotion, but don't follow the ankle trajectory of the human gait.

Our goal is to extend the work by Tsuge and McCarthy [18] by constraining a 3R chain with four bar linkages using path synthesis. Similar procedures for constraining serial chains have

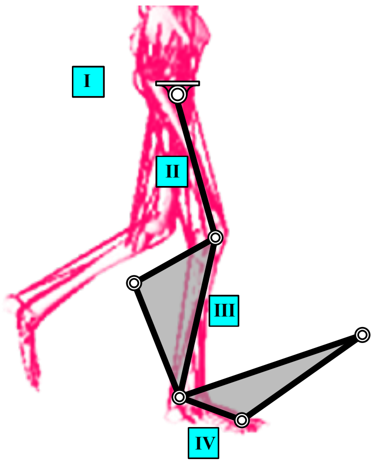

FIGURE 1. 3R SERIAL CHAIN

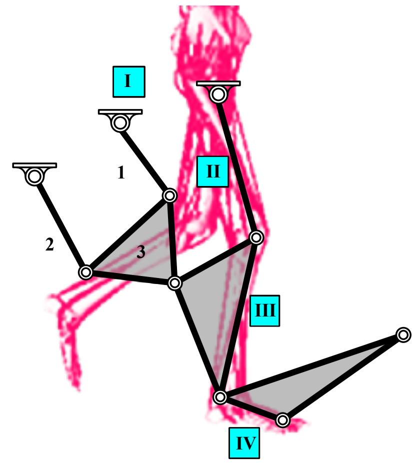

FIGURE 2. 3R CHAIN WITH ONE FOUR-BAR MODULE

also been presented by Soh and McCarthy [19] - [20] for motion generation. The path synthesis procedure for the four-bar linkage modules follows that of Wampler, Morgan, and Sommese [21] and the selection of precision points is completed using basis splines, similar to Unruh and Krishnaswami [22]. The result will be a linkage that forces the leg to move in the plane, just as it does in the LOKMAT rehabilitation device.

THE CONSTRAINED 3R CHAIN

The first step of the design procedure is to define an open serial chain. The joints and link lengths correspond with the dimensions of human leg, as shown in figure (1). The fixed pivot, between link I and II, corresponds with the hip joint, and the joints attached to links II, III, and IV correspond with the knee, ankle, and toe respectively. The designer also has the freedom of defining two additional joint locations so that links III and IV become triangular links. Assuming that the ankle joint locations and the foot angle are known, inverse kinematics can be used to determine the joint angles and locations.

Next a four-bar linkage is attached to link III of the serial chain to constrain the motion of the ankle, figure (2). The fixed pivots of this linkage and the fixed pivot of the 3R chain would all be connected to link I. Next an additional four-bar linkage is coupled to link IV to constrain the angle of the foot, as shown in figure (3). The ground pivots of this linkage are attached to link II. Since the ground pivots of this linkage are moving relative to link II, the coordinates of the precision points, for the path synthesis problem, must be in the same frame at link II. The result of constraining the serial chain with two separate four-bar linkages, using path synthesis, is a 10-bar linkage.

The path synthesis method used follows that of Wampler [21], where the links of the four-bar linkage are represented as a collection of vectors in complex form. The resulting formulation of the design equations consists of 12 equations with 12 unknown variables. This method requires that nine precision points are defined.

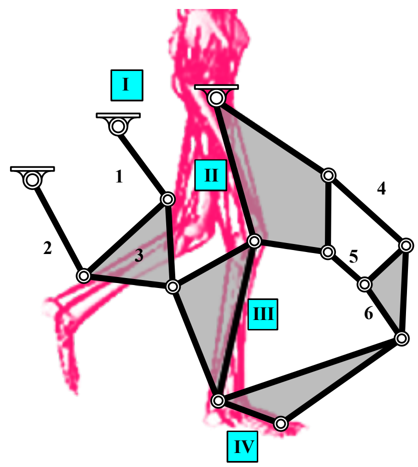

FIGURE 3. 10-BAR LINKAGE

SELECTION OF PRECISION POINTS

The precision points that are used in the path synthesis algorithm are derived from a set of data points collected from motion capture data of a walking subject. This data set marks the joint locations through 14 gait cycles. From this data, the lengths of the upper leg, lower leg, and foot can be determined. These lengths are determined by measuring the distances between the hip and knee, knee and ankle, and ankle and toe data points respectively. The motion capture data collected was in 3 dimensions, but only data in 2 dimensions were used in this procedure for simplicity. Figure (4) shows the coordinates of the ankle during multiple walking cycles; units are in centimeters. These points, however, were collected in the global frame, where the hip joint is moving. Since, the synthesis procedures require that the hip joint be stationary, a new set of data points relative to the hip is required. The coordinates of the ankle trajectory relative to the hip joint is shown in figure (5); the location of the hip joint was also moved to the origin for simplicity.

Next, one of the cycles was chosen to be the desired path for the synthesis procedure. This path consisted of 205 data points; since 9 points are required for the four-bar path synthesis, basis splines techniques were used. The trajectory created by the 205 data points can be represented by the equation of a basis spline. This is a parametric equation and the data points are used as the control points.

The parametric equation for a spline consists of a basis equation, as shown in equations 1 and 2, where t is the parameter of the curve, k is the order of the curve, i is the ith control point, and $\mathbf{x}_i$ are elements of the knot vector. The knot vector deals with the weighting of a particular control point.

$$N_{i,1}(t) = \begin{array}{l} 1 \ \text{if} \ \mathbf{x}_i \leq t < \mathbf{x}_{i+1} \\ 0 \ \text{otherwise} \end{array} \tag{1}$$

$$N_{i,k}(t) = \frac{(t - \mathbf{x}_i)N_{i,k-1}(t)}{\mathbf{x}_{i+k-1} - \mathbf{x}_i} + \frac{(\mathbf{x}_{i+k} - t)N_{i+1,k-1}(t)}{\mathbf{x}_{i+k} - \mathbf{x}_{i+1}} \tag{2}$$

These basis equations are then used to form the parametric equation for the coordinates of the basis spline curve evaluated at t_j, where j is the jth point along the curve. Equations 3 and 4 are the parametric equations that yield the coordinates of the spline curve for a given parameter t.

$$Px(t_j) = \sum_{i=0}^{n-1} Px_i N_{i,k}(t_j) \tag{3}$$

$$Py(t_j) = \sum_{i=0}^{n-1} Py_i N_{i,k}(t_j) \tag{4}$$

The 205 data points of the ankle trajectory can then be used as the control points in the B-spline equation so that the ankle trajectory can be represented by a single parametric equation. Nine values of t, between 0 and 1, were selected and substituted into the B-spline equations; these values of t were evenly distributed. The result was 9 points that were distributed about the curve.

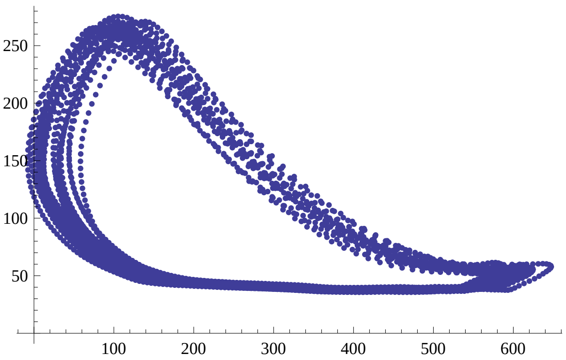

FIGURE 4. COORDINATES OF THE ANKLE DURING MULTIPLE GAIT CYCLES

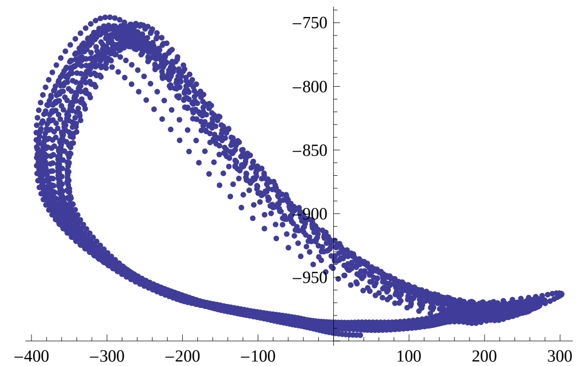

FIGURE 5. COORDINATES OF THE ANKLE RELATIVE TO THE HIP JOINT

The selected data points have a high density at the bottom of the curve because there was a higher density of data points in this area. With these 9 points of the ankle, the 9 precision points for the first four-bar linkage problem can be determined, as shown in figure (6).

This same procedure can also be used to choose precision points for the second four-bar linkage. However, these points will be in the same frame as the upper leg, or link II from figure (3).

SORTING SOLUTIONS

The solutions resulting for solving the 9 point path synthesis problem yield the joint locations, **O**, **A**, **B**, and **C**, shown in figure (7). The values of a, b, g, h, r, α, and θ_0 can be calculated from these joint locations. The equations for the coordinates of the

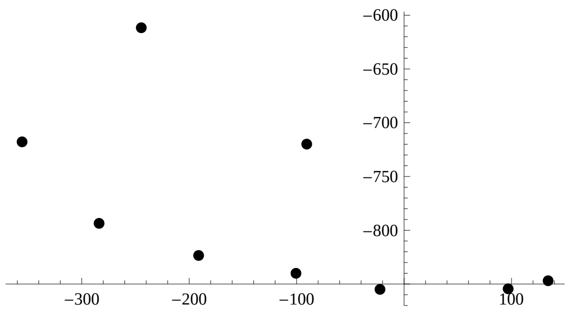

FIGURE 6. 9 PRECISION POINTS DERIVED FROM THE BASIS SPLINE

coupler point X are given by

$$x_0 + a\cos(\theta+\theta_0) + r\cos(\alpha+\theta+\theta_0+\phi) \qquad (5)$$
$$y_0 + a\sin(\theta+\theta_0) + r\sin(\alpha+\theta+\theta_0+\phi),$$

where, x_0 and y_0 are the coordinates of point **O**. The equation for ψ is given below [23].

$$\psi = \arctan\left(\frac{B}{A}\right) \pm \arccos\left(\frac{C}{\sqrt{A^2+B^2}}\right), \qquad (6)$$

where

$$A = 2ab\cos\theta - 2gb, \qquad (7)$$
$$B = 2ab\sin\theta$$
$$C = g^2+b^2+a^2-h^2-2ag\cos\theta.$$

Also, the equation for ϕ is

$$\phi = \arctan\left(\frac{b\sin\psi + a\sin\theta}{g+b\cos\psi - a\cos\theta}\right) - \theta. \qquad (8)$$

When the known variables are substituted into the equations for the coordinates of **X**, the equations become a parametric equation that varies with θ. A parametric plot of this function will produce the coupler curve of the linkage.

The coupler curves for all of the solutions are compared with the desired trajectory. Solutions that deviate too much from the desired path are eliminated. In addition, linkages that do not satisfy the Grashof condition are also removed.

Lastly, the values of the selected linkage can then be randomized about a relatively small tolerance zone, in order to increase the number of potential linkage solutions. The new set of randomized solutions is also compared with the original set of precision points. The overall process is repeated for the second four-bar linkage in the moving frame.

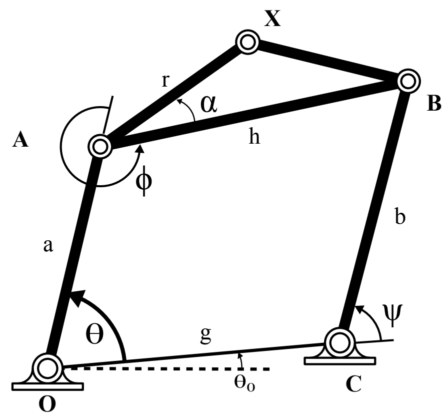

FIGURE 7. THE FOUR-BAR LINKAGE

TABLE 1. NINE PRECISION POINTS FOR THE FIRST FOUR-BAR LINKAGE

Point	x	y
1	-22.4077	-854.975
2	-100.615	-840.025
3	-191.23	-823.445
4	-283.884	-793.584
5	-355.541	-717.779
6	-244.59	-611.663
7	-90.6067	-719.892
8	134.077	-846.936
9	96.8837	-854.441

NUMERICAL EXAMPLE

The design procedure was applied to a single cycle of the gait cycle. The desired trajectories were acquired from a motion capture system. The two sets of precision points, derived from the b-splines, for the four-bar linkage problem are in tables 1 and 2. For the 3R chain, the hip joint was set at the origin and the lengths of the upper leg, lower leg, and foot were set to 510.908, 526.941, and 184.343 respectively. Figure (8) illustrates this chain. Plots of these two sets of precision points are shown in figures (6) and (9).

The two path synthesis problems were solved using the polynomial solver, Bertini [24]. What resulted were 240 solutions for the first four-bar linkage and 228 solutions for the second four-bar linkage. Figure (10) shows the coupler curve of a selected four-bar linkage. The linkage parameters were then randomized about a tolerance of 100cm, meaning that there was an allowed variance of the linkage dimensions in either direction of 100cm. A new linkage was found that had a more gradual curve, figure (11). Also, the linkage that was found for the second four-bar linkage followed an acceptable trajectory and did not require randomization. Figure (12) shows the coupler curve of this linkage

TABLE 2. NINE PRECISION POINTS FOR THE SECOND FOUR-BAR LINKAGE

Point	x	y
1	-74.7801	-488.537
2	-132.881	-479.35
3	-185.032	-463.913
4	-240.36	-433.901
5	-331.163	-352.701
6	-399.219	-258.914
7	-268.016	-408.732
8	13.3298	-486.262
9	-0.929189	-490.803

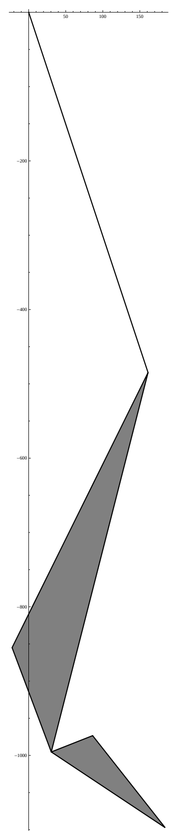

FIGURE 8. 3R CHAIN WITH TRIANGULAR LINKS

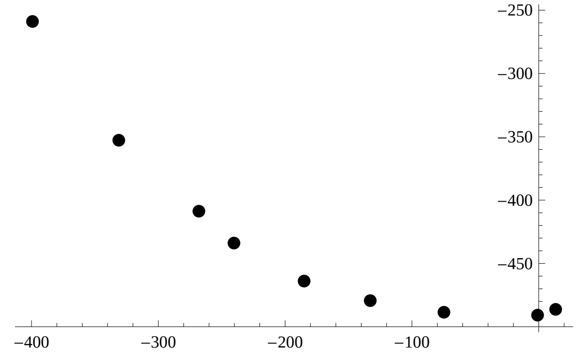

FIGURE 9. PRECISION POINTS FOR THE SECOND FOUR-BAR LINKAGE IN THE MOVING FRAME

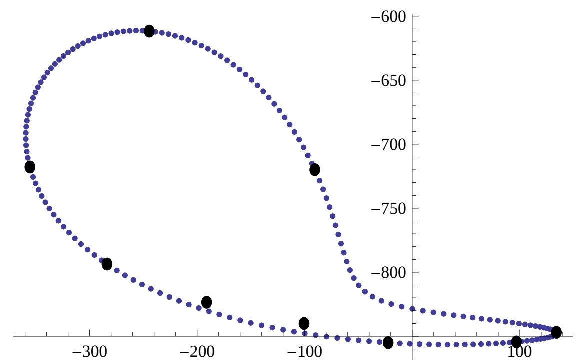

FIGURE 10. COUPLER CURVE OF THE FIRST FOUR-BAR LINKAGE

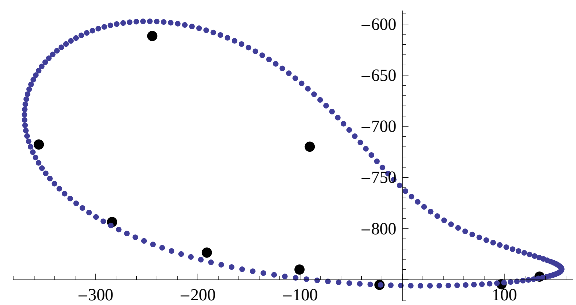

FIGURE 11. COUPLER CURVE OF THE RANDOMIZED FIRST FOUR-BAR LINKAGE

in the moving frame. The solutions for the first and second four-bar linkages are in tables 3 and 4.

The solutions that had the most desirable coupler cures are shown in figure (13). The first four-bar linkage is drawn in blue and the second linkage is drawn in red. While the resulting linkage had acceptable coupler curves, the packaging of the linkage did not. The overall size would make it difficult to attach this linkage to a human leg. Additional work still needs to be done to find solutions of reduced overall size. Also, while each of the four-bar linkages may be completely cyclic individually, the ten-bar linkage may not be; further work is required to find analysis techniques to determine if this is the case. Lastly, issues resulting from linkage singularities have yet to be addressed.

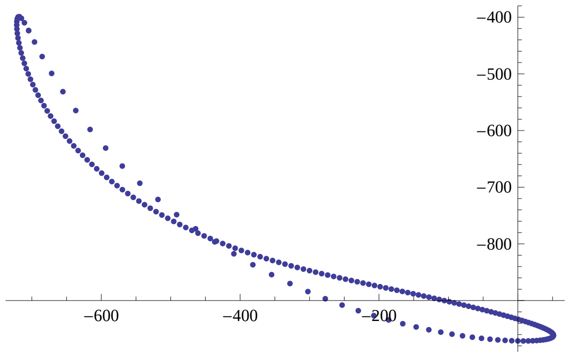

FIGURE 12. COUPLER CURVE OF SECOND FOUR-BAR LINKAGE

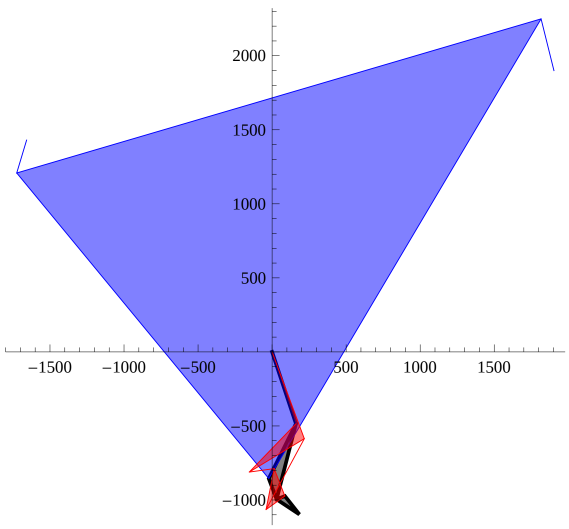

FIGURE 13. FINAL TEN-BAR LINKAGE SOLUTION

TABLE 3. First Four-Bar Solution

O	(-1657.82, 1430.38)
A	(-1724.49, 1208.35)
B	(1816.42, 2249)
C	(1903.3, 1899.64)

CONCLUSION

In this paper, we present a procedure to design a 10-bar linkage that is intended to guide the movement of the human leg during walking. The procedure consists of constraining a serial chain that matches the dimensions of a user's leg. The chain is constrained by two separate four bar linkages that are synthesized using path synthesis techniques.

The precision points for the path synthesis problem are acquired by deriving a basis spline equation from the desired leg trajectory. The nine points are selected by substituting 9, evenly distributed, values of the parameter into the basis spline equation. These 9 points are then used in the path synthesis problem and solved using a polynomial solver. Coupler curves of the resulting linkage solutions are compared with desired trajectories. Solutions are then randomized about a tolerance zone, in order to search for additional linkage candidates.

TABLE 4. Second Four-Bar Solution

O	(-154.111, -811.408)
A	(13.9757,-788.164)
B	(-40.8045, -1064.83)
C	(217.512, -585.897)

A numerical example was given using motion capture data. There were 240 solutions found for the first four-bar linkage and 228 solutions found for the second linkage. Additional solutions candidates could be found with further randomization of linkage parameters.

The design procedure provides a means to design a linkage that matches the gait pattern; the linkage can then be attached to a user's leg as an exoskeleton or rehabilitation device.

ACKNOWLEDGMENT

The authors gratefully acknowledge the support of the National Science Foundation award CMMI-1066082. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors, and do not necessarily reflect the views of the National Science Foundation.

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