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Analyses of Start-Stop Waves in Congested Freeway Traffic

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Publication Date
2002
Analyses of Start–stop Waves in Congested Freeway Traffic

by

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A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy
in
Engineering – Civil and Environmental Engineering
in the
GRADUATE DIVISION
of the
UNIVERSITY OF CALIFORNIA AT BERKELEY

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Spring 2002
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Spring 2002
Analyses of Start–stop Waves in Congested Freeway Traffic

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by

Michael Mauch
Abstract

Analyses of Start–stop Waves in Congested Freeway Traffic

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Doctor of Philosophy in Engineering – Civil and Environmental Engineering

University of California at Berkeley

Professor Michael J. Cassidy, Chair

Freeway traffic was observed over multiple days and was found to display certain regular features. Oscillations arose only in queues; they had periods of several minutes; and their amplitudes stabilized as they propagated upstream. They propagated at a nearly constant speed of about 20 to 24 kilometers per hour, independent of the location within the queues and the flow measured there; this was observed for a number of locations and for queued flows ranging from about 850 to 2,000 vehicles per hour per lane. The effects of the oscillations were not felt downstream of the bottleneck. Thus, the only effect on upstream traffic was that a queue’s tail meandered over time by small amounts. (For the long queues studied here, the tails deviated by no more than about 16 vehicle spacings, as compared with predictions that ignored the oscillations). Notably, the character of queued traffic at fixed locations did not change with time, despite the oscillations; i.e., traffic did not decay.

There were changes over space, however. New oscillations formed in moderately dense queues near ramp interchanges and then grew to their full amplitudes while propagating
upstream, even though the range of wave speeds was narrow. The formations of these new oscillations were strongly correlated with vehicle lane–changing. It thus appears that the oscillations were triggered by random vehicle lane–changing in moderately dense queues more than by car–following effects. But this pattern of formation and growth was less evident in a very dense queue (caused by an incident), although frequent lane–changing occurred near the interchanges.

Finally, kinematic wave theory was found to describe the propagation of the oscillatory (i.e., start–stop waves) to within small errors. For distances approaching one kilometer, and for two–hour periods, the theory predicted the locations of vehicles to within about 5 vehicle spacings. Further analysis showed that some of these small discrepancies are explained by differences in car–following behavior across drivers.

___________________________________

Professor Michael J. Cassidy,

Committee Chair
DEDICATION

This work is dedicated to my father and mother, who supported my efforts throughout the years, and who have always been there for me.

I also dedicate this work to Professor Gordon F. Newell. Gordon was one of my advisors and a mentor to me. I admired him greatly. A fatal car accident took him from us on February 16, 2001. He will never be replaced. Without his insights, this research never would have happened.

ACKNOWLEDGEMENTS

I am grateful and indebted to my advisor, Professor Michael Cassidy. Conducting this research and writing this dissertation would not have been possible without his patience, guidance, and tireless devotion to this work.

I also thank David Tsui and Mark Fox of the Ontario Ministry of Transportation for providing the data. Funding for this research was provided by the University of California Transportation Center.
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1 INTRODUCTION

This dissertation documents an empirical study of the observable features of oscillations or slow–and–go disturbances in queued freeway traffic. Although the driver interactions that initiated these disturbances were not completely identified, the visual inspection of cumulative vehicle count curves, measured from loop detectors and transformed in special ways, revealed a number of previously undetected and significant attributes of these disturbances.

Traffic oscillations are shown to originate within the traffic queue upstream of a bottleneck and propagate upstream, as waves, with predictable properties. These observations have several important implications. First, since these oscillations were always found to originate in the queue, they were a consequence of queuing and not the cause. Next, since the oscillations propagated in the upstream direction only, they did not affect bottleneck capacity and congestion related delays. Finally, since they propagated in a predictable manner, their effects on fuel consumption, engine emissions, driver comfort, and traffic safety can be modeled. Towards developing such a model, several of the observable features of these oscillatory disturbances were quantified, including the mean traffic flows carried by these oscillations, and their travel–times between detector stations.

Finally, kinematic wave theory was found to describe the propagation of the oscillations to within small errors. For distances approaching one kilometer, and for two–hour periods, the theory predicted the locations of vehicles to within about 5 vehicle spacings. Further analysis
showed that some of these small discrepancies are explained by differences in car-following behavior across drivers. In all, the findings indicate that the observed disturbances propagated in ways consistent with the simplest theories of traffic flow.

Related research is summarized in the following section (Section 2). The freeway site used for this study, and the many hours of traffic data collected there, are described in section 3. In section 4, some of the notable features observed in the oscillations are presented, along with some evidence that the oscillations were triggered by vehicle lane-changing. Section 5 shows that the oscillations propagated through queued traffic in ways that closely matched the descriptions of simple theories. Some of the implications of these findings are briefly discussed in the dissertation’s sixth and final section.
2 RELATED RESEARCH

Some of the earliest studies on disturbances propagating through traffic streams were conducted by L.C. Edie and R.S. Foote as part of an ongoing effort to increase the capacity of New York’s Holland and Lincoln tunnels.\(^1\) Curves of cumulative vehicle counts verses time were created using data collected within the tunnels. Using this graphical tool, the researchers were able to trace the paths of six backward moving disturbances that were observed in one traffic lane under congested conditions. In addition, they estimated the average backward moving wave speed (10.2 mph) and these propagated as per kinematic wave theory.\(^2\) In a later article that used the same Lincoln and Holland Tunnel data, Edie and Bavarez (1967) studied the motion of 12 stoppage waves and found that these waves were all generated upstream of the bottleneck area, and that the average stoppage–wave speed was comparable to earlier findings (10.86 mph).

In 1968, T.W. Forbes and M.E. Simpson showed several examples of waves (i.e., disturbances in the flow of traffic) in a single freeway lane, by graphing vehicle trajectories in the time–space plane. They observed that some of the waves changed speed as they propagated upstream through the traffic queue, but most propagated at a stable speed. Some of the deceleration and acceleration waves fanned out as they propagated, some converged, and some remained at parallel over time and space. The researchers also measured wave travel time between vehicles (i.e., driver response times) and found that these were in the range of 1.06 to

\(^1\) Edie, L.C. and Foote, R.S. (1961).
2.4 seconds for a moderately dense queue, and from 0.9 to 2.0 seconds for a more congested segment. The researchers concluded that “Deceleration wave propagation response time will approach true driver sensitivity (minimum perception–judgment–response time) when headways are very short, with longer headways the driver–and–vehicle response to deceleration ahead may include an additional lag.”

More recently, B.S. Kerner and H. Rehborn (1996, 1999) studied start–stop waves on a three–lane Autobahn between Frankfurt a. Main and Bad Homburg, Germany. They examined the motion of waves as they traveled upstream through congested traffic for over 13 kilometers. The researchers reported that these waves propagated with a “stable” velocity and that they traveled through multiple junctions without significant changes in their structure.

In 1998, J.D. Windover conducted an empirical study concentrating on wave speed and some associated properties (i.e., variance in wave speed). He showed that wave speed was nearly constant over the 2–kilometer homogeneous freeway section studied. Additionally, he observed that acceleration and deceleration disturbances had statistically similar travel–times between fixed observation points. He also verified that the wave speed was independent of the traffic conditions (e.g., traffic flow and density), which implies that the flow–density relation is linear over the observed range of traffic conditions and that this held true for both acceleration and deceleration waves. These findings were consistent with Newell’s simplified kinematic

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wave theory, which stated that the fundamental relationship between flow and density could be estimated by a triangular–shaped curve. Another significant contribution was that Windover showed that wave propagation could be modeled as Brownian motion.

Most recently, J.M. Del Castillo (2001) postulated that vehicle speed perturbations might be ultimately caused by vehicle maneuvers such as a lane change or merging. He also stated that geometrical and environmental changes of the road might disturb the speed of the traffic stream. He also showed an instance of the fanning of a speed perturbation in a single travel lane as it propagated upstream over about a 5 kilometer stretch of freeway. He also showed vehicle trajectory plots, i.e., time–space diagrams from Forbes (1968); among the sample of waves shown, some waves remained at constant spacing, while others fanned out, and still other waves were converging. This showed that the observed wave speeds (over relatively short time periods) may vary even though the expected (e.g., long–term) wave speed might be constant as found by Windover.

Next, the freeway site and the data collected there are described. Then, in Section 4, findings from the study of these data are presented.

---

3 STUDY SITE AND DATA DESCRIPTION

The data used in this study were collected during six weekday mornings from freeway loop detectors along a ten–kilometer eastbound section of the Queen Elizabeth Way (QEW) in Ontario, Canada (see Figure 3.1). The six mornings analyzed were April 6 1998, December 15 1998, September 15 1999, September 21 1999, September 22 1999, and September 23 1999. In Figure 3.1, the loop detectors are shown as small circles. However, in reality, each detector consists of two closely spaced loops in each travel lane. Each detector station records vehicle counts, detector occupancies, and time mean vehicle speeds over 20–second intervals for each travel lane. The detector station numbering scheme used by the Ontario Ministry of Transportation (MTO) is also shown in Figure 3.1 and this scheme was used to identify the detector stations for this research. Throughout the study area, traffic responsive metering restricts the on–ramp flows (with the exception of the on–ramp just downstream of station 53 which has no meter).

The freeway geometry and traffic conditions along this particular stretch of the QEW were well suited for studying start–stop waves. During the typical morning commute period, a bottleneck formed somewhere between detector stations 51 and 52, creating a long queue of congested traffic upstream. Plots of oblique $N$–curves (i.e., queueing diagrams revealing the accumulation of vehicles due to congestion) constructed for longer periods confirmed that this bottleneck remained active for more than two hours on each of the study days. There are three detector stations downstream of the bottleneck (stations 52, 53 and 54) and twelve stations upstream
(stations 40–51). Numerous start-stop waves are initiated within, and propagate upstream through this queue.

Moreover, the freeway contains no obvious inhomogeneities other than the ramp junctions and an occasional auxiliary lane. These limited and identifiable inhomogeneities proved quite helpful in determining that wave propagation speeds were independent of the observed traffic flow (or vehicular density), and in studying the effects of the ramp flows on the oscillations. Also as a result of the ramp flows, the observed flows within the queue varied between ramp junctions providing a wider range of flows and vehicular density than would have otherwise been available. The latter phenomena occurred because the inflows from each of the site’s on-ramps exceeded the exit flows to each off-ramp. The net inflows restricted freeway traffic arriving at the interchanges such that each became a bottleneck, although not an active one.\footnote{For more details, see Cassidy and Mauch, 2001.}

The range of observed traffic conditions was further extended, thanks to freeway incidents that occurred on two of the study days. The first such incident, a freeway accident, occurred on the morning of April 6\textsuperscript{th} between stations 46 and 47. This accident caused lane closures, resulting in a slow moving queue of dense traffic upstream of the incident. On September 21\textsuperscript{st}, a second incident occurred a short distance downstream of detector 50 and persisted for about 40 minutes. Like the April 6\textsuperscript{th} incident, it created a long queue that was denser than those typical of rush-hour traffic.
Figure 3.1
Queen Elizabeth Way
(Eastbound)
The average discharge flow from the bottleneck caused by the April 6\textsuperscript{th} incident was about 2,190 vehicles per hour (vph). The average discharge flow from the September 21\textsuperscript{st} incident induced bottleneck was less than 4,600 vph. Normally, the recurrent bottleneck’s discharge rate was in the range of 6,200 vph. Both incidents were confirmed from detailed incident records maintained by MTO, the regional transport authority.

As was stated earlier, a bottleneck activated somewhere between detectors 51 and 52 on each of the six mornings studied. On December 15\textsuperscript{th}, the queue extended upstream to somewhere between stations 40 and 41. On the other 5 mornings, the queues extended backward over this entire stretch of freeway (i.e., upstream of detectors 40). Additionally, observations described in Cassidy and Bertini (1999) show that a bottleneck consistently formed at this location on each of several other mornings studied.

Next (in Section 3.1), oblique $N$–curves constructed from the loop data are used to confirm bottleneck activation and that queue formation does indeed occur as previously described. The findings are discussed in Section 4, which follows.
3.1 BOTTLENECK ACTIVATION AND QUEUE FORMATION

On each of the six observation days, the freeway segment between stations 51 and 52 contained an active bottleneck; i.e., a bottleneck characterized by queues upstream and freely flowing traffic downstream. Figure 3.2 verifies this using the December 15th data and it serves to illustrate the method used for making this determination. Shown in this figure are specially transformed curves of cumulative vehicle count, \( N \), versus time, \( t \), measured across all travel lanes at detectors 50 through 53. The curves were constructed such that the vertical separations between any two of them are the excess vehicle accumulations between their respective detectors due to vehicular delays.

An oblique coordinate system was used in the figure to plot \( N - q_0 \times (t - t_0) \) versus \( t \) for each curve’s starting time, \( t_0 \), and some choice of a background flow, \( q_0 \); the latter was selected so that the range of \( N - q_0 \times (t - t_0) \) was small when compared to the \( N \), itself. This coordinate system magnified the figure’s vertical axis which, in turn, amplified not only the curves’ vertical separations but also the changing slopes of the curves themselves. Since each curve was drawn using piece-wise linear interpolations through the detectors’ 20-second vehicle counts, flow changes were made more visible by these amplifications in the slopes.

The oblique \( N \)-curves in Figure 3.2 reveal the bottleneck’s location. They show that traffic on, and downstream of, station 52 was initially freely flowing and that conditions remained in free-flow between detector stations 52 and 53. Curves 51 and 50, however, eventually diverge
from their two downstream counterparts. Increased vehicle accumulations appearing upstream of detector 52 while free–flow conditions prevailed immediately downstream indicates that a bottleneck activated between detectors 51 and 52 (at approximately 6:44 am). It is suspected that this activation was due to merging; apparently some vehicles from the Cawthra Road on–ramp merged after traveling a good distance on the freeway's shoulder.\textsuperscript{11}

The increased separations between curves 51 and 50 (beginning at about 6:47:20) reveal when the backward–moving queue arrived at detector 51. It is also clear that the tail of this queue arrived at detector 50 at approximately 6:54; that curves 50 and 51 become roughly parallel at this time indicates that the flow over the entire intervening segment had become constrained by the queue.\textsuperscript{12} By inspecting curves at upstream detectors in the manner just described, it was observed that the tail–of–the–queue propagated upstream and eventually passed detector 41.

While the bottleneck was active, the flows within the upstream queue were constrained by the bottleneck. These flows can be estimated in a simple way by adopting a suggestion made by Newell (1993): the interchanges are modeled as single points along the freeway, with any ramp entries and exits occurring at these points. Vehicles that enter the queued portion of the freeway from an interchange restrict the flow of vehicles arriving at this same interchange from upstream; i.e., the on–ramp vehicles take available road space from the upstream freeway arrivals. Since

\textsuperscript{10}Daganzo, C.F. (1997).
\textsuperscript{11}It is known that the activation occurred each day at approximately the same time and location. Following this, the bottleneck exhibited a reproducible average vehicle discharge rate (see Cassidy and Bertini, 1999a). These point to exogenous cause(s) for the bottleneck's activation.
\textsuperscript{12}Newell, G.F. (1993).
at each interchange, the rate that vehicles entered the freeway via its on-ramp(s) exceeded the
rate exiting from its off-ramp(s), the flows on each successive upstream segment steadily
diminished; i.e., the queued flow just upstream of the bottleneck equaled the bottleneck’s
capacity, while the flow upstream of the first ramp junction was this bottleneck capacity minus
the net inflow from that interchange, etc. This is important because it provided a range of flows
within the queue, with the highest of these occurring at the bottleneck and the lowest flows
occurring at the upstream end of the queue.

On September 21st, the recurrent bottleneck (between stations 51 and 52) activated sometime
between 6:15 and 6:20 am. Then at about 7:57 am, an incident occurred that restricted
vehicular flows causing an incident–induced bottleneck to activate somewhere between
detectors 50 and 51 (see Figure 3.3). According to the MTO’s incident records: “Debris was
hanging down over lane 2 from a pedestrian overpass east of Cawthra Road from 8:10 am to
8:40 am.” It is presumed that this debris caused a bottleneck to activate between detectors 50
and 51, which deactivated the downstream (recurrent) bottleneck between detectors 51 and
52. Although, the debris was first reported at 8:10 am, it is clear from Figure 3.3 that a
bottleneck formed at this location prior to 8:00 am. The bottleneck deactivated at about 8:40
am, after which time, the recurrent bottleneck downstream (between stations 51 and 52)
reactivated.
Figure 3.2
Oblique N-Curves - Horizontally Shifted
(December 15 1998, Recurrent Bottleneck)
Figure 3.3
Oblique N-Curves - Horizontally Shifted
(September 21 1999, Incident-induced Bottleneck)
During the time of the incident, the flows were much lower than those observed while the recurrent bottleneck was active, as the former was more restrictive than the latter. This provided an opportunity to observe start–stop waves under different conditions (i.e., with lower prevailing flows, and with the active bottleneck at a different location).

In Figure 3.3, oscillations in the flows (i.e., start–stop waves) can be seen forming near station 49, which is upstream of the incident–induced bottleneck. These oscillations are even more prominent upstream at station 48. In contrast, the flows through the incident–induced bottleneck (at station 50) are smooth, as they are at downstream stations 51 and 52. This shows that these start–stop waves were initiated within the queue, upstream of the bottleneck, and that these disturbances did not propagate downstream to affect the flows through the bottleneck.

The oblique $N$–curves constructed from the April 6th data, in Figure 3.4, reveal that a bottleneck activated somewhere between detectors 46 and 47 at about 6:15:40 am. This bottleneck caused a queue to form that extended upstream of detector 40. Interestingly, no start–stop waves were observed in the queue upstream of this incident. The incident (near station 46) was confirmed by MTO’s incident records, which stated:

The incident started out as a left lane blockage at 6:16:33. At 6:19:47, it was upgraded to 2 left lanes blocked. At 6:30:24, it went back to single left lane blockage, and then cleared at 6:43:41. The whole incident lasted for 27 minutes 8 seconds.
Also from the re–scaled $N$–curves in Figure 3.4, it can be seen that the incident cleared around 6:43:40 am, releasing the previously constrained queue of vehicles downstream. Minutes later, at approximately 6:52:40 am, the recurrent bottleneck (between detectors 51 and 52) activated (see Figure 3.5). Again, the queue grew and eventually extended upstream beyond detector 40.

Once the bottlenecks’ locations, durations, and associated queues were identified, the congested regime was known. This is important for studying start–stop waves, as start–stop waves proved to be a property of congested traffic and cannot exist in freely flowing conditions. More is said about this in the following section.
Figure 3.4

Oblique N-Curves - Horizontally Shifted

(April 6 1998, Incident-induced Bottleneck)
Figure 3.5
Oblique N-Curves - Horizontally Shifted
(April 6 1998, Recurrent Bottleneck)
4 RESEARCH FINDINGS

Section 4 presents some of the observations made in this study. In section 4.1, the oscillations are displayed by constructing specially transformed cumulative vehicle count curves from the measured data. Section 4.2 explores plausible causation between the formation and growth of these oscillations and vehicle lane-changing (i.e., the merging and diverging that arises at the freeway’s interchanges). Section 4.3 shows these oscillations in the individual travel lanes. Section 4.4 examines the propagation speeds of these oscillations. Section 4.5 shows that the propagation of the individual disturbances that comprise these oscillations can be modeled as Brownian motion. Section 4.6 combines the results presented in Sections 4.1 through 4.5 to construct some vehicle trajectories, illustrating the effects that these oscillations have on the vehicles traversing the queues.

4.1 OSCILLATIONS IN QUEUED TRAFFIC

Curves like those in Figure 4.1(a) reveal a number of details about oscillations. Each curve shown here is the difference between the observed cumulative vehicle counts to time \( t \), \( N(t) \), and a 15–minute moving average of the \( N(t) \) centered on time \( t \), \( \overline{N}_{15}(t) \). As such, the slope of each curve is the deviation (in flow) from the 15–minute average flow at that time. The vertical displacements between one of these \( N \)–curves and its horizontal trend line are denoted

\[ N(t) = \sum_{i=0}^{t} n_i \text{, where } n_i \text{ is the number of vehicles (measured across all travel lanes) that passed over the detector during the 20–second count period } i \text{.} \]

\[ \overline{N}_{15}(t) = \left[ N(t + 7.5 \text{ min}) + N(t - 7.5 \text{ min}) \right]/2 \text{.} \]
Analogously, these $N - \overline{N} \textsubscript{15}$ curves can be thought of as residual plots, e.g., from a regression analysis, where the residuals are simply the difference between an observed variable and some model predicting this variable. (Here, the variable of interest is $N$, and the predicting model is the 15–minute moving average of $N$.)

Each deviation curve in Figure 4.1(a) is vertically displaced from its neighbor in proportion to the distance actually separating their detectors. (And most of the freeway stretch is reproduced along the left edge of the figure as a convenience to the reader). The April 6$^{th}$ data were used to create Figure 4.1(a). The counts for these curves were measured across all lanes. These were taken when the queue from the active bottleneck had filled the entire upstream portion of the freeway stretch. Some of the average flows measured during the one–hour period shown in the figure are annotated to illustrate that flows diminished upstream of the junctions.

The wiggles made prominent on some of the deviation curves are the start–stop waves themselves. They are characterized by sequences of high and low flows with periods of several minutes each. The amplitude of each oscillation eventually stabilized; the $N - \overline{N} \textsubscript{15}$ are not more than 50 vehicles (or about 16 vehicles per lane) and a scale is provided in Figure 4.1(a) to verify this.

$^{15}$ A 15 minute time window was used for the moving average as 15 minutes is at least 2 times the duration of the oscillations, in accordance with the Nyquist sampling criterion. The duration of the oscillations are in the range of 4 to 7 minutes.
Also included in this figure are dashed lines tracing the motion of some of these oscillations. (These lines are shown connecting the peaks of wiggles, as this made for an uncluttered presentation). These dashed lines show that the oscillations propagated upstream, against the flow of traffic. That the lines are parallel indicates that the wave speeds were nearly constant, despite the presence of ramp interchanges and the reduced flows that prevailed upstream of each ramp junction.  

The oscillations did not affect freely flowing traffic upstream of a queue’s tail. As evidence of this, Figure 4.1(b) presents the $N - \overline{N}_{15}$ curves constructed using the December 15th count data. The features of these curves are much the same as the April 6th curves shown in the previous figure. But on this day, the bottleneck's queue did not quite fill the entire upstream freeway stretch; i.e., the tail of the queue propagated beyond detector 41 (as before), but it did not reach detector 40 further upstream. Figure 4.1(b) shows that the oscillations, in turn, moved through the queue, but they did not continue beyond its tail; i.e., the wiggles displayed on curve 41 and most of its downstream counterparts are not evident on curve 40.

Moreover, the oscillations originated within the queue and their effects did not propagate downstream beyond the head of the queue. That these oscillations are initiated upstream of the active bottleneck is consistent with the findings of Edie and Foote (1961), Edie and Bavarez

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16 Further analyses showed that average wave speeds on each segment ranged from about 20 to 24 kilometers per hour and that flow had no systematic effects on these (See Section 4.4 of this thesis). Small reductions in average wave speeds did occur near interchanges, but we suspect this was linked to vehicle lane-changing. More is said about lane-changing effects in Section 4.2 of this thesis.
Figure 4.1(b)

$N - N_{15}$ Curves at Each Detector

(December 15 1998, Recurrent Bottleneck’s Queue)
Figure 4.1(a)

$N-\overline{N}_{15}$ Curves at Each Detector

(April 6 1998, Recurrent Bottleneck’s Queue)
(1967) and Del Castillo (2001). In both Figures 4.1(a) and (b), the deviation curves at detector 52 describe the bottleneck’s discharge and these remained relatively smooth, despite the oscillations upstream. It follows that the oscillations had relatively little effect on the growth of queues; i.e., they caused a queue’s tail to meander over time by small amounts. The extent of this meandering did not exceed about 16 vehicle spacings.

Of further note, the effects of these oscillations (i.e., the flow variations they caused) did not change systematically over time. Figure 4.2 presents evidence of this for several queued locations. These plotted lines display root mean squared errors (RMSE) taken over 20–minute periods; i.e., shown for each detector interval ending at time \( t \) are the

\[
\left[ \sum_{t-10\text{min}}^{t+10\text{min}} \left( N(k) - \bar{N}_{15}(k) \right)^2 / \text{(20 minutes)} \right]^{1/2}. 
\]

The values in Figure 4.2 are actually averages taken over four days (April 6th, September 15th, September 22nd, and September 23rd). These show no systematic trends over time, revealing that the oscillations did not steadily grow in amplitude and cause traffic to decay. Also, visual inspection of the curves shown in Figures 4.1(a) and (b) at any location reaffirms that the start–stop waves did not grow systematically over time.

But the oscillations and their effects did change with location. For example, traffic did not oscillate between the same two flows at all points within the queue. Rather, both the high and low flows diminished with distance from the active bottleneck in much the same fashion as did

\[17\] On December 15th, station 43 was malfunctioning. On September 21st, station 42 was malfunctioning.
the longer-run average flows. Evidence of this is provided in Table 4.1, which lists some of the average flows (and vehicle speeds) measured during the queued portion of the April 6th rush.

<table>
<thead>
<tr>
<th>Detector Station Number</th>
<th>Average Flows</th>
<th>Deviations From the Long-Run Mean Flow</th>
<th>Average Vehicle Speeds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Long-run (vph)</td>
<td>High (vph)</td>
<td>Low (vph)</td>
</tr>
<tr>
<td>40</td>
<td>3,900</td>
<td>4,640</td>
<td>2,640</td>
</tr>
<tr>
<td>42</td>
<td>3,900</td>
<td>4,670</td>
<td>2,650</td>
</tr>
<tr>
<td>44</td>
<td>4,380</td>
<td>5,250</td>
<td>3,030</td>
</tr>
<tr>
<td>46</td>
<td>4,910</td>
<td>5,550</td>
<td>3,640</td>
</tr>
<tr>
<td>48</td>
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<td>5,800</td>
<td>3,960</td>
</tr>
<tr>
<td>50</td>
<td>5,030</td>
<td>5,460</td>
<td>4,400</td>
</tr>
<tr>
<td>52</td>
<td>6,060</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 4.1
Traffic Conditions in a Long Freeway Queue
(April 6 1998, Recurrent Bottleneck’s Queue)

Figure 4.2 also shows the oscillations’ effects increased in the upstream direction. This occurred because new oscillations arose at different locations in the queue and grew to their full amplitudes while propagating upstream. Numerous examples of this are apparent in Figures 4.1(a) and (b).

---

18 Vehicles–per–hour.
19 Kilometers–per–hour.
Figure 4.2
20-Minute Root Mean Squared Errors of Flow Deviation Curves
(Four Day Average, Recurrent Bottleneck’s Queue)

Still more evidence of this is shown in Figure 4.3(a). For each detector, it displays the root mean squared errors of the $N - \overline{N}_{15}$ curves; these are the averages measured for 2 hours on each of the four days. The figure reveals certain telling trends. Namely, the oscillations’ effects grew as they propagated over freeway segments near interchanges. However, this growth did not continue on segments located further upstream where no interchanges were present, i.e., segments 40–41 and 41–42.
Figure 4.3 (b) shows long-run average flows measured by the detectors in individual lanes; i.e., these flows were measured over two-hour periods for each day (from 7–9 am, while the recurrent bottleneck was active and the tail of the queue was upstream of station 40). The
curves displayed are averages over the four days.\textsuperscript{20} This figure clearly shows decreases in shoulder–lane flows downstream of the off–ramps and increases in shoulder–lane flows downstream of the on–ramps. This figure also illustrates that the observed freeway flows increase as one passes the ramp junctions (in the downstream direction) as was described in Section 3.

This growth might seem puzzling in light of the very regular ways the oscillations propagated through queued traffic. Since each traveled at a nearly constant speed, the occurrence of new oscillations cannot be caused by diverging waves; i.e., these new formations cannot be explained by theories of traffic instability (e.g., Newell, 1962). The answer instead appears to lie with the freeway’s geometric features (i.e., ramps) and the lane–changing these induced. Further evidence of this is provided in Section 4.2, which follows this section.

Interestingly, the oscillations observed in the queue during the September 21\textsuperscript{st} incident were visibly different than those upstream of the recurrent bottleneck on that same day. Figure 4.4 shows the $N - \bar{N}_{15}$ curves constructed from the data on this day. From this figure, it is clear that the oscillations that occurred during the incident were smaller in amplitude and period than those found prior to, and following, the time of the incident. Figure 4.5 further illustrates this by displaying the RMSE’s of the $N - \bar{N}_{15}$. These were calculated for each detector station over

\textsuperscript{20} The four days used were April 6 1998, September 15 1999, September 22 1999, and September 23 1999. The December 15 1998 data were not used as station 43 was malfunctioning and the tail of the queue did not reach detector 40. The September 21 1999 data were not used because of the previously described incident and because station 42 was malfunctioning.
Figure 4.4
N-\bar{N}_{15} Curves at Each Detector
(September 21 1999, Incident-induced Queue)
the duration of the incident. For comparative purposes, this graph also shows the RMSE’s of the $N - \overline{N}_{15}$ calculated for the same day while the recurrent bottleneck was active (i.e., prior to the incident and after the incident cleared). The latter oscillations and their associated RMSE’s are comparable to those found upstream of the recurrent bottleneck on the other study days.

![Graph showing RMSE and Average Measured Flows](image)

**Figure 4.5**
(a) Average RMSE of $N - \overline{N}_{15}$ Curves, and  
(b) Average Measured Flows  
(September 21 1999, Incident-induced Queue)
4.2 FREEWAY GEOMETRY AND VEHICLE LANE–CHANGING

Strong correlation between vehicle lane changing and the formation and growth of new oscillations is evident in Figures 4.3(a) and (b), previously shown in Section 4.1. The latter of these figures shows long–run average flows measured by the detectors in individual lanes; i.e., these flows were measured over two–hour periods and averaged over 4 days. Each lane's flow remained nearly fixed across detectors 40–42, indicating an absence of systematic lane changing on the two upstream–most freeway segments. The flows tell a different story for the downstream segments, however. There they changed over space in ways that reveal lane changing. As an example, vehicles entered the shoulder lane while traveling between detectors 45 and 46; the latter measured higher flow in the shoulder lane and corresponding flow reductions in the center and median lanes.

But lane–changing is described in Figure 4.3(b) with very coarse time scales and certain patterns of interest might not be revealed at this scale. Figures 4.6(a) and (b) verify that higher systematic lane–changing persisted near interchanges. Shown here are oblique N–curves measured while an oscillation propagated past detectors 46 and 45. The curves from downstream detector 46 were shifted horizontally (by the oscillation’s trip time on the segment) and vertically to superimpose the initial portion of each pair of curves.

Figure 4.6(a) displays N–curves measured in the shoulder lane. The detector 46 N–curve rises above the one for detector 45; i.e., higher flows were measured by the downstream detector. As in the long–run observations in Figure 4.3(b), lane–changers entered the shoulder lane.
Figure 4.6
Oblique N-curves
(a) Shoulder Lane
(b) Center and Median Lanes
(c) Shown for a Longer Period
(April 6 1998, Recurrent Bottleneck’s Queue)
Figure 4.6(a) shows this occurrence was also observed during shorter time periods, e.g., time periods comparable to this oscillation’s period.

The curves in Figure 4.6(b) describe the oscillation as measured in the center and median lanes together. By grouping these lanes, their oblique N-curves exhibit effects opposite to those in the shoulder lane; i.e., the curves in Figure 4.6(b) reveal the net defection of vehicles. Since there are no ramps between stations 45 and 46, this difference in flows between upstream and downstream detectors for a given lane group must be from vehicles that are lane–changing.

The curves in Figure 4.6(c) were constructed for a longer time period so that short–run lane–changing could be studied over more of the rush. These curve pairs were translated much like before; the downstream curves at 46 were shifted horizontally by the average trip time for oscillations traversing this segment, $T$ (measured here in minutes). The rate of change in the vertical separation between each pair of curves was computed for every 20–second time interval using $N$–curves like those in Figure 4.6(c), i.e.

$$r(t)' = \left[ N(t+T) - N(t-20\text{sec}+T) \right]_u - \left[ N(t) - N(t-20\text{sec}) \right]_d ,$$

where:

$r(t)'$ = the rate of change for the 20–second interval ending at $t$ for lane group $l$,

$l = (sh)$ for the shoulder lane or $l = (cm)$ for the center and median lanes, and 

$u$ and $d$ denote counts from the upstream and downstream detectors, respectively.
Figure 4.7
Rate of Lane-changing Vs. Time
(April 6 1998, Recurrent Bottleneck’s Queue)
Systematic vehicle lane–changing rates, \( r(t)^{sh} \) and \( r(t)^{cm} \), are shown in Figures 4.7(a) 4.7(b) for segments 40–41 and 45–46, respectively. Like Figure 4.3, Figure 4.7 shows consistently higher systematic lane–changing rates on the segment 45–46 (near the freeway interchanges) than on segment 40–41 (upstream of the interchanges).

The cross–correlation term is the product of \( r(t)^{sh} \) and \( r(t)^{cm} \). Finally, a 10–minute moving average was used to smooth fluctuations in these cross correlation terms. Values were less than zero when lane–changing occurred (and small positive values greater than zero were interpreted as noise). Figures 4.8(a) and (b) provide typical examples of these cross–correlation terms, measured in queued traffic. They indicate not only that lane–changing rates fluctuated sharply, but also that these rates were consistently higher on segments near interchanges. Not so on segments further upstream (i.e., segments 40–41 and 41–42), away from interchanges.\(^{21}\)

Having shown the correlation between the oscillations’ spatial growth and short–run lane–changing, this section concludes with a puzzling observation. Namely, the correlation was less evident in the very dense queue caused by the September 21\(^{st}\) incident (just downstream of detector 50), as is evident in Figures 4.9(a) and (b). These figures were constructed in the identical manner as Figures 4.7(a) and (b), but with data from the incident–induced queue. Detector 42 was not functioning during this time. Yet Figures 4.5(a) and (b) still show that the spatial growth in the oscillations’ effects was small. This trend held despite frequent lane–

\(^{21}\) Figures 4.6, 4.7, and 4.8 were created using the April 6\(^{th}\) data. Similar patterns were observed in the flow deviation curves, vehicle lane-changing rates, and the cross correlation terms on other study days while the recurrent bottleneck was active.
changing on some segments, as evident in Figure 4.9. Apparently, lane-changing had less of an effect in queues of very slow-moving vehicles.

Figure 4.8
Cross Correlation Term Vs. Time
(a) Upstream Freeway Segments
(b) Downstream Freeway Segments
(April 6 1998, Recurrent Bottleneck’s Queue)
Figure 4.9
(a) Average RMSE of $N-N_t$ Curves
(b) Average Long-Run Flows
(c) Cross Correlation Term Vs. Time; Downstream Segments During Incident
(September 21 1999, Incident-induced Queue)
4.3 THE OSCILATIONS IN INDIVIDUAL TRAVEL LANES

The $N - \bar{N}_{1x}$ curves shown in the previous section were constructed by summing the vehicle counts from all three individual travel lanes. Figure 4.10 presents deviation curves for the individual travel lanes. It is clear that the oscillations in a travel lane bear remarkable resemblance to those in the adjacent travel lanes (at the same location). And, these notable similarities exist at each of the detector stations where oscillations arose.

Furthermore, once an oscillation formed and started propagating upstream in any of the three travel lanes, it appeared in the other two lanes within about a kilometer or so. And once this happened, these disturbances propagated like single waves that spanned all three travel lanes (with some relatively minor stochastic fluctuations).

Close inspection of Figure 4.10, reveals instances of waves that fan (at least over short distances, e.g., between two adjacent detectors) as well as instances of waves that diverge. These observations are consistent with those of Forbes and Simpson (1968) and those of Del Castillo (2001). Yet, if viewed over longer distances, they remain roughly parallel, a finding consistent with the Kerner and Rehborn (1996), the Forbes and Simpson (1968), and Windover (1998).
Figure 4.10
Individual Travel-lane $N - N_{15}$ Curves at Each Detector
(April 6 1998, Recurrent Bottleneck’s Queue)
Using curves like those in Figure 4.10, the times that the individual disturbances arrived at each detector were recorded for each travel lane using the April 6th and December 15th data (while the recurrent bottleneck was active) and the September 21st data (while the incident bottleneck was active). To test for systematic variations in wave arrival times across travel lanes, the difference between the lane–2 wave arrival time and the lane–1 wave arrival time was measured as:

$$\Delta T_{2-1,i} = T_{2,i} - T_{1,i},$$

where:

- $T_{2,i}$ is the time that the wave arrived to detector station $i$ in lane–2, and
- $T_{1,i}$ is the time that the wave arrived to detector station $i$ in lane–1.

Likewise, the lane–2 and lane–3 differences in wave arrival times were measured and denoted

$$\Delta T_{2-3,j} = T_{2,j} - T_{3,j},$$

The lane–2 wave arrival times were compared to the lane–1 and lane–3 arrival times because lane–2 is adjacent to both lane–1 and lane–3. It was presumed that lane–changing or some other type of driver interaction (between adjacent lanes) was responsible for the observed wave alignment.

There were no statistically significant differences, at the 0.05 level, in the $\Delta T_{2-1}$ and $\Delta T_{2-3}$ across study–days. Therefore, the three days of data were pooled and treated as a single

---

22 Lane–1 is the median lane. Lane–2 is the center lane, and lane–3 is the shoulder lane.
sample drawn from a common population. Figure 4.11 shows the $\Delta T_{2-1}$ and $\Delta T_{2-3}$ by time and location (i.e., detector station). As can be seen by Figure 4.11(a) the $\Delta T_{2-1}$ and $\Delta T_{2-3}$ are not time dependent, while 4.11(b) shows that $\Delta T_{2-1}$ and $\Delta T_{2-3}$ do not change over space; they do not increase (or decrease) as the waves propagate upstream.

Table 4.2(a) and 4.2(b) display the mean $\Delta T_{2-1}$ and $\Delta T_{2-3}$ for each day, respectively. They also list the standard deviations and the 95% confidence intervals from their means. Table 4.2(b) shows that the start–stop waves, on average, arrived about 6 seconds later in the shoulder lane than the median and center lanes. This difference was statistically significant at the 0.05 level. Figure 4.12 shows the box plots for $\Delta T_{2-1}$ and $\Delta T_{2-3}$.

Additionally, F–tests conducted on the $\Delta T_{2-1}$ and $\Delta T_{2-3}$ revealed that the differences in the $\Delta T_{2-1}$ and $\Delta T_{2-3}$ measured at different detector stations were less than random noise; see Tables 4.3(a) and (b) for summary statistics and F–test results for $\Delta T_{2-1}$ and $\Delta T_{2-3}$, respectively. Thus, we can conclude that either location has no real effect on $\Delta T_{2-1}$ and $\Delta T_{2-3}$ or that the samples were too small to detect the differences.

There might thus be some aligning mechanism (e.g., vehicle interaction such as vehicle lane–changing) that keeps the start–stop waves in the individual travel lanes aligned, causing them to propagate like single waves that span all travel lanes. Moreover, any given single–lane wave tends to look much like its counter parts in the other two travel lanes. That is, at any given
detector station, the $N - \bar{N}_{15}$ curves for the median lane tended to look much like the $N - \bar{N}_{15}$ curves for the center lane and the shoulder lane and vice versa.

<table>
<thead>
<tr>
<th>Study Day</th>
<th>Number of Observations (N)</th>
<th>Mean $\Delta T_{2-1}$ (sec.)</th>
<th>Standard Deviation (sec.)</th>
<th>95% Confidence Limits for Mean (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 6 1998</td>
<td>109</td>
<td>1.7</td>
<td>22.6</td>
<td>-2.6</td>
</tr>
<tr>
<td>December 15 1998</td>
<td>78</td>
<td>-4.1</td>
<td>24.0</td>
<td>-9.5</td>
</tr>
<tr>
<td>September 21 1999</td>
<td>85</td>
<td>0.5</td>
<td>25.8</td>
<td>-5.1</td>
</tr>
<tr>
<td>Three–day Average</td>
<td>272</td>
<td>-0.4</td>
<td>24.1</td>
<td>-3.2</td>
</tr>
</tbody>
</table>

Comparisons significant at the 0.05 level are indicated by ***.

Table 4.2(a)

Summary Statistics for Differences in Wave Arrival–times ($\Delta T_{2-1}$) by Study Day

<table>
<thead>
<tr>
<th>Study Day</th>
<th>Number of Observations (N)</th>
<th>Mean $\Delta T_{2-3}$ (sec.)</th>
<th>Standard Deviation (sec.)</th>
<th>95% Confidence Limits for Mean (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 6 1998</td>
<td>109</td>
<td>-7.5 ***</td>
<td>29.9</td>
<td>-13.2</td>
</tr>
<tr>
<td>December 15 1998</td>
<td>78</td>
<td>-3.8</td>
<td>27.2</td>
<td>-10.0</td>
</tr>
<tr>
<td>September 21 1999</td>
<td>85</td>
<td>-5.2 ***</td>
<td>21.9</td>
<td>-9.9</td>
</tr>
<tr>
<td>Three–day Average</td>
<td>272</td>
<td>-5.7 ***</td>
<td>26.8</td>
<td>-8.9</td>
</tr>
</tbody>
</table>

Comparisons significant at the 0.05 level are indicated by ***.

Table 4.2(b)

Summary Statistics for Differences in Wave Arrival–times ($\Delta T_{2-3}$) by Study Day
Figure 4.11(a) Differences in Wave Arrival-times by Time

Figure 4.11(b) Differences in Wave Arrival-times by Detector Station
<table>
<thead>
<tr>
<th>Detector Station</th>
<th>Number of Observations (N)</th>
<th>Mean $\Delta T_{2-1}$ (sec.)</th>
<th>Standard Deviation (sec.)</th>
</tr>
</thead>
<tbody>
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<td>40</td>
<td>20</td>
<td>5.0</td>
<td>25.0</td>
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<td>27</td>
<td>4.4</td>
<td>36.5</td>
</tr>
<tr>
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<td>21</td>
<td>3.8</td>
<td>25.0</td>
</tr>
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<td>43</td>
<td>22</td>
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<td>-4.1</td>
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<td>4.7</td>
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<td>40 – 49</td>
<td>272</td>
<td>-0.4</td>
<td>24.1</td>
</tr>
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</table>

Table 4.3(a) Summary Statistics for Differences in Wave Arrival-times ($\Delta T_{2-1}$) by Location, and Analysis of Variance for $\Delta T_{2-1}$ by Date, Location, and Date*Location

Tests significant at the 0.05 level are indicated by **

$^{23}$ The Type III SS is the sum of squares that results when that variable is added last to the model (where each effect is adjusted for every other effect).
<table>
<thead>
<tr>
<th>Detector Station</th>
<th>Number of Observations (N)</th>
<th>Mean $\Delta T_{2-3}$ (sec.)</th>
<th>Standard Deviation (sec.)</th>
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<td>-6.0</td>
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<td>-10.6</td>
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<td>-4.2</td>
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<td>47</td>
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<td>-0.6</td>
<td>20.0</td>
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<td>-3.4</td>
<td>17.8</td>
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<tr>
<td>49</td>
<td>19</td>
<td>-1.1</td>
<td>30.9</td>
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<td>40 – 49</td>
<td>272</td>
<td>-5.7</td>
<td>26.8</td>
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<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
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<td>Corrected Total</td>
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<table>
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<th>Pr &gt; F</th>
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<td>641.2</td>
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<td>0.5592</td>
</tr>
<tr>
<td>Date*Station</td>
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<td>6201.4</td>
<td>413.4</td>
<td>0.56</td>
<td>0.9058</td>
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</table>

Tests significant at the 0.05 level are indicated by ***.

Table 4.3(b)

Summary Statistics for Differences in Wave Arrival–times ($\Delta T_{2-3}$) by Location, and Analysis of Variance for $\Delta T_{2-3}$ by Date, Location, and Date*Location

24 The Type III SS is the sum of squares that results when that variable is added last to the model (where each effect is adjusted for every other effect).
Figure 4.12
Box Plot of Differences in Wave Arrival-times ($\Delta T_{2,1}$, $\Delta T_{2,3}$) by Detector Station
4.4 DISTURBANCE PROPAGATION SPEEDS

The wave propagation speeds were determined using measured wave travel times between adjacent detectors and the corresponding distances between the respective detectors. The mean wave travel time for all oscillations propagating over a particular freeway section was approximated by the time shift, \( t' \), that minimized the sum of squared errors, \( \text{SSE}(t') \), between the upstream and downstream \( N - \overline{N}_{15} \) curves bounding the freeway section. That is, the mean wave travel–time was defined to be the time, \( t' \), that minimized

\[
\text{SSE}(t') = \sum_{t=b}^{e} (D_u(t) - D_d(t + t'))^2,
\]

Eq. 4.1

where:

- \( D_u(t) \) and \( D_d(t) \) denote the \( N - \overline{N}_{15} \) at the upstream and downstream detectors, respectively, and
- \( b \) and \( e \) are a beginning and ending time that bound the oscillations.

Normally, the recurrent bottleneck activated and a queue formed prior to 7 am and the queue did not dissipate until well after 9 am. So the two–hour period, from 7 to 9 am, was used to measure wave travel times upstream of the recurrent bottleneck. For determining wave speeds during the September 21st incident, the squared errors in Eq. 4.1 were summed over the duration of the incident.

Figure 4.13 illustrates this technique by showing deviation curves constructed from April 6th count data for stations 41 and 42 from 7 to 9 am. The time axis for the station 42 curve was
shifted by time, \( t' = 129 \) seconds. This corresponds to a mean wave speed of about 20.9 kilometers per hour (kph) between the two detectors.

The oscillations tended to travel slower over segments containing ramp junctions and the wave propagation speeds tended to decrease near the tails of the queues (at station 40).

The mean wave speeds are in Figure 4.14 for the four days when all detectors functioned.\(^{25}\)

\(^{25}\) These four days were April 6\(^{th}\), September 15\(^{th}\), September 22\(^{nd}\), and September 23\(^{rd}\). The December 15\(^{th}\) data were not used as station 43 was malfunctioning. The September 21\(^{st}\) data were analyzed separately to study differences between the incident-induced queues and those caused by the recurrent bottleneck.
While the recurrent bottleneck was active, the mean wave velocities were 22.3 kph for freeway segments containing ramp junctions and 23.5 kph otherwise. The mean wave propagation speed over all freeway segments for these four days was 22.7 kph.

**Figure 4.14**

**Mean Wave Propagation Speeds by Freeway Segment**

(Four-Day Average, Recurrent Bottleneck’s Queues)
It was observed that the wave’s amplitude and period were smaller during the September 21st incident than those upstream of the recurrent bottleneck. To study the incident’s effects on wave speeds, mean wave propagation speeds were measured while the recurrent bottleneck was active (prior to and after the incident); separate measurements of wave propagation speeds were taken from the incident–induced queue. On this day, the mean measured wave speeds prior to and after the incident (22.5 kph) were comparable to those on other study days while the recurrent bottleneck was active (22.7 kph).

In contrast, the oscillations found in the denser queue caused by the incident propagated at a slower speed (averaging 19.8 kph). Figure 4.15 shows the mean wave velocities observed during the incident and while the recurrent bottleneck was active; the April 6th mean wave propagation speeds are also shown for comparative purposes.

In the next section, wave propagation speeds are used to show that the propagation of these start–stop waves may be modeled as Brownian motion. Section 4 closes by illustrating the effects that these waves have on the individual vehicles traversing through the traffic queues upstream of the bottlenecks.

Detector 42 was malfunctioning on September 21 1999. As such, mean wave propagation speeds for segments 41–42 and 42–43 were not measurable.
Figure 4.15
Mean Wave Propagation Speeds by Freeway Segment
(Incident-induced Queue Vs. Recurrent Bottleneck’s Queue)
4.5 MODELING WAVE PROPAGATION AS BROWNIAN MOTION

Windover (1998) showed that the propagation of disturbances in congested freeway traffic can be modeled as a random walk. By measuring disturbance travel times between adjacent detectors, he found that the mean disturbance speed was relatively constant (19.5 kph). In addition, he showed that the deviations in these observed travel times were normally distributed, and that these deviations were independent over time and distance. To do this, Windover measured the travel times of several disturbances observed propagating over four adjacent freeway segments in one lane of a five-lane freeway near Hayward, California. Three of the four segments were 1,700 feet in length, and the fourth was a 1,450 foot segment.

The QEW data in the present work were used to confirm and extend Windover’s findings by showing that wave propagation could be modeled as a random walk across all travel lanes, and not just in a single travel lane. These findings even held over relatively long distances that spanned ramp junctions. For this part of the research, individual wave travel times were measured over two adjacent and relatively long segments of the QEW. The first segment, starting at station 41 and extending to station 45, is denoted segment 41–45. This segment spans the Mississauga Road ramp junction. The second segment was bounded by stations 45 and 48; it spans the Highway 10 ramp junction.

For wave propagation to be modeled as Brownian motion (i.e., a random walk), three assumptions must hold. First, the expected wave travel time for a segment must be constant.
Second, wave travel–times must have stationary and independent increments.\textsuperscript{27} Lastly, deviations in wave travel–times must be normally distributed with mean = 0 and variance = $\sigma^2 t$; the variance in wave travel times must be proportional to the wave travel–times. To test how well the data supported the Brownian motion prerequisites, individual wave travel times were measured in each of the three travel lanes for several disturbances propagating from station 48 to station 41 on three of the study days (April 6\textsuperscript{th} and December 15\textsuperscript{th} while the recurrent bottleneck was active, and September 21\textsuperscript{st} during the time of the incident).

Statistical testing revealed that the differences in the mean wave travel–times were not statistically significant, at the 0.05 level, between the April 6\textsuperscript{th} and the December 15\textsuperscript{th} samples for segments, 41–45, 45–48, nor for the combined segment 41–48. This finding is consistent with the Brownian motion prerequisite that the expected wave travel–time for a given segment length must be constant. Furthermore, analysis of variance (ANOVA) tests showed that there were no statistically significant differences, at the 0.05 level, in the variances in wave travel–times for each of these study segments. Therefore, the two days of data (when the recurrent bottleneck was active) were pooled and treated as a single sample for each of the study segments. Table 4.4 shows the summary statistics and results of the ANOVA tests for the April 6\textsuperscript{th} and December 15\textsuperscript{th} data.

\textsuperscript{27} Stationary (time) increments mean that the expected wave speed does not change over time. Independent (time) increments mean that the wave propagation speed for any given time period does not affect the wave propagation speed for any other non–overlapping time period.
The September 21st incident wave travel-times were analyzed as a separate sample because testing revealed statistically significant differences, at the 0.05 level, between the mean wave travel-times for these waves and those upstream of the recurrent bottleneck. Further testing revealed statistically significant differences, at the 0.05 level, between the variances in wave travel-times for waves upstream of the incident and the waves upstream of the recurrent bottleneck for each of the three study segments. Table 4.5 shows the summary statistics and the results of the ANOVA tests for the September 21st wave travel times (denoted Bottleneck–cause = Incident) when compared to the combined April 6th & December 15th waves (denoted Bottleneck–cause = Recurrent).

Next, the deviations in wave travel-times were plotted and found to be normally distributed.\textsuperscript{28} Figures 4.16(a), (b) and (c) display the distributions for the observed wave travel-times for segments 41–45, 45–48, and 41–48, respectively, for waves upstream of the recurrent bottleneck. Likewise, Figure 4.17 shows the deviations in travel-times for the waves upstream of the September 21st incident.

\textsuperscript{28} Chi-square Goodness-of-Fit test failed to reject the null hypothesis, at the 0.05 level, that there were statistically significant differences between the empirically obtained deviations in travel-times and a (discrete) normal distribution.
<table>
<thead>
<tr>
<th>Segment: Date</th>
<th>Number of Observations (N)</th>
<th>Mean Wave Travel–time (sec.)</th>
<th>Travel–time Std. Deviation (sec.)</th>
<th>95% Confidence Limits for Mean (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41–45; 04/06/98</td>
<td>35</td>
<td>364.6</td>
<td>32.2</td>
<td>353.9 – 375.3</td>
</tr>
<tr>
<td>41–45; 12/15/98</td>
<td>27</td>
<td>351.8</td>
<td>27.9</td>
<td>340.8 – 362.9</td>
</tr>
<tr>
<td>41–45; Both Days</td>
<td>62</td>
<td>359.0</td>
<td>30.8</td>
<td>351.2 – 366.9</td>
</tr>
<tr>
<td>45–48; 04/06/98</td>
<td>35</td>
<td>338.9</td>
<td>39.7</td>
<td>325.2 – 352.5</td>
</tr>
<tr>
<td>45–48; 12/15/98</td>
<td>34</td>
<td>339.4</td>
<td>28.1</td>
<td>329.6 – 349.2</td>
</tr>
<tr>
<td>45–48; Both Days</td>
<td>69</td>
<td>339.1</td>
<td>34.2</td>
<td>330.9 – 347.3</td>
</tr>
<tr>
<td>41–48; 04/06/98</td>
<td>35</td>
<td>703.4</td>
<td>43.5</td>
<td>688.5 – 718.4</td>
</tr>
<tr>
<td>41–48; 12/15/98</td>
<td>22</td>
<td>693.6</td>
<td>41.1</td>
<td>675.4 – 711.9</td>
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<tr>
<td>41–48; Both Days</td>
<td>57</td>
<td>699.6</td>
<td>42.5</td>
<td>688.4 – 710.9</td>
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**Segment: 41–45**

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<tr>
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<td>2197.1</td>
<td>2197.1</td>
<td>2.20</td>
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<td>999.9</td>
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**Segment: 45–48**

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<tr>
<td>Date</td>
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<td>118.4</td>
<td>118.4</td>
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<td>Error</td>
<td>55</td>
<td>62281.6</td>
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**Segment: 41–48**

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</table>

Tests significant at the 0.05 level are indicated by ***.

Table 4.4
Summary Statistics for Wave Travel–times by Date, and Analysis of Variance by Date
<table>
<thead>
<tr>
<th>Segment; Bottleneck–cause</th>
<th>Number of Observations (N)</th>
<th>Mean Wave Travel–time (sec.)</th>
<th>Travel–time Std. Deviation (sec.)</th>
<th>95% Confidence Limits for Mean (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41–45; Incident</td>
<td>32</td>
<td>409.4</td>
<td>35.6</td>
<td>396.6 422.2</td>
</tr>
<tr>
<td>41–45; Recurrent</td>
<td>62</td>
<td>359.0</td>
<td>30.8</td>
<td>351.2 366.9</td>
</tr>
<tr>
<td>45–48; Incident</td>
<td>31</td>
<td>356.1</td>
<td>25.5</td>
<td>346.8 365.5</td>
</tr>
<tr>
<td>45–48; Recurrent</td>
<td>69</td>
<td>339.1</td>
<td>34.2</td>
<td>330.9 347.3</td>
</tr>
<tr>
<td>41–48; Incident</td>
<td>31</td>
<td>765.8</td>
<td>43.3</td>
<td>749.9 781.7</td>
</tr>
<tr>
<td>41–48; Recurrent</td>
<td>57</td>
<td>699.6</td>
<td>42.5</td>
<td>688.4 710.9</td>
</tr>
</tbody>
</table>

**Segment : 41–45**

<table>
<thead>
<tr>
<th>Source</th>
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<th>Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNCause</td>
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<td>50255.7</td>
<td>50255.7</td>
<td>44.89</td>
<td>&lt;0001 ***</td>
</tr>
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<td>Error</td>
<td>86</td>
<td>96289.8</td>
<td>1119.6</td>
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**Segment : 45–48**

<table>
<thead>
<tr>
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<th>SS</th>
<th>Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
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<td>BNCause</td>
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<td>5223.6</td>
<td>5.48</td>
<td>0.0215 ***</td>
</tr>
<tr>
<td>Error</td>
<td>86</td>
<td>81935.5</td>
<td>952.7</td>
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</table>

**Segment : 41–48**

<table>
<thead>
<tr>
<th>Source</th>
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<th>SS</th>
<th>Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
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</thead>
<tbody>
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<td>BNCause</td>
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<td>87884.0</td>
<td>87884.0</td>
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<td>&lt;0001 ***</td>
</tr>
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<td>86</td>
<td>157347.8</td>
<td>1829.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tests significant at the 0.05 level are indicated by ***.

Table 4.5
Summary Statistics for Wave Travel–times by Bottleneck–cause, and Analysis of Variance by Bottleneck–cause
Figure 4.16
Distribution of Wave Travel-times
(Two-day Average, Recurrent Bottleneck’s Queue)
Figure 4.17
Distribution of Wave Travel-times
(September 21 1999, Incident-induced Queue)
If the Brownian motion prerequisite of independent increments holds, then the propagation speed of a wave in segment 41–45 should be uncorrelated with the propagation speed of the same wave in segment 45–48. To test this, the observed wave speeds over segment 41–45 were compared to those over segment 45–48 for each of the individual oscillations. Figure 4.18 shows the results of this comparison for the recurrent bottleneck and the incident bottleneck. Faster than average waves over 41–45 are not necessarily faster than average over 45–48, illustrating that the prerequisite of independent increments holds.

Figure 4.18
Wave Propagation Speeds Scatterplot
(Segment 41-45 Vs Segment 45-48)

The correlation coefficient was -0.041 when comparing wave speeds for segments 41-45 and 45-48 while the incident bottleneck was active; and -0.156 for waves upstream of the recurrent bottleneck.
Finally, if deviations in wave travel times are independent between segment 41–45 and 45–48, then variances in these travel times should be additive, i.e., the variance in wave travel–times for segment 41–45 plus this variance for segment 45–48 should equal the observed variance in wave travel–times for the combined segment 41–48. This was tested using the combined April 6th and December 15th wave travel time data, and the September 21st wave travel–time data.

For those waves measured while the recurrent bottleneck was active, listed in Table 4.6(a), the variance in wave travel times for segment 41–45 was 950 seconds$^2$ and 1,170 seconds$^2$ for segment 45–48; the sum of which is 2,120 seconds$^2$. The measured variance for segment 41–48 was 1,807 seconds$^2$, a 17% difference.

For the waves upstream of the September 21st incident, shown in Table 4.6(b), the variance in wave travel–times for segment 41–45 was 1,264 seconds$^2$ and 651 seconds$^2$ for segment 45–48, summing to 1,915 seconds$^2$. The measured variance for segment 41–48 was 1,872 seconds$^2$, which is only about a 2% difference.

Tables 4.6(a) and (b) list summary statistics for the wave travel times, for the recurrent queue, and the incident–induced queue. Statistical testing (i.e., F–tests for difference in sample variances) failed to reject the null hypothesis that these differences in variance were statistically significant. This shows that the assumption of additive variances held. Consequently, the

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30 For the values shown in Table 4.6(a), the p–value from the F–tests was 0.271. For the values shown in Table 4.6(b), the p–value from the F–tests was 0.475. Both p–values are larger than 0.05, the value required to reject the null hypothesis at the 0.05 level.
assumption of independence holds reasonably well, which is the third prerequisite for Brownian motion.

<table>
<thead>
<tr>
<th>Freeway Segment</th>
<th>Number of Waves (N)</th>
<th>Segment Length (km)</th>
<th>Mean Wave Travel–times (sec.)</th>
<th>Variance In Wave Travel–times (sec.$^2$)</th>
<th>Mean Wave–Speed (kph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41–45</td>
<td>62</td>
<td>2.27</td>
<td>359.0</td>
<td>950</td>
<td>22.8</td>
</tr>
<tr>
<td>45–48</td>
<td>69</td>
<td>2.09</td>
<td>339.1</td>
<td>1,170</td>
<td>22.2</td>
</tr>
<tr>
<td>Combined</td>
<td>62</td>
<td>4.36</td>
<td>698.2</td>
<td>2,120</td>
<td>22.5</td>
</tr>
<tr>
<td>41–48</td>
<td>57</td>
<td>4.36</td>
<td>699.6</td>
<td>1,807</td>
<td>22.4</td>
</tr>
</tbody>
</table>

Table 4.6(a)
Summary of Wave Travel–times
(Recurrent Bottleneck, April 6th & December 15th)

<table>
<thead>
<tr>
<th>Freeway Segment</th>
<th>Number of Waves (N)</th>
<th>Segment Length (km)</th>
<th>Mean Wave Travel–times (sec.)</th>
<th>Variance In Wave Travel–times (sec.$^2$)</th>
<th>Mean Wave–Speed (kph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41–45</td>
<td>32</td>
<td>2.27</td>
<td>409.4</td>
<td>1,264</td>
<td>20.0</td>
</tr>
<tr>
<td>45–48</td>
<td>31</td>
<td>2.09</td>
<td>356.1</td>
<td>651</td>
<td>21.1</td>
</tr>
<tr>
<td>Combined</td>
<td>31</td>
<td>4.36</td>
<td>765.5</td>
<td>1,915</td>
<td>20.5</td>
</tr>
<tr>
<td>41–48</td>
<td>31</td>
<td>4.36</td>
<td>765.8</td>
<td>1,872</td>
<td>20.5</td>
</tr>
</tbody>
</table>

Table 4.6(b)
Summary of Wave Travel–times
(September 21st Incident–induced Bottleneck)
4.6 PREDICTING VEHICLE MOTIONS IN FREEWAY TRAFFIC QUEUES

A simple model was developed using the previously discussed results to gain some insights into
the effects that these disturbances have on queued vehicles. To accomplish this, a few
simplifying assumptions were made. First, it was assumed that a single (piece-wise linear)
flow-density curve adequately described the fundamental relationship at all points along the
freeway, an assumption consistent with previous observations. Figure 4.19 shows the
empirically obtained flow-density curve (for congested traffic) for the homogeneous sections of
the study site. A mean vehicle speed of 100 kph was used for the freely flowing portions of the
freeway, consistent with the observed free flow vehicle speeds provided by the detectors.

The oscillations were modeled with a six minute periodicity, which is comparable to the average
observed wave frequency. From the mean observed flows carried by the waves at each
detector station (shown in Table 4.1 of this thesis) and the empirically obtained flow-density
curve (shown in Figure 4.19), average vehicle speeds and mean vehicle spacings were
determined for each freeway segment.

The resulting hypothetical vehicle trajectories are shown in Figure 4.20. In this illustration,
vehicle trajectories were not constructed through the merge areas near on-ramps, nor through
the diverge areas near the off-ramps; driver interactions and the associated vehicular
movements are not well understood in these areas. An occasional “through-moving” vehicle
trajectory has been included to provide continuity for the reader.
From this, it can be seen that vehicles traveling the entire freeway stretch are expected to travel through about six slow–and–go oscillations. Figure 4.20 illustrates that mean vehicular speeds are lowest near the tail of the queue (where vehicular densities are highest). Additionally, it shows that the deviations in vehicular speeds brought about by these waves are the greatest in the denser portions of the queue, near the tail–of–queue. These observations are all consistent with previous observations.

Figure 4.19
Empirical Density Vs Flow Relationship

Figure 4.20
Representative Vehicle Trajectories
(Each trajectory represents 15 vehicles, or 5 in each lane.)
5 TESTING SIMPLE TRAFFIC FLOW THEORIES

Section 5 uses the observed oscillations to validate a few simple traffic flow models. Section 5.1 tests how well Newell’s Simplified Kinematic Wave Theory, a macroscopic model, explains the observed variations in flow. In Section 5.2, Newell’s Simplified Car-following Theory, a microscopic or car-following model, is evaluated to see how well this model explains the differences between observation and those predicted by the simplified kinematic wave theory.

5.1 SIMPLIFIED KINEMATIC WAVE THEORY

The oscillations propagated upstream in ways consistent with the simplified version of kinematic wave theory proposed by Newell (1993). This was verified for freeway segments without intervening interchanges using N-curves from the segment’s upstream and downstream detectors. The curve from the latter was shifted horizontally by the oscillations’ average trip time on the segment and vertically in an effort to superimpose this curve on its upstream neighbor; (the vertical translation is the average number of vehicles through which the waves passed while traversing the segment). Figure 5.1 illustrates this technique by displaying the oblique N-curve for station 41 along with the time-shifted oblique N-curve for station 42 using the April 6th data. In this figure, the station 42 oblique N-curve was shifted by 130 seconds, which corresponds to a mean wave speed of 21.0 kph.

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32 This simplified version describes the occurrence of kinematic waves as per Lighthill and Whitham (1955), with wave speed presumed independent of flow.
This recipe was carried out for continuous one-hour periods on each of four days.\textsuperscript{33, 34} The resulting root mean squared errors (RMSE) are provided in column 2 of Table 5.1; these were computed from the deviations measured each 20-second interval and the values shown are four-day averages. For each segment studied, the RMSE’s were less than 12 vehicles (in all three lanes). Thus, for distances approaching one kilometer, kinematic wave theory predicted vehicle locations (in each lane, on average) to within 4 vehicle spacings or less.

\textsuperscript{33} The four days used were April 6th, September 15th, September 22nd, and September 23rd. The December 15\textsuperscript{th} and September 21\textsuperscript{st} data were not used because of detector malfunctions.

\textsuperscript{34} The one-hour time periods used for this test were periods of relatively constant flows, except for the oscillations, as seen in Figure 5.1.
<table>
<thead>
<tr>
<th>Freeway Section (1)</th>
<th>RMSE: Kinematic Wave Theory (vehicles) (2)</th>
<th>RMSE: Naïve Approach (vehicles) (3)</th>
<th>Correlation Coefficient $^{35}$ (4)</th>
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<tr>
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<td>8.47</td>
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<tr>
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<td>48-49</td>
<td>11.05</td>
<td>19.47</td>
<td>0.823</td>
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<tr>
<td>Average</td>
<td>10.29</td>
<td>24.49</td>
<td>0.896</td>
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Table 5.1

Test of Kinematic Wave Theory
(Four–day Average, Recurrent Bottleneck’s Queue)

Column 3 of the table provides the errors that resulted from a more naïve approach to predicting queued flows. Here the long–run (one–hour) average flow measured by a downstream detector served as the estimate of the neighboring $N$–curve upstream. This represents how one might estimate queued traffic in the absence of any theory. In Figure 5.1, the horizontal line along the x–axis represents this naïve approach; and it represents a model predicting a flow of 3,760 vehicles per hour across all three lanes. It is clear that the time–shifted station 42 curve is a much better predictor of the station 41 curve than is the horizontal trend line depicting the naïve approach.

$$^{35} \rho_{\text{Col-2, Col-3}} = \sqrt{\frac{(\text{RSME}_{\text{Col-2}})^2 - (\text{RSME}_{\text{Col-2}})^2}{(\text{RSME}_{\text{Col-3}})^2}}$$
Column 4 lists the correlation coefficients from the two recipes. These indicate that, overall, the kinematic wave theory explained about 90 percent of the variation that was not explained by the naïve model (i.e., the wiggles and other time dependent changes in flow). These findings show that queued flows are constrained from downstream much as described by this simple theory.

The kinematic wave theory performed remarkably well in the relatively dense traffic, upstream of station 43 (i.e., segments 40–41 and 41–42) where the oscillations were not growing (as shown in Figure 4.3). This, in spite of the slow-and-go driving conditions found there (see Figures 4.1(a) and 4.1(b) and Table 4.1). Even in the less dense traffic observed over segments 45–46 and 48–49 where the oscillations were growing (see Figure 4.3), kinematic wave theory performed reasonably well explaining between 80 and 90 percent of the variation not explained by the naïve model.

It turns out that some of these small discrepancies between the flows predicted by kinematic wave theory and those provided by direct measurement can be explained by driver differences; i.e., by variations in the ways that different drivers respond to traffic downstream. This finding is a logical one. After all, the drivers passing through a wave at some downstream location are different from those who pass through the wave further upstream. And different drivers choose different spacings for following the vehicle ahead of them.36

5.2 SIMPLIFIED CAR–FOLLOWING THEORY

A model of driver differences was proposed by Newell (1999). According to this simple theory, the trajectory of some \( j^{th} \) vehicle is obtained by translating the \((j - 1)^{th}\) trajectory horizontally by duration \( \tau_j \) and vertically by distance \( d_j \) in the manner shown in Figure 5.2.

The \( \tau_j \) and \( d_j \) vary with each \( j^{th} \) driver such that the kinematic waves propagate from one vehicle to the next as a random walk. The wave’s average speed is equal to the ratio of \( d \) to \( \tau \), the expected values of the vertical and horizontal translations required to superimpose two neighboring trajectories.

![Figure 5.2](image)

**Figure 5.2**

Hypothetical Vehicle Trajectories
Traveling Between Two Detectors In a Single Freeway Lane
Given the above, the total time, $T(n)$, and distance, $D(n)$, covered by a wave propagating through $n$ vehicles is a bivariate process with independent increments, and therefore described by a bivariate normal distribution (BVN) with mean and covariance matrix proportional to $n$; i.e.,

$$[T(n), D(n)] \sim \text{BVN}\left(\begin{bmatrix} \tau \times n, d \times n \end{bmatrix} \begin{bmatrix} \sigma_{\tau}^2 & \sigma_{\tau d} \sigma_{\tau d} \sigma_{d}^2 \end{bmatrix} \times n \right)$$

Eq. 5.1

The present data were obtained from detectors at fixed locations. Therefore, the wave’s trip time on segment $i$, $T_i$, and the number of vehicles through which it propagated, $N_i$, (see Figure 5.2) were measured in each lane using oblique $N$–curves.$^{37}$

The $[T_i, N_i]$ are described by a bivariate normal distribution with mean and covariance matrix proportional to the segment’s length, $L_i$; i.e.

$$[T_i, N_i] \sim \text{BVN}\left(\begin{bmatrix} L_i/d \end{bmatrix} \begin{bmatrix} \sigma_{\tau d}^2/d \sigma_{\tau d}^2 \sigma_{d}^2/d \end{bmatrix} \times L_i \right)$$

Eq. 5.2

The joint distribution of $[T_i, N_i]$ would thus be independent of exogenous factors, including the flow and the freeway segment from which measurements came.

This was tested by plotting joint observations from queued freeway segments with different flows. For example, measurements from segments with moderately dense queues typical of a rush were compared with those from the very dense queue upstream of the incident. These
observations were transformed such that, if the theory holds, samples drawn from different $L_i$ would still come from a common distribution.\textsuperscript{38} To this end, we note that

$$\begin{bmatrix} T_i - E\{T_i\} \; N_i - E\{N_i\} \end{bmatrix} \sim \text{BVN}\left(\begin{bmatrix} 0, 0 \end{bmatrix}, \frac{1}{E\{N_i\}} \Sigma L_i \right) \equiv \text{BVN}\left(\begin{bmatrix} 0, 0 \end{bmatrix}, \Sigma d\right), \text{ Eq. 5.3}$$

where $\Sigma$ is the covariance matrix in (5.2). And since

$$\begin{bmatrix} T_i - \tau \left(\frac{L_i}{d}\right) \; N_i - \frac{L_i}{d} \end{bmatrix} \equiv \begin{bmatrix} \tau \left(\frac{T_i}{d} - \frac{L_i}{d}\right) \; \left(\frac{N_i}{d} - \frac{L_i}{d}\right) \end{bmatrix}, \text{ Eq. 5.4}$$

only the braced terms on the right–hand–side of (Eq. 5.4) were plotted and denoted as $[T^*, N^*]$. These $[T^*, N^*]$ are dimensionless and normally distributed about mean $[0, 0]$.\textsuperscript{39}

Figure 5.3(a) presents observations of $[T_i^*, N_i^*]$ measured in moderately dense queues on segments 40–41 and 41–42. Figure 5.3(c) shows another set of observations taken from segment 40–41; these from the denser September 21\textsuperscript{st} incident–induced queue.\textsuperscript{40} Segments 40–41 and 41–42 (the upstream–most segments) were selected because lane–changing was infrequent there.

\textsuperscript{37} The $N_i$ in each lane were estimated using cumulative curves that were constructed from counts simultaneously measured across all travel lanes. The $T_i$ came from curves separately constructed for each lane.

\textsuperscript{38} Without the transformations, different segment lengths would affect comparisons, since the mean values of $N_i$ and $T_i$ increase linearly with $L_i$ while their standard deviations increase with the square root of $L_i$.

\textsuperscript{39} Chi-square Goodness-of-Fit test failed to reject the null hypothesis, at the 0.05 level, that there were statistically significant differences between the empirically obtained $T^*$ and $N^*$ distributions and (discrete) normal distributions.

\textsuperscript{40} Observations from segment 41–42 were not available during the incident because detector 42 malfunctioned.
Despite the different flows (2,500 vph on segment 40–41 during the incident compared to 4,000 vph on the same segment while the recurrent bottleneck was active), the similar scatter in Figures 5.3(a) and 5.3(c) indicate that both samples of \([T_i^*, N_i^*]\) were drawn from similar distributions, if not from a common one. Statistical testing supported the null hypothesis that these samples were drawn from a common population, i.e., there were no statistically significant differences (at the \(p = 0.05\) level) when comparing the BVN parameter estimates describing these two data sets.

Furthermore, there are no statistically significant differences (at the 0.05 level) between the distribution in Figure 5.3(d) with the ones shown in Figure 5.3(a) and 5.3(c). Figure 5.3(d) displays the \([T_i^*, N_i^*]\) from the September 21st incident–induced queue on segments 45–46 & 48–49 (with mean flows of 3,600 and 3,900 vph, respectively). Therefore, the incident–induced queue (at upstream segment 40–41), the incident–induced queue (at downstream segments 45–46 and 48–49), and the recurrent queues (at upstream segments 40–41, and 41–21) all appear to come from very similar distributions.

In contrast, the \([T_i^*, N_i^*]\) shown in Figure 5.3(b) appear to come from a different distribution; these data were taken from the moderately dense queues upstream of recurrent bottlenecks on segments 45–46 & 48–49 (with mean flows of 5,100 and 5,400 vph, respectively). This is confirmed by the \(p\)-values shown in Table 5.2 for difference of sample variances tests. Thus, Newell’s simple car–following theory provided a reasonable description of the driver
differences in relatively dense traffic streams, and it explains some of the additional effects not captured by kinematic wave theory.

Newell’s theory does not describe the effects of lane–changing and Figure 5.3(b) presents observations of $[T^\ast, N^\ast]$ from segments in moderately dense queues that are marked by frequent lane–changing. The wide scatter of these data indicates the variance of the $\tau_j$ and the $d_j$ increased along the wave paths. The insertion and defection of (lane–changing) vehicles evidently disrupted traffic streams.

Yet these effects were not visible in the incident’s very dense queue. Figure 5.3(d) displays measurements of $[T^\ast, N^\ast]$ during the incident. One of the two freeway segments (45–46) used for this figure was marked by frequent lane–changing while the other (48–49) was not; the reader can refer back to Figure 4.10(c) to verify this. But the scatter of the $[T^\ast, N^\ast]$ from both segments is small.
### Freeway Segment, Bottleneck–cause
<table>
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<th>Variable</th>
<th>t-test</th>
<th>F–test</th>
<th>ANOCOVA</th>
</tr>
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<tr>
<td>N*</td>
<td>0.644</td>
<td>0.000 ***</td>
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<td>T*</td>
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### Recurrent
<table>
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<th>Freeway Segment, Bottleneck–cause</th>
<th>Variable</th>
<th>t-test</th>
<th>F–test</th>
<th>ANOCOVA</th>
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</thead>
<tbody>
<tr>
<td>(40-41) &amp; (41-42)</td>
<td>(45-46) &amp; (48-49)</td>
<td>N*</td>
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<tr>
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<td>Recurrent</td>
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<tr>
<td>Recurrent</td>
<td>Incident</td>
<td>T*</td>
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<td>0.596</td>
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</tr>
<tr>
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<td>(45-46) &amp; (48-49)</td>
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<td>0.400</td>
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<td>(45-46) &amp; (48-49)</td>
<td>(45-46) &amp; (48-49)</td>
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<td>0.000 ***</td>
<td>0.955</td>
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<tr>
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<td>Incident</td>
<td>T*</td>
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</table>

Tests significant at the 0.05 level are indicated by ***.

Table 5.2
p–values for Statistical Testing for Differences in \( [T^*, N^*] \) Distributions
Figure 5.3

T* vs. N*

(a) Upstream Segments in Recurrent Queues
(b) Downstream Segments in Recurrent Queues
Figure 5.3

(c) Upstream Segments in Incident-induced Queues
(d) Downstream Segments in Incident-induced Queues
6 CONCLUDING REMARKS

Perhaps, the most significant single finding from this research is that these oscillations appeared and grew only when vehicle lane changing was present in relatively fast moving queues. The oscillations were never found outside queues (i.e., in freely flowing traffic). And, they didn’t grow (appreciably) when initiated in dense traffic queues. Table 6.1 summarized the findings relating oscillation growth, vehicle lane changing, and vehicular density. (Table 6.1 pertains only to queued traffic on multi-lane traffic where vehicle lane changing is permitted.)

<table>
<thead>
<tr>
<th></th>
<th>Systematic Vehicle Lane Changing Present</th>
<th>Systematic Vehicle Lane Changing Not Present</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Moderately Dense Queued Traffic</strong></td>
<td>Measurable Wave Growth</td>
<td>No Wave Growth</td>
</tr>
<tr>
<td><strong>Very Dense Queued Traffic</strong></td>
<td>Nominal Wave Growth</td>
<td>No Wave Growth</td>
</tr>
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</table>

Table 6.1
Summary of Findings

The observed oscillations displayed regular features that did not cause the characteristics of queued traffic to change over time. The propagation of these oscillations was described well by kinematic wave theory. Driver differences added small effects, but marked improvements in traffic flow theories will likely come by incorporating lane–changing effects. After all, it appears that in moderately dense queues, oscillations were due more to lane–changing (including those
caused by merging and diverging) than to endogenous car-following effects. Any new theories aimed at improving traffic prediction can be tested against the present findings. The predictions of any new theories need to be at least as good as those of the simple models examined in Section 5.

That lane-changing effects were less evident in the incident’s very dense queue is puzzling. But this finding is consistent with previous reports that speeds, flows, densities, etc. exhibit greater variation when measured in moderately dense queues, as compared with measurements in queues of higher density.41

But this finding does not necessarily imply that queued traffic is composed of two distinct classes; i.e., synchronized and jammed. In the present study, data were drawn from moderately dense queues that formed daily and from one very dense queue created by an incident. There are ranges of density for which no observations were available. It may be that data scatter (i.e., correlation) changes continuously with density.

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REFERENCES


