Title
White Paper: Measuring the Neutrino Mass Hierarchy

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Publication Date
2018-03-06

Peer reviewed
White Paper: Measuring the Neutrino Mass Hierarchy

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Lawrence Berkeley National Laboratory
September 30, 2013

†Our colleague Stuart Freedman was a member of the Neutrino Hierarchy Working Group. His sudden passing, on November 10, 2012, was a terrible loss to the Nuclear Science and Physics Divisions at LBNL and to the entire international scientific community. It was a great personal loss to those of us who were Stuart’s friends. Stuart set extraordinarily high standards for scientific inquiry and integrity. We hope that these are reflected in our report.

*Co-chair
Abstract

This whitepaper is a condensation of a report by a committee appointed jointly by the Nuclear Science and Physics Divisions at Lawrence Berkeley National Laboratory (LBNL). The goal of this study was to identify the most promising technique(s) for resolving the neutrino mass hierarchy. For the most part, we have relied on calculations and simulations presented by the proponents of the various experiments. We have included evaluations of the opportunities and challenges for these experiments based on what is available already in the literature.

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1 Executive Summary

The neutrinos remain the most enigmatic of the fundamental fermions and we still don’t know the answers to several basic questions: what is their absolute mass scale? Do neutrinos violate CP? Are neutrinos Dirac or Majorana? Knowledge of the neutrino mass hierarchy can help to inform each of these questions and is thus a fundamental step towards completion of the Standard Model of particle physics. Moreover, it may lead to hints of physics beyond the Standard Model, since neutrinos may obtain their mass in a different way than other fundamental fermions. The neutrino mass hierarchy has implications, as well, for cosmology and for neutrinoless double beta decay. Though these are undeniably fundamental questions, it was outside the scope of this study to evaluate the importance of determining the neutrino mass hierarchy relative to other opportunities on a similar timescale.

Of the experiments considered, only the long baseline technique has demonstrated the ability to measure the mass hierarchy independent of oscillation parameters. LBNE, in combination with T2K/NOνA, promises to resolve the mass hierarchy with a significance of more than $3\sigma$ by 2030. Hyper-Kamiokande can achieve a similar significance on a similar timescale by combining a shorter baseline measurement with atmospheric neutrino data. The European LAGUNA-LBNO project promises an exceptional sensitivity of greater than $(5\sigma)$ on a short timescale ($\approx 1$ year of data) due to its very long baseline, but the project status remains in question.

A variety of other experiments have been proposed with some sensitivity to the mass hierarchy. These are of interest primarily because some could be completed much more rapidly than long-baseline projects and careful attention to the design of the experiments could give them a reasonable chance of measuring the mass hierarchy.

The most viable approaches appear to be reactor neutrinos (JUNO, formerly known as Daya Bay II) and neutrinos in ice (PINGU at IceCube). While requiring significant technological advances in detector design and performance, JUNO promises a potential sensitivity of more than $3\sigma$ ($4\sigma$) assuming current (future 1.5%-level) uncertainties on $\Delta m^2_{32}$. This challenging experiment appears to be on the fast track to approval in China. PINGU offers excellent statistical sensitivity to the hierarchy, with the primary challenge lying in controlling and evaluating systematic effects. Sensitivity estimates vary and are subject to the choice of oscillation parameters and hierarchy. In a favorable scenario, a $4\sigma$ measurement could be achieved with 3 years of data; a more conservative analysis finds a $1 - 5\sigma$ range in sensitivity. At the time of composing this report, these studies are still being refined.

Future dark energy experiments such as MS-DESI (formerly BigBOSS), Euclid, and LSST have the capability to measure the sum of the neutrino masses with precision relevant to the mass hierarchy. Should the hierarchy be normal and the neutrino masses minimal, MS-DESI could provide an early indication and other dark energy experiments could discern this at a several-sigma level from the power spectrum on a timescale comparable to that for LBNE.

While none of these other experiments, nor current long-baseline oscillation measurements (T2K, NOνA), is certain to be able to measure the mass hierarchy, one or more of them could do so if oscillation parameters are favorable. With more probability, one might find an indication of the hierarchy at, say, a two-sigma level.
2 Introduction

The now well-accepted picture of neutrino mixing involves three underlying mass states, with three mixing angles defining the linear superpositions that make up each of the three weak, or flavor states. The magnitude of the mass-squared splitting between states $\nu_1$ and $\nu_2$ is known from the KamLAND reactor experiment, and the much-larger splitting between the third, $\nu_3$ state and the $\nu_1 - \nu_2$ pair is known from atmospheric and long-baseline experiments. However, pure neutrino oscillations are sensitive only to the magnitude of the mass splitting, not the sign. Defining the $\nu_1$ state as having the largest admixture of the electron flavor eigenstate, the sign of the mass splitting between states $\nu_2$ and $\nu_1$ is determined to be positive ($\Delta m^2_{21} > 0$) using the pattern of neutrino oscillations through the varying-density solar medium. However, the corresponding sign of $\Delta m^2_{32} \approx \Delta m^2_{31}$ remains unknown. That is, there are two potential orderings, or “hierarchies”, for the neutrino mass states: the so-called “normal hierarchy”, in which $\nu_3$ is the heaviest, and the “inverted hierarchy”, in which $\nu_3$ is the lightest (as shown in Fig. 1).

Figure 1: Pictorial representation of the possible neutrino mass hierarchies. Note: $\Delta m^2_{atm}$ is equivalent to $\Delta m^2_{32}$ and $\Delta m^2_{sol}$ is equivalent to $\Delta m^2_{21}$. [1].

2.1 Status of Neutrino Mixing

The relationship between neutrino flavor $\{\nu_e, \nu_\mu, \nu_\tau\}$ and mass $\{\nu_1, \nu_2, \nu_3\}$ eigenstates is described by the PMNS mass matrix [2, 3]:

$$
\begin{pmatrix}
|\nu_e\rangle \\
|\nu_\mu\rangle \\
|\nu_\tau\rangle
\end{pmatrix} =
\begin{pmatrix}
\begin{bmatrix}
c_{12}c_{13} \\
-s_12c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} \\
s_12s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}}
\end{bmatrix} &
\begin{bmatrix}
s_{12}c_{13} \\
-c_{12}s_{23}s_{13}e^{i\delta_{CP}} \\
-s_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}}
\end{bmatrix} &
\begin{bmatrix}
-s_{12}c_{13} \\
-c_{12}s_{23}s_{13}e^{i\delta_{CP}} \\
-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}}
\end{bmatrix}
\end{pmatrix}
\begin{pmatrix}
e^{i\alpha_1/2}|\nu_1\rangle \\
e^{i\alpha_2/2}|\nu_2\rangle \\
|\nu_3\rangle
\end{pmatrix}
\tag{1}
$$
where \( c_{ij} \equiv \cos \theta_{ij} \) and \( s_{ij} \equiv \sin \theta_{ij} \). This matrix depends on: three mixing angles \( \theta_{12}, \theta_{13}, \) and \( \theta_{23} \), of which the first and last are the dominant angles for solar and atmospheric oscillations, respectively; a Dirac phase \( \delta_{CP} \) that can induce CP-violating differences in the oscillation probabilities for conjugate channels such as \( \nu_\mu \to \nu_e \) versus \( \bar{\nu}_\mu \to \bar{\nu}_e \); and two Majorana phases \( \alpha_1 \) and \( \alpha_2 \) that will affect the interference among mass eigenstates in the effective neutrino mass probed in the lepton-number-violating process of neutrinoless double \( \beta \) decay.

The current best knowledge of the oscillation parameters is given in Table 1.

### Table 1: Neutrino parameters from \([4]\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best-fit value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin^2 2\theta_{12} )</td>
<td>( 0.857 \pm 0.024 )</td>
</tr>
<tr>
<td>( \sin^2 2\theta_{23} )</td>
<td>( &gt; 0.95 )</td>
</tr>
<tr>
<td>( \sin^2 2\theta_{13} )</td>
<td>( 0.098 \pm 0.013 )</td>
</tr>
<tr>
<td>( \Delta m^2_{21} )</td>
<td>( 7.50 \pm 0.20 \times 10^{-5} \text{ eV}^2 )</td>
</tr>
<tr>
<td>(</td>
<td>\Delta m^2_{32}</td>
</tr>
</tbody>
</table>

Using these values, we find that the first maximum of oscillation, which occurs when

\[
\Delta = 1.27 \frac{\Delta M^2 (\text{eV}^2) L(\text{km})}{E(\text{GeV})} = \frac{\pi}{2}
\]

(2)

determines the oscillation distance according to

\[
L(\text{km}) = 16 \times 10^3 E(\text{GeV})
\]

(3)

\[
= 16 \ E(\text{MeV})
\]

(4)

in the “solar” case while

\[
L(\text{km}) = 540 \ E(\text{GeV})
\]

(5)

in the “atmospheric” case. With reactor neutrinos, whose typical energy is 4 MeV, the preferred distance is about 65 km for “solar” oscillations, while for an accelerator experiment with a typical energy of 3 GeV, we would want a distance of about 1500 km for “atmospheric” oscillations, or those needed to determine the sign of the \( \Delta m^2_{31} \) mass splitting.

### 2.2 Motivation for Determining the Neutrino Mass Hierarchy

Progress in neutrino physics has been impressive over recent decades, with the discovery of atmospheric neutrino disappearance by the Super-Kamiokande experiment, observation of solar neutrino flavor change by the Sudbury Neutrino Observatory (SNO), and final confirmation of the phenomenon of neutrino oscillation by the KamLAND reactor neutrino experiment, which observed both disappearance and reappearance. More recently, the final mixing angle, \( \theta_{13} \), has been determined by the Daya Bay collaboration to be large:
\(\sin^2 2\theta_{13} = 0.089 \pm 0.010\) (stat) \(\pm 0.005\) (syst) \([5]\), an observation subsequently confirmed by RENO \([6]\) and others.

Once we understand the ordering of the neutrino mass states, the uncertainty on a measurement of the CP-violating phase, \(\delta_{CP}\), is significantly reduced. Knowledge of the mass hierarchy would define the scope for future neutrinoless double beta decay (0\(\nu\beta\beta\)) experiments, seeking to resolve the mass nature of the neutrino, by limiting the domain for observation of a signal. In combination with cosmological measurements, which are sensitive to the sum of neutrino masses, knowledge of the mass hierarchy could also be used to determine the absolute mass scale of neutrinos. The mass hierarchy could also further understanding of core-collapse supernovae. For these many reasons, determination of the neutrino mass hierarchy is thus a fundamental step towards completion of the Standard Model of particle physics.

3 Related Experiments that will not Resolve the Hierarchy

There are a variety of experiments subtly influenced by the neutrino mass hierarchy. However, the anticipated sensitivity of these experiments is unlikely to provide useful information about the hierarchy. These include solar neutrino experiments, measurements of galactic supernovae, neutrinoless double beta decay, and direct measurement of neutrino masses. These are summarized in Table 2.

Solar neutrino measurements are sensitive to the sign of \(\Delta m^2_{21}\). While the sensitivity of solar neutrino oscillations to the sign of the remaining mass splitting, \(\Delta m^2_{31}\), is negligible, precision probes of the solar sector can provide a more detailed understanding of the interaction of neutrinos with matter, and thus inform terrestrial experiments.

The observation of neutrinos from core-collapse supernovae would be sensitive to the mass hierarchy via MSW effects in two ways: “standard” adiabatic level-crossing effects, and collective effects in the core. The former is well understood, as evidenced by observations in the solar neutrino sector, and predicts large rate and systematic shape distortions, affecting the relative rates observed in different detectors. The collective effects are less well understood, with large uncertainties and many open questions. Further work is needed to fully understand these effects, and to determine with confidence that they do not smear out the standard MSW effects to the point at which a hierarchy determination becomes impossible. As such, due to the large model uncertainties and the non-predictive nature of the source, there is an inherent difficulty in using these observations to measure the hierarchy. Because the rate of supernovae in the galaxy is about one per century, it is also difficult to anticipate the next experimental opportunity in this field. Instead, knowledge of the neutrino mass hierarchy would provide valuable input to supernova modeling.

Current direct neutrino mass measurements do not have sufficient sensitivity to reach either the normal or the inverted hierarchy regimes. It is not anticipated that these experiments could reach mass scales below 0.1 eV in the foreseeable future. The primary purpose for current experiments is to probe the mass scale in the quasi-degenerate region in a model independent way. They will also complement neutrinoless double beta decay experiments
and cosmological studies down to the 0.2 eV level.

Neutrinoless double beta decay experiments are sensitive to the neutrino mass hierarchy, but taken alone cannot provide a definitive measurement unless a clear observation is made in an unambiguous region of parameter space. The unambiguous region, unfortunately, occurs at an extremely low value of the decay rate, orders of magnitude below that accessible to current experiments. As a consequence, this is not considered a viable method by which to determine the neutrino mass hierarchy; instead, the hierarchy would provide a valuable input to this field, defining the scope for future experiments. The next-generation experiments, planned for late 2010s to early 2020s, will aim to have sensitivity sufficient to completely explore the inverted hierarchy region, independently of nuclear effects.

Table 2: Comparison of mass hierarchy (MH) experiments.

<table>
<thead>
<tr>
<th>Technique</th>
<th>MH sensitivity</th>
<th>Timescale for results</th>
<th>Major concerns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>Zero</td>
<td>Ongoing</td>
<td>No sensitivity to sign of $\Delta m^2_{32}$</td>
</tr>
<tr>
<td>Supernova</td>
<td>Model dependent</td>
<td>Unpredictable</td>
<td>Unpredictable timescale, astrophysical uncertainties</td>
</tr>
<tr>
<td>All</td>
<td>Zero (unless degenerate)</td>
<td>$\sim$2020</td>
<td>Only sensitive in degenerate region</td>
</tr>
<tr>
<td>Neutrinoless double beta decay</td>
<td>Limited by Nature</td>
<td>$\sim$2025</td>
<td>No scope for definitive mass hierarchy measurement</td>
</tr>
</tbody>
</table>

4 Long Baseline Experiments

The long-baseline experiments have sensitivity to the neutrino mass hierarchy, due to the interaction of neutrinos with matter as they pass through the Earth. The baseline itself is a critical factor in the hierarchy sensitivity, and thus we consider the experiments in multiple categories: near term and relatively short baseline (T2K and NO$\nu$A); the US-based long-baseline experiment (LBNE); and alternatives outside the US (primarily HyperK and LBNO). While T2K has very little sensitivity to the hierarchy, due to the short baseline, NO$\nu$A has the potential to make a measurement at the $2-3\sigma$ level, dependent on the value of the CP phase parameter, $\delta_{CP}$. LBNE has demonstrated $2\sigma$ ($3\sigma$) sensitivity over 100% (80%)
of the values of $\delta_{CP}$ with 10 years of data. LBNO has the potential to achieve a high sensitivity on a shorter timescale, due to the longer baseline (therefore increased sensitivity to the matter effects). HyperK can achieve a similar sensitivity to LBNE on a similar timescale by combining the somewhat shorter baseline measurement with an independent atmospheric measurement in the same detector. A combination of several experiments at different baselines (e.g. T2K+NO$\nu$A, or T2K/NO$\nu$A+LBNE, etc) can disentangle the competing effects of CP violation and matter-induced neutrino-antineutrino differences, and thus improve the constraints in the hierarchy significantly beyond any single measurement.

4.1 T2K and NO$\nu$A

4.1.1 Introduction

Nearly all the neutrino parameters are already well measured, as shown in Table 1. In the absence of the matter effect, the amplitude for $\nu_\mu \rightarrow \nu_e$ oscillation is given by

$$\langle \nu_e|\nu_\mu(t) \rangle = \Delta_{21} \sin 2\theta_{12} \cos \theta_{23} + e^{-i(\Delta_{31}+\delta)} \sin \Delta_{31} \sin 2\theta_{13} \sin \theta_{23}$$

where we have assumed that $\Delta_{31} \approx \pi/2$ so that $\Delta_{21} \approx \Delta_{31}/30$ is small.

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta_{31}$$

$$+ \Delta_{21} \sin 2\theta_{13} \sin 2\theta_{23} \sin \Delta_{31} \cos(\Delta_{31} + \delta_{CP})$$

$$+ \Delta_{21}^2 \cos^2 \theta_{23} \sin^2 2\theta_{12}$$

where $\Delta_{ij} = \Delta m_{ji}^2 L/(4E)$ and where $\sin 2\theta_{13}$, $\Delta_{12}$ and $|\Delta m_{21}^2/\Delta m_{31}^2|$ are treated as small.

In practice, experiments are designed so that oscillations are maximal, i.e. $\Delta_{31} \approx \pi/2$. For $\nu_\mu \rightarrow \nu_e$ the sign of $\delta_{CP}$ is reversed. Notice that changing $\Delta_{31}$ to $-\Delta_{31}$ and $\delta_{CP}$ to $\pi - \delta_{CP}$ leaves $P$ unchanged so measuring $P(\nu_\mu \rightarrow \nu_e)$ and the corresponding probability for $\bar{\nu}$ cannot alone determine the hierarchy.

If we adopt as polar coordinates

$$r = \sin 2\theta_{13}; \quad \theta = \delta_{CP}$$

then a result $P_\nu$ for $P(\nu_\mu \rightarrow \nu_e)$ gives a circle with radius squared proportional to $P_\nu$.

Similarly, the result $P_{\bar{\nu}}$ for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ gives a circle with a different center and radius squared proportional to $P_{\bar{\nu}}$.

Perfect measurements of $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ would give two intersecting circles. The plots for normal and inverted hierarchies would be mirror images of each other.

When the matter effect is included, the result is

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2(1-x)\Delta_{31}}{(1-x)^2}$$

$$+ \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin[(1-x)\Delta_{31}] \sin x\Delta_{31}}{1-x} \frac{x}{x} \cos(\Delta_{31} + \delta_{CP})$$

$$+ \left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2}\right)^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(x\Delta_{31})}{x^2}$$

(9)
where $x = 2\sqrt{2}G_FN_eE/\Delta m^2_{31}$ and where non-leading terms in $\Delta m^2_{21}/\Delta m^2_{31}$ and $\theta_{13}$ have been neglected.

For antineutrino scattering, again the sign of $\delta_{CP}$ changes, but also the sign of $x$ is reversed because the effective potential for neutral current scattering of $\bar{\nu}_e$ is the negative of that for $\nu_e$.

For any given experiment with data for both $P(\nu_\mu \rightarrow \nu_\tau)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$, we can draw two circles to represent these data. Of course, in practice the circles would have thicknesses indicative of the uncertainties in the measurement. In addition, a circle could be drawn, with center at the origin, for the measurement of $\sin^2 2\theta_{13}$ from reactor experiments. Two plots would be made, one on the assumption of normal hierarchy, the other assuming inverted hierarchy. In at least one of the two there should be a solution where the three circles intersect.

As we see in the examples below, the centers of the circles are such that $\delta_{CP} = \pi/2$ and $\delta_{CP} = 3\pi/2$ form the extreme situations, making either maximal or minimal separation between the circles. With increasing length $L$, the discriminating power increases as the radii of the circles grow and shrink in the different hierarchies.

4.1.2 T2K

T2K runs 295 km from J-PARC in Tokai to Super-Kamiokande. In Fig. 2 we see that the low energy of the T2K beam makes it impossible to determine the hierarchy if $\delta_{CP} = \pi/2$, and even for $\delta_{CP} = 3\pi/2$, superb precision would be required.

4.1.3 NOνA

NOνA uses an off-axis beam, which is rather monochromatic. The far detector has a mass of 14 kt and is located at Ash River, Minnesota, 810 km from the beam source at Fermilab. The experiment is scheduled to take data starting in 2014 and to run for six years. In addition to the far detector, there is a 330-t near detector. The NOνA website describes the detectors as being “made up of 344,000 cells of extruded, highly reflective plastic PVC filled with liquid scintillator. Each cell in the far detector measures 3.9 cm wide, 6.0 cm deep and 15.5 meters long.” Fig. 3 shows the statistical separation power of NOνA; we see that the separation is easiest when $\delta_{CP} \approx 3\pi/2$. The reach as determined by the NOνA collaboration is shown in Fig. 4. These figures are consistent with Fig. 3 and show that for about a third of a range of $\delta_{CP}$, centered around $\delta_{CP} = 3\pi/2$, NOνA can determine the hierarchy with a confidence of $2 - 3\sigma$. A combination of NOνA and T2K (Fig. 4, right) has an ability to resolve the hierarchy at $1 - 3\sigma$ significance for all values of $\delta_{CP}$. This comes about because of the difference in baselines: T2K is primarily sensitive to the CP asymmetry between neutrinos and antineutrinos (effect of $\delta_{CP}$), which can be subtracted from the NOνA asymmetries to extract the matter-induced hierarchy signal.

4.1.4 Prospects

NOνA has a much better chance of measuring the hierarchy than T2K, but this would only be possible for a very favorable value of $\delta_{CP}$. It could find an indication of the hierarchy and
Figure 2: Plots for T2K. Upper, with true hierarchy being normal, with $\delta_{CP} = \pi/2$. Lower, again with true hierarchy being normal, but now with $\delta_{CP} = 3\pi/2$. The dot dash circles show the constraints from $\nu_\mu$ scattering, while the solid circles are for $\bar{\nu}_\mu$. The dashed circles show the constraints from a hypothetical perfect measurement of $\sin^2 2\theta_{13}$.

This could reduce the impact of later experiments if they ended up confirming, say, a two-sigma NO$\nu$A result. The sensitivity of T2K and NO$\nu$A could be improved with additional run time, or upgrades to the detectors or the beam intensity \[7\].
Figure 3: Plots for NOνA. Upper, with true hierarchy being normal, with $\delta_{CP} = \pi/2$. Lower again, with true hierarchy being normal, but now with $\delta_{CP} = 3\pi/2$. The dot dash circles show the constraints from $\nu_{\mu}$ scattering, while the solid circles are for $\overline{\nu}_{\mu}$. The dashed circles show the constraints from a hypothetical perfect measurement of $\sin^2 2\theta_{13}$.
Figure 4: Significance with which (Left) NOνA and (Right) T2K+NOνA can resolve the mass hierarchy for $\sin^2 2\theta_{13} = 0.095$ and $\sin^2 2\theta_{23} = 1$, as a function of $\delta_{CP}$. This assumes a nominal 3+3 year run plan. The blue/solid (red/dashed) curve shows the sensitivity given a normal (inverted) hierarchy. Figures from NOνA document [8].

4.2 LBNE

4.2.1 Introduction

The purpose of the Long Baseline Neutrino Experiment is to measure all the neutrino oscillation parameters including the currently unknown values of the the CP violating phase $\delta_{CP}$ and the neutrino mass hierarchy. Matter effects compete with the CP violation effects, and the baseline needs to be optimized to be able to disentangle them.

To detect these neutrino oscillations, a broad-band neutrino beam of mean energy 3.5 GeV is proposed to originate at FNAL and to be directed underground 1300 km away to the Sanford Underground Research Facility in South Dakota. The advantage of the long baseline and correspondingly high neutrino energy at maximal mixing is apparent in Fig. 5, analogous to those shown for T2K and NOνA.

To maximize the event rate, the beam is typically tuned so that the peak of the neutrino energy is at the first oscillation maximum for a given distance, see Fig. 6. This is a broad-band beam, optimized to cover both the first and second oscillation maxima. Narrow band beams pick out a single energy and typically do not sample the full distribution, but only a single energy bin. A broad-band beam allows a measurement of the shape of the spectra (including multiple maxima) favorable for the measurement of $\delta_{CP}$ while a narrow band beam only allows a counting experiment at a single energy.

4.2.2 Phasing the Long Baseline Neutrino Experiment

Due to budgetary constraints, the experiment is to be phased. The stated goal of the first stage of LBNE, as summarized in the recent HEPAP Major Facilities report, is to determine the hierarchy to greater than $3\sigma$ for all values of the CP phase $\delta_{CP}$ (when constraints from other experiments are included), and to set the stage for a world-class US neutrino program that will include Project X, and physics goals such as a comprehensive search for CP violation.
The Phase I experiment consists of the beamline and a 10 kT liquid argon (LAr) detector located on the surface at the SURF facility. The beamline at FNAL includes a target, horn and decay tunnels consistent with accepting a 2.3 MW beam, and a set of instruments (an array of high pressure gas muon Cherenkov counters, four layers of stopped muon detectors, and two or three layers of hadron ionization detectors) at the absorber complex to measure the flux, angle and centerline of the beam. There is no near neutrino detector in the first phase. The final phase includes a 34 kT liquid argon detector, underground at the far site and a substantial near detector most likely with both a liquid argon detector and a magnetized,

![Illustrative plots for a LBNE](image)

Figure 5: Illustrative plots for a LBNE. Upper, with true hierarchy being normal, with $\delta_{CP} = \pi/2$. Lower, again with true hierarchy being normal, but now with $\delta_{CP} = 3\pi/2$. The dot-dash circles show the constraints from $\nu_\mu$ scattering, while the solid circles are for $\bar{\nu}_\mu$. The dashed circles show the constraints from a hypothetical perfect measurement of $\sin^2 2\theta_{13}$. The power of LBNE is enhanced by combining with results from T2K (and to lesser extent NO$\nu$A) because the “wrong” solution occurs at different values of $\delta_{CP}$ for the three experiments. Note that the circles correspond to a fixed beam energy of 3.5 GeV, whereas the planned neutrino beam is broadband.
Figure 6: The black shaded areas are the unoscillated charged current spectra for the Homestake site (left-hand y-axis scale) at 1300 km for $\nu_\mu \rightarrow \nu_e$ (top) and $\nu_\mu \rightarrow \bar{\nu}_e$ transitions (bottom). The colored lines are oscillation probabilities for different oscillation parameters, as given in the legend (right-hand y-axis scale). The maxima of the beam energy has been optimized to correspond to the maximum of the first “node,” i.e. maximum of the probability of $\nu_\mu \rightarrow \nu_e$ transitions [9].

high pressure Ar [straw] tracker, and EM calorimeter to measure the absolute neutrino flux via neutrino-electron scattering.

4.2.3 Detector Performance

The detector performance is evaluated in the context of the “GLoBES” software package [10], which includes the energy spectrum of the beam, interaction cross sections, the corresponding detector efficiencies as well as specific backgrounds and uncertainties e.g. $\nu_\tau$ backgrounds
have recently been added, see below. Table 3 summarizes the properties of the detector as coded into GLoBES.

Fig. 7 shows the range of sensitivity to the hierarchy for LBNE Phase I, both alone and in combination with T2K+NOνA. The left panel of Fig. 8 shows the 2σ and 3σ sensitivity limits on the hierarchy as a function of detector mass at the SURF site, and the right panel shows the 3σ and 5σ sensitivities to δCP.

The value of \( \Delta m_{32}^2 \) is one of the limiting factors in the ability of second generation reactor experiments to measure the mass hierarchy. As a further example of the capabilities of the experiment, LBNE will have an unprecedented capability to measure \( \Delta m_{32}^2 \). The precision with which \( \Delta m_{31}^2 \sim \Delta m_{32}^2 \) can be measured is shown in Fig. 9, approaching \( 10^{-5} \text{eV}^2 \) for a 10 year exposure of a 34 kT detector.

Table 3: Estimated range of the LAr-TPC detector performance parameters for the primary oscillation physics. The expected range of signal efficiencies, background levels, and resolutions from various studies (middle column) and the value chosen for the baseline LBNE neutrino-oscillation sensitivity calculations (right column) are shown. For atmospheric neutrinos this is the misidentification rate for events below 2 GeV; the misidentification rate is taken to be zero events above 2 GeV.

<table>
<thead>
<tr>
<th>Identification of ( \nu_e ) CC events</th>
<th>( \nu_e ) CC efficiency</th>
<th>70-95%</th>
<th>80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_\mu ) NC mis-identification rate</td>
<td>4-20%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>( \nu_\mu ) CC mis-identification rate</td>
<td>0-2-20%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>Other background</td>
<td>0%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>Signal normalization error</td>
<td>1-5%</td>
<td>1%</td>
<td></td>
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Figure 7: Mass hierarchy sensitivity for Phase I of LBNE alone (red band) and in combination with T2K+NO\(\nu\)A (blue band) for (Left) normal and (Right) inverted mass hierarchies [11].

Figure 8: (Left) 2- and 3-\(\sigma\) sensitivity to the mass hierarchy versus the fraction of \(\delta_{CP}\) coverage and detector mass when located at SURF, under the assumption of a normal hierarchy. Solid points are 2\(\sigma\) limits, while open circles are 3\(\sigma\) limits. Black is for LBNE alone, with 5\(\nu\) + 5\(\bar{\nu}\) years at 700 kW, and red is combined with results from NO\(\nu\)A and T2K. [9]. (Right) 3- and 5-\(\sigma\) sensitivity to \(\delta_{CP}\) versus the fraction of the CP phase \(\delta_{CP}\) as a function of the mass of the detector at the SURF site, under the assumption of a normal hierarchy. Solid points are 3\(\sigma\) limits, while open circles are 5\(\sigma\) limits. Black is for LBNE alone, with 5\(\nu\) + 5\(\bar{\nu}\) years at 700 kW, and red is combined with results from NO\(\nu\)A and T2K. [9].
Figure 9: Estimated error in $\Delta m^2_{31} \sim \Delta m^2_{32}$ versus detector exposure at 1300 km for three values of $\sin^2 2\theta_{23}$ [indicated in the legend only as $\theta_{23}$], for a 700 kW beam. Note the vertical scale ($\Delta m^2_{32}$ resolution) has a scale factor of $10^{-3}$. The plot assumes a near detector for these precision measurements. Neutrino and anti-neutrino running are combined in the ratio of 1:1, The mass hierarchy is assumed to be known [9].

4.2.4 Open Concerns

There are four major concerns for the Phase I LBNE detector program:

1. The surface location of the LAr detector;
2. The lack of a near detector;
3. The small mass (10 kT) of the far detector;
4. Uncertainties in nuclear effects.

With a 10µsec beam window, the detector is live for approximately 100 sec/year and assuming an overburden of 3m of rock the background from cosmogenically induced events is negligible, less than 1% in the $\nu_e$ appearance experiment. Measuring $\delta_{CP}$ and the mass hierarchy can be accomplished by a LAr detector on the surface. Monte Carlo studies have shown that these measurements are statistics limited, and a simplified set of flux monitors at the end of the decay pipe at FNAL are sufficient to monitor the beam intensity. For longer
exposures (above 100 kT \times \text{years}), a near detector is needed, and collaborators from India have expressed interest in building such a detector.

A larger detector is attractive, and an excellent opportunity for foreign investment. Some potential foreign collaborators have also expressed interest in technical components of the beamline that would help offset portions of the total project cost, and free DOE funds to increase the scope of the US project. A smaller detector can also be offset via higher beam intensity, and current plans at FNAL call for Phase I of Project X to be completed by the start of LBNE, boosting the beam power from 700 kW to \sim 1.1 \text{ MW}. This would effectively remove all ambiguity in the 3\sigma mass hierarchy measurement for all values of \delta_{CP} for a 10 year run of a 10kT LBNE LAr detector.

Putting the detector underground would permit a broader physics program, including proton decay searches and the detection of neutrinos from supernova events. As a LAr detector can easily identify charged kaons (which are below threshold in a water Cherenkov detector), its sensitivity to proton decay quickly exceeds that of SuperK by an order of magnitude summed over all decay modes, despite SuperK’s extrapolated 30yr integrated running history.

The CD1 approval letter states that LBNE may increase the scope of the project if additional resources can be found. It is important to recognize that unlike other DOE/HEP projects where the total project cost (TPC) is usually capped, including both non-DOE domestic or foreign contributions, in the LBNE case the project has been given a “hunting license” to seek outside funding to increase the scope of the project. Such negotiations are ongoing.

Construction of an unprecedentedly massive liquid-argon detector presents significant technological challenges. It will require a rigorous R&D effort before the technology is mature enough for this endeavor. If cost is not an issue, one approach in realizing this scheme is to modulize the detector so that it can be staged.

Nuclear effects will smear the reconstruction of the incident neutrino energy [12]. A LAr detector is a “tracking detector” and this may help in identifying low energy protons or gammas ejected from the nucleus, but properly identifying neutrons may be problematic, and extracting elementary reaction amplitudes for meson production is difficult. The issue of the energy reconstruction in the presence of these effects needs to be addressed.

4.3 Non-US Options

Both the European community and Japan have proposed deep-underground accelerator-based neutrino oscillation experiments, which overlap significantly in motivation, technology, and physics scope with the US neutrino program. There are two main proposals on the table: LAGUNA-LBNO, a European initiative with the CERN proton beam and several potential sites for the far detector; and T2HK, an oscillation experiment with the JPARC beam and the HyperK target. It is important to compare the sensitivity and timescale of these experiments with the US options.
4.3.1 Hyper-Kamiokande

The Japan Subcommittee on Future HEP projects (2012) [13] has recommended that: “Japan should aim to realize a large-scale neutrino detector ...This new large-scale neutrino detector should have sufficient sensitivity to allow the search for proton decays”. The emphasis of the program is on CP violation and proton decay, not determination of the mass hierarchy.

The neutrinos for Japan’s long-baseline program are produced at the 50 GeV proton synchrotron at J-PARC accelerator laboratory, in Tokai on the east coast of Japan. The J-PARC development plan calls for a staged increase in proton beam power, from 300 kW presently to 1-2 MW by the middle of the next decade [14].

Two primary options have been considered for the far detector: Hyper-Kamiokande (HyperK) [15], a Megaton-scale water Cherenkov detector in the Kamioka region [15], 295 km away from J-PARC; and a 100 kTon liquid argon detector at Okinoshima island at a baseline of 660 km. As of this writing, HyperK appears to be the front-runner.

HyperK is a proposed very large water Cherenkov detector with a total mass of 1 MTon. It is proposed to be located in a new detector cavity in the Tochibora mine, about 8 km from the Super-Kamiokande location, under an overburden of about 1750 mwe.

The HyperK design is based on the very successful Super-Kamiokande (SuperK) experiment, and represents a factor of 20 increase in the fiducial mass. The extrapolation to the Megaton-scale detector will certainly present technological challenges; however, the detector technology and, in particular, systematic effects in water Cherenkov detectors are well understood. Water Cherenkov technology offers excellent particle ID capabilities and high reconstruction efficiency for the sub-GeV electrons and muons relevant for the oscillation program.

The total cost of the detector is estimated to be 50-75 Billion yen (≈ $500 - 700M) [14]. The cost of the neutrino beamline upgrades to provide 1-2 MW of beam power at J-PARC is estimated at 38B yen. Thus, the long-baseline neutrino program falls in the medium range of the future projects being considered at Japan, between SuperKEKB/SuperBelle and the ILC.

At the baseline of 295 km, the matter-induced neutrino mixing is relatively small. This means a weak sensitivity to the neutrino mass hierarchy (see Fig. 2 in Section 4.1). On the other hand, the sensitivity to the CP angle $\delta_{CP}$ is enhanced compared to the matter-induced effects. Thus, CP measurements in HyperK would require an independent determination of the mass hierarchy. Such determination is possible with an atmospheric neutrino campaign in HyperK, as discussed below.

The short baseline also requires a lower neutrino energy to maintain the detector at the first oscillation maximum. The T2HK (Tokai to HyperK) project will utilize an off-axis, narrow-band beam similar to that of T2K, with the $\nu_\mu$ energy peaking at around 600 MeV. At that energy, backgrounds from $\nu_e$ and in particular $\nu_\tau$ are suppressed. This configuration requires large beam power and long run time. The HyperK proposal assumes 10 years of running at a beam power of 750 kW (3 years in neutrino and 7 years in anti-neutrino modes).

The very large mass and significant overburden of HyperK provides the potential for a measurement of the neutrino mass hierarchy with atmospheric neutrinos.

The sensitivity of atmospheric neutrino oscillations to the neutrino mass hierarchy is
discussed in detail in Section 6. The sensitivity comes from the $\nu_\mu \to \nu_e$ appearance induced by the Earth matter, and thus the effect is strongest for upward-going neutrinos. In a charge-symmetric detector like HyperK, the measurement requires excellent identification of neutrino flavor and precise reconstruction of neutrino energy and azimuthal angle, at the energies of a few GeV. A water Cherenkov detector like HyperK is well matched to these requirements.

As discussed in Section 6, understanding of the energy and angular resolutions, as well as the systematics of the overall energy scale are extremely important for extracting the mass hierarchy signal from the atmospheric neutrino data. Unlike PINGU and other large atmospheric neutrino detectors, with SuperK experience these systematic effects are fairly well understood in a water Cherenkov detector. Most importantly, HyperK will have a beam-based calibration signal (the beam energy spectrum) and the ability to detect both upward- and downward-going neutrinos. Those factors make the potential for observing the neutrino mass hierarchy with atmospheric neutrinos in HyperK quite robust.

After 10 years of operation starting at the end of this decade (i.e. results ready by 2028-2030), HyperK projects to be able to determine the mass hierarchy with a confidence level of $2 - 3\sigma$ with the atmospheric measurements, and greater than $1\sigma$ with the beam-based measurements alone (Fig. 10). The sensitivity of the beam-based measurement depends strongly on the value of $\delta_{CP}$ (Section 4.1), while the atmospheric sensitivity depends somewhat on the value (octant) of $\theta_{23}$ and, to a small extent, $\delta_{CP}$. The two measurements are complementary, and adding them together allows HyperK to project greater than $3\sigma$ sensitivity to the mass hierarchy for the entire range of $\delta_{CP}$.

![Beam Map](image1)

![Atm. ν Map](image2)

Figure 10: Projected sensitivity in units of $\Delta \chi^2$ of HyperK to the neutrino mass hierarchy from the long-baseline beam measurements (left plot, blue histogram) and from the atmospheric measurements (left plot, dark-green histogram). The combined sensitivity is shown in the right plot. The plot is from Ref. [16].
4.3.2 LAGUNA-LBNO

The European version of the long-baseline neutrino oscillation experiment is known as LAGUNA-LBNO \[17, 18\] (Large Apparatus studying Grand Unification and Neutrino Astrophysics and Long-Baseline Neutrino Oscillations). The current cycle of design studies (2011-2014) is focused on the conventional neutrino beam originating at CERN, aiming at a deep underground site in Pyhäsalmi, Finland, at a baseline of 2300 km. Fig. 11 illustrates the advantages of such a long baseline.

Three detector concepts are being developed for that site: GLACIER (Giant Liquid Argon Charge Imaging ExpeRiment), a LAr dual-phase TPC scalable to a total mass of up to 100 kTon; MIND, a 25 kTon magnetized iron-scintillator calorimeter similar to MINOS \[19\]; and LENA (Low Energy Neutrino Astronomy), a \( \sim \) 50 kTon liquid scintillator detector. In addition, a MTon-scale water Cherenkov detector MEMPHYS (MEgaton Mass PHYSics) is being considered for a site in an extended Modane Laboratory at a very short baseline of 130 km from CERN. All cases include a near detector (upgraded SHINE/NA61), a deep

Figure 11: Plots for a hypothetical site 2400 km from the source. Upper, with true hierarchy being normal, with \( \delta_{\text{CP}} = \pi/2 \). Lower, again with true hierarchy being normal, but now with \( \delta_{\text{CP}} = 3\pi/2 \). The dot dash circles show the constraints from \( \nu_{\mu} \) scattering, while the solid circles are for \( \bar{\nu}_{\mu} \). The dashed curve shows the constraint from reactor neutrino experiments.
underground location (2500 mwe for GLACIER, 4000 mwe for LENA, 4800 mwe for MEMPHYS), and a range of neutrino beam options, starting with an 800 kW wide-band beam (for GLACIER and LENA) to an upgrade to 4 MW (for MEMPHYS), to high-power $\beta$-beams at CERN (for MEMPHYS), and ultimately to a neutrino factory.

Each detector offers a different set of complementary physics measurements: GLACIER and LENA, with a very long baseline and large underground detector, will be able to resolve the neutrino mass hierarchy quickly, while also providing excellent CP reach and opportunities to search for proton decay and supernova neutrinos. The physics case for such detectors is well described in Section 4. In addition, LENA, by virtue of its low energy threshold, would be a large-scale solar neutrino experiment. MEMPHYS offers a different tradeoff: a clean measurement of the CP phase $\delta_{CP}$ without significant matter effects, increased detector mass (compared to SuperK) for proton decay searches, and sensitivity to the mass hierarchy via measurements of atmospheric neutrinos. The physics reach of MEMPHYS is similar to that of HyperK.

GLACIER, by the virtue of its excellent tracking capabilities, large mass, and a very long baseline, can offer an extremely sensitive measurement of the mass hierarchy (Fig. 12). It would be able to resolve the mass hierarchy with a significance of more than $5\sigma$ in a few years of operation with a conventional SPS beam ($\approx 2 \times 10^{20}$ protons on target). If a decision on construction is reached by 2015, the proponents estimate a $5\sigma$ measurement by 2023 [18] [19]. The high sensitivity would possibly enable a staged approach to the project: a smaller (20 kTon) detector that starts operating quickly, followed by a larger detector, near detector upgrades, beam intensity upgrades, etc, aiming at a 30-year program for precision measurement of $\delta_{CP}$, neutrino astrophysics, and proton decay searches. LENA would require about 10 years of operations to reach $>5\sigma$ sensitivity to the mass hierarchy, with the completion timescale similar to that of LBNE and HyperK.

While LAGUNA-LBNO claims the best sensitivity to the mass hierarchy among the options we have considered, it is not yet a fully fleshed-out project. The current study of the detector configuration options is funded through 2014, at which point the proponents are planning to submit a full proposal to CERN [17] [18]. As of this writing, no cost estimates

![Figure 12: LAGUNA sensitivity to the mass hierarchy as a function of $\delta_{CP}$ (left) [19]. Sensitivity to $\delta_{CP}$ (right) for staged detector construction.](image)
are available, although one could scale from LBNE and HyperK estimates and come up with numbers in the range of a few billion Euro. At that scale, the neutrino projects will be in direct competition with the LHC upgrades, any proposals for a high-energy $e^+e^-$ collider, etc. The recently updated 2013 European Strategy for Particle Physics document lists neutrino physics as priority #4 (last) and states:

“CERN should develop a neutrino programme to pave the way for a substantial European role in future long-baseline experiments. Europe should explore the possibility of major participation in leading long-baseline neutrino projects in the US and Japan”.

Taken at face value, this indicates a lukewarm interest for an expensive neutrino project in Europe, although other opinions exist.

4.3.3 Open Concerns

The biggest concern for the long-baseline projects in Japan and Europe is the fact that these projects are not fully funded in their host countries, and so they remain virtual for now. It is difficult to imagine the world-wide scientific community supporting more than one expensive long-baseline effort, especially in the face of other priorities in high energy physics. The European Strategy document seems to openly acknowledge this fact. However, should the US LBNE effort falter, or should the political and economic climates change, the off-shore projects may provide viable alternatives (and in some scenarios stiff competition) to LBNE in the race to unambiguously and decisively determine the neutrino mass hierarchy.

5 Reactor Neutrinos

A signature of the neutrino mass hierarchy is present in the oscillation of anti-neutrinos from nuclear power reactors. A very precise, high-statistics measurement of the reactor anti-neutrino energy spectrum is required to determine the hierarchy. JUNO (formerly known as Daya Bay II) is the only experiment proposed to measure the hierarchy by this method.

The three-flavor electron anti-neutrino survival probability is given by:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

$$- \sin^2 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}), \hspace{1cm} (10)$$

where $\Delta_{ij} = 1.27 \Delta m_{ij}^2 [\text{eV}^2] \cdot [\text{L}[\text{m}] / \text{E}[\text{MeV}]]$.

The hierarchy signature is contained in the term proportional to $\sin^2 2\theta_{13}$ and can be written as

$$\frac{1}{2} \left[ 1 - \cos(\Delta_{31} + \Delta_{32}) \cos \Delta_{21} + \cos 2\theta_{12} \sin(\Delta_{31} + \Delta_{32}) \sin \Delta_{21} \right] \hspace{1cm} (11)$$

The quantity $\Delta_{21}$ is unambiguous. If the hierarchy is normal, then $\Delta_{31} + \Delta_{32}$ is positive, while it is negative for the inverted hierarchy. But this quantity is only the phase, whose sine must be evaluated. By increasing or decreasing $\Delta_{31} + \Delta_{32}$ slightly, one hierarchy can be
made to emulate the other over a modest range of energy, outside which the two solutions will no longer be in phase. Thus to distinguish the inverted from the normal hierarchy we must measure the oscillations, suppressed by $\sin^2 2\theta_{13}$ over many cycles. Additional discrimination can come from precision measurements of combinations of $\Delta m^2_{31}$ and $\Delta m^2_{32}$.

Disappearance measurements determine the effective mass-squared differences for different channels of neutrino oscillation:

$$\Delta m^2_{ee} \approx \cos^2 \theta_{12} \Delta m^2_{31} + \sin^2 \theta_{12} \Delta m^2_{32}$$

$$\Delta m^2_{\mu\mu} \approx \sin^2 \theta_{12} \Delta m^2_{31} + \cos^2 \theta_{12} \Delta m^2_{32} + \sin 2\theta_{12} \sin \theta_{13} \tan \theta_{23} \cos \delta_{CP} \Delta m^2_{21},$$

where terms of order $\sin^2 \theta_{13} \Delta m^2_{21}$ have been neglected for simplicity. Improvements here could constrain the fits to JUNO data.

Reactor anti-neutrinos are commonly detected via the inverse beta-decay reaction, $\bar{\nu}_e + p \rightarrow e^+ + n$. The outgoing positron energy preserves information of the original anti-neutrino energy, $E_{e^+} \simeq E_{\bar{\nu}_e} - 0.8\text{MeV}$. Fig. 13 shows the positron energy spectrum (including annihilation) for an ideal reactor experiment (20 kton detector, 40 $GW_{th}$ reactor power at 58 km, 5 years of operation, $\Delta m^2_{32} = \pm 2.32 \times 10^{-3} \text{eV}^2$). The primary deficit between 2 and 4 MeV is due to solar oscillation at 58 km. The high-frequency oscillation is due to the atmospheric mass difference $\Delta m^2_{3X}$. A choice of the mass hierarchy appears as a difference in the phase of the oscillation.

JUNO is a liquid-scintillator detector very similar to KamLAND in design, but twenty times larger in mass (20 kton) \[21\]. It is proposed to be built in the Jiangmen city of Guangdong province in southern China, roughly 70 km from Macau. The Yangjiang (17.4 $GW_{th}$) and Taishan (18.4 $GW_{th}$) reactor facilities serve as the electron anti-neutrino source. The

\[\text{Figure 13: Estimated positron energy spectrum of the inverse-beta decay reaction for an ideal reactor anti-neutrino experiment (20 kton detector, 40 $GW_{th}$ reactor power at 58 km, 5 years operation). The solar oscillation ($\Delta m^2_{31}$) causes the broad deficit between 2-4 MeV. The normal (blue) versus inverted (red) hierarchy results in an effective shift in the phase of the high-frequency oscillation. In this example, $\Delta m^2_{31}$ has been changed to $-\Delta m^2_{31}$ in inverting the hierarchy.}\]
detector is located at the first solar oscillation minimum, approximately 58 km from both reactor facilities.

An effort based in Korea also intends to build a large scintillating reactor anti-neutrino experiment (RENO-50). The original design had a marginal sensitivity to the mass hierarchy, due to a smaller 10 kton target mass and poorer detector resolution than JUNO. Recent changes to the design goals \cite{22} aim at a detector size of $O(20)$ kton and sensitivity similar to JUNO. RENO-50 plans to submit a Letter of Intent to funding agencies in 2013.

To determine the neutrino mass hierarchy, JUNO and RENO-50 will need to improve over current standards for energy resolution and calibration. The JUNO collaboration’s goal is to achieve a detector resolution of $\sigma_E/E = 3\%/\sqrt{E_{e+}[\text{MeV}]}$ in order to measure the hierarchy. For KamLAND, the energy resolution was limited to $6.5\%/\sqrt{E}$ by photoelectron statistics. An improved resolution requires more photons per unit of positron energy be detected. The JUNO experiment aims to increase photon statistics by increasing the scintillator light yield ($\times1.5$), increasing the total photocathode coverage ($\times2.3$), and by increasing the efficiency of each photomultiplier ($\times2.0$) relative to those of KamLAND. Improving the light yield and PMT efficiency have not yet been demonstrated; both require significant technological advances. Increasing the photocathode coverage is straightforward, only requiring sufficient funds to purchase $\sim15,000$ 20”-diameter PMTs.

The current state of the art in liquid scintillator is $\sim2\%$ for gamma-ray calibration, as demonstrated by the KamLAND experiment, although sub-percent level precision has been achieved in water Cherenkov detectors such as SNO. In liquid scintillator, particle-dependent scintillator quenching introduces a comparable systematic uncertainty when estimating the positron calibration from gamma-ray data. The JUNO group has addressed the problem of non-linearity by fitting the actual observed energy spectrum and parameterizing the non-linearity, a process of self-calibration.

The JUNO group has produced an estimate of their sensitivity in Ref. \cite{23}. Fig. 14 shows the $\Delta \chi^2$ distributions obtained for both the normal and inverted hierarchy models as a function of $\Delta m^2_{ee}$. The degeneracy due to uncertainty in the mass-squared difference is visible as the false minimum for the inverted hierarchy at a value of $\Delta m^2_{ee}$ 0.5% from the true value. The collaboration finds a sensitivity to the hierarchy of more than $3\sigma$ under the assumption of a $3\%/\sqrt{E}$ detector energy resolution, $2\%$ correlated and $0.8\%$ uncorrelated uncertainties in reactor flux, $1\%$ uncertainty in the reactor flux spectrum, and $1\%$ uncertainty in the detector response. This analysis takes the best-fit oscillation parameters from the most recent global analysis, and takes into account the true spatial distribution of reactor cores. When allowing for potential future improvements in the uncertainty in $\Delta m^2_{32}$ this significance can be improved to more than $4\sigma$. This sensitivity has been confirmed by an independent study \cite{24} under the same assumptions for detector performance and future precision on oscillation parameters.

The effect of finite detector resolution is to smear out the oscillation signal at lower positron energies. Fig. 15 shows the expected positron spectra, including a detector resolution of $\sigma_E/E = 3\%/\sqrt{E_{e+}[\text{MeV}]}$ and a maximally-ambiguous $5\%$ shift in $\Delta m^2_{32}$ ($\sim1.2\sigma$ based on the current global best fit). The residual differences in the spectra of the two hierarchies are limited to the 2-3 MeV region. Ref. \cite{23} considers the effect of an improved uncertainty in $\Delta m^2_{32}$ and finds that reduction of the uncertainty to $1.5\%$ would result in a hierarchy
Figure 14: $\Delta \chi^2$ distributions obtained for the normal (black) and inverted (red) hierarchy models as a function of $\Delta m^2_{ee}$, taken from Ref. [23]. The degeneracy due to uncertainty in the mass-squared difference is visible as the false minimum for the inverted hierarchy at a value of $\Delta m^2_{ee}$ 0.5% from the true value. Including a penalty based on a 1.5% global uncertainty in $\Delta m^2_{32}$ disfavors the inverted hierarchy at an additional $\sim 0.5\sigma$ (solid red versus dashed red).

Figure 15: Estimated positron energy spectrum of the inverse beta-decay reaction for an ideal reactor anti-neutrino experiment (20 kton detector, 40 GW$_{th}$ reactor power at 58 km, 5 years operation). A detector energy resolution of $\sigma_E/E=3%/\sqrt{E_{e+}[\text{MeV}]}$ has been applied, reducing the oscillation signal below 2 MeV. The mass difference for the inverted case has been shifted by 1.2$\sigma$ from the current MINOS best estimate (a shift of 5%), removing the hierarchy discrimination above 3 MeV.
significance of $3.7\sigma$, and $4.4\sigma$ if a 1% uncertainty could be achieved. The additional penalty to $\Delta \chi^2$ is illustrated in Fig. 14.

Current accelerator neutrino experiments (T2K, NO$\nu$A) will, it is hoped, reduce the uncertainty on $\Delta m^2_{32}$ by a factor of two. Estimates give a global uncertainty of 1.5% in $\Delta m^2_{32}$ by 2025. Further reduction in this uncertainty will come from future long-baseline experiments (Section 4).

The JUNO project has an aggressive schedule [21]. It has already received the Chinese equivalent of CD-1 approval. Civil construction is targeted for 2014-2017. Detector assembly is planned for 2018-2019. Data taking would commence in 2020, with a target of 5-6 years of operation for the hierarchy measurement.

The main challenges for JUNO are technological. To obtain sufficient detector resolution requires multiple factors of two improvements in detector technology: improved scintillator light yield, attenuation length, and PMT efficiency. Constraints on detector uniformity and linearity are demanding, and will likely require the development of new methods to calibrate the detector response to positrons. These challenges must be met successfully if the subtle effect of the neutrino mass hierarchy on the observed positron signal is to tell us whether the neutrino mass hierarchy is normal or inverted.

6 Atmospheric Neutrinos

Atmospheric neutrinos have the potential to resolve the neutrino mass hierarchy via matter-enhanced oscillation within the Earth. Resonant oscillation occurs either for neutrinos in the case of the normal hierarchy, or antineutrinos for the inverted hierarchy. Determination of the hierarchy requires measurement of the energy and direction of Earth-crossing atmospheric neutrinos with energies in the range of 2 to 10 GeV. Massive detectors ($\gtrsim$Mton) are required to obtain sufficient signal statistics within a few years of operation. Existing proposals use either water Cherenkov (PINGU, ORCA, HyperK), liquid Argon TPC (LBNE, LBNO), or magnetized iron calorimeter (INO) detectors. Discrimination of neutrinos from antineutrinos enhances hierarchy sensitivity. Hierarchy determination has some dependence on the oscillation parameters, in particular $\Delta m^2_{31}$, but is largely insensitive to the Earth density profile. Primary concerns are detector properties such as total mass, energy resolution, and angular resolution.

6.1 Signature of the neutrino mass hierarchy

The two possible neutrino mass hierarchies predict distinctly different oscillation probabilities for Earth-crossing neutrinos. Due to interactions with electrons within the Earth, resonant flavor conversion occurs at a specific pattern of neutrino energies and Earth-crossing paths. This resonant conversion only occurs for neutrinos in the case of the normal hierarchy, while only for antineutrinos for the inverted hierarchy. A detector capable of discriminating $\nu$ interactions relative to $\bar{\nu}$ needs only to demonstrate for which state the resonance occurs. Detectors which only distinguish neutrino flavor rely on the intrinsic difference in the atmospheric flux between $\nu$ and $\bar{\nu}$ as well as differences in interaction cross-sections to discriminate the hierarchy.
Figure 16: The estimated oscillation survival probability for Earth-crossing $\nu_\mu$ (left) and $\bar{\nu}_\mu$ (right), assuming the normal neutrino mass hierarchy. Resonant oscillation in the mantle is most significant in the region $\sim 6$ GeV and $\cos \theta_Z \simeq -0.7$, as indicated by the difference between the two panels. For the inverted hierarchy, the resonance occurs for $\bar{\nu}_\mu$ instead of $\nu_\mu$.

Resonant oscillation of Earth-crossing atmospheric neutrinos has been extensively examined [25, 26, 27]. Oscillation probabilities are commonly presented as Earth oscillograms, which show probabilities as a function of neutrino energy and zenith angle $\theta_Z$. Fig. 16 shows the calculated oscillation probabilities assuming the normal hierarchy.

Assuming a 1 Mton detector, $\sim 4000$ muons per year are generated in the energy range of 2 to 10 GeV. Fig. 17 shows the rate of muon production per Mton-yr assuming the normal hierarchy. The major feature of the hierarchy is the resonant excess of muons at $\sim 6$ GeV and $\cos \theta_Z \simeq -0.7$. The intrinsic $\nu/\bar{\nu}$ ratio of $\sim 1.3$ in the resonance region provides an observable signal of the hierarchy, even for a detector insensitive to muon charge.

Sensitivity to the hierarchy is primarily driven by how well the initial neutrino energy and direction can be determined. It is clear that as the energy resolution increases beyond a few GeV, the resonance feature is obscured. Intrinsic resolution is introduced by the kinematics of the outgoing muon and nuclear effects for GeV $\nu$ interactions. Of concern is the uncertainty in $\Delta m^2_{31}$, which has the strongest correlation with the predicted oscillation pattern for muon-charge insensitive detectors.

6.2 Proposed Experiments

6.2.1 INO and MIND

The India-based Neutrino Observatory (INO) will be located in the Bodi-West Hills in Pottipuram village near Bodinayakanur in the Tamil Nadu State, with an overburden of about 1.3 km [28]. Three detector modules will form a 50-kT magnetic iron calorimeter. The prominent features of the INO detector are its capability of determining the charge and momentum of the the muon in the charged-current interaction of an atmospheric muon.

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1Downward-going neutrinos have $\cos \theta_Z = 1$, while upward-going neutrinos which have crossed the Earth's core have $\cos \theta_Z = -1$. 

28
(anti-)neutrino, and the energy of the hadronic shower. These features would allow for discrimination between neutrinos and anti-neutrinos as well as their energy.

MIND [19] is a proposed 25 kT magnetized iron-scintillator calorimeter, to be constructed in conjunction with the European long-baseline effort in an underground lab in Pyhäsalmi, Finland. Its sensitivity to atmospheric neutrino oscillations can be inferred from the INO numbers below.

From a GEANT4-based simulation, the energy resolution of the hadronic component, $\sigma/E$, is about 40% for hadronic energies greater than 4 GeV and is quite independent of the zenith angle of the incident (anti-)neutrino. The efficiency of correctly identifying the charge of a track is better than 95% for $\cos \theta > 0.35$ and momentum greater than 1 GeV. For momentum greater than 1 GeV, the momentum resolution, $\sigma_p/p$, is better than 22% and the resolution in $\cos \theta$ is better than 0.045 for for $\cos \theta > 0.35$.

For $\sin^2(2\theta_{13})=0.1$, the hierarchy can be resolved at 2 (2.7) standard deviations with 5 (10) years of running. The sensitivity is limited by statistics.

Civil construction for INO began in February 2013. The first three detector modules will be installed and commissioned in the underground experimental hall by 2017. According to the schedule presented in Ref. [28], excavation will take 1.5 years for a 2-km-long 7.5-m-wide tunnel, which is quite an aggressive schedule when taking uncertainties in the geotechnical conditions into account.

6.2.2 PINGU

Precision IceCube Next Generation Upgrade (PINGU) is a proposed multi-megaton detector to be located below the dust layer in the inner core of the existing IceCube and DeepCore detectors. PINGU consists of closely separated vertical strings of Digital Optical Modules
(DOMs) for detecting atmospheric neutrinos with energies down to a few GeV. In this configuration, IceCube and DeepCore serve as vetoes for rejecting cosmic-ray muons both online and in the offline analysis. The DOM-to-DOM spacing is shorter than that of DeepCore. The design of the DOM is taken to be the one used in DeepCore. The trigger efficiency is expected to be much higher at lower energies for PINGU than for DeepCore. The cost of constructing PINGU is about $10 M for start up and $1.25 M per string based on the IceCube experience.

**Sensitivity** The sensitivity for determination of the mass hierarchy by PINGU has been estimated both by proponents of the experiment [29] and independently [7], as shown in Figs. 18 and 19 respectively. According to the proponents, PINGU can achieve a sensitivity of $\sim 2.1-3.4\sigma$ per year of data taking, resulting in a $3\sigma$ discrimination between the normal and inverted hierarchies by 2020, assuming initial deployment in 2016/17. Details on the underlying assumptions and statistical techniques used to evaluate the sensitivity are somewhat limited at this stage, although a more detailed Letter of Intent is in preparation [30]. The conclusions of Ref. [7] are more modest, presumably because of differing assumptions in detector performance or statistical methodology. Ref. [7] points out the complementarity between PINGU and the current accelerator and reactor oscillation experiments (NO$\nu$A, T2K, Daya Bay, RENO, Double Chooz). As identified in Ref. [7], the key factors influencing the experimental sensitivity are the current uncertainty on $\Delta m^2_{31}$, the total active detector mass, energy threshold, energy and angular resolution, and mis-identified cascades. The existing studies have not incorporated possible muon charge discrimination via inelasticity of the neutrino interaction.

It will take 1.5 years to procure and ship the detector components to the South Pole. Installation of all strings is expected to complete in 2-3 years. If the proposal is submitted to the funding agencies in a year, the full PINGU detector could be deployed by 2020. Note

![Figure 18](image)

**Figure 18:** Significance of determination of the mass hierarchy using PINGU as a function of run time, for different detector configurations, by the proponents of the experiment [29].
that PINGU can begin taking data as PMT strings are installed, slowly reaching the full target mass by 2020.

Besides having relatively poor energy and directional resolutions, lack of charge determination and minimal particle identification could be an issue in achieving the scientific goals. Of primary concern is whether PINGU can achieve the required detector performance characteristics. These concerns with detector performance may be mitigated by improvements in detector design, or even with incremental extension of the detector after initial data has been obtained.

### 6.2.3 ORCA

Oscillation Research with Cosmics in the Abyss (ORCA) is a proposed dense instrumentation of the central region of the KM3NeT detector in the Mediterranean Sea [31]. In principle, this is similar in concept to PINGU, except the target is water instead of ice. Funding of 40M Euros for KM3NeT phase 1 has been secured, and 50-70 lines of sensors are expected to be ready for deployment by the end of 2016. This comprises a total of 1200 pressure-vessel sensor modules, each containing 31 3” photomultiplier tubes.

### 6.2.4 Hyper-Kamiokande

Hyper-Kamiokande (HyperK) is a proposed multi-purpose water Cherenkov detector, described in detail in Section 4.3. The detector is based on well-known technology, and is
primarily a scale up of the existing Super-Kamiokande detector. In this respect, the expected performance in event reconstruction and particle identification in the sub-GeV region is well-characterized. The detector is sensitive to and can discriminate both muons and electrons. However, it will have limited capability to determine the sign of the lepton generated by the neutrino charged-current interaction.

With excellent particle identification, HyperK can tackle the mass hierarchy problem using both disappearance of muon neutrinos and appearance of electron neutrinos. The expected statistical significance in settling the mass hierarchy of HyperK is presented in Fig. 20. For $\sin^2 \theta_{23} = 0.5$ and $\sin^2 2\theta_{13} = 0.1$, the wrong hierarchy could be ruled out at almost three standard deviations with about five years of running. The discriminating power is weakly dependent on the value of the CP-phase $\delta_{CP}$. Qualitatively, a small $\delta_{CP}$ will yield better discrimination for a given value of $\sin^2 \theta_{23}$.

In January 2013, HyperK was included in the future plan of KEK [32]. If funded, the project would start in JPY2016, with the two-year construction of access tunnels beginning in 2016. Excavation of the underground cavern is expected to start in 2018 and beneficial occupancy would take place in 2021. Operation of the HyperK detector modules would start in January 2023. The main concern with HyperK is cost, which is estimated at $500-700M (US). It is unclear if Japan will have sufficient funding for this experiment. Given the drive for liquid argon-based long-baseline accelerator programs in both the US and Europe, it is unlikely that foreign partners will contribute significantly to the cost.

6.2.5 Atmospheric neutrino experiments using liquid argon detectors

Besides providing excellent particle identification, liquid-argon detectors are superior to iron-scintillator calorimeters, water Cherenkov detectors, or ice-based detectors in energy and
angular resolutions over the energy range of MeV to multi-GeV. Furthermore, since the energy of the hadronic component of the charge-current interaction is measured, the energy of the incident (anti-)neutrino can be determined. One could even imagine magnetizing a large liquid-argon detector for charge discrimination. Thus, liquid-argon detectors could be an ideal tool for addressing the mass hierarchy problem with atmospheric neutrinos.

Assuming the energy resolution to be $\sqrt{(0.003)^2 + (0.15)^2}/E_{\text{had}}$ for hadronic showers, 0.01 for charged leptons in the GeV range, and the angular resolution to be 0.03 (0.04) rad for electrons (muons or hadronic showers), an estimated sensitivity in delineating the mass order as a function of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ for an exposure of 250 kT-yr is given in Fig. 21. This exposure corresponds to: 25 years of operation of a Phase-I sized LBNE detector; 7.5 years of the full-scale 34-kT detector (either would need to be located underground); 12.5 years of Phase I of LBNNO-GLACIER; and only 2.5 years of the full GLACIER detector. Discrimination power of over $3\sigma$ for the neutrino mass hierarchy seems achievable using liquid argon detectors.

7 Cosmology

Forseeable cosmological measurements can determine the sum of the neutrino masses, but not directly the separate contributions of the three species. However, if the hierarchy is normal with the sum of masses near the minimum of about 0.06 eV, strong evidence for this should accumulate through the next decade, as expected large-scale structure/gravitational lensing experiments (Euclid, LSST, MS-DESI/BigBOSS, etc) come online. However, should the sum turn out to be about 0.10 eV or more, no conclusion could be drawn about the hierarchy. Fortunately, the constraints on the neutrino masses are a byproduct of experiments primarily motivated by dark energy studies, so they will happen independent of neutrino-science considerations.

Neutrinos decouple from other particles very early and become non-relativistic quite late, $z_{\nu r} \sim 83 (m_\nu/0.05 \text{ eV})$, relative to the time CMB was imprinted, $z = 1100$. The masses of the neutrinos affect the fundamental cosmological observable, the power spectrum. The
power spectrum is the Fourier transform of the two-point correlation function between mass density at one point and mass density at another point. To be sure, we are talking about cosmological separations. The density of galaxies can stand as a proxy for the density of matter, both ordinary and dark.

Fig. 22 shows the ratio of power for $\Sigma m_\nu = 0.11$ eV to $\Sigma m_\nu = 0.06$ eV as a function of $k$, the Fourier transform variable measured in Mpc$^{-1}$. Fig. 22 illustrates the small differences in power spectra for different distributions of the total mass between particles, but these are not expected to be observable on the time scale under consideration [34]. The dominant effect that can be measured is the suppression in power shown in Fig. 22. Note, however, that it is not necessary to rely entirely on measuring the effect on the \textit{shape} of the power spectrum shown in Fig. 22. Redshift-space distortions, lensing, and other methods can be used to measure the overall suppression in amplitude relative to the CMB measurement of the primordial perturbations (this relies, however, on GR calculations of the growth of structure).

There are numerous ways to detect the large-scale power suppression by neutrinos, with varying degrees of statistical power and expected systematic uncertainties. The focus here is on galaxy redshift surveys, which appear to offer the best combination of statistical power and projected control of systematic effects.

The first major redshift survey that may seriously test the hierarchy is MS-DESI (approximately equivalent to BigBOSS [35]). Led by LBNL, the survey will cover $\sim 14000$ square degrees, taking spectra of $\sim 20$ million galaxies and quasars at $z \sim 0 - 3$. MS-DESI is likely to run on the Mayall 4 m telescope at Kitt Peak National Observatory near Tucson, AZ, over a $\sim 5$ year period from 2018 to 2022. It is possible that the spectrograph could then be moved to the twin Blanco telescope in Chile to cover another $\sim 10000$ square degrees.

Another big redshift survey that appears certain to happen is the European-led Euclid satellite [36]. Euclid will measure redshifts for $\sim 50$ million galaxies over $\sim 15000$ square degrees. The target survey period is $\sim 2020 - 2026$. In addition to redshifts, Euclid will do

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig22.png}
\caption{Ratio of linear power for $\Sigma m_\nu = 0.11$eV in the inverted (black) or normal (blue) hierarchy, to $\Sigma m_\nu = 0.06$eV in the normal hierarchy.}
\end{figure}
imaging for gravitational lensing measurements.

While not a redshift survey (meaning it has limited radial resolution), LSST will be a major US cosmological experiment running in the 2022-2032 time period [37]. It will image \( \sim 20000 \) square degrees for gravitational lensing, which can add power to the redshift survey measurements of neutrino masses. Before LSST, and even before MS-DESI, there will be a smaller, 5000 sq. deg. lensing survey, DES (http://www.darkenergysurvey.org). We add DES to all projections that do not include LSST.

Another experiment that might happen is the NASA/WFIRST satellite, intended for a 2022 launch [38]. It would be qualitatively similar to MS-DESI and Euclid, but complementary (i.e. adding statistical power) because it would take a different strategy of going deeper over a smaller area.

Projections are given in Table 4. As shown, cosmology can generally achieve or at least approach the 0.01 eV RMS error level needed to probe the hierarchy. These calculations are consistent with similar projections made for Euclid [39].

As seen in Table 4, cosmology is likely to reach the 2 - 2.5 \( \sigma \) level for distinguishing the minimal normal from minimal inverted hierarchies by the end of MS-DESI (in the North at least) in 2022. At that point Euclid and LSST will be running, and significance will accumulate, probably reaching the \( \sim 3.5 \sigma \) level around 2026 (LSST will only be half-done at that point, but most of the gain from it will likely be extractable already). Whether these measurements will determine the neutrino mass hierarchy is contingent on the hierarchy being normal and the masses minimal.

### Table 4: Potential constraints on \( \Sigma m_\nu \) for the minimal parameter set.

<table>
<thead>
<tr>
<th>Combination</th>
<th>( \frac{k_{\text{max}}}{[\text{Mpc}^{-1}]} )</th>
<th>( \sigma_{\Sigma m_\nu} ) [eV]</th>
<th>( \sigma_{0.04 \text{ eV}} )</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>P+BigBOSS14+DES</td>
<td>0.07</td>
<td>0.021</td>
<td>1.9</td>
<td>2022</td>
</tr>
<tr>
<td>P+Euclid+DES</td>
<td>0.07</td>
<td>0.019</td>
<td>2.1</td>
<td>2026</td>
</tr>
<tr>
<td>P+BigBOSS24+DES</td>
<td>0.07</td>
<td>0.019</td>
<td>2.1</td>
<td>2026</td>
</tr>
<tr>
<td>P+BB24+Euc+DES</td>
<td>0.07</td>
<td>0.016</td>
<td>2.5</td>
<td>2026</td>
</tr>
<tr>
<td>P+BB24+Euc+LSST</td>
<td>0.07</td>
<td>0.014</td>
<td>2.9</td>
<td>( \lesssim 2030 )</td>
</tr>
<tr>
<td>P+BB14+DES</td>
<td>0.14</td>
<td>0.017</td>
<td>2.4</td>
<td>2022</td>
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<td>P+Euclid+DES</td>
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<td>0.015</td>
<td>2.9</td>
<td>2026</td>
</tr>
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<td>0.14</td>
<td>0.015</td>
<td>2.7</td>
<td>2026</td>
</tr>
<tr>
<td>P+BB24+Euc+DES</td>
<td>0.14</td>
<td>0.013</td>
<td>3.1</td>
<td>2026</td>
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<tr>
<td>P+BB24+Euc+LSST</td>
<td>0.14</td>
<td>0.011</td>
<td>3.6</td>
<td>( \lesssim 2030 )</td>
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</table>
8 Conclusions

This report has considered several approaches to a measurement of the neutrino mass hierarchy, including long-baseline experiments (Section 4), reactor neutrinos (Section 5), atmospheric neutrinos (Section 6) and cosmology (Section 7). It was outside the scope of this study to evaluate the importance of determining the neutrino mass hierarchy relative to other opportunities on a similar timescale.

The question of the confidence level needed to “decisively” determine the mass hierarchy is a subjective one. One approach is to consider the impact of such a determination, for example on the field of neutrinoless double beta decay. Were the hierarchy determined to be inverted, the next generation of experiments, with the ability to cover the inverted-hierarchy region of parameter space, will become decisive. To motivate these experiments, a $2-3\sigma$ indication of an inverted hierarchy would be sufficient. However, before claiming a Dirac nature of neutrinos on the basis of no signal in such experiments, a much higher significance determination of the hierarchy would be required. Likewise, if cosmological and terrestrial determinations disagree, a high significance determination will be needed before interpreting such a discrepancy as a violation of cosmological theories.

We note that many current studies rely on simplified calculations of confidence levels, assuming $\chi^2$ distributions of test statistics. These assumptions are not always valid, and care must be taken to correctly interpret quoted significance levels, and future experimental results. In the following, we quote sensitivities as stated by the authors in each case, with the here-noted caveat that these are not always directly comparable.

Table 5 summarizes the potential sensitivity, timescale, and open questions for each approach. The claimed sensitivity and timescale are also summarized in Fig. 23. The spread in the displayed sensitivity includes both the projected experimental uncertainties (where evaluated by the proponents), and also the underlying limitations due to currently unknown physics parameters, such as the CP-violating phase and the overall neutrino mass scale.

Of the experiments we have considered, only the long-baseline experiments (LBNE in combination with T2K/NO$\nu$A, the European-based LBNO, and HyperK’s combined long-baseline and atmospheric data) have demonstrated the ability to measure the mass hierarchy with a statistical significance of at least $4\sigma$ regardless of the value of $\delta_{CP}$ and other oscillation parameters. This statement is not without some caveats. So far, LBNE sensitivities are based on parameterized (“toy”) simulations, rather than a complete understanding of the complicated physics involved in neutrino-nucleus interactions at low energies, or even detailed reconstruction algorithms in LAr TPC. Addressing these deficiencies is very important. Also, there is a finite risk that with the long timescale of LBNE construction (the 10-year run is planned to start in about 2020), some other experiment(s) will determine the hierarchy before LBNE produces a result. This would put a costly project in an unfortunate position to confirm someone else’s discovery.

In addition to LBNE, both Japan and Europe are considering long-baseline neutrino oscillation experiments. HyperK is a Megaton-scale water Cherenkov detector, to be constructed in the Kamioka mine, with the same baseline (about 300 km) from the neutrino source at J-PARC as T2K. With the short baseline, HyperK needs atmospheric neutrino data to break the degeneracy between the matter effects and $\delta_{CP}$, and to determine the mass hierarchy with a significance of at least $3\sigma$. The timescale and costs for HyperK are
comparable to that of LBNE.

European long-baseline projects (LAGUNA-LBNO) involve an intense neutrino source at CERN, a near detector, and a (phased) 100 kT underground LAr detector at Pyhäsalmi in Finland, at a baseline of 2300 km. The long baseline, large detector mass, underground location, near detector, and a broad-band neutrino beam from a 2 MW proton source make LAGUNA-LBNO an ultimate neutrino oscillation experiment, with outstanding sensitivity to both the neutrino mass hierarchy and $\delta_{CP}$. However, the timescale, costs, and priority to host such an experiment in Europe are not well defined at present.

JUNO is a 20 kT liquid scintillator detector to be located at the solar oscillation maximum, approximately 60 km away from two nuclear power plants in China. This experiment plans to exploit subtle distortions in the neutrino energy spectrum sensitive to the sign of $\delta_{CP}$.

Figure 23: Summary of sensitivities to the neutrino mass hierarchy for various experimental approaches, with timescales, as claimed by the proponents in each case. In the case of PINGU, for which multiple studies exist, the proponents’ stated sensitivity [29] is shown in the dark blue region, with the larger blue region representing the independent analysis of Ref. [7]. One difference between the two is the consideration of a wider range of oscillation parameters in [7] (see Section 6 for details). The vertical scale of each region represents the spread in the expected sensitivity after the full exposure. We do not attempt to project the natural increase in sensitivity over time. Note: the “long baseline” region represents the inclusive range of sensitivities for individual long-baseline experiments (LBNE, HyperK, and LBNO) rather than a combined sensitivity.
Δm^2_{32}. RENO-50 is a similar experiment proposed in South Korea. This measurement is extremely challenging, both technologically and in terms of required experimental precision. Successful determination of the neutrino mass hierarchy depends critically on achieving unprecedented energy resolution and controlling the energy scale systematics to about 1%. If these challenges can be met, then the hierarchy could be measured to > 3σ (> 4σ) assuming current (future 1.5%-level) uncertainties on Δm^2_{32}.

Another proposed experiment that could in principle rapidly determine the neutrino mass hierarchy is PINGU, a dense array of phototube strings in the middle of the IceCube detector at the South Pole. The measurement relies on polar-angle dependent distortions in the neutrino energy spectrum due to matter-induced neutrino oscillations. With the copious samples of upward-going electron and muon neutrinos and large target mass, PINGU has excellent statistical sensitivity to the mass hierarchy. However, disentangling the hierarchy-dependent effects from the data requires an excellent energy resolution and understanding the energy scale systematics in the detector. The sensitivity also depends on Nature’s choice of oscillation parameters, including the hierarchy itself. As evaluated by the proponents, the sensitivity of PINGU could be ~ 4.8-7.6σ with 5 years of data. While the potential for a decisive, inexpensive, and fast measurement is there, we do not feel the systematic issues have been fully addressed by the proponents to date. These studies are ongoing as of this writing. An independent study finds a larger range of potential sensitivities (1−5σ), including a lower bound for the case of an inverted hierarchy, which can be partially mitigated by combination with T2K/NOνA data.

Future dark energy experiments such as MS-DESI (formerly BigBOSS), Euclid, and LSST, in combination with the Cosmic Microwave Background measurements, have the capability to measure the sum of the neutrino masses with a precision relevant to the neutrino mass hierarchy. Should the hierarchy be normal and the neutrino masses minimal, global cosmological fits could discern the neutrino mass hierarchy on a timescale comparable to that of LBNE. Early indication can be obtained from currently running or near-future measurements. These measurements rely on the present best cosmological model (ΛCDM), which is supported by the wealth of astrophysical data.

While any individual measurement in the next decade may be susceptible to large uncertainties, either statistical or systematic, a combination of results from multiple experiments and techniques could yield a greater confidence in a determination of the mass hierarchy. We have not explicitly considered the potential improvements from such combinations, although independent studies exist [7, 40]. It is our hope that several of the experiments here described will be pursued. Ultimately, a cross check between multiple techniques will be required for any decisive, unambiguous determination of the mass hierarchy.

9 Acknowledgments

We would like to thank Mark Strovink for contributions to this document, and many members of the Nuclear Science and Physics divisions at LBNL for enlightening conversations.
Table 5: Comparison of mass hierarchy experiments. “NH” refers to the normal hierarchy. In the case of PINGU, for which multiple studies exist, both the sensitivities claimed by the proponents [29] and the independent analysis of Ref. [7] are presented.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Experiment</th>
<th>MH sensitivity</th>
<th>Timescale for results</th>
<th>Major concerns</th>
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<tr>
<td>Accelerator</td>
<td>T2K+NOνA</td>
<td>1–3σ</td>
<td>~2020</td>
<td>Non-optimal baselines</td>
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<td></td>
<td>HyperK</td>
<td>&gt; 3σ with atmospheric</td>
<td>~2030</td>
<td>Likelihood of going ahead</td>
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<td></td>
<td>LAGUNA-LBNO</td>
<td>&gt; 5σ</td>
<td>~2025</td>
<td>Likelihood of going ahead</td>
</tr>
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<td></td>
<td>LBNE phase I</td>
<td>3σ (2σ) over 80% (100%) of δCP</td>
<td>~2030</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LBNE 34 kT</td>
<td>&gt; 6σ</td>
<td>~2030</td>
<td>Assuming Phase-I timescale</td>
</tr>
<tr>
<td>Reactor</td>
<td>JUNO</td>
<td>3–4σ</td>
<td>~2025</td>
<td>Energy scale &amp; resolution</td>
</tr>
<tr>
<td>Atmospheric</td>
<td>PINGU</td>
<td>4.8 – 7.6σ [29]</td>
<td>~2023</td>
<td>Energy scale &amp; resolution, correlated parameters</td>
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<td></td>
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<td>1 – 5σ [7] (dependent on oscillation parameters)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosmology</td>
<td>All</td>
<td>0–4σ (3–4σ for NH and minimal masses)</td>
<td>~2025</td>
<td>Measures sum of masses – can only determine hierarchy for minimal masses</td>
</tr>
</tbody>
</table>
References


[31] P. Coyle et al. [The Km3Net collaboration], contribution to the European Strategy Preparatory Group Symposium, September 2012 Krakow, Poland.


