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VIRTUAL DIVESTITURES, WILL THEY MAKE A DIFFERENCE?
COURNOT COMPETITION, OPTION MARKETS AND EFFICIENCY

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Abstract Antitrust authorities in Europe and the U.S. oblige dominant generators to virtually divest generation capacity as a way to mitigate market power. This paper analyzes the implementation of such a divestiture of Virtual Power Plants (VPPs), and distinguishes two types: financial VPPs, which are pure insurance contracts on the price for electricity, and physical VPPs, which are contracts for physical delivery of electricity. Our findings show that in a monopoly framework both contracts have the same outcomes whilst in an oligopoly setting both contracts have different effects on the strategic behavior of the players, affecting their competitiveness.

Keywords: Futures markets, Options markets, Cournot, Market power, Electricity, Arbitrage

JEL: C72, D43, G13, L13, L50, L94

1 INTRODUCTION

In the last few decades, several countries have liberalized electricity markets and organized competition in electricity generation. It was assumed that economies of scale and entry barriers in the generation sector were sufficiently small to make competition viable.

1 Financial support from the UC Energy Institute's Center for the Study of Energy Markets, the K.U.Leuven institute for energy transport and environmental economics ETE, and the Florence School of Regulation is gratefully acknowledged. The author is indebted to Severin Borenstein, Jim Bushnell, Lea-Rachel Kosnik, Steven Puller, Ana Rivas, Jens Weinmann, Giibert Zwart and the seminar participants in Berkeley, Boston, Leuven and Lodtz for their valuable comments.
In most countries the process of liberalization was accompanied by a structural reform of the electricity sector: firms were privatized and split up into smaller firms, in order to create a healthy and competitive market. On the European continent, a forced restructuring of the electricity industry did not take place. The rationale was that in a large integrated European market no generator would have a dominant position, and that it was therefore unnecessary to split up the former monopolists.

Integration of the European electricity market has however proceeded more slowly than expected for a number of reasons. International trade in electricity has developed very slowly due to, among other factors, the shortage of cross-border transmission capacity and the lack of harmonized rules. Politicians have often tried to create national champions, something which has not been beneficial to spur competition. Moreover, a wave of mergers has created a very concentrated electricity sector. As a result, abuse of market power has become a serious concern for the European Commission and for national electricity regulators.\(^2\)

The obvious response to these concerns would be to change the structure of the industry, i.e., split up firms into smaller entities. In the current political climate, such an approach seems to be impossible. Instead, regulators have to rely on virtual divestitures, i.e., they have to design a set of long-term contracts which mimic a real divestiture, but which do not require the legal transfer of ownership of assets.

The aim of this paper is to develop a theoretical framework to study virtual divestitures in a coherent way. We have three goals. The first goal (section 1) is to describe virtual divestitures, and to explain the difference between financial and physical Virtual Power Plants. The second goal (section 2) is to study the effect of a virtual divestiture on the spot price. The third goal (section 3) is to look at whether generators would like to virtually divest when they are not obligated by anti-trust authorities, and how much they would sell. In a follow-up paper we will study the practical implementation of virtual divestitures in Europe.

1.1 Market power and long run contracts in electricity markets

Market power is a serious issue in all but a few electricity markets because of the typical characteristics of the electricity market: the non-storability of electrical energy and the inelasticity of demand. Generators therefore often succeed in driving up prices significantly above competitive levels.\(^3\) This is especially true during periods with high demand and in areas which have weak transmission links with the rest of the market.

The amount of market power that is exercised depends both on the horizontal and the vertical structure of the electricity market. With the horizontal structure we mean, roughly speaking, the number of firms active in the generation sector. With the vertical structure we look at the contractual relation of generation and retail firms: a generation firm can integrate vertically with a retail firm, sign long term contracts with a retailer, or sell its energy directly on the spot market.

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\(^2\) The European Commission, DG competition, has published its “Preliminary Findings of the Energy Sector Inquiry” on 16 February 2006. In its preliminary report it “indicted the barriers currently impeding the development of a fully functioning open and competitive EU wide energy market”.

The effect of the horizontal market structure on market power is undisputed: if more firms are active in the market, then prices go down. The effect of vertical integration is less clear-cut.

Several studies have underlined the important role of the vertical market structure in electricity markets. Bushnell et al. (2004) compare different electricity markets in the US, and study both the vertical and horizontal market structure. They show that California had a relatively unconcentrated generation market in the summer of 2000 but that the lack of long-term contracts led to high price-cost margins. With long term contracts, generators sell part of their electricity ex-ante, at a locked-in price. As a result, generators will behave more competitively in the spot market\textsuperscript{4}.

The intuition is that of the durable goods’ monopolist in Coase’s conjecture (Coase, 1972). See Figure 1. Graph A shows the profit maximizing price $p^M$ for a monopolist who sells only in the spot market, has production costs $C(q)$ and faces an inverse demand function $P(q)$. The monopolist will set the price such that marginal revenue equals marginal cost. Graph B shows the same situation for a monopolist who signed long term contracts for $k$ units of electricity. In the spot market (the second stage), $k$ units will therefore disappear both at the demand and the supply side. The profit maximizing price then equals $	ilde{p}^M$ and is lower than $p^M$.

Figure 1 Effect of an ex-ante contract on the behavior of a monopolist

Historically, policy makers have been opposed to long term contracting. They feared that long term contracts between incumbent generators and retailers might slow down entry both in generation and in retail. They also assumed that long term contracting would decrease the transparency and the liquidity of the spot markets. Illiquid spot markets would lead to inefficient real time production decisions, and would also make entry more difficult.

Currently, policy makers are becoming more favorably inclined towards long term contracts. They hope that long term contracts will ease entry in the generation market by reducing the risk for entrants and will reduce market power in the spot market. Long term contracts will also help retailers who sell electricity at fixed regulated prices to hedge their price risks.

\textsuperscript{4} In the remainder of the text, we will use the term spot market for what is often called the day-ahead market in electricity markets. This is the market which resembles most closely to a real-time spot market.
Nowadays, the policy debate is whether the regulator should impose the usage of some long term contracts or whether generators and retailers will sign the right amount of long term contracts on their own. See for example Creti and Fabra (2004).

1.2 Virtual Power Plants

In this new philosophy of imposing long term contracts on incumbent generators to mitigate market power, several European regulators have relied upon a specific type of contract: Virtual Power Plants (VPPs). Such a system is currently used in several European countries. With a VPP, the incumbent generator sells part of its production capacity to other market participants. This divestiture of generation capacity remains virtual as no production capacity changes hand. Legally, the incumbent generator remains the owner of all its generation plants.

The buyer of the virtual production plants can use its virtual production capacity to generate electricity, but will need to pay the virtual production costs to the seller of the VPPs. These payments represent a refund for the variable production costs of the plant.

Regulators often prefer a virtual divestiture with VPPs over a real divestiture, because the latter is irreversible, might be more costly and is politically difficult to implement. Moreover, as European markets become more integrated, the need for mechanisms to reduce market power might diminish, so reversible mechanisms are preferred.

In economic terms, a Virtual Power Plant is a call option on the production of electrical energy, where the virtual production cost is equal to the strike price of the option. The buyer of the option has the right but not the obligation to consume electricity of his VPP.

Options might have some advantages compared with futures contracts, which are contracts in which the buyers of the contracts are obliged to use their Virtual Power Plants always at full capacity:

- Options allow generators and retailers to hedge quantity risks, while futures can only be used to hedge price risks. Given that electricity cannot be stored very easily, quantity risks are very important in the electricity market, and options therefore play an important role. See Oum et al. (2005).
- Market power is most pronounced during periods of peak demand. Retailers might buy virtual peaking capacity (call options with a high strike price) to counter the market power of generators during these periods.

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5 A system of VPPs is used in Belgium, France, Italy, Denmark, and the Czech Republic. When all programs will be fully operational, more than 11000 MW of VPPs will be sold. Other countries are setting up similar systems: its introduction is being discussed in Spain; and The Netherlands are studying its re-introduction. Ireland has a similar system but will abolish it when it integrates its market with Northern Ireland. Irish VPPs are known as Virtual Independent Power Producers (VIPPs).

6 Also in the U.S. virtual divestitures have been used. FERC—the federal electricity regulator – allowed the merger of Exelon and PSEG, under the condition that the new company would sell 4000 MW of fossil fuel plants, and virtually divest 2600 MW of nuclear capacity. The implementation of the virtual divestiture is different than in the European systems, as standard futures contracts were used instead of options.
The implementation details of VPPs vary widely between countries. Usually, the incumbent generator is obliged to sell off a specific mix of VPPs with different virtual production costs (strike prices) which represent peak and base load generation. This is inspired by the idea that retailers, which are typically the buyers of VPPs, need both peak and base-load power to provide energy to their consumers. In some designs, buyers can use their VPPs freely, while in others they need to reach a minimal total output with their virtual plant over a whole year. This last rule is meant to deter withholding of production capacity from the market by the buyers of the VPPs. Anti-trust authorities typically impose certain conditions, such as, buyers of the VPPs must be viable entrants, an upper bound on the number of contracts one player might obtain, and the generator and its affiliates cannot buy back their own VPPs. Most VPP contracts are designed as standard options, but sometimes they replicate the characteristics of a specific plant: they include provisions for start-up costs, minimal up-times, ramping constraints, and an indexation of the strike price on a fuel price index.

1.3 Financial and Physical VPPs

In this text we will not look at the broad range of all possible VPPs. Instead we will distinguish two types of contracts: financial and physical VPPs.

We will define a financial VPP as a pure financial contract. Hence, it is not linked with the delivery of electrical power, or with the presence of physical generation capacity. If the spot price increases above a certain level, then a retailer, who bought such a contract, will receive a payment from the generator, which is equal to the difference between the spot price and the virtual production cost of the VPP, multiplied by its generation capacity. If the spot price is below that level, the retailer does not receive anything. The retailer remains a passive observer in the spot market.

Summarizing:

A financial VPP is a pure insurance contract against increases in the price of the spot market. The owner of a financial VPP is a passive participant in the spot market.

We will define a physical VPP as a contract for physical delivery of electrical energy. The buyer of a Virtual Production Plant can sell this energy to final consumers in a bilateral agreement, or sell it on the spot market. The buyer of the physical VPP becomes an active participant in the spot market, contributing to the price discovery process in the market and adding liquidity.

If the buyer of the VPP sells power into the power exchange, it can bid power up to the capacity of its virtual power plant. When the spot market clears, the retailer will learn its accepted production schedule, and will ask the seller of the VPP to provide the electrical power physically.

The seller of a physical VPP is responsible for providing the electrical energy scheduled by the retailer at the moment the spot market closes. The seller of the VPP pays a penalty if he does not deliver the power scheduled by the VPP owner.

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7 Also the regulator can use options to aim its regulation more precisely at periods of high demand, minimizing its intervention in the market.

8 In Belgium and in France one fourth of the capacity was sold as peak load capacity, and three fourth as base load plants.
A physical VPP is an option contract for physical delivery of energy. The energy can be resold by the buyer of the option. The owner of a physical VPP is an active participant in the spot market.

1.4 Physical VPPs: Delegation of production decisions

The main difference between financial and physical VPPs is one of the delegation of production decisions. With a physical VPP the seller delegates (part of) its production decision to the buyer. The buyer of the physical VPP will obtain the right to use, or not use its production capacity. The rational behind this is the following:

In order to meet the demand of the retailer, the seller of the VPP would normally keep some generation capacity in reserve. This capacity will be used when the buyer of the VPP exercises his option. The incumbent generator can sell in the spot market itself as well. However, in order to avoid the punishment of not delivering scheduled energy, he will only bid his unreserved capacity in the market, i.e., total generation capacity minus the capacity reserved for VPPs.

The comparison of financial and physical VPPs can be brought back to the discussion about financial and physical transmission rights (Joskow and Tirole, 2000). Financial and physical transmission rights are both a form of property rights but there is a difference with respect to the decision right on the usage of the line. A physical transmission right entitles its owner to withhold transmission capacity from the market, and by doing so, he can change the value of the transmission line, and become an active participant in the market.

The owner of a financial transmission right is a passive player in the market, who cannot influence the use of the transmission line. The same is true with a physical VPP: the owner of the VPP obtains the right to use or not to use its virtual power plant.

Summarizing:

The main difference between financial and physical contracts (let it be transmission rights or Virtual Power plants) is whether the buyer of the contract has the right to decide about the actual physical use of the underlying asset (the transmission lines and the production plants).

Other differences between physical and financial VPPs are often claimed to exist. We will now discuss them briefly.

- As the punishment for non-delivery might be higher with a physical VPP, it is often argued that a physical contract is more “firm”, i.e., the probability that electricity is physically delivered is larger.

- Physical VPPs have as a disadvantage that the counterparty risk is larger, as it is a bilateral contract between a generator and a retailer, which is not guaranteed by an independent clearing house.

- Another disadvantage of using physical VPPs is that production efficiency might decrease, when generation plants are scheduled in a more decentralized way. With financial VPPs, on the other hand, production decisions remain with the incumbent generator, who will utilize its portfolio of generation plants in a cost-efficient manner.
We will not study these claims here, but it is worthwhile to mention that their validity will depend on the micro-structure of the electricity market: How are responsibilities allocated and who is the supplier of last resort?

1.5 An Example: The Belgian and the Dutch VPPs

In the previous subsections, we defined two categories of contracts: financial and physical VPPs. In this subsection we will look at the actual implementation of the Belgian and the Dutch VPPs. We will show that VPPs, which are used in practice, do not always fit clearly into one of the two categories of contracts we defined above. A follow-up paper will look into implementation issues in more detail.

In our view, if we want to classify VPPs, we should look at which agent has the effective control over the physical production output: the buyer or the seller of the VPP. In the rest of this subsection we will argue that the Belgian VPPs have characteristics which make them “financial”, while the Dutch VPPs are “physical”.

The Dutch VPPs are designed for a specific production plant. The contract specifies for instance some virtual ramping constraints, and determines that the strike price depends on a fuel price index. The deadline for nomination of the Dutch VPPs is after gate closure of the Dutch spot market APX, allowing the buyer to bid into the spot market. The generator, which would need to sell VPPs, is a rather small player, which does not have a lot of generation capacity. Therefore the generator is obliged to use that specific power plant to fulfill most of its VPP related obligations. The generator will be obliged to keep this power plant in reserve for VPP related production. Hence, the owners of the VPPs decide effectively on the production capacity of the plant. The Dutch VPPs can therefore be considered a physical VPP.

In Belgium, the incumbent generator produces about 85% of total national production. The Virtual Power Plants it had to sell account for 6% of its production capacity. The Belgian VPPs are not designed for one specific generation plant, but are standard option contracts. They have a fixed strike price and do not consider ramping constraints. Given the large portfolio of generation assets, the small share of the VPPs in total production, and the non-specificity of the contract, the incumbent generator has a lot of substitution possibilities in deciding how it will generate the power associated with the VPP contracts.

In Belgium there is no liquid spot-market for electricity. This implies that a buyer of VPPs is not likely to wait until market clearing to contract its energy. It will decide about using its (virtual) power plants before market closure. The owner of the VPPs is therefore no longer an active participant in the spot market, and the seller of the VPPs retains full control of the total production of its generation portfolio.

Furthermore, as the incumbent generator is the only provider of emergency power in Belgium, total production of the incumbent might be determined more by emergency conditions and the total demand for electricity in Belgium, than by the scheduling decisions of the retailer. Retailers who only own VPPs, are

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9 Often the Dutch spot price is used as a reference price for the Belgian market. The Belgian VPPs need to be nominated before gate closure of the Dutch market, making it difficult for the owners of the VPPs to become an active participant in the spot market.

10 With emergency power in the real time market we mean the balancing market. In the electricity sector this is the market where the network operator will buy power when the market players deviate from the production schedules they agreed upon ex-ante. If the retailers schedule too little power, then the incumbent generator will have to provide less energy in
not allowed to actively participate in the market for emergency power, as only players with physical generation capacity can provide electricity here. We conjecture that VPPs will have a smaller impact on the behavior of generators that are dominant in the market for emergency power.

We think therefore that the Belgian incumbent generator stays fully in charge of its own production decisions, and that the VPPs should be considered as financial VPPs.

1.6 Research Methodology and Literature review

We will study Virtual Power Plants in a two stage Cournot game. We assume that there are only two markets: a VPP market in the first stage of the game, where retailers buy virtual generation capacity and a spot market in the second stage of the game. In section 2 we look at the second stage of the game, and study the strategic effects of the VPPs on the behavior of the generators in the spot market.

In section 3, we look at the first stage of the game, where firms sell VPPs. We study whether and how much VPPs firms would like to sell, without the obligation by anti-trust authorities to do so. The amount of VPPs which will be sold forms a reference point for the regulator when it would like to oblige the sales of VPPs.

In the model we assume away uncertainty, so hedging is not an issue. The number of generators is assumed to be fixed. Hence, we do not look at the entry decision of new generation firms. Both simplifications will underestimate possible beneficial effects of VPPs.

Our paper is an extension of Allaz and Vila (1993). They show that, in a Cournot game, firms have a strategic reason to sell futures contracts, because futures contracts serve as a commitment device for the firms to be more aggressive in the spot market, and hence help them to obtain a larger market share. Selling futures leads to a prisoners’ dilemma type of problem. All firms sell futures, and as a result the spot price will decrease. We will use a similar framework as Allaz and Vila to analyze a general class of contracts, i.e., VPPs instead of futures contracts.

Several papers have criticized the assumptions of the Allaz and Vila paper. As we make the same assumptions, these criticisms are also applicable to the current paper.

Allaz and Vila assume Cournot competition in the spot market. The actions of the players are strategic substitutes. Mahenc and Salanie (2004) show that in a Bertrand game, players will take opposite positions in the spot market: they buy their own output instead of selling it.

The result of Allaz and Vila depends also on the assumption that the number of futures contracts a firm signs is observable by all firms. Hughes and Kao (1997) show that if the contract position is not observed by other players, then firms no longer have an incentive to sell futures contracts. ¹¹

the spot market, but provide more emergency power in the balancing market. The total production of the incumbent is thus independent of the decisions of the retailers.

¹¹ They show, however, that under uncertainty and risk aversion, there are again strategic reasons to buy or sell futures contracts. The behavior of the firms will depend on the type of uncertainty that the firms face. If demand levels are uncertain, generators will sell futures contracts. If cost levels are uncertain, generators will buy futures contracts.
A third key assumption in the model is perfect arbitrage between the contracting stage and the spot market. In practice, arbitrage in the electricity market is, however, far from perfect. The precise reason for imperfect arbitrage and how one should model its effects remains for further investigation.\footnote{Arbitrage might be hindered by lack of information, perceived or real regulatory restrictions on arbitrage, and entry barriers in the arbitrage market.}

Le Coq (2003) looks at whether long term contracts make tacit collusion between generators more likely. As generators interact which each other on an hourly basis, tacit collusion is certainly an issue that has to be addressed. Le Coq shows that some long term contracts might stimulate tacit collusion leading to higher prices.

Finally, Thille (2003) shows that the results of Allaz and Vila are weakened when storage is possible. However, storage is not possible in electricity.

The paper is closely related to recent work of Chao and Wilson (2004). They argued that generators should be obliged to sell physical VPPs to retailers, but they did not distinguish financial from physical VPPs. Our paper looks at a similar problem to Chao and Wilson but makes different assumptions. They assume perfect regulation of the number of VPPs that generators have to sell and free entry in the contracting stage. However, with these two assumptions a lot of contract types will give the perfect competitive outcome. A comparison of contract types (futures, financial and physical VPPs) is not possible. In our paper we assume a fixed number of firms, and that generators decide themselves about the number of VPPs they will sell.

Also on the modeling side there is a difference. Chao and Wilson assume that generators bid linear supply functions in the spot market while we assume that they behave à la Cournot.

2 THE EFFECT OF VPPS ON THE SPOT MARKET

In this section we investigate the effect of VPPs on the behavior of generators in the spot market. We will build a simple model in which we define financial and physical Virtual Power Plants and show how they are different.

Our paper considers an oligopoly with two identical firms $i, j \in \{1, 2\}$. Firm $i$ produces $Q_i$ units at a production cost $C(Q_i)$. Total production of both firms is equal to $Q_1 + Q_2$, and the spot price $p$ is given by the inverse demand function:

$$p = P(Q_1 + Q_2). \quad (1)$$

We assume that generators sell a mixture of VPPs with different strike prices. This is what happened in Belgium: The incumbent generator sold 400 MW virtual peak-load power with a strike price of 29€/MWh, and 800 MW of virtual base-load power with a strike price of 9 €/MWh. A similar solution was chosen in France: the incumbent generator sold peak-load and base-load capacity. For mathematical tractability, we assume that generators sold VPPs with a continuous distribution of strike prices, and that the relative proportion of peak-load plants and base-load plants is the same for both generators. In other words, the
generators sell bundles of VPPs with fixed composition -- each bundle contains some base-load and peak load plants -- but the number of bundles generators sell can be different.

Mathematically we will express a bundle of VPPs by the cumulative distribution function \( \Gamma(S) \) and the associated density function \( \gamma(S) = \Gamma'(S) \). The standard VPP bundle will contain \( \gamma(S)dS \) MW of VPP with a strike price between \( S \) and \( S + dS \).

### 2.1 Financial Virtual Power Plants

A financial VPP is a bundle of financial call options. A financial call option is a one-sided insurance contract which insures retailers against price increases above the strike price \( S' \). If the spot price \( p \) is above the strike price, then the generator will refund the retailer the difference between the spot price and the strike price: \( p - S' \). When the spot price is below strike price, then there is no payment. In short, the generator pays the retailer the amount \( \max\{P - S', 0\} \).

If a retailer buys \( k \) bundles, it owns \( k\gamma(S')dS' \) financial call options with a strike price \( S' \). The generator pays the retailer the amount \( (k\gamma(S')dS') \cdot \max\{p - S', 0\} \).

The total insurance payment by the generator to the retailer is equal to sum of the payments for all options with different strike price within the bundle:

\[
\int_{0}^{\infty} k \max\{0, p - S'\} d\Gamma(S') = k \int_{0}^{P} \Gamma(S)dS. \tag{2}
\]

The profit of firm \( i \) who sells \( Q_i \) units in the spot market and sold \( k_i \) bundles of financial VPP is equal to the spot market revenue, minus the production costs and the insurance paid to the owners of the financial VPPs.

\[
pQ_i - C(Q_i) - k_i \int_{0}^{P} \Gamma(S)dS \tag{3}
\]

The profit of generator \( i \) is shown in Figure 2. The dotted downward sloping line represents the residual demand function, the horizontal line the marginal production cost, and the upward sloping function specifies the amount of VPPs which are in the money. The insurance premium paid to the owners of the financial VPPs is equal to the hatched area, and the profit of generator \( i \) is equal to the colored area.
Figure 2 Profit $\Pi_i$ of generator $i$, who sold $Q_i$ units of electricity in the spot market, and sold $k_i$ financial Virtual Power Plants. For simplicity the figure is drawn with constant marginal costs and linear demand functions.

In the spot market, firms choose their production in a Cournot fashion, i.e., they set their production quantity $Q_i$.

The Nash Equilibrium in the spot market is determined by the two first order conditions of the firms: $i=1,2$:

$$p'(Q_i - k_i \Gamma(p)) + p = C'(Q_i)$$  

with $p = P(Q_1 + Q_2)$ and $p' = P'(Q_1 + Q_2)$. Each firm sets its marginal revenue minus its marginal insurance payment equal to its marginal production costs.

Equation (4) describes the reaction functions of both players, and determines the spot price. If a generator has sold financial VPPs, then it will have an incentive to sell more in the spot market, and this will lower the spot price. If generator $i$ sells financial VPPs, it will shift its reaction function outside, but it will not affect the reaction function of generator $j$.

2.2 Physical options

Physical options are more difficult to model than financial options. The reason is that we have to model the production decisions of the buyers of virtual production capacity.

If a retailer bought $k$ bundles of physical VPPs, then it reserved $k\gamma(S')dS'$ MW of a virtual plant with virtual production costs between $S'$ and $S'+dS'$.

The retailer will buy electricity in the spot market at a price $p$. It is optimal for him to use the virtual production plants as long as their virtual production cost $S'$ is below the spot price $p$. The easiest way for him to do this is by bidding all virtual generation capacity into the spot market at its virtual production cost. The retailer will produce $q_v$ MW from its virtual production plant:
and will pay the seller of the VPP the virtual production cost:

\[ C_V = k \int_0^p Sd\Gamma(S) \]  

Firm \( i \) bids \( q_i \) MW in the spot market and has sold \( k_i \) bundles of physical virtual power plants. Total production of firm \( i \) is equal to

\[ Q_i = q_i + k_i \Gamma(p) \]  

Here, we assume that firms play Cournot in the spot market, \( i.e. \), they bid the quantity \( q_i \) in the spot market at a price 0. Equation (7) shows that the total production of the firm is no longer under its full control. Total production consists of what the firm bids in the spot market, plus the amount it has to produce due to its VPP commitments. The production of the virtual production plant is not constant, but depends on the spot price \( p \).

The price in the spot market is, as before, determined by the inverse demand equation:

\[ P(Q_i + Q_j) = p \]  

Equations (7) and (8) describe the spot market price \( p \), and the physical generation \( Q_i \) of both firms as a function of the bids of the firms \( q_i \).

Firm \( i \)'s profit

\[ p \cdot q_i - C(q_i) - k_i \int_0^p Sd\Gamma(S) \]  

is equal to the revenue of selling in the spot market, minus the total production cost (VPP related and direct bids in the spot market), plus compensation for the virtual production costs the firm receives from the retailers. Firm \( i \) maximizes its profit (9) by setting its bid \( q_i \) in the spot market, taking into account the way how prices and actual production are related (equations (7) and (8)).

By elimination of \( q_i \) using equation (7) and after partial integration, equation (9) is identical to equation (3):

\[ p \cdot Q_i - C(Q_i) - k_i \int_0^p \Gamma(S)dS \]  

Equation (10) shows that the total profit of a firm, written as function of the spot price and total production quantity is the same with financial and physical VPPs. However, the firms do not have the same say over their production quantity. With financial VPPs, the firm sets its total production quantity directly. With physical VPPs, the firm can change its total production level \( Q_i \) only indirectly by changing its bid \( q_i \).

However, by changing its bid, it will not only change its own production level \( Q_i \), but also the production quantity of its competitor \( Q_j \) through the price effect. In particular, if firm \( i \) reduces its bid \( q_i \) then it will decrease its total production \( Q_i \), leading to a higher price \( p \). As a response to the higher price, the production by the VPP of its competitor will increase. Hence, if firm \( i \) changes its total production, then
also total production of firm \( j \) will change. We present this relation by the function \( \hat{Q}_j(Q_i;k_j) \). The marginal effect can be described by the parameter \( \beta_i \):

\[
\beta_i = \frac{d\hat{Q}_j(Q_i,k_j)}{dQ_i} = \frac{dQ_j}{dQ_i} = \frac{k_j\gamma(p)p'}{1 - k_j\gamma(p)p'} < 0
\] (11)

We can interpret equation (11) as the “\( \beta \)-index” in a standard model of conjectural variations. It describes how much a firm expects that the other players will increase their production in response to a decrease of its production. An index equal to zero corresponds to a Cournot outcome. An index equal to minus one corresponds to Bertrand competition. Here we find a solution in between Bertrand and Cournot.

\[ p = P(Q + \hat{Q}_j(Q)) \]

\( k_i\Gamma(p) = Q \)

\( p = P(Q) \)

\( \hat{Q}_j(Q) \)

\( a \)
\( b \)
\( c \)
\( d \)
\( e \)
\( f \)
\( q_i \)
\( Q_i \)
\( \Pi_i \)

**Figure 3** Profit \( \Pi_i \) of generator \( i \), who sold \( Q_i \) units of electricity in the spot market, and sold \( k_i \) physical Virtual Power Plants.

The colored area in Figure 3 shows the profit of a generator who sold physical VPPs. The dotted downward sloping line is again the residual demand function. Compared with the profit function of the generator with financial VPPs, we see that the residual demand function is flatter. This corresponds with the fact that the output of firm \( j \) increases when firm \( i \) decreases its output. The area \( abfe \) is equal to the payments firm \( i \) receives from the owners of the VPP, while the area \( cdef \) is equal to the extra production cost it incurs to produce the VPP-related quantity \( k_i\Gamma(p) \).

The first order condition of equation (10) is:

\[
p_i^{**}(Q_i - k_i\Gamma(p)) + p - C'(Q_i) = 0
\] (12)

where

\[
p_i^{**} = p'(1 + \beta_i) = \frac{p'}{1 - k_j\gamma(p)p'}
\] (13)
is the slope of the residual demand function of firm $i$, \textit{i.e.}, taking into account the “conjectural variation”
term. Equations (12) and (13) show that the production of firm $i$ depends on the amount of options which
are in the money of the firm itself $k_i \Gamma(p)$, and the number options of its competitor which are on the
margin $k \gamma(p)$. The own options are important because they reduce the infra-marginal profits of a firm.
The number of marginal options of its competitor is important, as it determines the aggressiveness of the
competitor in the equilibrium, \textit{i.e.}, the slope of the residual demand function.

2.3 Comparing the different types of contracts

Financial VPPs versus physical VPPs

We will compare the effects of financial and physical VPPs on the spot market.

Equation (4) shows that a firm who sold financial VPPs has only an impact on its own reaction function. Firms who sold financial options are more aggressive in the spot market. Equation (12) shows that this effect is also present with Physical VPPs. A firm who sold physical options becomes more aggressive in the spot market. There is also a second effect of selling physical options: a firm will also flatten the slope of the residual demand function of its competitor (as shown in Figure 3). This makes competition more a la Bertrand, inducing generators to behave more competitively.

Summarizing:

\textit{If firms sold the same number of VPPs, then a market with physical VPPs is more competitive than
one with financial VPPs.}

Financial and physical VPPs in a monopoly.

Given the derivations above we can easily study the effect of financial and physical VPPs in a monopoly setting by simply putting the output of firm $j$ to zero: $q_j = Q_j = k_j = 0$. We obtain that financial and physical VPPs will lead to the same outcome. Delegating production decisions has an effect on the market outcome, because it flattens the residual demand function of strategic competitors. If there are no strategic competitors, then physical VPPs have the same effect as financial VPPs.

Summarizing:

\textit{In a monopoly framework, financial and physical VPPs give identical outcomes.}

Financial and Physical Futures contracts

A financial (physical) future contract can be modeled as a financial (physical) call option with a strike
price equal to 0. Hence, $\Gamma(S) = 1$ and $\gamma(S) = 0$ for all strike prices. As $\gamma(S) = 0$ for all strike prices,
there are never options which are on the margin. Hence the slope of the residual demand of the competitor
does not change, and financial and physical futures contracts have the same effects. If firm $i$ sold $k_i^f$
future contracts, both equations (12) and (4) simplify to:

$$ p_i^r (Q_i - k_i^f) + p - C'(Q_i) = 0 $$

which describes the reaction functions of the firms.

Summarizing:
Financial and physical futures contracts are equivalent.

Futures contracts and VPPs
How do financial and physical VPPs compare with futures contracts?

Futures and financial VPPs have the same effect on the spot market when the number of contracts which are in the money are identical, hence when:

\[ k_i \Gamma(p) = k_i^F \] (15)

If a financial option is in the money, it does not matter which strike price it has.

Comparing futures and physical VPPs is more complicated as the equilibrium does not only depend on the number of contracts in the money, but also on how many contracts of the competitor are marginal. Futures and physical VPPs have the same effect when

\[ k_i \Gamma(p) = k_i^F + (Q_i - k_i^F) k_i \gamma(p)p^1 \] (16)

If there are contracts which are on the margin, then less physical VPPs have to be sold (note: \( p^1 < 0 \)), because physical contracts make markets more competitive than financial VPPs, so we need less of them to obtain the same result.

Summarizing:

The impact of financial contracts on the spot market equilibrium depends on the number of contracts in the money.

The impact of physical contracts on the spot market equilibrium depends on the number of contracts in the money and the number of contracts on the margin.

3 THE NATURAL CONTRACTING POSITION OF FIRMS

In this section we study what kind of contracting structure will develop naturally between generators and retailers. We look at the case where generators can freely sign contracts with retailers before the spot market operates.\(^{13}\) This is modeled as a two-stage game: in the first stage, the contracting stage, generators sell VPPs to retailers, in the second stage generators bid into the spot market.

This section consists of 4 subsections. In the first subsection we determine the demand for long term contracts by retailers, and discuss why generators would like to sell such contracts. The second and third subsections look at the equilibrium level of contracting with financial and physical VPPs and the resulting price levels. The last subsection gives additional intuition by referring to the literature on delegation games.

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\(^{13}\) It is interesting to observe that RWE, a German electricity producer, started selling VPPs on a voluntary basis from February 2006 onwards.
3.1 Incentives for Retailers and Generators to contract

In order to model the demand for VPPs by retailers, we assume that the contract price \( f \) is determined by inter-temporal arbitrage between the contract market and the spot market. This means that a retailer is indifferent between buying VPPs and not buying them. This is the same assumption as taken by Allaz and Vila (1993). As we are looking at a model without uncertainty, arbitrage seems to be a valid assumption.

**THEOREM** Given inter-temporal arbitrage, the contracting price \( f \) is equal to:

\[
 f = \int_0^p \Gamma(S) dS \tag{17}
\]

This is true both for physical and financial VPPs.

**PROOF**

A retailer that buys a financial VPP should make zero profit. The price of a VPP has to be equal to the expected insurance payment. Hence:

\[
 f = \int_0^p \Gamma(S) dS \tag{18}
\]

A retailer that buys a physical VPP should not be able to make a profit by reselling its power on the spot market. The price of the VPP is equal to the expected value of the produced electricity, minus the virtual production costs. Hence:

\[
 f = p \Gamma(p) - \int_0^p S d\Gamma(S) \tag{19}
\]

Using partial integration, it can be shown that the expressions in (19) and (17) are identical.

**QED**

Equation (17) implicitly describes the demand function for VPPs by retailers.

The total profit of a generation firm is the sum of the profit it makes in the spot market and the revenue from selling VPPs in the contracting stage. For both physical and financial VPPs we find:

\[
 \frac{p \cdot Q_i - C(Q_i)}{\text{profit in spot market}} - \frac{k_i \int_0^p \Gamma(S) dS + k_i f}{\text{revenue in contract market}} \tag{20}
\]

Using arbitrage condition (17), the profit of the firm \( i \) (20) can be rewritten as:

\[
 p Q_i - C(Q_i) \tag{21}
\]

Equation (21) shows that the profit of firm \( i \) depends on the spot price \( p \) and on its production level \( Q_i \) and only indirectly on the number of contracts it has signed. Therefore, the reason for firm \( i \) to sell VPPs is strategic. By selling contracts, the firm hopes to obtain a larger market share \( Q_i \) in the spot market, without driving down the price \( p \) too much.

In the case that arbitrage is imperfect, the generation firm would have an additional incentive to contract as it might want to price discriminate between the two periods.
Firm $i$ chooses the amounts of contracts $k_i$ that maximize its profit (21), taking into account how the spot market equilibrium depends upon the contracting position of the players. By selling VPPs, firm $i$ can change the equilibrium production quantities of both firms. The first order condition of (21) with respect to the contracting position $k_i$ is

$$[p'Q_i + p - C'(Q_i)]\frac{dQ_i}{dk_i} + [p'Q_j]\frac{dQ_j}{dk_i} = 0$$

where $\frac{dQ_i}{dk_i}$ and $\frac{dQ_j}{dk_i}$ describe the impact of contracting on the own production quantity and on the production quantity of the competitor. They can be derived from equation (4) for financial VPPs and from equation (12) for physical VPPs. Dividing equation (22) by $\frac{dQ_j}{dk_i}$ it can be rewritten in conjectural variational terms:

$$p'(1 + \beta_i^*Q_i + p - C'(Q_i) = 0$$

Equation (23) describes the incentives for firm $i$ to contract. The parameter $\beta_i^* = \frac{dQ_j}{dQ_i}$ is found by a local sensitivity analysis of the spot market equilibrium conditions. It describes how much the equilibrium production quantities $Q_i$ and $Q_j$ will change in relative terms when firm $i$ sells more VPPs. For each type of contract, the value will be different.

$\beta_i^*$ is also a measure for the competitiveness of the contracting market. Values close to minus one lead to Bertrand competition, while values close to zero lead to Cournot outcomes.

In the next two subsections we will look at financial and physical VPPs.

### 3.2 Financial VPPs

For financial VPPs we can easily derive the conjectural parameter $\beta_i^*$. As the sale of VPPs by firm $i$ will only change firm $i$’s reaction function, keeping the reaction function of firm $j$ constant, it follows that the equilibrium production quantities will “move” along the reaction function of firm $j$. Hence, $\beta_i^*$ is equal to the slope of the reaction function of firm $j$.

$$1 + \beta_i^* = \frac{p' - C''(Q_j)}{-\frac{p''}{p^2}(p - C'(Q_j)) + 2 - \frac{C''(Q_j)}{p'}}$$

Equation (24) shows how the conjectural variations parameter depends on first and second order derivatives of the inverse demand functions and of the production costs. The equation looks rather

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14 Note that $\beta_i^*$ is the conjectural variations parameter in the contracting stage, this is a different parameter than $\beta_i$ which was defined before as the conjectural variations parameter in the spot market.
complex. Luckily, we are only interested in how contracting affects the conjectural parameter $\beta_i^e$. For a given price $p$ and production quantity $Q$, firm $i$ has more incentives to sell VPPs, the more VPPs of its competitor are marginal (the larger $k_j\gamma(p)$).

We will now determine for which type of financial VPP the spot price will be lowest, i.e., the contract $\Gamma(S)$ which leads to the largest prisoners’ dilemma. Using equations (24), (23) and (4) we will show that an increase in the elasticity of the supply function of the VPP

$$\varepsilon(p) = \frac{p\gamma(p)}{\Gamma(p)}$$

(25)

makes markets more competitive.

**THEOREM:** If there is a long-term market for financial VPPs, then the market is more competitive when the virtual supply function of the VPPs is more elastic.

**PROOF**

Consider a VPP described by the distribution function $\Gamma(S)$. Assume that for this VPP the price in equilibrium, is given by $p^e$, the production quantities by $Q^e$ and contracting positions by $k^e$.

Consider now an alternative VPP described by the distribution function $\Gamma(S)$. Variables which are related with the alternative will be underlined. We will check under which conditions the equilibrium with the alternative VPPs is more competitive. We do this in three steps. First we calculate the equivalent contract quantity $\tilde{k}$ of alternative VPPs which gives the same result in the spot market as $k^e$. Then we check when firms would sell more than the equivalent amount of contracts. Finally we combine the two results.

(Step 1) From equation (4) we can derive the equivalent contract quantity $\tilde{k}$ of alternative VPPs which gives the same result in the spot market, i.e., the same production quantities $Q^e$ and price $p^e$:

$$\tilde{k} = k^e \frac{\Gamma(p^e)}{\Gamma(p^e)}$$

(26)

The intuition behind equation (26) is that the spot market outcome is identical when the same number of options is in the money.

(Step 2) We now check which of the two bundles gives the largest marginal incentive to contract: $\tilde{k}$ bundles of $\Gamma(S)$ or $k^e$ bundles of $\Gamma(S)$. If at $\tilde{k}$ the marginal incentive to contract is larger than at $k^e$, then more contracts will be sold in the “alternative” equilibrium $k^e > \tilde{k}$. As more contracts are sold,

---

15 We only consider symmetric equilibria, and drop therefore the index $i$ in the equilibrium quantity and contract position.
prices will be lower. Equation (24) tells us that contracting incentives are given by the number of marginal VPPs \( k\gamma(p) \). Hence, more than the equivalent number of contracts will be sold when:

\[
\frac{\hat{\gamma}(p^f)}{k} > k\gamma(p^f)
\]  \hspace{1cm} (27)

(Step 3) Combining equation (27) and (26) we see that the markets will be more competitive with the alternative VPP when it is more elastic:

\[
\varepsilon(p^f) > \varepsilon(p^f)
\]  \hspace{1cm} (28)

QED

The intuition of Equation (28) is simple: The incentive to sell VPPs is given by the number of marginal VPPs (27). The effect on the spot market depends on the amount of infra-marginal VPPs (28). The combination of the marginal and infra-marginal effects determines the impact on the spot price, and is described by the elasticity of the VPP.

From this result we can derive the following corollary:

If long term contracting is not regulated, then markets are more competitive when firms sell financial VPPs than when they sell futures contracts.

The corollary follows directly from the fact that futures contracts have zero elasticity (\( \gamma(p) = \varepsilon(p) = 0 \)). Allaz and Vila (1993) obtain the result that futures contracting makes markets more competitive. Here we show that this effect is reinforced with financial options.

3.3 Physical VPPs

In appendix I we give the analytical expression for the conjectural parameter \( \beta^f \) for physical VPPs. Given the complexity of the expression, we cannot derive a simple rule as we did for financial VPPs.

In order to comprehend whether physical VPPs will lead to a more competitive market than financial VPPs and futures contracts, we rely on a simplified analytical model which assumes higher order derivatives to be zero. In the simplified model the inverse demand function is linear and normalized to \( P(q) = 1 - q \), the firms have identical quadratic production cost functions \( C_i(q) = .5q^2 \) and the VPPs have linear cumulative distribution functions \( \Gamma(S) = S \).

In appendix II we derive the equilibria of this game for three types of long term contracts: futures contracts, financial VPPs, and physical VPPs. The equilibria of these games are a function of the cost parameter \( c \in [1,0] \).

Figure 4 shows how many VPPs the firms will contract as a function of the cost parameter \( c \). For the whole range of the cost parameter, firms sell more financial than physical VPPs. Note that generators do not sell physical VPPs if the production cost is below \( 1/4 \).
As physical VPPs have a larger pro-competitive effect on the spot market than financial VPPs (as shown in section 2.3), we cannot conclude, on the basis of Figure 4, that physical contracts would lead to higher prices. Less physical contracts are sold, but each contract has a larger pro-competitive effect.

Figure 5 shows the final price as function of the production cost of the generators for the different types of long term contracts. Note that the 45 degrees line would present perfect competition.

Prices are lower with financial VPPs than with physical VPPs. Even though physical VPPs have a larger pro-competitive effect than financial VPPs, the fact that less of them are sold outweighs this effect.

Figure 5 shows also that the price is higher with physical VPPs than with futures. When production costs are low, firms will not sell physical VPPs, and the final price is equal to the standard Cournot outcome.
The figure confirms the results we obtained for financial VPPs in the previous subsection: the final price is lower with financial VPPs than with futures.

Summarizing:

A regulator should support the development of financial option markets as this will increase market efficiency. A market for physical options is unlikely to develop on a voluntary basis.

3.4 Delegation games

In the previous subsections we looked at the contracting structure that will develop naturally between generators and retailers. In this subsection we provide additional insight into the incentives of generators to contract, by comparing the results with the literature on delegation games.

Delegation games are two-stage oligopoly games, where in the first stage the owners of the firm delegate decisions to the managers, and in the second stage the managers compete. In this literature the term “delegation” means something else than what we have used throughout the text: Two types of delegation decisions are commonly considered. In the first type, the owners of the firm decide on an incentive scheme for the managers, i.e. how the managers will be paid as a function of the market outcome. In the second type, the owners decide which strategic variables the managers should set.

Setting the incentive schemes

Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) consider incentive schemes where the wage of managers is determined on the basis of a weighted sum of revenue and costs. Fumas (1992) looks at the weighted sum of own profit and its rival’s profit. They show that in a Cournot game the owners of a firm will set up an incentive scheme such that the managers behave more aggressive.

Financial VPPs are, in fact, good instruments to change the incentive schemes of firms. If a firm sells Financial VPPs it will change its preferences in the spot market, making itself more aggressive.

Changing the strategy sets

Some authors have studied delegation games where the firm owners decide on the strategy set of the managers. For instance, Singh and Vives (1984) look at a game where owners specify in the first stage whether managers set quantities or prices, and then the managers of the firms compete with each other. They show that it is a dominant strategy for the owners of a firm to let managers set quantities when the products are substitutes.

Physical VPPs can be considered as a combination of both types of delegation games. When a firm sells physical VPPs, it will not only change its incentive schemes, making it more aggressive, but it will also make competition more a la Bertrand. As Singh and Vives have shown that it is a dominant strategy for firms to play Cournot, it comes not as a surprise that firms are reluctant to sell physical VPPs as they dislike Bertrand competition.

Summarizing:

Generators will only sell limited amounts of physical contracts, because physical VPPs contain a commitment to play more à la Bertrand, which is a dominated strategy.
CONCLUSION

4.1 Main contribution and results of the paper

A virtual divestiture is a set of contracts which intend to mimic a real divestiture of a firm, without requiring the legal transfer of assets. Virtual divestitures have been used in Europe as a way to mitigate market power of electricity firms whenever real divestitures were deemed to be too costly or politically infeasible.

The main contribution of this paper is the development of a model to study virtual divestitures. Essentially, it extends the two-stage Cournot model developed by Allaz and Vila (1993), who look at only one type of contractual relationship between retailers and generators: the standard futures contract. Here, we look at different types of contractual relations between retailers and generators, and look at their impact on competition.

With a virtual divestiture the regulator obliges the generator to sell Virtual Power Plants. The buyer of the Virtual Power Plant obtains the right to “generate electricity” with its Virtual Power Plant, but will have to pay the seller of the plant a refund for the “variable costs”. In economic terms, a Virtual Power Plant can be seen as a bundle of call options.

The main conclusions of our model are the following:

1. We show that two types of Virtual Power Plants exist: financial and physical VPPs. Financial VPPs are pure price insurance contracts against upward jumps in the spot price. Physical VPPs are contracts for the physical delivery of energy, which give the buyer of the contract the right to resell the energy in the spot market. Financial and physical VPPs are different because physical contracts involve the delegation of production decisions, while financial contracts leave generators fully in charge of their production decisions.

2. In an oligopoly setting, physical and financial divestitures have different effects on the spot market. By selling physical VPPs, generators can commit to compete more a la Bertrand in the spot market. Hence, the spot market is more competitive with a physical divestiture than with financial divestiture. In a monopoly setting, a monopolist will perfectly foresee the production decisions of the buyers of the physical VPPs, and therefore financial and physical divestitures will give the same market outcome.

3. In the contracting stage, firms will sell financial VPPs for strategic purposes. This will make the market more competitive. Firms will only sell a limited amount of physical VPPs, marginally increasing the spot market competitiveness.

4.2 Policy recommendations

The most efficient way to mitigate market power problems in a highly concentrated generation market is to restructure the sector and to split up the existing firms. When such a horizontal restructuring is too costly or politically infeasible, regulators could regulate the vertical structure of the electricity sector specifying the contractual relationship of retailers and generators. One way of organizing this is to impose a virtual divestiture.
A virtual divestiture will only work when the full contracting positions of the firms can be monitored. Otherwise, firms will have an incentive to buy some VPPs in order to off-set the VPPs they had to sell. Without close monitoring by the regulator a virtual divestiture will not work.

As a market power mitigation measure, regulators should oblige generators to sell physical VPPs, i.e., VPPs which can be resold in the spot market. Compared with financial VPPs, they ensure the active participation of retailers in the spot market, increase the liquidity of the spot market, and reduce market power of the generators. A virtual divestiture by means of physical VPPs would give similar results as a real divestiture.

In mature electricity markets with healthy competition, regulators should promote markets for long term financial contracts which contain a flexible component (i.e., options). The existence of a market for financial options is not only beneficial in terms of risk management, but will also make spot markets more competitive. The pro-competitive effect of the existence of an option market is larger than that of a futures market.

4.3 Limitations and possible extensions of the model

Several reservations have to be made with respect to the outcome of the model:

First, the paper omits hedging issues and focuses purely on the effects of long term contracts on market power. More work is needed to analyze the effects of uncertainty in a world with risk-averse agents.

Second, we model the spot market as a Cournot game. We could have extended our discussion to include other types of competition, such as Bertrand competition. We conjecture that firms will buy VPPs under Bertrand competition as this would soften competition in the spot market. Firms are also more likely to buy physical VPPs than financial VPPs, as physical VPPs make the game more à la Cournot. It is a dominant strategy for players to commit to play Cournot.

Third, instead of looking at bundles of options, we could also look at a standard option contract with one fixed strike price $T$. Such a contract can be considered as a VPP with the cumulative distribution function:

$$
\Gamma(S) = \begin{cases} 
0 & \text{if } S < T \\
1 & \text{if } S > T
\end{cases}
$$

(29)

Solving a model with standard options is complicated, as the discrete distribution leads to discontinuities in the reaction functions of the firms in the spot market. Willems (2004) studies financial contracts of this type. The intuition developed for financial VPPs carries through for standard financial options:

- We have shown that the effect of financial VPPs on the spot market depends on the number of contracts which are in the market (i.e., infra-marginal). This result also holds for standard options. The number of contracts which are in the market depends on the strike price. If the strike price is

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16 Physical options are a way to delegate production decisions. In a Cournot game, this delegation will make competition more à la Bertrand, as the supply function of the traders is more elastic than the bids of the oligopolists (which are inelastic). In a Bertrans game, this delegation will make competition more à la Bertrand, as the supply function of the traders is less elastic than the bids of the oligopolists (which is perfectly elastic).
very low, then all options are in the market and financial options are similar to futures contracts. If the strike price is very high, then the options are not in the market and they have no effect on the competitiveness of the market.

- We have shown that the amount of financial VPPs that the firms sell depends on the number of contracts which are marginal. If a lot of contracts (of the competitor) are marginal, then a firm will sell more contracts itself. For standard options, the number of marginal options depends on the spot price and the strike price. If the spot price is equal to the strike price of the option, the number of marginal options is infinite. If the spot price is different from the strike price, the number of marginal options is zero. The incentive to sell options is therefore not constant, and a whole range of equilibria is possible. Markets can coordinate on equilibria where a lot of options are sold and the spot price is equal to the strike price (a lot of contracts are marginal and hence the incentive to sell is large as well), or they can coordinate on equilibria where they sell less options and no options are marginal.

Fourth, we only studied VPPs which have a fixed percentage of peak-load and base-load capacity. Firms have only one degree of freedom in the contracting stage. If firms can choose freely the type of long term contracts they offer, then we would obtain a multitude of equilibria. Miller and Pazgal (2001) have shown that when firms have sufficient degrees of freedom in setting the type of long term contracts, all equilibria between the Cournot outcome and the Bertrand outcome can be reached, independent of the type of competition in the spot market.\(^\text{17}\) We think therefore that we would not obtain a lot of interesting insights in a model where generators can sign any type of long term contract.

Fifth, we assume that there is perfect arbitrage between the contracting stage and the spot market, and that there is perfect information on the contracting positions of the generators. In order to relax these assumptions, future studies would need a more detailed description of the market micro-structure and deal with asymmetric information in the contracting stage and in the spot market.

**APPENDIX I: CONJECTURAL VARIATIONS PARAMETER FOR PHYSICAL VPPS**

This appendix gives the expression of \( \beta_i^* \), the conjectural variations parameter for physical VPPs:

\[
\frac{dQ_i}{du_i} / \frac{dQ_i}{du_i} = \beta_i^* = \frac{A - B}{B} \tag{30}
\]

where

---

\(^{17}\) Miller and Pazgal studied delegation games. As shown above, long term contracting and delegation games are equivalent. They show that the set of equilibria which can be achieved with delegation is identical to the set of equilibria in a one stage game where firms compete by setting Supply Functions (See Klemperer and Meyer 1989).
\[ A = (C_j - p)p'\gamma(p' + C'_j(-1 + k_jp'\gamma) - p'(p' + C'_j(-1 + k_jp'\gamma))\Gamma \]

\[ B = p' + C'_j(-1 + k_jp'\gamma)\Gamma \]

\[ + (C'_j - p)\gamma [p' + p''(Q_1 + k_j(C'_j - p)\gamma - k_j\Gamma) - p'^2(k_j\gamma + k_j(\gamma - C'_j\gamma' + p\gamma'))] \]

\[ + \Gamma[p' + p''(Q_j + k_j(C'_j - p)\gamma - k_j\Gamma) - p'^2(k_j\gamma + k_j(\gamma - C'_j\gamma' + p\gamma'))] \]

(31)

The conjectural parameter depends on the first and second order derivatives of the cost functions \( C', C'' \), the cumulative distribution function \( \Gamma'(p) = \gamma(p) \), \( \Gamma''(p) = \gamma'(p) \) and the inverse demand function \( p' = p'(Q_1 + Q_2) \), \( p'' = p''(Q_1 + Q_2) \).

**APPENDIX II: SOLUTION OF THE LINEARIZED MODEL**

**Standard Cournot game**

For the standard Cournot game without long term contracts, we obtain a Nash Equilibrium where equilibrium production quantities and spot price \( \bar{q}^C(c) \) and \( P^C(c) \) are given by

\[ q_i^C = \frac{1 - c}{3} \quad (32) \]

\[ p^C = \frac{1 + 2c}{3} \quad (33) \]

where \( \bar{q} \) is shorthand for the vector \((q_1, q_2)\).

**Financial VPPs**

For the Cournot game with financial VPPs, the Nash equilibrium in the spot market is given by:

\[ q_i(\vec{k}, c) = \frac{1 + k_i + c(k_i - k_j - 1)}{3 + k_i + k_j} \quad (34) \]

The price in the spot market is equal to:

\[ p(\vec{k}, c) = \frac{1 + 2c}{3 + k_1 + k_2} \quad (35) \]

In the first stage the firms will sell the following amount of VPPs:

\[ \vec{k}(c) = \frac{8 + \sqrt{17c}}{4} - \frac{5}{4} \quad (36) \]

The equilibrium price for electricity is equal to

\[ p = \frac{2\sqrt{\pi}(1 + 2c)}{\sqrt{\pi} + \sqrt{8 + 17c}} \quad (37) \]

**Physical VPPs**

For the Cournot game with physical VPPs, the second stage Nash Equilibrium is:
The equation describes how the generators will produce in the second stage of the game.

The price in the spot market is equal to

\[
p(k, c) = \frac{1 + k + c(2 + 2k + k^2)}{3 + 2k}
\]

with \( k = k_1 + k_2 \)

The generators sell their VPPs in a non-cooperative way. The equilibrium contracting position in the first stage is

\[
k(c) = \max(0, k^*(c))
\]

where \( k^*(c) \) is the root of a polynomial \( A \) of the third order:

\[
A = (1 + 5c - 4c^2) - (4 - 10c + 21c^2)k
- 8c(-1 + 4c)k^2 - 16c^2k^3
\]

According to equation (40) generators will sell no physical VPPs when the production cost is below \( 1/4 \).

For production costs above \( 1/4 \) generators will sell a positive amount.

The equilibrium price is given by

\[
p(c) = \frac{1 + 2c(1 + k(c))}{3 + 4k(c)}.
\]

REFERENCES


