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On the Transmission Mechanism of International Business Cycles

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Daniel Felles Farhat

June 2009

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To my mother and my sister.
Models that feature endogenously determined trade patterns are able to capture many features of international business cycle transmission. Although these models are "Ricardian" in the sense that countries trade labor services, changes in employment are often omitted (labor is supplied inelastically). The effect of foreign booms and recessions on domestic labor markets can therefore not be analyzed, and hence these models overlook an important mechanism of business cycle transmission across countries. Further, these models often omit capital in the production of goods. "Investment" in the model is defined as new firm construction, which does not compare well to the empirical measure of investment. This dissertation develops a two-country dynamic stochastic general equilibrium model featuring an endogenously determined trade pattern. Employment dynamics are added to the model by allowing households to choose how much labor to supply every period. The model generates high output volatility, procyclical employment and positive correlation of employment across countries. Capital is then added to the production of non-traded final goods. The model produces high international correlations of output, consumption
and investment when shocks to the production of final non-traded goods are considered.

Further, the model is able to reproduce high volatility of imports, exports and the exchange rate, features that past literature had difficulty capturing.
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Chapter 1

Introduction

Business cycle fluctuations among industrialized countries are positively correlated. Most would say that trade and financial linkages between countries are responsible for transmitting booms and recessions across countries. Efforts to develop models in which these linkages serve as a mechanism for business cycle transmission generally end in failure more often than not. Through the 1990’s, extensions of the closed economy neoclassical business cycle model to multiple countries consistently predicted negatively correlated business cycles or required abhorrent features inconsistent with the modern trend of globalization in order to achieve matches to the stylized facts. Recently, research has shown that changes in the pattern of trade (a feature omitted from the previous body of international real business cycle research) can greatly affect business cycle transmission. Models with foundations in trade theory that allow for shifting trade patterns show promise in succeeding where the previous models had failed. These models, when compared to the rest of the real business cycle literature, are quite primitive. To simplify the framework, elements
necessary for the study of business cycles are often omitted.

One such model developed by Ghironi and Melitz (2005) has become quite popular among international macroeconomists. The model features monopolistic competition, love of variety and intra-industry trade. Endogenizing the number of firms and the size of the export sector allow for changes in the trade pattern over time. Persistent trade deficits, persistent deviations in purchasing power parity and bilateral trade flows are all possible within the model. In calibrated experiments, the model is capable of matching many stylized facts at the firm level. The original framework, however, oversimplifies many sectors of the economy relevant to the study of business cycles. For example, labor in the model is supplied inelastically which makes it impossible to study employment dynamics. Since the model requires labor in the production of traded goods and to pay costs associated with exporting, fluctuations in employment are certainly relevant to changes in the trade pattern which governs business cycle transmission. Also, the model omits capital. "Investment" is simply new firm construction, which is not how investment is typically defined in the data. Further, capital accumulation and changes in capital holdings can exacerbate the persistence of shocks to the economy. The trade off between capital and labor services in the production of goods is also relevant to the pattern of trade and can influence the direction and intensity of business cycle synchronization.

In chapter 2, I will perform a survey of the literature and a reassessment of the stylized facts associated with the international business cycle research program. Chapter 3 and Chapter 4 attempts to extend the Ghironi and Melitz framework in two directions. In chapter 3, a more complete labor market will be developed within the model. By incor-
porating endogenously determined labor supply, the model becomes capable of generating fluctuations in employment which increase the volatility of output. Further, the labor market reacts in such a way as to produce procyclical employment and positive correlation of employment across countries. Labor market outcomes result in improved international correlations of output, consumption and investment. Trying to improve business cycle correlation further, as some researchers have done, by restricting asset trade results in distortions in the labor market and the international market.

"Investment" continues to be misspecified in versions of the model that omit capital accumulation. In chapter 4, the definition of capital will be improved by incorporating capital into the production of non-traded final goods. Including capital in this fashion provides a more realistic concept of investment while leaving the fundamental structure of trade undisturbed. When business cycles are driven by productivity shocks to intermediate goods production, the inclusion of capital in final goods production succeeds in capturing the relationship between imports, exports and GDP, but fails to adequately capture international business cycle correlation and many features of the international market. When business cycles are driven by productivity shocks to final goods production, capital plays a crucial role in business cycle synchronization. The model is able to produce more realistic series for the volatility of imports, exports, and the real exchange rate as well as capture the relationship between international prices and output.

\footnote{When traded goods require only labor, the underlying structure of trade is "Ricardian" in the sense that countries trade labor services.}
Chapter 2

A Survey of the International Real Business Cycle Research Program

2.1 Introduction

A review of the current research and a reexamination of the stylized facts associated with international business cycles is necessary. Many reoccurring themes in the international real business cycle (IRBC) research program inspire the structure of the models presented in chapter 3 and 4. Themes of particular note include trade pattern endogeneity, love of variety, trade in intermediate goods, asset market frictions and demand shocks. These will be highlighted in section 2.2. Since many of the models in the research program omit data from the last 20 years, an update of the statistics used to evaluate the fit of the model’s simulation results is also a necessity. This is done in section 2.3. Section 2.4 briefly concludes.
2.2 Recent Literature

Prior to the 1990’s, theoretical explanations of business cycle transmission and synchronization could be classified into two groups: real models and money models. The money models argued that in the presence of sticky prices, unanticipated changes in the money supply by one country induced economic changes in general. The real models focused on preference or technology shocks transmitted through trade linkages between countries. Either sort of model was loosely based on the Mundell-Flemming open economy model which focuses on the dynamic relationship between investment/savings (foreign direct investment) and the trade balance.

In the mid-1980’s, a wide-spread revival of classical ideas (coupled with a rejection of Keynesian approaches) disseminated throughout the literature. Many researchers such as Dellas (1986), Stockman and Svensson (1987), and Cantor and Mark (1988) began extending the newly developed real business cycle models of Kydland and Prescott (1982) and Long and Plosser (1983) to multiple countries. These early studies focused on the theoretical underpinnings of the transmission mechanism of business cycles. Dellas (1986) for example, generates a dynamic model in which either correlated productivity shocks or trade linkages (in inputs) could generate business cycles. Cantor and Mark (1988) and Stockman and Svensson (1987) on the other hand, suggest that international sales of assets transmits business cycles through risk pooling or the revaluation of assets. Canova and Dellas (1993) develop a DSGE including domestic and foreign consumption as well as trade in intermediate goods to explicitly analyze the role of international trade in transmitting and synchronizing cycles. Their model’s results suggest that business cycles could become correlated through
input trade, depending on the magnitude of trade linkages. Using standard time-series techniques, they show that trade linkages are an important medium for cyclical correlations opposed to common productivity shocks.

After the essential framework began to emerge, researchers began applying calibration techniques to test the developing models in lieu of econometric analysis. Backus, Kehoe and Kydland (1992) and Mendoza (1991) proposed two-country DSGE models featuring complete financial markets (which facilitate international risk sharing), correlated productivity shocks, frictionless trade and capital adjustment costs. Their central purpose was to find how accurately a simulated model reflects the real economy. Unfortunately their answer was "not much". Several anomalies came to light from the initial pass of research.

Many inconsistencies between the simulated results and the real economy were derived by Backus, Kehoe and Kydland and persist in the face of several experiments. Unlike the real economy, simulation results often suggest consumption among countries has high correlation while output has negative or low correlation (dubbed the "consumption-output" or C-O anomaly). This result occurs essentially from using financial markets to pool risk. Consider, for example, two countries: A and B. A favorable shock in country A causes assets (in the form of capital, let’s say) to be diverted to that country. Production of output is reduced in country B and increases in country A. Since consumers purchase output from income stored in assets, consumption for everyone increases. Hence, consumption across countries, within the model, becomes too highly correlated while output becomes negatively or slightly correlated. To a somewhat lesser extent, international correlations between investment, savings and employment were also difficult the match. Resolving the
C-O anomaly, however, became the most sought after target for the next wave of research.

Continual advancements in the research program were made as economists tried to resolve the mystery. Some researchers attempt to augment the way financial markets are modeled to achieve better results. Baxter and Crucini (1995) and Kollmann (1996) include incomplete asset markets to try to retard the effect of risk pooling. Heathcote and Perri (2000) close financial markets completely and compare the results to those that occur in complete markets. Kose and Yi (2001, 2002, 2005) follow Heathcote and Perri using a three-country model to analyze the impact of trade linkages on cyclical synchronization under both open and closed financial markets. They, however, include transportation costs to induce variation in trade linkages. Kehoe and Perri (2002) look not at exogenously constrained financial markets, but endogenously constrained financial markets by including imperfectly enforceable lending agreements. Olivero (2004) models non-competitive lenders which reduces the effects of risk-pooling via counter-cyclical mark-ups for loans.

Other researchers make changes to the way non-financial components within the model interrelate. Stockman and Tesar (1995) introduce a non-traded goods sector to allow for high correlation of traded goods, but not necessarily for non-traded goods. Baxter and Farr (2005) include variable capital utilization which reduces the size of productivity shocks necessary to match several moments in the international data. As these sorts of augmentations continued to result in limited improvements, the direction of the research program began to shift towards relying on demand-side sources of cyclical fluctuations.

Stockman and Tesar (1995) and Wen (2002) illustrate the importance of demand-side shocks to international comovement by incorporating persistent demand shocks into
an IRBC framework. The intuition is that when one country has a technology shock, assuming perfect risk pooling, investment into that country’s capital stock rises at the expense of investment in the other country’s capital stock. Output in the shocked economy rises while output in the other falls (hence the counterfactually negative or low correlation of output across countries that is predicted by models relying on technology shocks). With demand shocks, the international market works as a medium for cyclical synchronization. When one country experiences a persistent demand shock, consumption of both domestically produced and imported goods rise. Thus, output and investment in both countries rise to compensate (hence correlated output and investment). Since demand shocks are idiosyncratic, we would expect the correlation of consumption across countries to be less than the correlation of output. However, when demand shocks are transitory, investment is crowded out to increase consumption temporarily in the shocked country, resulting in low or negative investment correlation.

Building on this idea, Guo and Sturzenegger (1998), Xiao (2003, 2004) and Fukuda (2004) incorporate sunspots within an IRBC context. To do so, the model is constructed to exhibit a sink-stable steady state to which an indeterminate number of equilibrium paths converge. The incorporation of this feature resolves the consumption-output anomaly, but also stresses self-fulfilling beliefs as an important mechanism generating cyclical fluctuations. Xiao (2004) includes variable capital utilization to reduce the degree of increasing returns necessary to achieve indeterminacy. Fukuda (2004) develops a model with indeterminacy within the context of a monetary economy. In general, IRBC models featuring indeterminacy outperform previous models on multiple dimensions and remain at the forefront of the
In the vast majority of aforementioned research, focus has not been on the international market. Imports, exports, terms of trade and exchange rates are treated, for the most part, as an aside. Focus on the dynamic properties of these variables is done within a somewhat separate but related strand of research. Backus, Kehoe and Kydland (1994), for example, introduced a new property of international business cycles: the j-curve. "J-curve" describes the shape of the cross-correlation function between the trade balance and the terms of trade. The trade balance tends to be negatively correlated with current and future movements in the terms of trade while positively correlated with past movements of terms of trade. They produce a DSGE model which can replicate this property. Cuñat and Maffezzoli (2002) also focus on matching moments of exchange rates, the trade balance and terms of trade within a two-country DSGE model. Productivity shocks drive the model, as in Backus, Kehoe and Kydland (1992), so the C-O anomaly persists in the Cuñat and Maffezzoli model. However, more focus is put on nominal and real exchange rates, terms of trade and net exports.

Mendoza (1995) and Zimmerman (1999) include exogenous terms of trade shocks and exchange rate shocks respectively within IRBC frameworks. Mendoza focuses on the Harberger-Larsen-Metzler effect and capturing the transmission mechanism of terms of trade. Zimmerman focuses on matching import and export volatility. These models differ from earlier research in the source of shocks, but relate to Backus, Kehoe and Kydland’s interest in the properties of the international market.

Closely related to the work by Backus, Kehoe and Kydland (1994) and the demand-
side models of Stockman and Tesar (1995) and Wen (2002) is a strand of research incorporating of monetary shocks. Schlagenhauf and Wrase (1995) focus on the role of exogenous monetary policy shocks when consumers trade both goods and currencies. Chari, Kehoe and McGrattan (1997) develop a model in which prices are sticky and consumers face stochastic shocks to the money growth rate. Both models focus on matching the volatility and persistence of the exchange rate and terms of trade.

In all of the models described so far, the evolution and determination of the trade pattern has been exogenous. A small but separate sect of IRBC literature has developed within the rubric of 'international trade', which leaves the trade pattern endogenous. In a broad sense, this is done by first endogenizing firm entry, then allowing firms to choose to be exporters. Thus, the effect of the calibration on the results are two-fold: one effect coming from endogenous firm entry while the other comes from endogenous export decision.

Head (2002) and Cook (2002) incorporate endogenous firm entry in monopolistically competitive input markets. Production of final goods within the model exhibits international increasing returns to scale. The C-O anomaly persists in Head’s calibrated results, although correlated productivity shocks are no longer required to achieve correlated output across countries. The C-O is resolved qualitatively in Cook’s model when firms enter input markets sequentially then engage in Cournot competition. Alessandria and Choi (2004) and Ghironi and Meltiz (2005) incorporate both endogenous firm entry and export decision. In Alessandria and Choi, consumer dislike of foreign variety allows for reduction in the C-O anomaly. The Ghironi and Melitz framework succeeds in reducing the C-O anomaly as well, but focuses on generating deviations from purchasing power parity and the Harrod-
Balassa-Samuelson effect. Models of this type are structured to reflect modern perceptions of international trade: non-competitive frameworks incorporating intra-industry trade.

This new class of models is still quite underdeveloped when compared to resent research on closed-economy real business cycles. The original Ghironi and Melitz model assumed labor is supplied inelastically to simplify the models already complex structure. The drawback of this feature is a lack of employment dynamics in the model. Since labor is used in the production of traded goods, to construct new firms and to pay export costs, changes in employment can dramatically affect the trade pattern. Further, the model distorts the definition of "investment" typically used in real business cycle models by omitting capital accumulation. Since investment exodus has been shown to be a central cause of low output correlation across countries, and since the presence of capital can prolong the effects of business cycle movements, incorporating capital into the model is certainly not a trivial exercise.

2.3 A Review of the Stylized Facts

Many researchers rely on the set of stylized facts generated by Backus, Kehoe and Kydland (1992) in order to test the fit of their models. This data covers 1954-1989 for the U.S. business cycle statistics and 1960-1989 for international business cycle statistics\(^1\). With almost 20 years of additional data available, it is important to update the set of stylized facts used to compare the fit of our models. We might expect that accounting

for changes in the economy during the past two decades would impact the set of statistics quite dramatically. On one hand, the sample period originally analyzed by Backus, Kehoe and Kydland puts heavy weight on the oil shock era (in which business cycles between the US and Europe were strongly related) which might provide us with an overestimate of the degree of international correlation between countries. On the other hand, the presence of globalization and the relaxing of trade restrictions that have occurred in recent history might have improved business cycle correlations. Further, the more moderate business cycle fluctuations among the industrialized countries will most certainly change measures of volatility.

I organize the data on international comovement into three categories: domestic business cycle statistics, international correlations and international market statistics. First, to test how well my model reproduces internal (or "intra-national") business cycles, I will compare the results of my model to the data on domestic business cycles for the United States reported in table 2.1\(^2\). As is commonly found, consumption, investment and labor hours are procyclical. Consumption and labor hours are less volatile than output while investment is more volatile.

To make appropriate international comparisons, I use annual data from the Penn World Tables which is adjusted for changes in purchasing power parity. I construct a "European Aggregate" following Backus, Kehoe and Kydland (1992) from 1970-2004. After

---

\(^2\)Table 2.1 - Quarterly data for the United States (1957.1-2007.1) is extracted from the International Financial Statistics database maintained by the World Bank. Data for investment includes gross fixed capital formation plus changes in inventories. Labor data is generated from civilian employment measures from the OECD database. Logs for deflated measures of output (Y), consumption (C), investment (I), and labor (L) are detrended using the HP filter.
business cycle comovement, however, proves to be time-dependent. Figure 2.1 shows how the correlation coefficients described in Table 2.2 change over time by looking at 5-year and 10-year rolling window estimates\(^4\). Data from the entire sample suggests output is more highly correlated across countries than consumption (the data used to isolate the consumption-output anomaly). Looking at the 10-year window, we see that output is more correlated across countries than consumption before 1985, but consumption is more correlated than output after 1985. In general, we can say that output correlation and consumption correlation are "close" and generally positive. Investment correlation, however, is low and occasionally negative.

The third set of data I consider looks at international market statistics for the

---

\(^3\)Table 2.2 - The "European Aggregate" consists of Austria, Finland, France, Germany, Italy, Switzerland and the United Kingdom. Annual labor data is taken from the OECD database and omits Finland due to missing data. Data availability for Germany constrains the sample to 1970-2004.

\(^4\)Figure 2.1 - At any time, \(t\), the figure plots the international correlations of output, consumption and investment using data from the preceding \(j\) years, where \(j = 5\) and 10 respectively.

<table>
<thead>
<tr>
<th>International Comovement Correlation</th>
</tr>
</thead>
</table>
| $\rho_{Y_{US},Y_{EU}}$ | 0.56  
| $\rho_{C_{US},C_{EU}}$ | 0.45  
| $\rho_{I_{US},I_{EU}}$ | 0.37  
| $\rho_{L_{US},L_{EU}}$ | 0.39  

United States. These measures include imports (IM), exports (EX), the trade balance as a fraction of output (Net Exports/Output = NX/Y), the terms of trade (TOT, measured as the ratio of import prices to export prices) and the real exchange rate index (Q) and are reported in table 2.3. The data suggests high volatility of quantities (imports and exports) and prices (terms of trade and the real exchange rate). Exports and imports are both procyclical, net exports is counter-cyclical and prices are approximately acyclical. This data will be used to test how well the model captures the behavior of trade linkages (an important source of business cycle transmission across countries) over the business cycle.

2.4 Conclusion

There are many reoccurring themes in the review of current research on international business cycle transmission. First, trade pattern endogeneity, love of variety, trade in intermediate goods and non-competitive markets add a dimension of realism and flexibility to modern international business cycle models. Second, open goods markets lead to positive comovement across countries while open asset markets lead to risk-pooling and

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5Table 2.3 - The quarterly data is extracted from 1957.1-2007.1 from the International Financial Statistics Database for imports, exports, the terms of trade (index) and the real exchange rate (index). Log measures are detrended using the HP filter.
negative comovement. The imposition of asset market can abate the negative international correlation, although this feature is inconsistent with the modern trend of globalization.

Third, international market statistics are important if trade linkages drive business cycle transmission, but are often overlooked. Since trade is considered the mechanism responsible for international comovement, matching stylized facts of the international market are important. Finally, demand-side shocks are potentially vital in transmitting business cycle fluctuations across countries.

The domestic business cycle statistics are as expected: consumption, employment and investment are all procyclical and investment is more volatile than GDP while consumption and employment are less volatile. In general, the updated empirical evidence suggests that consumption, output, investment and employment are generally correlated across countries. Although output is more correlated across countries than consumption for the period prior to 1985, consumption is more correlated across countries than output for the modern period. On average, output and consumption correlations are "close" and positive. In the international market, imports, exports, the terms of trade and the real exchange rate are all highly volatile relative to GDP. Imports and exports are both pro-

---

Table 2.3: **U.S. International Market Statistics (1957q1-2007q4).**

<table>
<thead>
<tr>
<th>International Market</th>
<th>Volatility (Standard Deviation)</th>
<th>Correlation with GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{IM} / \sigma_Y)</td>
<td>3.31</td>
<td>(\rho_{IM,Y})</td>
</tr>
<tr>
<td>(\sigma_{EX} / \sigma_Y)</td>
<td>3.74</td>
<td>(\rho_{EX,Y})</td>
</tr>
<tr>
<td>(\sigma_{NX/Y})</td>
<td>0.40</td>
<td>(\rho_{NX/Y,Y})</td>
</tr>
<tr>
<td>(\sigma_{TOT/Y})</td>
<td>1.69</td>
<td>(\rho_{TOT,Y})</td>
</tr>
<tr>
<td>(\sigma_Q / \sigma_Y)</td>
<td>3.37</td>
<td>(\rho_{Q,Y})</td>
</tr>
</tbody>
</table>
cyclia, but imports are more closely related to output than exports. The trade balance and the real exchange rate are countercyclical while the terms of trade is acyclical. The analyses presented in Chapter 3 and 4 will utilize this updated set of statistics in evaluating the fit of the models.
Figure 2.1: U.S.-European Comovement Statistics (1970-2004) - Rolling Window.
Chapter 3

Endogenous Labor Supply, International Trade and Macroeconomic Dynamics

3.1 Introduction

The Ghironi-Melitz framework has quickly become a standard model, bridging traditional macroeconomic theory and new models of international trade\(^1\). Although the focus of the original paper was on generating deviations from purchasing power parity and the Harrod-Balassa-Samuelson effect\(^2\), the model can also help reduce the consumption-output anomaly. While the model is able to match many stylized facts, it oversimplifies the

\(^1\)The model is based in new trade theory, featuring non-competitive markets incorporating intra-industry trade.

\(^2\)The Harrod-Balassa-Samuelson effect states that prices are generally higher in wealthier (or more productive) countries.
labor market by assuming inelastic labor supply and cannot be used to study the dynamics of employment. Hence, the basic model cannot predict how downturns in the economies of our trading partners affect our employment rate. Since the model is "Ricardian" in spirit\(^3\), a development of the model has the potential to understand how employment can influence the trade pattern, enhancing our understanding of international business cycles. I carry out this development by adding a richer theory of the labor market to the original Ghironi-Melitz framework.

My model succeeds in simulating more realistic output volatility, procyclical employment and positive international correlation of employment when labor supply is endogenous. When there is a positive shock to aggregate productivity, output and consumption increase as does the demand for labor as new firms enter the market. Although increases in consumption reduce the amount of labor services the households are willing to supply, the effect of an increase in labor demand dominates. In equilibrium, the model predicts higher wages and procyclical employment. Output increases further as employment expands, generating high volatility of output. Capital market adjustments induce an exodus of investment and an economic contraction in the other country which causes foreign workers to choose to supply more labor in equilibrium, resulting in positive international correlation of employment. Domestic expansions result in an increase in employment as do foreign upswings (although foreign consumption may be contracting).

To induce stronger international correlation, past research suggests imposing strong asset market frictions. With higher foreign bond adjustment costs, there is less transfer of

\(^3\)In the Ghironi-Melitz model, countries trade intermediate products for which labor is the only input. Effectively, the countries are trading labor services.
investment spending from the foreign country to the home country when the home country experiences a positive productivity shock. Output, consumption and investment all become more correlated across countries. However, the purchase and sale of foreign assets allow a country to run a trade deficit or surplus. Asset frictions of this sort result in reduced volatility of imports, exports and the trade balance as each country attempts to maintain more balanced trade. When labor supply is endogenously determined, positive consumption correlation that arises from changes in the trade pattern cause a reduction of labor supply in the foreign country whenever the home country experiences a positive shock. Employment correlation across countries is reduced. Imposing strong asset market frictions improves international correlation at the expense of the international market and the labor market.

The remainder of the chapter is as follows. I construct the model in section 2. I present and discuss a series of numerical experiments in section 3. Section 4 presents a short conclusion.

3.2 The Model

The theoretical model is heavily influenced by Ghironi and Melitz (2005). I diverge, however, from their framework by including endogenously determined labor supply in the household’s decision. The model described here considers two countries, home and foreign (denoted by *). Both countries are large and assumed to be structurally identical. With that in mind, I construct the framework for one country (the home country) knowing that a symmetric framework exists for the foreign counterpart.
3.2.1 The Consumer’s Problem

The most relevant changes I make to the original framework are in the household’s problem. I assume that there exists a representative household that chooses to work, consume and save. A variety of products are available to the household. Denote the universe of intermediate input varieties available to both countries as $\Omega$. At any time, $t$, a subset $\Omega_t \subseteq \Omega$ is actually available to home consumers. $\Omega_t$ contains both domestically produced and imported goods. It need not be the case that $\Omega_t = \Omega_s$ for $t \neq s$ nor $\Omega_t = \Omega_t^*$. Composite consumption, $C_t$, is produced using Dixit-Stiglitz technology:

$$C_t = \left( \int_{\omega \in \Omega_t} c_t(\omega)^{(a-1)/a} \, d\omega \right)^{a/(a-1)}$$

(3.1)

where $c_t(\omega)$ denotes the quantity of variety $\omega$ used in the production of the composite, and $a > 1$ denotes the elasticity of substitution across varieties. The framework suggests that, ceteris paribus, the amount of composite commodity produced is increasing in the number of available varieties (referred to in the literature as "love of variety" (Dixit and Stiglitz (1977))). This set-up allows me to clearly incorporate imperfect competition on the producer’s side of the model. Recent advancements in the trade literature with respect to imperfect competition and trade patterns make this formulation desirable\(^4\).

Denote $P_t(\omega)$ as the price for variety, $\omega$, and $P_{Ct}$ as the price index for consumption. It is straightforward to construct conditional demand equations for individual varieties, $c_t$\(^5\).

$$c_t(\omega) = C_t(P_t(\omega)/P_{Ct})^{-a}$$

\(^4\)Helpman and Krugman (1985) illustrate the emergence of this strand of literature.

\(^5\)We solve a series of cost-minimization problems to construct demand equations and price indices.
I also construct the associated price index:

\[ P_{Ct} = (\int_{\omega \in \Omega_t} P_t(\omega)^{1-a} d\omega)^{1/(1-a)} \]  

(3.2)

The household is endowed with 1 unit of time that it can divide between labor, \( l_t \), and leisure. Labor earns the real wage, \( w_t \). The household can choose to save by purchasing bonds, \( B_{t+1} \), which each cost one unit of consumption, but yield \((1 + r_{t+1})\) units in the next period. Both domestic and foreign bonds \((B_s)\) are available to the household and are subject to a quadratic transaction cost which is rebated, lump-sum \((\Gamma_t)\), to the household. The household can also purchase shares of a mutual fund, \( x_{t+1} \), which entitle the owner to a fraction of the profits \((\hat{d}_t)\) of the producing firms. Arbitrage prices the shares at the firms expected discounted value \((N_{ht}\hat{v}_t\) for new shares and \(N_{dt}\hat{v}_t\) for old shares, where \(N_{ht}\) is the number of existing firms, \(N_{dt}\) is the number of producing firms, and \(\hat{v}_t\) is the average value of a firm). Households are obligated to hold bonds and stocks for only one period before they are resold in asset markets\(^6\). The consumer’s real period budget constraint is then given by:

\[
C_t + B_{t+1} + Q_t B_{st+1} + \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{st+1}^2) + (N_{ht}\hat{v}_t)x_{t+1} = \\
w_t l_t + (1 + r_t)B_t + Q_t(1 + r^*_t)B_{st} + \Gamma_t + x_t(N_{dt}\hat{v}_t + N_{dt}\hat{d}_t)
\]

where \(Q_t\) is the real exchange rate, \(n_2 \geq n_1 > 0\) are the scale parameters on foreign and domestic bond adjustment costs respectively, and the lump-sum rebate \(\Gamma_t = \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{st+1}^2)\) in equilibrium. Allowing for differences in domestic and foreign bond adjustment costs allow me to adjust the magnitude of asset market frictions in international bond markets. As \(n_2\) increases, domestic consumers will be less willing to adjust their holdings

\(^6\)All savings instruments are denoted with the time subscript in which they yield a return.
of foreign bonds which will limit the extent to which a country is willing to finance a trade deficit.

I will construct two different formulations for the agent’s instantaneous utility function. When considering perfectly inelastic labor supply, as in the original Ghironi-Melitz framework, I assume the period utility function is of the form:

$$U_t(C_t, l_t) = \log C_t$$

When considering endogenously determined labor supply, the instantaneous utility function takes the form:

$$U_t(C_t, l_t) = \log C_t - \frac{t^{1+\lambda}}{1+\lambda}$$

with $\lambda > 0$. The agent’s maximization problem can then be written as:

$$\max_{\{C_s, l_s, B_{s+1}, x_{s+1}\}_{s=t}^\infty} \mathbb{E}_{t} \sum_{s=t}^\infty \beta^{s-t} [U_s(C_s, l_s)] \quad s.t.
\begin{align*}
C_s + B_{s+1} + Q_s B_{s+1} + \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{t+1}^2) + (N_{hs} \bar{v}_s) x_{s+1} = \\
w_s l_s + (1 + r_s) B_s + Q_s (1 + r_s^*) B_{s+1} + \Gamma_s + x_s (N_{ds} \bar{v}_s + N_{ds} \bar{d}_s)
\end{align*}$$

The first-order conditions for the consumer’s problem generate an equation that guides labor supply in addition to three Euler equations. The labor supply equation is:

$$l_t = C_t^{-1/\lambda} w_t^{1/\lambda}$$

with elasticity of labor supply equal to $1/\lambda$ when labor supply is endogenous and

$$l_t = 1$$

---

7In their original paper, Ghironi and Melitz use a CRRA utility function: $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, with $\gamma = 2$. I have chosen log consumption ($\gamma = 1$) for both cases to ensure balanced growth properties of the model when endogenous labor supply is considered.
when labor is supplied inelastically. The Euler equation for domestic bonds is:

\[ C_t^{-1}(1 + n_1 B_{t+1}) = (1 + r_{t+1})\beta E_tC_{t+1}^{-1} \]  

(3.5)

Similarly, the Euler equation for foreign bonds is:

\[ Q_tC_t^{-1}(1 + n_2 B_{*t+1}) = (1 + r_{t+1}^*)\beta E_tQ_{t+1}C_{t+1}^{-1} \]  

(3.6)

The Euler equation for stocks is:

\[ \tilde{v}_t = \beta(1 - \delta)E_t(\frac{C_{t+1}}{C_t})^{-1}(\tilde{d}_{t+1} + \tilde{v}_{t+1}) \]  

(3.7)

### 3.2.2 The Firm’s Problem

I now consider the problem faced by a typical intermediate good producer in the home country. I assume that there exists an unbounded mass of firms in the economy that may begin production at any time. As in Ghironi and Melitz (2005), these firms are monopolistically competitive and produce for domestic and foreign markets separately (i.e. markets are segmented). Given the demand for their products derived in the previous section, the firm’s problem is as follows:

- **Step 1:** Decide whether or not to enter.
- **Step 2:** Upon entry, choose how much output to produce and what price to set.
- **Step 3:** Death or exit.

In practice, firms will first derive the solutions to step 2 and step 3 before choosing whether or not it’s worth it to enter the industry.

Consider a potential domestic producer, \(j\). If firm \(j\) decides to enter the market, they must pay a one-time entry fee in terms of effective units of labor. Once the entry cost
is paid, they receive a firm-specific productivity draw, $z_{jt}$. The firm then produces output according to the production function:

$$y_{jt} = Z_t z_j L_{jt}$$

where $y_{jt}$ denotes the quantity of output firm j produces, $Z_t$ denotes an economy-wide technology variable, and $L_{jt}$ is the quantity of labor firm j hires in the production of output.

The firm will choose how much to produce for the domestic market ($c_{djt}$) and how much to produce for export ($c_{xjt}$). Firms that choose to export must pay a per-period export fee in terms of effective units of labor. Exports are subject to an iceberg cost, $\tau \geq 1$. Thus, total output for the firm is given by $y_{jt} = c_{djt} + \tau c_{xjt}$.

The firm’s expenses include a wage bill, a one-time start-up cost, and exporter fees. The wage bill is simply $w_t L_{jt}$. Start-up costs are a fixed cost paid in terms of labor once during the period the firm begins production. The “production” of entry is given by $F_{Et} = Z_t L_{Ejt}$ which suggests a total fixed cost of $w_t F_{Et}/Z_t$\(^8\). Exporter fees are paid each period the firm chooses to export ($c_{xjt} > 0$). The production of the exporter fee is given as $F_{Xt} = Z_t L_{Xjt}$. Thus, the per-period exporter cost is $w_t F_{Xt}/Z_t$. Dropping the start-up cost, I form the firm’s real instantaneous profit maximization problem:

$$\max_{c_{xjt}, c_{djt}} \frac{\Pi_{jt}}{P_{Ct}} = \frac{P_{djt}}{P_{Ct}} c_{djt} + \epsilon_t \frac{P_{xjt}}{P_{Ct}} c_{xjt} - \frac{w_t}{Z_t z_j} (c_{djt} + \tau c_{xjt}) - I_{xt} \frac{w_t F_{Xt}}{Z_t}$$

where $I_{xt} = 1$ if $c_{xjt} > 0$, else $I_{xt} = 0$. $\epsilon_t$ denotes the nominal exchange rate. It is important to note that $P_{xjt}$ is measured in terms of foreign currency. For any variable, X, we denote real prices as $\rho_X = P_X/P_C$ and substitute the demand functions from the

\(^8\)The presence of a fixed entry fee induces constantly falling average cost, ensuring each firm produces a unique variety.
previous section into the firm’s problem (as is standard with monopolistic competition) to generate the first order conditions for the firm:

\[ c_{djt} = C_t \left( \frac{a}{a-1} \frac{w_t}{Z_t z_j} \right)^{-a} \]

\[ c_{xjt} = C_t^* \left( \frac{a}{a-1} \frac{w_t}{Z_t z_j Q_t} \right)^{-a} \]

where the real exchange rate reflects the relative price of consumption between the home and foreign countries, \( Q_t = \epsilon_t P^*_t / P_t \). For simplicity, it is assumed that \( \epsilon_t = 1 \). The demand equations are used to find the optimal real prices charged by the firm:

\[ \rho_{djt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j} \]

(3.8)

\[ \rho_{xjt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j Q_t} \]

(3.9)

The prices suggest a constant markup over marginal cost \((w_t/Z_t z_j)\).

Profits for the firm can be divided into "domestic production profits" and "export profits". They are given as:

\[ d_{djt} = \frac{1}{a} C_t^* \rho_{djt}^{1-a} \]

(3.10)

\[ d_{xjt} = \begin{cases} \frac{1}{a} C_t^* Q_t \rho_{xjt}^{1-a} - \frac{w_t F}{Z_t} & \text{if } c_{xjt} > 0 \\ 0 & \text{if } c_{xjt} = 0 \end{cases} \]

(3.11)

For a continuum of potential firms, there exists some productivity draw, \( z_{xt} \), such that the firm with that draw earns zero profits whether it exports or not (the exporter cutoff). For this firm, \( \frac{1}{a} C_t^* Q_t \rho_{xjt}^{1-a} = \frac{w_t F}{Z_t} \).

\(^9\)It is assumed that firms do not observe the impact of price setting on the aggregate price level when solving the firm’s maximization problem.
Without knowing their productivity draw prior to entry, firms form an expectation of their profits before deciding to enter the market. To do this, I employ the "special averages" developed by Melitz (2003). First, it is assumed that the productivity draws follow a Pareto distribution with CDF $G(z) = 1 - (z_{\text{min}}/z_j)^k$ and PDF $g(z) = k z_{\text{min}}^k z^{-k-1}$ where $k$ denotes the shape parameter of the distribution. Next, I define the special productivity averages as:

$\tilde{z}_{D}^{1-a} = \int_{z_{\text{min}}}^{\infty} z_j^{1-a} g(z) dz$

$\tilde{z}_{xt}^{1-a} = \int_{z_{xt}}^{\infty} z_j^{1-a} \frac{g(z)}{1 - G(z_{xt})} dz$

Finally, I substitute these definitions in constructing average prices: $\tilde{\rho}_{Dt}^{1-a} = \int_{z_{\text{min}}}^{\infty} \rho_{djt}^{1-a} g(z) dz = (\frac{a}{a-1}\frac{w_t}{Z_tz_{D}^{1-a}})^{1-a}$ and $\tilde{\rho}_{xt}^{1-a} = \int_{z_{xt}}^{\infty} \rho_{xjt}^{1-a} \frac{g(z)}{1 - G(z_{xt})} dz = (\frac{a}{a-1}\frac{w_t}{Z_tz_{xt}^{1-a}})^{1-a}$. We define $N_{dt}$ as the number of firms that produce for the domestic market and $N_{xt}$ as the number of producing firms that produce for export. The model is recast in terms of average prices:

$\tilde{\rho}_{Dt} = \left(\frac{a}{a-1}\frac{w_t}{Z_tz_d}\right)$

$\tilde{\rho}_{xt} = \left(\frac{a}{a-1}\frac{w_t}{Z_tz_xt\tau}\right)$

$\tilde{a}_{Dt} = \frac{1}{a}C_t\tilde{\rho}_{Dt}^{1-a}$

$\tilde{a}_{xt} = \frac{1}{a}C_tQ_t\tilde{\rho}_{xt}^{1-a} - \frac{w_tF_xt}{Z_t}$

Completing the integral for the special productivity averages suggests that $\tilde{z}_{D} = \left(\frac{k}{k+1-a}\right)^{1/(a-1)}z_{\text{min}}$ and $\tilde{z}_{xt} = \left(\frac{k}{k+1-a}\right)^{1/(a-1)}z_{xt}$ where $k > a - 1$ for boundedness. Knowing that all existing firms produce for the domestic market, and a fraction of those become exporters, the expected per-period profit for a potential firm, on average, is:

$\tilde{d}_t = \tilde{d}_{Dt} + (1 - G(z_{xt}))\tilde{d}_{xt}$

(3.16)
There are two important features guiding the expected value of the firm’s lifetime profits. The first is a lag in production. A firm that enters in period \( t \) starts producing at period \( t + 1 \). The entering firm, however, is still counted as a firm in period \( t \). The total number of firms that exist at period \( t \), \( N_{ht} \), is given by the number of producing firms that already exist plus the number of new firms, \( N_{et} \).

\[
N_{ht} = N_{dt} + N_{et}
\]

Second, firms are subject to an exogenous exit shock. The number of firms that "survive" to produce in period \( t + 1 \) is given by:

\[
N_{dt+1} = (1 - \delta)N_{ht} = (1 - \delta)(N_{dt} + N_{et})
\]  

(3.17)

Therefore, the expected value of the firm’s lifetime profit stream, \( \tilde{v}_t \), is given by:

\[
\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [1 - \delta]^{s-t} \left[ \beta^{s-t} \left( \frac{C_s}{C_t} \right)^{-\gamma} \right] \tilde{d}_s
\]

where \( \left[ \beta^{s-t} \left( \frac{C_s}{C_t} \right)^{-\gamma} \right] \) is the stochastic discount factor of the household and \( (1 - \delta) \) is the firm’s survival probability. Notice that repetitive forward substitution of the Euler equation for stocks generates the expected value of the firm’s lifetime profit stream. Firms continue to enter as long as the discounted value of their profit stream exceeds the cost of entry. Therefore, the entry cutoff is determined by:

\[
\tilde{v}_t = w_t F_{Et}/Z_t
\]  

(3.18)
3.2.3 Market Clearing and Equilibrium

I now turn to important market clearing conditions implied by the model. The most important of these conditions for this discussion is labor market clearing. As determined in the consumer’s problem, labor supply is given as $l_t = C_t^{-1/\lambda} w_t^{1/\lambda}$ when labor supply is endogenous and $l_t = 1$ when labor is supplied inelastically. Labor demand comes from three sources: production of intermediate inputs, exporter costs and start-up costs. Labor demand for intermediate goods production is derived from the labor requirement curve, the demand equations and average prices. I calculate total labor demand for production of goods as $N_{xt} \tilde{d}_{xt} (a-1) w_t + N_{xt} a w_t \tilde{d}_{xt} + N_{xt} (a-1) F_{Xt}/Z_t$. Labor demand from the payment of exporter fees is simply $N_{xt} F_{Xt}/Z_t$. Labor demand for the payment of start-up fees is $N_{Et} F_{Et}/Z_t$. Total labor demand is then:

$$L_t = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{Dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et})/Z_t$$

(3.19)

The equation that governs equilibrium in the labor market when labor supply is endogenously determined is:

$$C_t^{-1/\lambda} w_t^{1/\lambda} = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{Dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et})/Z_t$$

(3.20)

When labor is supplied inelastically, the labor market clearing condition is:

$$1 = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{Dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et})/Z_t$$

(3.21)

The second market clearing condition ensures market clearing in the mutual fund market (shares of the portfolio sum to unity):

$$x_{t+1} = x_t = 1$$

(3.22)
The third market clearing condition ensures zero net supply of bonds. The representative agent in each country is either a borrower or a lender, but not both. If the agent produces bonds to sell, they must be purchased by agent in the foreign country: $B_{t+1} = -B^*_{t+1}$. Thus, the zero net supply conditions are:

$$B_{t+1} + B^*_{t+1} = 0$$
$$B_{st+1} + B^*_{st+1} = 0$$

The presence of adjustment costs ensure that the unique steady state bond issuance is zero ($B = 0$)\textsuperscript{10}.

The final market clearing condition ensures balance of payments. I impose mutual fund market clearing and lump-sum transfers of bond adjustment costs\textsuperscript{11}:

$$2(1 + r_{t+1})B_t + 2(1 + r^*_{t+1})Q_tB_{st} = [C_t - Q_tC_t^*] + [N_{et}\tilde{v}_t - Q_tN_{et}^*\tilde{v}_t^*] + 2[B_{t+1} + Q_tB_{st+1}]$$
$$-\left[w_t\tilde{l}_t - Q_tw_t^*\tilde{l}_t^*\right] - \left[N_{dt}\tilde{d}_{dt} - Q_tN_{dt}^*\tilde{d}_{dt}^*ight]$$
$$-\left[N_{xt}\tilde{d}_{xt} - Q_tN_{xt}^*\tilde{d}_{xt}^*ight]$$

The full equation system that characterizes the model is described in the appendix. 29 non-linear equations are generated and then log-linearized around a symmetric steady state (denoted by $\hat{\cdot}$)\textsuperscript{12}. In addition to these equations, the stochastic processes for the technology innovations must be specified. Productivity shocks are assumed to take the

\textsuperscript{10}This can be seen by comparing the home agent’s Euler equation for home bonds and foreign agent’s demand for home bonds at the steady state then applying the zero net supply of bonds condition. For the home agent, $\beta(1 + r) = (1 + nB)$. For the foreign agent, $\beta(1 + r) = (1 + nB^*)$. The zero net supply condition implies $B = -B^*$. Thus, $(1 + nB) = (1 - nB) \rightarrow B = 0$.

\textsuperscript{11}A description of how the balance of payments equation is generated is in the appendix.

\textsuperscript{12}Since the variable, $B_t$, has an expected value of zero in equilibrium, it is linearized with respect to steady state consumption: $\hat{B}_t = \frac{B_t - 0}{C} = \frac{B_t}{C}$. 

30
following VAR form:

\[
\begin{bmatrix}
\hat{Z}_t \\
\hat{Z}_t^*
\end{bmatrix} = \begin{bmatrix}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{bmatrix} \begin{bmatrix}
\hat{Z}_{t-1} \\
\hat{Z}_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
\xi_{zt} \\
\xi_{zt}^*
\end{bmatrix}
\]

(3.25)

\[
E_t(Z_t) = E_t^*(Z_t^*) = \bar{Z}
\]

\[
\sigma_{\xi_{zt}} = \sigma_{\xi_{zt}^*} = \sigma_{\xi}
\]

\[
\rho_{\xi_{zt}\xi_{zt}^*} = \rho_{\xi z \xi z}^*
\]

Additional definitions are also required. Following Ghironi and Melitz (2005), income (GDP) is defined as:

\[
Y_t = w_t l_t + N_{dt} \tilde{d}_{dt}
\]

(3.26)

Total imports is constructed using the average revenue foreign exporters earn from their sales abroad:

\[
IM_t = N_{xt}^* C_t \tilde{\rho}_{xt}^{1-a}
\]

(3.27)

Total exports is constructed using the average revenue domestic exporters earn from their sales abroad:

\[
EX_t = Q_t N_{xt} C_t^* \tilde{\rho}_{xt}^{1-a}
\]

(3.28)

Terms of trade is defines as the ratio of import prices to export prices:

\[
TOT_t = \tilde{\rho}_{xt}^* / Q_t \rho_{xt}
\]

(3.29)

Investment is defined as expenditures on new firm entry:

\[
I_t = N_{et} \tilde{v}_t
\]

(3.30)
In the model, the price of consumption \( (P_{Ct}) \) is measured as a welfare-based price index following Feenstra (2003). This price index accounts for changes in the variety of goods, while the price index in the data does not. It is thus important to transform this welfare-based index into one which closer matches the price index calculated in the data. To do so, average prices are substituted into the definition of the consumption price index:

\[
1 = \left[ N_{dt} \tilde{\rho}_{dt}^{1-a} + N_{xt}^* \tilde{\rho}_{xt}^{1-a} \right]^{1/(1-a)}
\]

Note that the price index is sensitive to the number of varieties. If we assume that all prices are the same on average, \( \tilde{\rho} = \tilde{P}_t/P_{Ct} \), we construct:

\[
\frac{P_{Ct}}{\tilde{P}_t} = [N_{dt} + N_{xt}^*]^{1/(1-a)}
\]

(3.31)

Any variable measured in terms of real consumption, \( X_t \), is adjusted to this index: \( \tilde{X}_t = \frac{P_{Ct}X_t}{\tilde{P}_t} \). Further, since the real exchange rate is constructed using the welfare-based price indices \( (Q_t = P_{Ct}^*/P_{Ct}) \), an adjusted real exchange rate is constructed with average prices:

\[
\tilde{Q} = \frac{\tilde{P}_t^*}{\tilde{P}_t} = Q_t \left[ \frac{N_{dt} + N_{xt}^*}{N_{dt}^* + N_{xt}} \right]^{1/(1-a)}
\]

(3.32)

### 3.3 Numerical Experiments

The stylized facts described in chapter 2 will be used to evaluate the fit of the model. I organize the data on international comovement into three categories: domestic business cycle statistics, international correlations and international market statistics. These statistics are summarized in table 3.1. There are three important notes regarding the comparison between the model’s results and the stylized facts. First, as has been shown
in chapter 2, positive and close international correlations of GDP and consumption will be considered a successful fit, although the data suggests output is more correlated across countries. Second, the data for labor is civilian employment, not labor hours\textsuperscript{13}. Finally, investment in the data comprises firm construction, investment in plant and equipment, residential investment and changes in inventories. "Investment" in the model described above is only new firm construction\textsuperscript{14}.

I start by employing a calibration that most closely matches the original Ghironi and Melitz (2005) framework. Many parameter values appear in past literature and are widely accepted. They are summarized in table 3.2:

The values for the discount factor (0.99 for quarterly simulations, 0.96 for annual simulations) are standard in the literature. The values for the probability of death are calibrated to match data on US job destruction (2.5% quarterly, 10% annually). The elasticity of substitution between intermediate inputs, the shape parameter on the Pareto distribution, the lower bound on the Pareto distribution, the iceberg cost on intermediate inputs and the costs of entry and export are set to match Ghironi and Melitz (2005) and are reported in table 3.2.

\textsuperscript{13}This matters in some experiments described in the appendix.
\textsuperscript{14}Although the measure of investment in the data and the measure of investment in the model are not the same, the data is relied on nonetheless. In chapter 4, investment is redefined in the model to be more consistent with the data.
Table 3.1: **Stylized Facts.**

<table>
<thead>
<tr>
<th>Domestic Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
</tr>
<tr>
<td>( \sigma_Y(%) )</td>
<td>1.54</td>
</tr>
<tr>
<td>( \sigma_C/\sigma_Y )</td>
<td>0.75</td>
</tr>
<tr>
<td>( \sigma_I/\sigma_Y )</td>
<td>3.41</td>
</tr>
<tr>
<td>( \sigma_L/\sigma_Y )</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>Correlation with GDP</strong></td>
<td></td>
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<tr>
<td>( \rho_C,Y )</td>
<td>0.88</td>
</tr>
<tr>
<td>( \rho_I,Y )</td>
<td>0.90</td>
</tr>
<tr>
<td>( \rho_L,Y )</td>
<td>0.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>International Comovement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Correlation</strong></td>
<td></td>
</tr>
<tr>
<td>( \rho_Y,Y* )</td>
<td>0.56</td>
</tr>
<tr>
<td>( \rho_C,C* )</td>
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</tr>
<tr>
<td>( \rho_I,I* )</td>
<td>0.37</td>
</tr>
<tr>
<td>( \rho_L,L* )</td>
<td>0.39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>International Market Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
</tr>
<tr>
<td>( \sigma_{IM}/\sigma_Y )</td>
<td>3.31</td>
</tr>
<tr>
<td>( \sigma_{EX}/\sigma_Y )</td>
<td>3.74</td>
</tr>
<tr>
<td>( \sigma_{NX}/Y )</td>
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</tr>
<tr>
<td>( \sigma_{TOT}/\sigma_Y )</td>
<td>1.69</td>
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<tr>
<td>( \sigma_Q/\sigma_Y )</td>
<td>3.37</td>
</tr>
<tr>
<td><strong>Correlation with GDP</strong></td>
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</tr>
<tr>
<td>( \rho_{IM,Y} )</td>
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</tr>
<tr>
<td>( \rho_{EX,Y} )</td>
<td>0.28</td>
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<tr>
<td>( \rho_{NX/Y,Y} )</td>
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</tr>
<tr>
<td>( \rho_{TOT,Y} )</td>
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<tr>
<td>( \rho_{Q,Y} )</td>
<td>-0.18</td>
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</table>
Table 3.2: **Calibrated Parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.99, 0.96</td>
</tr>
<tr>
<td>$\delta$ Probability of death shock</td>
<td>0.025, 0.10</td>
</tr>
<tr>
<td>$\tau$ Iceberg costs associated with trade</td>
<td>1.3</td>
</tr>
<tr>
<td>$a$ Elasticity of substitution between inputs</td>
<td>3.8</td>
</tr>
<tr>
<td>$k$ Shape parameter on Pareto distribution</td>
<td>3.4</td>
</tr>
<tr>
<td>$z_{\min}$ Lower bound on Pareto distribution</td>
<td>1</td>
</tr>
<tr>
<td>$\tilde{z}_D$ ”Special” average productivity draw</td>
<td>$z_{\min}(\frac{k}{k+1-a})^{1/(a-1)}$</td>
</tr>
<tr>
<td>$F_{Et}$ Fixed cost of entry</td>
<td>1</td>
</tr>
<tr>
<td>$F_{Xt}$ Fixed exporter costs</td>
<td>0.235$(F_{Et})^{1-\beta(1-\delta)/(\beta(1-\delta))}$</td>
</tr>
</tbody>
</table>

The productivity process is calibrated as

\[
\begin{bmatrix}
\hat{Z}_t \\
\hat{Z}_t^*
\end{bmatrix} = \begin{bmatrix}
0.99 & 0 \\
0 & 0.99
\end{bmatrix} \begin{bmatrix}
\hat{Z}_{t-1} \\
\hat{Z}_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
\xi_{Zt} \\
\xi_{Zt}^*
\end{bmatrix}
\]

(3.33)

\[\sigma_{\xi_{Zt}} = \sigma_{\xi_{Zt}^*} = \sigma_{\xi} = 0.00852\]

\[\rho_{\xi_{Zt},\xi_{Zt}^*} = \rho_{\xi_{Zt}Z_{t}^*} = 0.258\]

The other model parameters, $(n,\bar{Z},\lambda)$, will be thought of as ”free parameters” and will be calibrated under different experiments discussed shortly.

I solve the model numerically using the brute force algorithm developed by Uhlig (1999). Simulations are performed in order to generate a set of statistics to compare to the data. In each experiment, a 200-period model is simulated 200 times by drawing a random

\[\text{Backus, Kehoe and Kydland (1992) calibrate technology shocks as a VAR process with spillovers:}\]

\[
\begin{bmatrix}
\hat{Z}_t \\
\hat{Z}_t^*
\end{bmatrix} = \begin{bmatrix}
0.906 & 0.088 \\
0.088 & 0.906
\end{bmatrix} \begin{bmatrix}
\hat{Z}_{t-1} \\
\hat{Z}_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
\xi_{Zt} \\
\xi_{Zt}^*
\end{bmatrix}
\]

\[\sigma_{\xi_{Zt}} = \sigma_{\xi_{Zt}^*} = \sigma_{\xi} = 0.00852\]

\[\rho_{\xi_{Zt},\xi_{Zt}^*} = \rho_{\xi_{Zt}Z_{t}^*} = 0.258\]

Baxter and Crucini (1995) note that this specification is not statistically different from a near-unit-root process without spillovers.

\[\bar{Z} \text{ is typically set to 1.}\]
vector of innovations. During each simulation, I make the appropriate price adjustments as described above then apply the HP filter\textsuperscript{17}. I calculate each summary statistic (volatility, correlation with output, etc.) for each simulation and then report the average statistics across simulations in the results tables.

\subsection*{3.3.1 The Benchmark Model}

To match the original Ghironi and Melitz (2005) framework as closely as possible, I set \( n_1 = n_2 = 0.0025 \) and omit endogenous labor supply from the model. Simulation results are reported in column (2) of table 3.3. The fundamental shortcomings of the model are under predicted output volatility, low correlation of output across countries and low correlation of consumption across countries. Also, the benchmark model generates negative international correlation of investment.

The model generates low international correlation of output (0.16), consumption (0.05) and negative correlation of investment (-0.43). To understand why these results occur, I inspect the impulse response functions for the endogenous variables of the model\textsuperscript{18}. When the home country experiences a positive shock to aggregate productivity, output and profits for existing firms rise immediately. This drives up the discounted value of an average firm’s profit stream and thus the return to mutual fund shares, spurring investment in new firm construction. With increased production by domestic firms, composite consumption rises. At the aggregate level, there is more demand for labor to pay start-up costs and to expand production which puts upward pressure on wages. Over time, new firm entry

\textsuperscript{17}Since the model exhibits a high degree of persistence, we take out the low-frequency trend using the HP filter.

\textsuperscript{18}Impulse response functions for the endogenous variables of the model are available upon request.
### Table 3.3: Simulation Results: Inelastic and Endogenous Labor Supply.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(4)</th>
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<td>Volatility</td>
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<tr>
<td>$\sigma_Y$ (%)</td>
<td>1.54</td>
<td>0.92</td>
<td>1.17</td>
<td>1.74</td>
<td>0.97</td>
<td>1.23</td>
<td>1.74</td>
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<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.75</td>
<td>0.63</td>
<td>0.56</td>
<td>0.48</td>
<td>0.63</td>
<td>0.57</td>
<td>0.47</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>3.41</td>
<td>3.91</td>
<td>4.13</td>
<td>4.56</td>
<td>3.11</td>
<td>3.49</td>
<td>4.07</td>
</tr>
<tr>
<td>$\sigma_L/\sigma_Y$</td>
<td>0.61</td>
<td>—</td>
<td>0.28</td>
<td>0.53</td>
<td>—</td>
<td>0.28</td>
<td>0.55</td>
</tr>
<tr>
<td><strong>Correlation with GDP</strong></td>
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<tr>
<td>$\rho_{C,Y}$</td>
<td>0.88</td>
<td>0.99</td>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
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<td>0.96</td>
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<tr>
<td>$\rho_{I,Y}$</td>
<td>0.90</td>
<td>0.94</td>
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<td>0.95</td>
<td>0.98</td>
<td>0.98</td>
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<tr>
<td>$\rho_{L,Y}$</td>
<td>0.81</td>
<td>—</td>
<td>0.98</td>
<td>0.97</td>
<td>—</td>
<td>0.97</td>
<td>0.97</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>$\rho_{Y,Y}$*</td>
<td>0.56</td>
<td>0.16</td>
<td>0.18</td>
<td>0.22</td>
<td>0.28</td>
<td>0.26</td>
<td>0.25</td>
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<tr>
<td>$\rho_{C,C}$*</td>
<td>0.45</td>
<td>0.05</td>
<td>0.08</td>
<td>0.17</td>
<td>0.34</td>
<td>0.31</td>
<td>0.30</td>
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<tr>
<td>$\rho_{I,I}$*</td>
<td>0.37</td>
<td>-0.43</td>
<td>-0.31</td>
<td>-0.17</td>
<td>0.07</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$\rho_{L,L}$*</td>
<td>0.39</td>
<td>—</td>
<td>0.52</td>
<td>0.47</td>
<td>—</td>
<td>0.23</td>
<td>0.22</td>
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<tr>
<td><strong>International Market Statistics</strong></td>
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<tr>
<td>Volatility</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{IM}/\sigma_Y$</td>
<td>3.31</td>
<td>0.79</td>
<td>0.72</td>
<td>0.62</td>
<td>0.58</td>
<td>0.51</td>
<td>0.43</td>
</tr>
<tr>
<td>$\sigma_{EX}/\sigma_Y$</td>
<td>3.74</td>
<td>0.84</td>
<td>0.77</td>
<td>0.63</td>
<td>0.57</td>
<td>0.50</td>
<td>0.43</td>
</tr>
<tr>
<td>$\sigma_{NX/Y}$</td>
<td>0.40</td>
<td>0.25</td>
<td>0.30</td>
<td>0.36</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma_{TOT}/\sigma_Y$</td>
<td>1.69</td>
<td>0.30</td>
<td>0.27</td>
<td>0.21</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>$\sigma_{Q}/\sigma_Y$</td>
<td>3.37</td>
<td>0.07</td>
<td>0.06</td>
<td>0.06</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>Correlation with GDP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{IM,Y}$</td>
<td>0.60</td>
<td>0.89</td>
<td>0.87</td>
<td>0.83</td>
<td>0.82</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
<td>$\rho_{EX,Y}$</td>
<td>0.28</td>
<td>0.17</td>
<td>0.18</td>
<td>0.22</td>
<td>0.80</td>
<td>0.79</td>
<td>0.77</td>
</tr>
<tr>
<td>$\rho_{NX/Y,Y}$</td>
<td>-0.36</td>
<td>-0.48</td>
<td>-0.45</td>
<td>-0.42</td>
<td>-0.22</td>
<td>-0.21</td>
<td>-0.22</td>
</tr>
<tr>
<td>$\rho_{TOT,Y}$</td>
<td>0.07</td>
<td>0.58</td>
<td>0.57</td>
<td>0.58</td>
<td>0.55</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td>$\rho_{Q,Y}$</td>
<td>-0.18</td>
<td>-0.58</td>
<td>-0.55</td>
<td>-0.51</td>
<td>0.41</td>
<td>0.40</td>
<td>0.29</td>
</tr>
</tbody>
</table>

(1) Data
(2) Benchmark, $n_2 = 0.0025$
(3) $\lambda = 1$, $n_2 = 0.0025$
(4) $\lambda = 0.25$, $n_2 = 0.0025$
(5) Benchmark, $n_2 = 1$
(6) $\lambda = 1$, $n_2 = 1$
(7) $\lambda = 0.25$, $n_2 = 1$
contributes to an expansion of product variety, inducing persistent increases in consumption, output and wages. As increases to the real wage overpower the productivity shock, the marginal cost of production rises, leading to increases in domestic prices.

The economic expansion immediately increases the demand for imports, which drives up the average price for imported goods (total imports increase). The increase in production in the export sector contributes to an immediate fall in export prices. As export production becomes less profitable, the size of the home country’s export sector contracts.

As the return to mutual fund shares in the home country rises, the return to home bonds does as well via the Euler equation. Foreign agents adjust their asset portfolios in favor of home bonds which now yield a higher return, reducing investment in new firm construction in their own country ("investment exodus"). On one hand, less labor is demanded for new firm construction. On the other hand, more labor is demanded to expand production in the foreign export sector to satisfy the increased import demand in the home country. The model predicts an overall decline in labor demand and a fall in the real wage. Output and consumption decline in the foreign country and business cycles across countries becomes contemporaneously negatively correlated. Any positive relationship across countries that occurs in simulation is driven by the underlying correlation of the productivity innovations and changes in the real exchange rate used to generate comparable measures.

Although the home country’s demand for imports remains high for several periods, the increase in demand drops off quickly as new varieties are produced in the home country. Also as the variety of home products increases, a larger variety of goods are exported to
the foreign country although export prices remain low. The overall value of exports for the home country increases over time. With a larger variety of goods coming into the country, the foreign consumers experience a boost in consumption in the future. Over time, the effect of an increase in labor demand in the foreign export sector overpowers the effect of a decline in labor demand for new firm construction and overall wages in the foreign country increase. Foreign national income increases over time. Increasing foreign consumption leads to a re-expansion of the home export sector, furthering the persistence of the positive shock in the foreign country. The international market in the model allows the home country business cycle to "lead". When one country has an expansion, the other has an immediate contraction followed by an expansion over time (in excess of recovery).

Since import prices rise while export prices fall in the home country when there is a positive productivity shock, the terms of trade rises. Import prices remain high while export prices rebound as the foreign country enters an expansionary period over time; the terms of trade declines quickly before smoothly returning to the steady state. Upon receiving a positive productivity shock, the home country begins to run a trade deficit. As the export sector expands, the trade deficit declines. When the foreign country experiences their future expansion, the home country runs a trade surplus.

In the home country, increases in domestic prices along with increases in export prices push up the aggregate price level while increases in the foreign export sector pushes the aggregate price level down. In the foreign country, decreases in prices for domestically produced goods along with falling import prices push the price level down while a contraction of the home export sector pushes the price level up. The model predicts the impact
of the changing trade pattern is dominate leading to an increase in the real exchange rate \( Q_t = P_{Ct}^* / P_{Ct} \). Over time, increases in the domestic variety of goods and foreign firm attrition lead to further increases in the exchange rate.

Since foreign variables are multiplied by the real exchange rate to make foreign measures comparable, changes in the exchange rate affect the degree of business cycle correlation. Increases in the real exchange rate induce statistical increases in foreign output and consumption in response to a positive productivity shock. Exchange rate changes are not powerful enough to statistically reverse the fall in foreign investment. Also, increases in the exchange rate make exports even less profitable to the home country, contributing to the contraction of the home export sector and decline of domestic exports.

### 3.3.2 Endogenous Labor Supply

I now turn to the model with endogenous labor supply. Previous macroeconomics literature suggests that endogenously determined labor supply has the potential to increase the volatility of output if employment is procyclical. Since the two countries described in the economy essentially trade labor services, changes in the labor market can also affect the pattern of trade, altering the international correlation of business cycles. I calibrate the elasticity of labor supply to match Fiorito and Zanella (2008) \( \lambda = 1 \) and consider low bond adjustment costs, \( n_1 = n_2 = 0.0025 \). The results are reported in column (3) of table 3.3. In the model I have constructed, supply-side changes in the labor market buffers against negatively correlated business cycles, results in positive international correlation of employment and generates more volatile output.
At first glance, the volatility of output (1.17) is improved. Employment in the model is procyclical (0.97) and the international correlation of employment is appropriate (0.52). Also, comparing the model to the case when labor is inelastically supplied (column (2) of table 3.3), the correlation of output and consumption across countries are increased and the negative correlation of investment has been reduced. Again, I inspect the impulse response functions for the endogenous variables to understand the impact of the labor market on the model’s results. The effect of a positive productivity shock on the domestic economy when labor is supplied inelastically is already known: more labor is demanded in the home country, driving up the real wage (shown in diagram a of figure 3.1: labor demand shifts from \(LD_1\) to \(LD_2\)), while in the foreign country less labor is demanded immediately after the shock due to firm attrition and more labor is demanded in future periods due to export expansion.

When the labor supply decision is endogenous, the increase in composite consumption from firm entry (more variety) and expansion (more goods produced) in the home country compels households to supply less labor. This is clear from the first order condition governing labor supply: 

\[
l_t = C_t^{-1/\lambda}w_t^{1/\lambda} = w_t/C_t \quad \text{for} \quad \lambda = 1
\]

(shown in graph b of figure 3.1: the labor supply curve shifts from \(LS_1\) to \(LS_2\) in response to a positive productivity shock). The increase in labor demand resulting from the positive productivity shock offsets the decline in labor supply, resulting in more employment and higher wages in equilibrium. The boost in employment further increases aggregate output, leading to the higher output volatility seen in the results table.

The positive international correlation of employment is due directly to trade and
Figure 3.1: **Home Country Labor Market: Positive Productivity Shock.**

financial linkages. As previously described, a positive productivity shock in the home country leads to firm attrition in the foreign country due to open asset markets. In foreign labor markets, there is a slight decline in the foreign demand for labor (as seen in graph b of figure 3.2: labor demand shifts from $LD_1^*$ to $LD_2^*$). Because the foreign economy has started to contract ($C^*_t$ is falling), foreign households increase their supply of labor according to the first order condition governing foreign labor supply: $l_t^* = w_t^*/C_t^*$ (shown in graph b of figure 3.2: the foreign labor supply curve shifts from $LS_1^*$ to $LS_2^*$). The slight decline in labor demand is offset by the increase in labor supply, resulting in more employment in the foreign country at lower wages. There is positive international correlation of employment. Further, the model predicts an increase in overall income which induces positive output correlation across countries. While output and employment become more positively correlated across countries due to the response of the labor market to fluctuations in trade that occur with the business cycle, the correlation of consumption and investment across countries found in
simulation remains due to the underlying correlation of the productivity innovations.

As the effects of the productivity shock on the home country wear off, employment smoothly returns to its steady state level. In the foreign country, however, workers shift from supplying a high level of labor during their recessionary period to supplying low levels of labor as the economy enters an expansionary period\(^{19}\).

Increasing the elasticity of labor supply can result in improved output volatility with little to no adverse effects on the rest of the model. Following Guo and Sturzenegger (1998), I increase the elasticity of labor supply to 4 (\(\lambda = 0.25\)) and report the results in column (4) of table 3.3. The more elastic labor supply curve generates a larger increase in employment in the home country when there is a positive productivity shock, raising the volatility of output. International correlation of output, consumption and employment either remain the same as in the model with unit-elastic labor supply elasticity or slightly improve. The model continues to predict a negative international correlation of investment\(^{20}\) and under predict the volatility of imports, exports, the terms of trade and the real exchange rate.

### 3.3.3 High Bond Adjustment Costs

The benchmark model replicates international correlations and international market statistics poorly. One might believe that the failure of the model to generate realistic

\(^{19}\)The impulse response function for foreign employment is "humped-shaped" with an increase immediately after the shock followed by a decline below the steady state level during the expansionary phase before returning to the steady state over time.

\(^{20}\)The international correlation of investment does improve (from -0.31 when labor supply is unit-elastic to -0.17). This is due to the increased response of investment to the productivity shocks which are positively correlated.
business cycle comovement is due to the openness of financial markets, as past literature has suggested. Imposing stronger asset market frictions by increasing the scale parameter on foreign bond adjustment costs ($n_2 = 1$) would make agents less willing to change their bond market portfolios. The results from this experiment are shown in column (5) of table 3.3. Consumption and output become more positively correlated across countries (0.26 and 0.31 respectively). Investment correlation remains low but is positive (0.08).

Why business cycles become more positively correlated when bond market adjustment costs increase is quite straight-forward. Foreign agents are no longer so willing to adjust their holdings of bonds issued by the home country when there is a positive shock to home productivity. There is no longer a dramatic fall in foreign firm construction and foreign labor demand. Consumption, output and investment increase in the home country, as does their demand for imported goods. Prices for imported goods rise due to the
increased demand, increasing the profitability of foreign exporters. To maintain more balanced trade, exports to the foreign country also rise. In the foreign country, the increase in the variety of imported goods leads to an increase in foreign consumption. Labor supply declines, resulting in a fall in employment, but an increase in the real wage. The model predicts a rise in foreign income. The wage increase in the foreign country does result in firm attrition. Even though more firms die than enter, each entering firm becomes more profitable (resulting in increased investment). The positive international correlations of output, consumption and investment are further influenced by the underlying correlation of the productivity innovations.

Column (6) of table 3.3 reports the results from the model with endogenous labor supply ($\lambda = 1$) and high asset market frictions ($n_2 = 1$). With both economies having expansions when one experiences a shock to aggregate productivity, labor supply in the two countries shift in the same direction. While employment in the home country increases in equilibrium due to a large rise in labor demand, employment in the foreign country declines. The model predicts an increase in the real wage in both countries along with duel increases in aggregate income. As can be seen from the simulation results, employment correlation across countries declines. Any correlation of employment found in simulation is due to the correlation of the underlying productivity innovations.

Although the inclusion of strong asset market frictions has improved the synchronization of business cycle fluctuations across countries, it has done so at the expense of both the international market and the labor market. With high bond adjustment costs, agents are less willing to finance trade deficits by issuing debt. A positive productivity
shock in the home country increases output and consumption, leading to an increase in imports. An increase in exports partially offsets the need to issue costly bonds to cover the resulting trade deficit. The model predicts that increased demand for home’s exports by foreign households and higher export prices (due to higher domestic wages) result in a larger value of exports in equilibrium. This results in less volatility in the international market and an over predicted correlation between exports and GDP. The volatility of the trade balance (as well as imports and exports separately) decline. The increase in the variety of exports shipped to the foreign country induces a rise in composite consumption and a powerful decline in labor supply. This results in a decline in foreign employment. Any positive correlation of employment generated in simulation is due to the underlying correlation of the productivity innovations.

3.4 Conclusion

The Ghironi and Melitz (2005) model is capable of capturing many features of international business cycles. By assuming labor is supplied to the market inelastically, the model cannot account for international employment dynamics. Incorporating endogenous labor supply increases the volatility of output to a realistic level. Procyclical employment and positive international correlation of employment are successfully reproduced by the model. Further, changes in labor supply improve the international correlations of output, consumption and investment.

Past researchers suggest that the correlation of business cycles produced by the model can be further improved by slowing the rate of investment exodus from one country
to the other through asset market frictions. By increasing costs associated with purchasing foreign bonds, the model can in fact generate stronger correlations of output, consumption and new firm construction. However, this is achieved at the expense of the international market and the labor market. Increased asset market frictions boost the volatility of exports and reduce the volatility of the trade balance as countries try to maintain more balanced trade. Changes in labor supply when strong bond market frictions are present induce a reduction in employment in the foreign country, worsening the correlation of employment across countries.

Future work includes incorporating unemployment into the model. When I test whether or not the framework with the developed labor market can accommodate involuntary unemployment as in Hansen (1985) and Rogerson (1988)\textsuperscript{21}, I find that output volatility, the international correlation of output, investment and employment and the correlation between exports and GDP all worsen. Since the transmission of job loss is of concern to policy makers, it is important to include a concept of unemployment in the model without distorting the international correlations and disturbing the international market.

Other potential extensions to the model include adding capital to the model in order to provide a more accurate measure of investment, potentially resolving the problem of negatively correlated investment across countries. The inclusion of demand shocks for final goods may lead to increased volatility of imports, exports and international prices. These are done in chapter 4.

\textsuperscript{21}Results for this model are reported in the appendix.
Chapter 4

Capital Accumulation, Non-traded Goods and International Macroeconomic Dynamics

4.1 Introduction

The original model of international business cycles proposed by Ghironi and Melitz (2005) lacks capital. When capital is omitted from the framework, "investment" is defined only as new firm construction. Investment in the data, however, includes plant and equipment purchases for both new firm creation and existing firm expansion. Without capital, comparing "investment" in the model and investment in the data is erroneous.

In this chapter, I will include capital accumulation into framework proposed by Ghironi and Melitz (2005) in such as way as to leave the underlying mechanism of trade
undisturbed. By including capital in the production of non-traded goods, I can improve the definition of "investment" as well as explore an alternative source of business cycle fluctuations: productivity shocks to final goods production\textsuperscript{1}. I show that the model can generate more realistic relationships between imports, exports, and GDP when business cycles are driven by shocks to the production of intermediate goods. Changes in the capital stock prolong the expansion, potentially strengthening the flow of investment from abroad. As seen in chapter 3, this reduces business cycle correlation across countries. Further, an increase in capital prices in response to a positive productivity shock to intermediate goods production leads to an increase in the domestic price level and a fall in the exchange rate. Declines in the exchange rate negatively impact foreign variables when transforming them into comparable terms. Although the inclusion of capital improves some of the international market variables, the model fails to generate positive business cycle correlation and fails to capture the volatility of the international market when business cycles are driven by shocks to intermediate goods production.

When shocks to the production of final goods are present, the model can produce positive correlation of output, consumption and investment. Further, the model can more closely match the data by generating increased volatility for imports, exports and the real exchange rate since shocks to final goods production affect the demand for intermediate inputs. Fluctuations in the exchange rate along with changes in the trade pattern in response to shocks to final goods production generate the improved results. Productivity shocks to final goods production alone are not sufficient to generate high output volatility,

\textsuperscript{1}Shocks to final goods production behave as "demand shocks" for intermediate goods in the model.
so a model in which there are shocks to both intermediate and final goods production is proposed. The improved results suggest both sources of business cycle fluctuations are necessary in the model.

I further extend the model by adding a developed labor market in the same fashion that was done in chapter 2. For relatively high elasticities of labor supply ($\lambda = 1$), output volatility more closely matches the data. In the presence of shocks to final goods production, however, employment becomes negatively correlated across countries. With lower labor supply elasticities\(^2\) ($\lambda = 5$) output volatility is similar to the benchmark framework without capital and positive employment correlation is achieved.

In summary, including capital in the production of final goods induces improved relationships between imports, exports and GDP, but results in negative business cycle correlation and cannot capture the volatility of the international market. Shocks to final goods production, which act as demand shocks for intermediate inputs, is capable of generating positive international correlation of consumption, output and investment along with increased volatility of the international market. Adding fluctuations to employment increase output volatility, but result in negative correlation of employment when labor supply elasticity is too high.

The remainder of the chapter is as follows. I construct the model in section 2. I present and discuss a series of numerical experiments in section 3. Section 4 presents a short conclusion.

\(^2\)Fiorito and Zanella (2008) note that labor supply elasticities are empirically lower at the individual level than at the aggregate level.
4.2 The Model

The theoretical model is heavily influenced by Ghironi and Melitz (2005) and is quite similar to the model developed in Chapter 3. Details of the model appear in the appendix. I will, however, outline the parts of the model that I augment when adding capital accumulation. The model described here considers two countries, home and foreign (which is denoted by *). Both countries are large and assumed to be structurally identical. That in mind, I construct the framework for one country (the home country) knowing that a symmetric framework exists for the foreign counterpart.

4.2.1 The Consumer’s Problem

Denote the universe of intermediate input varieties as $\Omega$. At any time, $t$, a subset $\Omega_t \subseteq \Omega$ are actually produced by firms. $\Omega_t$ contains both domestically produced and imported goods. Further, it need not be the case that $\Omega_t = \Omega_s$ for $t \neq s$ nor $\Omega_t = \Omega^*_t$. A composite intermediate good, $M_t$, is produced using Dixit-Stiglitz technology:

$$M_t = \left( \int_{\omega \in \Omega_t} m_t(\omega)^{(a-1)/a} d\omega \right)^{a/(a-1)} \quad (4.1)$$

where $m_t(\omega)$ denotes the quantity of variety $\omega$ used in the production of the composite, and $a$ denotes the elasticity of substitution across varieties. The composite intermediate good is then combined with capital, $K_t$, to produce consumption using a simple Cobb-Douglass production process:

$$C_t = A_t M_t^b K_t^{1-b} \quad (4.2)$$
At denotes total factor productivity at time, $t$. At $A_t = b = 1$, the model reverts to the structure described by Ghironi and Melitz (2005).

It is straightforward to construct demand equations and price indices for $m_t$, $M_t$, $K_t$ and $C_t$. Denote $P_{Xt}$ as the price index for good X. I construct the following conditional demand equations by solving a straight-forward cost-minimization problem$^3$:

$$ m_t(\omega) = M_t(P_t(\omega)/P_{Mt})^{-a} $$

$$ M_t = C_t \left( \frac{P_{Mt}}{P_{Kt}} \right)^{1-b} b^{-1} $$

$$ K_t = C_t \left( \frac{P_{Mt}}{P_{Kt}} \right)^b $$

I also construct the associated price indices:

$$ P_{Mt} = \left( \int_{\omega \in \Omega_{Dt}} P_t(\omega)^{1-a} d\omega \right)^{1/(1-a)} $$

$$ P_{Ct} = \frac{P_{Mt}^b P_{1-b}^{1-b}}{A_t(1-b)^{1-b} b^b} $$

Next, I turn to the household’s problem faced by the representative agent. It is assumed that there exists a representative household that chooses to work, consume and save. The household is endowed with 1 unit of time that it can divide between labor, $l_t$, and leisure. Labor earns the real wage, $w_t$. The household can choose to save by purchasing bonds, $B_{t+1}$, which each cost one unit of consumption, but yield $(1 + r_{t+1})$ units in the next period. Both domestic and foreign bonds ($B_*$) are available to the household and are

$^3$The production function for $C_t$ is essentially a multi-step CES production function. First, I find the lowest-cost way to produce one unit of the intermediate index, $M_t$. Then, I solve a cost-minimization problem for $C_t$ over $M_t$ and $K_t$. 

52
subject to a quadratic transaction cost which is rebated, lump-sum ($\Gamma_t$), to the household. The household can also purchase shares of a mutual fund, $x_{t+1}$, which entitle the owner to a fraction of the profits of the producing firms. Arbitrage prices the shares at the firms expected discounted value ($N_{ht}\bar{v}_t$ for new shares, $N_{dt}\bar{v}_t$ for old shares). Households are obligated to hold bonds and stocks for only one period before they are resold in asset markets. Finally, the households have the option of saving real wealth in the form of savings accounts, $S_{t+1}$. Savings are used to construct capital in the next period and earn a real return of $R_t$. The consumer’s real period budget constraint is then given by:

\[ C_t + B_{t+1} + Q_t B_{st+1} + \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{st+1}^2) + (N_{ht}\bar{v}_t)x_{t+1} + S_{t+1} = \]

\[ w_t l_t + (1 + r_t)B_t + Q_t(1 + r_st)B_{st} + \Gamma_t + x_t(N_{dt}\bar{v}_t + N_{dt}\bar{d}_t) + R_t S_t \]

where $n_2 \geq n_1 > 0$ are scale parameters on foreign and domestic bond adjustment costs, $\Gamma_t = \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{st+1}^2)$ in equilibrium and $Q_t$ is the real exchange rate.

The instantaneous utility function for the agent is assumed to have the form:

\[ U_t(C_t, l_t) = \log C_t - \frac{1}{1+\lambda} \]

The agent’s maximization problem can then be written as:

\[ \max_{\{C_s,l_s,B_{s+1},B_{ss+1},x_{s+1},S_{s+1}\}_{s=t}} \sum_{s=t}^{\infty} E_t \beta^{s-t} \left[ \log C_s - \frac{1}{1+\lambda} \right] \text{ s.t.} \]

\[ C_s + B_{s+1} + Q_s B_{ss+1} + \frac{n_1}{2}(B_{s+1}^2) + \frac{n_2}{2}(Q_s B_{ss+1}^2) + (N_{hs}\bar{v}_s)x_{s+1} + S_{s+1} = \]

\[ w_s l_s + (1 + r_s)B_s + Q_s(1 + r_{st})B_{ss} + \Gamma_s + x_s(N_{ds}\bar{v}_s + N_{ds}\bar{d}_s) + R_s S_s \]

\[ (4.8) \]

\[ (4.9) \]

All savings instruments are denoted with the time subscript in which they yield a return.
The first-order conditions for the consumer’s problem generate an equation that guides labor supply in addition to three Euler equations. The labor supply equation is:

\[ l_t = C_t^{-1/\lambda}w_t^{1/\lambda} \]  

(4.10)

which suggests that $1/\lambda$ represents elasticity of labor supply. In the original Ghironi and Melitz (2005) framework, labor was supplied inelastically ($l_t = 1$). The Euler equation for domestic bonds is:

\[ C_t^{-1}(1 + n_1B_{t+1}) = (1 + r_{t+1})\beta E_t C_{t+1}^{-1} \]  

(4.11)

Similarly, the Euler equation for foreign bonds is:

\[ Q_t C_t^{-1}(1 + n_2B_{^*t+1}) = (1 + r_{^*t+1})\beta E_t Q_{t+1}C_{t+1}^{-1} \]  

(4.12)

The Euler equation for stocks is:

\[ \tilde{v}_t = \beta(1 - \delta)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}(\tilde{d}_{t+1} + \tilde{v}_{t+1}) \]  

(4.13)

Notice that repetitive forward substitution of the Euler equation for stocks generates the expected value of the firm’s lifetime profit stream. Finally, the Euler equation for savings is:

\[ C_t^{-1} = \beta E_t R_{t+1}C_{t+1}^{-1} \]  

(4.14)
4.2.2 The Firm’s Problem

An elaborate reconstruction of the firm’s problem when capital is relevant in the production of final goods is available in the appendix and follows symmetrically from Chapter 3. There are two important characteristics of the firm’s problem with capital accumulation that are of note. First, the indexed price of intermediate goods (which equals the aggregate price of consumption in the model without capital, but not in the model with capital) affects the equilibrium quantity produced by any individual firm:

\[
\begin{align*}
    m_{djt} &= bC_t \rho_{Mt}^{-1} (\frac{a}{a - 1} \frac{w_t}{Z_t z_j})^{-a} \\
    m_{xjt} &= bC_t^* \rho_{Mt}^*^{-1} (\frac{a}{a - 1} \frac{w_t}{Z_t z_j} \frac{\tau}{Q_t})^{-a}
\end{align*}
\]  (4.15)

Second, the domestic and export profits are strongly related to the level of consumption:

\[
\begin{align*}
    d_{djt} &= \frac{b}{a} C_t \rho_{Mt}^{-1} \rho_{djt}^{1-a} \\
    d_{xjt} &= \begin{cases} 
    \frac{b}{a} C_t^* \rho_{Mt}^*^{-1} Q_t \rho_{xjt}^{1-a} - \frac{w_t F_X t}{Z_t^*} \text{ if } c_{xjt} > 0 \\
    0 \text{ if } c_{xjt} = 0
    \end{cases}
\end{align*}
\]  (4.17)

Increasing the volatility of consumption or the volatility of the aggregate price of labor services will result in increased volatility of firm profits and thus more volatility in the decision to export. Changes in export decisions affect the size of the export sectors as well as the trade pattern, which influences the transmission of business cycle fluctuations across countries.
4.2.3 Market Clearing and Equilibrium

The market clearing conditions and definitions needed to solve the model are also
follow symmetrically from the model in Chapter 2 and are available in the appendix. The
central differences when capital accumulation is added to the framework are the existence
of a market for capital, the redefinition of investment and the redefinition of the aggregate
price index.

In the capital market, savings today become the foundation for tomorrow’s capital
stock \(S_{t+1} = K_{t+1}\). In equilibrium, the gross return to savings \(R_t F_t = R_t K_t\) should be
equal to total payments to capital plus the return of non-depreciated capital \((\rho_{Kt} K_t + (1 -
\delta_K) K_t)\). This suggests the equilibrium price of capital is given by:

\[
\rho_{Kt} = R_t - 1 + \delta_K
\]  

\[(4.19)\]

Capital demand for the production of consumption in period \(t\) is given by:

\[
K_t = \frac{C_t \rho_M t}{A_t \rho_{Kt}} \left(1 - \frac{1}{b}\right)^b
\]

\[(4.20)\]

while supply of capital in the next period is determined by the Euler equation:

\[
C_{t-1} = \beta E_t R_{t+1} C_{t+1}^{-1}
\]

\[(4.21)\]

Capital market clearing suggests \(S_{t+1} - R_t S_t = K_{t+1} - (\rho_{Kt} K_t + (1 - \delta_K) K_t) = K_{t+1} - (1 -
\delta_K) K_t - \rho_{Kt} K_t\). I impose mutual fund market clearing and lump-sum transfers of bond
adjustment costs to generate the balance of payments condition:
\[2(1 + r_{t+1})B_t + 2(1 + r_{t+1}^*)Q_t B_{st} = \left[C_t - Q_t C_t^*\right] + \left[N_{et} \tilde{v}_t - Q_t N_{et}^* \tilde{v}_t^*\right] + 2\left[B_{t+1} + Q_t B_{st+1}\right]
\]
\[-[w_t l_t - Q_t w_t^* l_t^*] - [N_{dt} \tilde{d}_{dt} - Q_t N_{dt}^* \tilde{d}_{dt}^*]
\[-[N_{xt} \tilde{d}_{xt} - Q_t N_{xt}^* \tilde{d}_{xt}^*] + [K_{t+1} - Q_t K_{t+1}^*]
\[-(1 - \delta_K)[K_t - Q_t K_t^*] - [\rho_{Kt} K_t - Q_t \rho_{Kt}^* K_t^*] \quad (4.22)\]

Investment is defined as expenditures on new firm entry plus the purchases of new capital:

\[I_t = N_{et} \tilde{v}_t + K_{t+1} - (1 - \delta_K)K_t \quad (4.23)\]

In the model, the price of consumption \((P_{Cl})\) is measured as a welfare-based price index following Feenstra (2003). It is thus important to transform this welfare-based index into one which closer matches the price index calculated in the data. To do so, we redefine the price index for aggregate consumption:

\[1 = \frac{[N_{dt} \tilde{\rho}_{dt}^{1-a} + N_{st}^* \tilde{\rho}_{st}^{1-a}]b/(1-a) \rho_{Kt}^{1-b}}{A_t(1-b)^{1-b}b^b} \]

If we assume that all prices are, on average, \(\tilde{\rho} = \tilde{P}_t/P_{Cl}\), we construct:

\[\frac{P_{Cl}}{\tilde{P}_t} = \frac{[N_{dt} + N_{st}^*]^{b/(1-a)}}{A_t(1-b)^{1-b}b^b} \quad (4.24)\]

Any variable measured in terms of real consumption, \(X_t\), is adjusted to this index: \(\tilde{X}_t = P_{Cl} X_t / \tilde{P}_t\). Further, since the real exchange rate is constructed using the welfare-based price indices \((Q_t = P_{Cl}^*/P_{Cl})\), we construct an adjusted real exchange rate:
\[ \tilde{Q} = \tilde{P}_t^* / \tilde{P}_t = Q_t \left[ \frac{A_t^*}{A_t} \right] \left[ \frac{N_{dt} + N_{xt}^*}{N_{dt}^* + N_{xt}} \right]^{b/(1-a)} \]  

(4.25)

4.3 Numerical Experiments

Many of the model’s parameter values are familiar from Chapter 3. They are listed in table 4.1. The value of the depreciation rate of capital is a new parameter to the model and is set to 2.5% quarterly and 10% annually (a standard in the literature). The other model parameters, \((b, n_1, n_2, \lambda, \text{and the parameters of the shock processes})\), will be thought of as "free parameters" and will be calibrated under different experiments discussed shortly. The model is solved numerically using the brute force algorithm developed by Uhlig (1999). Simulations are performed in order to generate a set of statistics to compare to the data. In each experiment, a 200-period model is simulated 200 times by drawing a random vector of innovations. During each simulation, I make the appropriate price adjustments as described above, then apply the HP filter. Summary statistics (volatility, correlation with output, etc.) are calculated for each simulation and I report the average statistics across simulations in the results tables.

---

5 The algorithm used in experimentation is described as follows. After calibrated parameters are chosen, I restructure the linearized equation system: \((E_t[FX_{t+1} + GX_t + HX_{t-1} + MZ_t] = 0; Z_t = NZ_{t-1} + \epsilon_t, \text{where} X \text{ is a vector of variables,} Z \text{ is a vector of stochastic processes guided by a VAR and } \epsilon \text{ is a vector of innovations). Using Uhlig’s brute force method of undetermined coefficients, I find a policy rule: } (X_t = PX_{t-1} + QZ_t). \text{ In each experiment, a 200-period model is simulated 200 times by drawing a random vector of innovations.}

6 Since the model exhibits a high degree of persistence, we take out the low-frequency trend using the HP filter.
Table 4.1: Calibrated Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Probability of death shock</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Iceberg costs associated with trade</td>
</tr>
<tr>
<td>$a$</td>
<td>Elasticity of substitution between inputs</td>
</tr>
<tr>
<td>$k$</td>
<td>Shape parameter on Pareto distribution</td>
</tr>
<tr>
<td>$z_{\text{min}}$</td>
<td>Lower bound on Pareto distribution</td>
</tr>
<tr>
<td>$\tilde{z}_D$</td>
<td>&quot;Special&quot; average productivity draw</td>
</tr>
<tr>
<td>$F_{Et}$</td>
<td>Fixed cost of entry</td>
</tr>
<tr>
<td>$F_{Xt}$</td>
<td>Fixed exporter costs</td>
</tr>
</tbody>
</table>

4.3.1 The Benchmark Model

To match the original Ghironi and Melitz (2005) framework as closely as possible, I set $n_1 = 0.0025$, $b = 1$ and omit endogenous labor supply from the model. Since the inclusion of capital accumulation may lead to a highly persistent and extreme capital exodus if bond markets are fluid, I limit the amount of international asset flow by imposing high adjustment costs for foreign bonds ($n_2 = 1$). I calibrate the technology shocks to intermediate goods production as a near-unit-root processes with correlated innovations$^7$:

$^7$Backus, Kehoe and Kydland (1992) calibrate technology shocks as a VAR process with spillovers:

$$
\begin{bmatrix}
\bar{Z}_t \\
\bar{Z}^*_t
\end{bmatrix} =
\begin{bmatrix}
0.906 & 0.088 \\
0.088 & 0.906
\end{bmatrix}
\begin{bmatrix}
\bar{Z}_{t-1} \\
\bar{Z}^*_{t-1}
\end{bmatrix} +
\begin{bmatrix}
\xi_{zt} \\
\xi^*_{zt}
\end{bmatrix}
$$

(4.26)

$$
\sigma_{\xi z_t} = \sigma_{\xi^* z_t} = \sigma_{\xi} = 0.00852
$$

(4.27)

$$
\rho_{\xi z_t, \xi^* z_t} = \rho_{\xi z, \xi^* z} = 0.258
$$

(4.28)

Baxter and Crucini (1995) note that this specification is not statistically different from a near-unit-root process without spillovers.
\[
\begin{bmatrix}
\dot{Z}_t \\
\dot{Z}_t^*
\end{bmatrix} =
\begin{bmatrix}
0.99 & 0 \\
0 & 0.99
\end{bmatrix} \begin{bmatrix}
\dot{Z}_{t-1} \\
\dot{Z}_{t-1}^*
\end{bmatrix} +
\begin{bmatrix}
\xi_{Zt} \\
\xi_{Zt}^*
\end{bmatrix}
\]  
(4.29)

\[
\sigma_{\xi_{Zt}} = \sigma_{\xi_{Zt}} = \sigma = 0.00852
\]  
(4.30)

\[
\rho_{\xi_{Zt}\xi_{Zt}^*} = \rho_{\xi_{Zt}\xi_{Zt}^*} = 0.258
\]  
(4.31)

with covariance of \((\xi_{Zt}, \xi_{Zt}^*)\) 0.0019. Technology shocks are calibrated as such in all the following experiments. I set \(A_t = 1\) for all \(t\). Results from simulations are reported in column (2) of table 4.2. The model fails to predict high correlation of investment across countries and high volatility of imports, exports, the trade balance, the terms of trade and the real exchange rate.. Further, the model over-predicts the correlation of exports, the terms of trade and the real exchange rate with GDP.

The failure to capture the appropriate investment dynamics is possibly due to how investment is defined. In the model without capital, investment is only new firm construction. When there is a positive productivity shock to intermediate goods production, each individual firm in the home country earns increased profits. Since firms now earn a higher return, there is as surge of new firm construction which leads to an increase in domestic investment. In the foreign country, agents adjust their investment portfolio in favor of home bonds (since the productivity shock leads to an increase in home’s interest rate via the Euler relationship). Although high bond adjustment costs have limited the extent to which foreign investors substitute, there is still an exodus of investment dollars out of the foreign country. There is firm attrition in the foreign economy and a net reduction in
real investment\(^8\). Changes in individual firm size are omitted from this idea of investment. The inclusion of capital may improve the international correlation of investment provided investment in capital is sufficiently procyclical in both countries.

The failure of the model to appropriately reproduce features of the international market suggests that international trade in goods is failing to act as the appropriate mechanism for the transmission of business cycles across countries. Inspection of the impulse response functions for the endogenous variables of the model suggest the relationship between the international market statistics and relatively docile endogenous variables are responsible for the low volatilities of imports, exports, the trade balance and international prices. Aggregate productivity shocks in the benchmark model lead to increases in foreign consumption, the size of the domestic export sector, export prices and the real exchange rate. Each of these are less volatile than domestic income, which results in low volatility of exports relative to GDP. A similar story holds for imports. Since prices in the model are marked-up over marginal costs, and marginal costs in the model are less volatile than GDP, both the terms of trade and the real exchange rate are predicted to have low volatility.

The correlations between the endogenous variables and GDP are driven by the presence of asset market frictions and the relationship between international prices and procyclical endogenous variables of the model.

\(^8\)The reduction in foreign investment is reduced after applying the real exchange rate to transform foreign denominaions into domestic terms. The positive productivity shock to home intermediate good production leads to a fall in the price of domestic goods and expansion of the foreign export sector (both of which push down the domestic price level in spite of rising import prices). Increases in the home export sector are not sufficient to outweigh increases in the prices of domestically produced goods in the foreign country; the foreign price level rises. Changes in aggregate prices brought about by the productivity shock cause the exchange rate to rise.
### Table 4.2: Simulation Results: Capital Accumulation.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic Statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_Y(%)$</td>
<td>1.54</td>
<td>0.97</td>
<td>0.76</td>
<td>0.15</td>
<td>0.77</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.75</td>
<td>0.61</td>
<td>0.50</td>
<td>1.78</td>
<td>0.57</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>3.41</td>
<td>3.11</td>
<td>2.19</td>
<td>43.45</td>
<td>8.12</td>
</tr>
<tr>
<td>$\sigma_L/\sigma_Y$</td>
<td>0.61</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Correlation with GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{C,Y}$</td>
<td>0.88</td>
<td>0.99</td>
<td>0.99</td>
<td>0.48</td>
<td>0.86</td>
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<tr>
<td>$\rho_{I,Y}$</td>
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<td>0.98</td>
<td>0.59</td>
<td>-0.17</td>
<td>0.14</td>
</tr>
<tr>
<td>$\rho_{L,Y}$</td>
<td>0.81</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td><strong>International Comovement</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{Y,Y^*}$</td>
<td>0.56</td>
<td>0.28</td>
<td>-0.15</td>
<td>0.67</td>
<td>0.45</td>
</tr>
<tr>
<td>$\rho_{C,C^*}$</td>
<td>0.45</td>
<td>0.34</td>
<td>-0.27</td>
<td>0.31</td>
<td>0.29</td>
</tr>
<tr>
<td>$\rho_{I,I^*}$</td>
<td>0.37</td>
<td>0.07</td>
<td>-0.75</td>
<td>0.77</td>
<td>0.72</td>
</tr>
<tr>
<td>$\rho_{L,L^*}$</td>
<td>0.39</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td><strong>International Market Statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{IM}/\sigma_Y$</td>
<td>3.31</td>
<td>0.58</td>
<td>0.68</td>
<td>8.87</td>
<td>1.70</td>
</tr>
<tr>
<td>$\sigma_{EX}/\sigma_Y$</td>
<td>3.74</td>
<td>0.57</td>
<td>0.71</td>
<td>11.73</td>
<td>2.18</td>
</tr>
<tr>
<td>$\sigma_{NX}/\sigma_Y$</td>
<td>0.40</td>
<td>0.02</td>
<td>0.12</td>
<td>0.56</td>
<td>0.58</td>
</tr>
<tr>
<td>$\sigma_{TOT}/\sigma_Y$</td>
<td>1.69</td>
<td>0.12</td>
<td>0.21</td>
<td>4.22</td>
<td>0.77</td>
</tr>
<tr>
<td>$\sigma_{Q}/\sigma_Y$</td>
<td>3.37</td>
<td>0.10</td>
<td>0.14</td>
<td>3.19</td>
<td>0.58</td>
</tr>
<tr>
<td>Correlation with GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{IM,Y}$</td>
<td>0.60</td>
<td>0.82</td>
<td>0.79</td>
<td>0.14</td>
<td>0.31</td>
</tr>
<tr>
<td>$\rho_{EX,Y}$</td>
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<td>0.80</td>
<td>0.40</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>$\rho_{NX/Y,Y}$</td>
<td>-0.36</td>
<td>-0.22</td>
<td>-0.34</td>
<td>-0.002</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\rho_{TOT,Y}$</td>
<td>0.07</td>
<td>0.55</td>
<td>0.56</td>
<td>-0.03</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho_{Q,Y}$</td>
<td>-0.18</td>
<td>0.41</td>
<td>-0.55</td>
<td>0.17</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

(1) Data  
(2) Benchmark, ($K_t = 0$), shocks to $Z_t$ only  
(3) Capital Accumulation, shocks to $Z_t$ only  
(4) Capital Accumulation, shocks to $A_t$ only  
(5) Capital Accumulation, shocks to $Z_t$ and $A_t$
4.3.2 Capital Accumulation

I now turn to the model with capital in the production of final consumption. There are two important characteristics of the model worth noting when capital is used in the production of domestic consumables. First, since capital is not used to produce the traded intermediate inputs, the two countries are in effect trading labor services as in the original framework. Including capital in this fashion allows me to bypass a potentially complicated discussion of changes in relative factor intensities. Second, since the production of final consumption entails its own production process, I can introduce an additional source of business cycle fluctuations, $A_t$. While $A_t$ acts as a technology shock to the domestic production of final consumption, it acts as an input demand shock to intermediate goods producers.

To proceed with experimentation, I first calibrate the parameter $b$ so that the steady state capital-to-output ratio matches the average value for the US (approximately $3.10^{10}$). I then analyze the model with aggregate shocks to intermediate goods production only, followed by the model with shocks to the production of composite consumption. Both models fail to reproduce sufficiently volatile output, so as an aside I analyze a version of the model with endogenous labor supply.

---

9Stockman and Tesar (1995) and Wen (2002) show that demand shocks are important to international business cycle synchronization. Although the formulation I adopt here does not impose demand shocks on behalf of consumers, demand shocks for intermediate inputs are potentially equally important.

10Annual data for the US, 1990-2007, from the Bureau of Economic Analysis for real total fixed assets plus consumer durables (Table 9.1) and real GDP (Table 1.1.6) are used to construct this average. Matlab has difficulty numerically determining $b$ in the model, so it is calibrated by hand. For quarterly simulations, $b = 0.88$ corresponds to a capital-output ratio of 3.09. For annual simulations, $b = 0.57$ corresponds to a capital-output ratio of 2.84. When labor supply is endogenous, $b = 0.60$ is used for annual calibrations which corresponds to a capital-output ratio of 2.77.
Capital Accumulation and Shocks to the Production of Intermediate Goods

I start with a model whose only deviation from the original framework is the significance of capital in the production of final consumption. Labor is supplied inelastically, \( A_t = 1 \) for all \( t \), and the underlying process driving business cycles are is shocks to the production of intermediate inputs. As described above, the costs associated with purchasing foreign bonds is high (\( n_2 = 1 \)) to limit the amount of investment exodus that can occur with open bond markets. The simulation results are reported in column (3) of table 4.2.

At first glance, the model with capital results in slightly improved volatilities of imports and exports as well as improved correlations between imports, exports and GDP. The model produces worse estimates of output and investment volatility and reduced correlation of consumption and investment across countries. Inspection of the impulse response functions link the worsening results of the simulation to changes in international prices once capital is incorporated into the model\(^{11}\).

When there is a positive productivity shock to the production of input goods in the home country, consumption increases due to an expansion of firm output. The production of intermediate goods becomes more profitable and new firms enter the market. More workers are demanded to expand output and to build new firms which leads to an increase in the domestic wage and higher income. The equilibrium price of capital (\( \tilde{p}_{Kt} \)) rises via the Euler equation for savings leading to further increases in income. The home country experiences an economic expansion.

Home consumers demand more intermediate goods, leading to an increase in the

\(^{11}\)Impulse response functions for the endogenous variables of the model are available upon request.
demand for imports. In the foreign country, the export sector expands leading to an increase in the demand for labor and pushing up foreign wages and foreign income. Although open asset markets lead to firm attrition and a reduction in labor demand for new firm construction, the model predicts wages to rise in equilibrium which implies the effect of an expanding export sector on labor demand outweighs the effect of firm attrition. Capital prices in the foreign country are also higher via the Euler equations for savings and home bonds.

Although wages rise in both countries, the productivity shock in the home country pushes down the marginal cost of labor while the marginal cost of labor abroad rises. This results in a fall in the price of domestically produced goods in the home country and a rise in the price of domestically produced goods abroad. An expansion of the export sector in both countries occurs to meet increased demand while export prices both at home and abroad rise. Changes in the trade pattern along with changes in domestic and export prices result in a fall in the intermediate good price index (\(\hat{p}_{Mt}, \hat{p}_{Mt}^*\)) for both countries. The fall in \(\hat{p}_{Mt}\) results primarily from a decline in domestic prices coupled with an increase in the number of foreign exporters. The fall in \(\hat{p}_{Mt}^*\) seems to result only from the increase in home’s export sector and is of smaller magnitude than the change in \(\hat{p}_{Mt}^*\). The input price ratio (\(\hat{p}_{Kt}/\hat{p}_{Mt}\)) rises in both countries inducing households to use relatively more labor services to produce final consumption. In equilibrium, households actually reduce their capital holdings\(^\text{12}\) in favor of new firm investment. Total investment in the home country is higher due to new firm construction although investment in capital has fallen. Investment

\(^{12}\)Although less capital is purchased, investment in the model \((K_{t+1} - (1 - \delta_K)K_t + N_e\tilde{v}_t)\) still increases due to new firm construction.
in the foreign country, however, is lower due to a fall in both new firm construction and capital purchases.

The impulse response functions for consumption denoted in national terms show increases in both foreign and domestic consumption. The simulation results, however, predict negative correlation. This is due to changes in the real exchange rate which is used to convert foreign consumption to comparable terms. In the model without capital, the real exchange rate increases when there is a positive productivity shock. A positive productivity shock leads to a fall in the price for domestically produced goods and an increase in the variety of imported goods, which offsets increases in import prices. In the foreign country, only an increase in the number of imported goods offsets increases in the prices of all goods. Although prices in both countries are likely to decrease, prices in the home country would decrease more than the foreign country, leading to a rise in the real exchange rate\textsuperscript{13}.

In the model with capital, changes in the price of capital influence changes in the real exchange rate. The percentage changes in the real exchange rate, when there are no technology shocks to the production of final goods, can be written as:

\[
\hat{Q}_t = b(\hat{\rho}_{M_t}^* - \hat{\rho}_{M_t}) + (1 - b)(\hat{\rho}_{K_t}^* - \hat{\rho}_{K_t})
\]

Declines in the price index for intermediate goods are more severe in the home country, \((\hat{\rho}_{M_t}^* - \hat{\rho}_{M_t} > 0)\), which pushes up the real exchange rate. Increases in capital prices are more severe in the home country \((\hat{\rho}_{K_t}^* - \hat{\rho}_{K_t} < 0)\), which pushes the real exchange

\textsuperscript{13}This conclusion is based on impulse response functions for \(\bar{P}_C/\bar{P}_t\), which decline with a positive productivity shock, suggesting that changes in the number of firms are more influential than changes in the prices of intermediate goods.
rate down. Changes in the price for capital in the home country is sufficiently strong 
\((\hat{\rho}_{Kt} > \frac{b}{1-b}(\hat{\rho}_{Mt}^* - \hat{\rho}_{Mt}) + \hat{\rho}_{Kt})\) to induce in a fall in the real exchange rate when there is 
a positive productivity shock. The correlation between output and the real exchange rate 
is therefore low in simulation, which corresponds more closely to the data. However when 
foreign variables have small positive changes, converting them to domestic terms using the 
reduced real exchange rate results in a negative movement. Consumption in the model, 
which increases in both countries, becomes negatively correlated after converting the foreign 
measure to comparable terms\(^{14}\). Output in the foreign country, which changes only slightly 
in real terms, declines after applying the exchange rate.

**Capital Accumulation Shocks to the Production of Aggregate Consumption**

By developing a production process for final goods that includes both capital 
and labor services, it is possible to extend the model by incorporating shocks to final 
goods production. Shocks of this nature can also be thought of as demand shocks for 
intermediate inputs. To analyze versions of the model with this feature, I will allow the 
final goods technology variables \(A_t, A_t^*\) to follow a VAR process. Since \(A_t\) is also related 
to technology, I use the same near-unit root shock process as was used for the shocks to the

\(^{14}\)The home export sector expands as the real exchange rate falls, leading to an increase in foreign con-
sumption. This increase in consumption, however, is not enough to overpower the fall in the exchange rate 
after converting foreign variables into comparable terms.
production of intermediate goods:

\[
\begin{bmatrix}
  \hat{A}_t \\
  \hat{A}_t^* 
\end{bmatrix} =
\begin{bmatrix}
  0.99 & 0 \\
  0 & 0.99 
\end{bmatrix}
\begin{bmatrix}
  \hat{A}_{t-1} \\
  \hat{A}_{t-1}^* 
\end{bmatrix} +
\begin{bmatrix}
  \xi_A_t \\
  \xi^*_A_t 
\end{bmatrix}
\]

(4.32)

\[
\sigma_{\xi_A t} = \sigma_{\xi^*_A t} = \sigma_{\xi_A} = 0.00852
\]

(4.33)

\[
\rho_{\xi_A t, \xi^*_A t} = \rho_{\xi_A, \xi^*_A} = 0.258
\]

(4.34)

As an intermediate step, I allow only $A_t$ to follow a stochastic process while $Z_t = 1$ for all $t$. The results for this calibration are reported in column (4) of table 4.2. Although shocks to final goods production alone are not sufficient to capture the dynamics of the real economy, the behavior of many economic variables are improved. Imposing shocks of this nature results in increased volatility of investment, increased international correlation, increased volatility of imports and exports, increased volatility of terms of trade, and increased volatility of the exchange rate index.

Investment volatility increases since input demand shocks have a more direct impact on capital than the productivity shocks of input producers. A positive productivity shock to the production of final goods will directly increase the marginal product of capital along with the marginal product of intermediate labor services. The higher marginal product of capital will increase the demand for capital in the current period. Since the supply of capital is predetermined, this leads to an immediate increase in capital prices. Further, since the shock is persistent, consumers expect future returns to savings to be higher. They supply more savings to capital markets and expand future capital holdings. The part of investment that is held as capital dramatically increases\textsuperscript{15}.

\textsuperscript{15}Formal intuition for this result appears in the appendix.
Inspection of the impulse response functions\textsuperscript{16} suggests that a positive productivity shock to final goods production results in firm attrition in the home country ($N_{et}$ declines). An increase in the return to capital makes savings relatively more profitable as an investment tool than new firm construction in the current period, resulting in fewer entering firms. Further, the productivity shock to final goods production leads to an immediate expansion of consumption and higher demand for input products, resulting in increased demand for labor and higher real wages. As real wages rise, it becomes more expensive to pay the fixed entry cost of starting a firm which causes the number of entrants to decline further. The discounted value of existing firms ($\tilde{v}_t$) rises since each existing firm now earns more profits, but this increase is more than offset by the decline in firm entry. The part of investment that is comprised by new firm construction ($N_{et}\tilde{v}_t$) decreases, but only slightly. There is an overall increase in investment resulting from capital purchases.

Although there is firm attrition in the presence of a positive productivity shock to final goods production, each existing firm expands production to meet increased demand for their product. The reduction in new firm construction puts downward pressure on labor demand, while the expansion of production for existing firms forces labor demand higher. The model predicts an increase in labor demand in equilibrium, resulting in a higher real wage. Higher labor income, coupled with an increased price of capital, leads to an increase in aggregate income. In the home country, aggregate output, investment and consumption all increase.

There is a movement away from issuing domestic bonds, purchasing foreign bonds

\textsuperscript{16}Available upon request.
and building new firms towards capital accumulation in the home country, since investment in capital earns a higher return. A reduction in the supply of home bonds results in lower bond issuance and higher bond returns. A reduction in home’s demand for foreign assets leads to a fall in foreign bond purchases and a reduction in the foreign interest rate. As the return to foreign bonds falls, foreign agents adjust their investment portfolios in favor of capital and new firm construction. There is a wave of firm entry in the foreign country coupled with rising capital prices, leading to higher foreign investment. As home’s demand for imports expands, the pressure in the foreign export sector to expand production coupled with an increase in labor demand for new firm construction leads to higher foreign wages. Aggregate foreign income rises as does foreign consumption. The two countries exhibit synchronous business cycle fluctuations.

Final goods production shocks have a more direct effect on aggregate prices, resulting in a higher volatility of the exchange rate index. We can define the real exchange rate as:

$$Q_t = \frac{P_{Ct}^*}{P_{Ct}} = \left[ \frac{A_t}{A_t^*} \right] \left[ \frac{P_{Mt}^* P_{Mt}^{(1-b)}}{P_{Mt}^* P_{Mt}^{(1-b)}} \right]$$

The effect of a movement in $A_t$ on $Q_t$ is strong and positive. Sharp increases in the real exchange rate increase the profitability of exporting in the home country and a fall in the profitability of exporting in the foreign country. There is an expansion of the home export sector and a contraction of the foreign export sector. In addition, increases in

\footnote{This is confirmed in the impulse response function for $Q_t$ for a 1 standard deviation shock to $A_t$. A positive productivity shock to final goods production results in an increase in the real interest rate, which suggests $\hat{A}_t > b(\hat{P}_{Mt} - \hat{P}_{Mt}^*) + (1 - b)(\hat{P}_{Kt} - \hat{P}_{Kt}^*)$.}
consumption in both countries lead to higher demand for intermediate inputs and thus higher prices for both imported and exported goods. The volatility of the terms of trade rises in part due to an increase in intermediate good price volatility as well as increases in the volatility of the real exchange rate. Although intermediate goods prices rise, the expansion of the domestic export sector and the contraction of the foreign export sector induce home’s exports to sharply increase while home’s imports change negligibly. This, however, implies a procyclical trade balance.

Changes in aggregate prices are not trivial. The factor used to transform welfare-based variables to variables that more closely match the data \( \left( \frac{P_{ct}}{P_t} \right) \) sharply declines in the home country in response to a positive shock to final goods production\(^\text{18}\). All home variables subject to the price adjustment are slightly reduced. Home country variables that exhibit small or marginal increases, such as consumption and imports, can even show declines after applying the price adjustment. Foreign variables subject to the price adjustment are also subject to changes in the real exchange rate\(^\text{19}\). Since the real exchange rate is rising, the negative effect of changes in the price adjustment factor is more than made up for. Foreign variables are seemingly more responsive to shocks in the home country than home variables due to changes in the exchange rate.

Timing is an important factor when connecting the intuition gleaned from the impulse response functions to the simulation results. In the cases of correlation with GDP, consumption and imports seem contemporaneously negatively correlated with GDP after

\[
\frac{P_{ct}}{P_t} = \frac{\left( N_{dt} + N_{st} \right)^{b/(1-a)}}{A_t(1-b) - b}. \quad \text{A positive shock to } A_t \text{ coupled with a net increase in } N_{dt} + N_{st} \text{ lead to a sharp fall in } \frac{P_{ct}}{P_t}.\]

\(^{18}\)In addition to converting foreign welfare-based variables to measures that more closely match the data, the real exchange rate must also be used to make the variables comparable to home measures.
applying the price adjustment while exports and investment seem contemporaneously positively correlated with GDP. Over time, after a positive productivity shock, GDP rises before returning to the steady state. Consumption and imports follow the same pattern, they rise then gradually return to the steady state. Imports and exports, however, continuously fall to the steady state after the initial shock. Consumption and imports are positively correlated with GDP when considering the entire transition period, while movements investment and exports along with the trade balance become negatively correlated with GDP. These results are consistent with predictions made in the simulation results.

The central benefits of productivity shocks to final goods production are an increase in investment volatility, high international correlation of output, investment and consumption, high volatility of exports and imports, high volatility of international prices. This stochastic feature can not, however, capture correctly the volatility of output and consumption. Further, changes in aggregate prices, changes in the exchange rate and timing induce counter cyclical investment and exports. Since productivity shocks to intermediate goods production had many successes on these dimensions, I now turn to the model with both sources of shocks.

The model in which both $A_t$ and $Z_t$ follow a near unit root process is reported in column (5) of table 4.2. The model captures output and consumption volatility as well as the benchmark model, but also succeeds in generating highly correlated consumption, output and investment across countries, higher volatility of imports and exports, higher volatility of the real exchange rate and low correlation between international prices and

\footnote{20}{Although there is underlying correlation between $Z_t$ and $Z_t^*$, it is assumed that $Z_t$ and $A_t$ are uncorrelated.}

\footnote{21}{The "benchmark" being the model without capital accumulation.}
GDP. These features are not captured by the model with capital accumulation and productivity shocks to intermediate goods production alone. The volatility of the terms of trade is improved, but still falls short of unity. The model over-predicts the volatility of investment and slightly under-predicts correlation between exports, imports and GDP as well as investment and GDP.

When capital is used in the production of non-traded final goods, the benchmark framework is unable to reproduce a fully functioning international market. The mechanism that correlates business cycles across countries becomes distorted when the model is driven by productivity shocks to intermediate goods production alone. When we subject final goods production to productivity shocks, we can improve the functioning of the international market. These sorts of productivity shocks can be interpreted as demand shocks for intermediate inputs. A model in which business cycles are driven by both sorts of shocks can capture a larger number of stylized facts more accurately while allowing us to represent investment in a manner that is consistent with the rest of the real business cycle literature.

**Endogenous Labor Supply**

I try to push the model further by incorporating endogenously determined labor supply. As in the previous chapter, I calibrate the elasticity of labor supply to be consistent with empirical measurements ($\lambda = 1$) and impose the strong costs to adjusting foreign bond holdings ($n_2 = 1$). I allow for capital accumulation as in the previous sections. The results from the model with intermediate goods shocks as the source of business cycle fluctuations, the model with shocks to final goods production and the model with both types of shocks
are reported in columns (6), (7) and (8) in table 4.3 respectively. The incorporation of
the labor-leisure decision increases output volatility and captures procyclical employment
and positive correlation of employment across countries. Further, some of the successes
that were achieved in the previous sections (such as increased volatility of imports and
exports, more correlated consumption, higher volatilities of international prices) can still
be achieved.

When business cycles are driven by shocks to the production of intermediate goods,
both the impulse response functions and many of the simulation statistics between the
model with capital accumulation and endogenous labor supply (table 4.3) and the model
with only capital accumulation (table 4.2) are strikingly similar. The model with a more
complete labor market, however, can achieve a higher estimate of output volatility, can
generate procyclical employment and can reproduce positive correlation of employment
across countries. In the model with endogenously determined labor supply, an positive
productivity shock leads to firm entry as well as firm expansion which results in an increase
in labor demand. Increased composite consumption that results from both firm entry (more
variety) and expansion (more goods produced) in the home country compels households to
supply less labor. The increase in labor demand resulting from the positive productivity
shock offsets the decline in labor supply, resulting in more employment and higher wages in
equilibrium. The boost in employment further increases aggregate output, leading to the
higher output volatility seen in the results table.

As income and consumption rise, the demand for imported goods rises inducing an
expansion of the foreign export sector and a boost in foreign labor demand. Increases in the
Table 4.3: Simulation Results: Capital Accumulation with Endogenous Labor Supply.

<table>
<thead>
<tr>
<th>Domestic Statistics</th>
<th>(1)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_Y$ (%)</td>
<td>1.54</td>
<td>1.05</td>
<td>0.20</td>
<td>1.08</td>
<td>0.83</td>
<td>0.15</td>
<td>0.86</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.75</td>
<td>0.44</td>
<td>1.11</td>
<td>0.47</td>
<td>0.48</td>
<td>1.55</td>
<td>0.55</td>
</tr>
<tr>
<td>$\sigma_L/\sigma_Y$</td>
<td>3.41</td>
<td>2.36</td>
<td>30.86</td>
<td>6.32</td>
<td>2.26</td>
<td>40.81</td>
<td>7.48</td>
</tr>
<tr>
<td>Correlation with GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{C,Y}$</td>
<td>0.88</td>
<td>0.98</td>
<td>-0.05</td>
<td>0.86</td>
<td>0.99</td>
<td>0.30</td>
<td>0.86</td>
</tr>
<tr>
<td>$\rho_{I,Y}$</td>
<td>0.90</td>
<td>0.60</td>
<td>0.12</td>
<td>0.28</td>
<td>0.60</td>
<td>-0.06</td>
<td>0.18</td>
</tr>
<tr>
<td>$\rho_{L,Y}$</td>
<td>0.81</td>
<td>0.98</td>
<td>0.69</td>
<td>0.94</td>
<td>0.99</td>
<td>0.36</td>
<td>0.91</td>
</tr>
</tbody>
</table>

| International Comovement |     |     |     |     |     |       |      |
| Correlation             |     |     |     |     |     |       |      |
| $\rho_{Y,Y^*}$         | 0.56| -0.05| 0.87 |0.42 | -0.11| 0.72  | 0.46 |
| $\rho_{C,C^*}$         | 0.45| -0.26| 0.10 |0.07 | -0.32| 0.21  | 0.18 |
| $\rho_{I,I^*}$         | 0.37| -0.59| 0.48 |0.41 | -0.73| 0.69  | 0.64 |
| $\rho_{L,L^*}$         | 0.39| 0.33 | -0.31|0.12 | 0.42 | 0.01  | 0.14 |

| International Market Statistics |     |     |     |     |     |       |      |
| Volatility                 |     |     |     |     |     |       |      |
| $\sigma_{IM}/\sigma_Y$    | 3.31| 0.64| 5.70 |1.24 | 0.69 | 7.95  | 1.53 |
| $\sigma_{EX}/\sigma_Y$    | 3.74| 0.66| 7.42 |1.54 | 0.70 | 10.45 | 1.89 |
| $\sigma_{NX}/\sigma_Y$    | 0.40| 0.17 |0.53 |0.57 | 0.13 | 0.54  | 0.56 |
| $\sigma_{TOT}/\sigma_Y$   | 1.69| 0.20 |2.64 |0.52 | 0.21 | 3.72  | 0.67 |
| $\sigma_{Q}/\sigma_Y$     | 3.37| 0.13 |2.01 |0.38 | 0.14 | 2.80  | 0.51 |
| Correlation with GDP       |     |     |     |     |     |       |      |
| $\rho_{IM,Y}$             | 0.60| 0.76 | -0.50|0.28 | 0.78 | -0.10 | 0.33 |
| $\rho_{EX,Y}$             | 0.28| 0.35 | 0.70 |0.28 | 0.36 | 0.34  | 0.17 |
| $\rho_{NX,Y,Y}$           | -0.36| -0.32| 0.62 |0.03 | -0.35| 0.24  | -0.05 |
| $\rho_{TOT,Y}$            | 0.07| 0.56 | -0.62|0.07 | 0.57 | -0.27 | 0.14 |
| $\rho_{Q,Y}$              | -0.18| -0.54| 0.68 |0.04 | -0.54| 0.38  | -0.06 |

(1) Data
(6) $\lambda = 1$, shocks to $Z_t$ only
(7) $\lambda = 1$, shocks to $A_t$ only
(8) $\lambda = 1$, shocks to $Z_t$ and $A_t$
(9) $\lambda = 5$, shocks to $Z_t$
(10) $\lambda = 5$, shocks to $A_t$
(11) $\lambda = 5$, shocks to $Z_t$ and $A_t$
return to home bonds leads to less firm construction in the foreign country resulting in a drop in labor demand. Export expansion increases foreign labor demand while firm attrition causes it to decrease, making deducing changes in labor demand indeterminate. Slight increases in the home country’s export sector in response to the improved productivity of labor lead to marginal increases in foreign consumption\textsuperscript{22}, causing an immediate reduction in foreign labor supply.

Although home employment rises and foreign employment falls contemporaneously, implying negative international correlation, the lagged response to a positive productivity shock matter in generating the positive number seen in the simulation results. After the initial positive shock, employment in the home country gradually falls back to the steady state. In the foreign country, employment continues to fall after the initial shock for several periods before increasing back to the steady state. Since the home country is expanding, increased varieties and quantities of exports to the foreign country result in increasing foreign consumption over time. Firm attrition and expansion of the foreign export sector continue to keep changes in labor demand marginal. Employment falls and wages rise in the foreign country for several periods after the initial shock. As both the foreign and domestic employment series move in the same direction for several periods (in addition to employment changes in the foreign country being quite small), the simulation statistics report a positive correlation. The analysis of output, consumption and investment correlations across countries, as well as the other variables reported after applying the appropriate price adjustments follow from the model with capital and inelastic labor supply described

\textsuperscript{22}This is due to changes in the variety of goods available to foreign customers.
above.

In models in which business cycles are driven only by shocks to the production of final consumption, major differences between the model with only capital accumulation and the model with both capital accumulation and complete labor markets lie in the dynamic response to employment. A positive shock to final goods production in the home country leads to an immediate increase in employment. After the initial shock, employment gradually returns to the steady state. With shocks to final goods production, there is no prolonged increase in the quantity or variety of goods that are exported to the foreign country. Although foreign consumption immediately increases in response to the shock, there is a smooth return to the steady state. Foreign employment then immediately decreases then rises back to its steady state level. The dynamic negative correlation between domestic and foreign employment are observed in the simulation results.

The model with both shock processes achieves the same successes as it had in the model with inelastic labor supply. A noticeable drawback is the negative international correlation of employment. The effect of final goods production shocks on the variety of products is dominate, resulting in a dynamic negative correlation of employment across countries.

When comparing international correlations between the model with developed labor markets (column (8) of table 4.3) and the model with inelastic labor supply (column (5) of table 4.2) we notice an inverted relationship between output volatility and business cycle synchronization. When we incorporate endogenous labor supply, any productivity shock results in a dramatic increase in output in the home country. This triggers a larger degree
of investment exodus out of the foreign country and reduces the international correlations of consumption, output and investment. To test the implications of this result, I experiment with a lower labor supply elasticity ($\lambda = 5$ corresponds to a labor supply elasticity of 0.2, which is a common estimate for men according to Fiorito and Zanella (2008)). The results are reported in columns (9), (10) and (11) of table 4.3 for the three different specifications of the shock processes driving the business cycles in the model.

The results for output volatility and the international correlations are as expected. Output volatility declines since labor supply is less responsive to economic shocks. The reduction in output volatility increases the international correlation of output, consumption, investment and employment since there is less investment exodus from the foreign country in response to a positive productivity shock. Reductions in labor supply elasticity increase the volatility of imports, exports and international prices relative to GDP since GDP volatility decreased. A better statistical fit for all the other endogenous variables save output volatility is achieved with a lower labor supply elasticity.

4.4 Conclusion

Without any notion of capital, the definition of "investment" in the original Ghironi and Melitz (2005) is inconsistent with the real business cycle literature. Although new firm construction is an important component, changes in plant and equipment by existing firms is included in the statistical measure of investment. By including capital into the production of non-traded goods, more reasonable comparisons between the data and the model’s results can be made.
At first, the inclusion of capital into the model improves the correlation between imports, exports and GDP when foreign bonds purchases are costly and economic fluctuations are driven by productivity shocks to intermediate goods production. The presence of capital, however, results in negative international correlations of output, investment and consumption and inadequately captures the volatility of the international market. Increases in the price of capital when there are positive productivity shocks leads to decreases in the real interest rate which induces statistical declines in foreign variables after transforming them into comparable terms.

When productivity shocks to final goods are incorporated into the model, dramatic improvements in international correlations along with higher volatilities of international market variables are achieved. Positive shocks to final goods production directly increase the exchange rate, making positive international correlations of output, consumption and investment statistically achievable. Since shocks to final goods production also acts as demand shocks for intermediate goods, there is increased volatility of trade measures and international prices.

When endogenous labor supply is incorporated into the model, increased output volatility along with procyclical employment and positive correlation of employment across countries is achieved when business cycles are driven by shocks to intermediate goods production (productivity shocks to firms that demand labor). When shocks to final goods production are added, the model can generate positive international correlation of employment with low labor supply elasticities. This is achieved, however, at the expense of output.

23 As in the benchmark model.
volatility.
Chapter 5

Conclusion

Current research surveyed in chapter 2 has suggested that changes in the pattern of trade, a feature omitted from the previous body of international real business cycle research, can affect the transmission of economic fluctuations across countries. Models with foundations in trade theory that allow for shifting trade patterns, such as the Ghironi and Melitz (2005) model, succeed in matching many empirical regularities but oversimplify many sectors of the economy relevant to the study of business cycles. In the Ghironi and Melitz model, labor in the model is supplied inelastically which makes it impossible to study employment dynamics. Also, the model omits capital which distorts the concept of "investment". This dissertation added more developed labor and capital markets to the Ghironi and Melitz model and was able to match a richer set of stylized facts using several features (such as asset market frictions and "demand shocks") consistent with previous literature.

Prior to developing the model, I first updated the set of stylized facts commonly
used to evaluate a model’s fit. Many of the stylized facts are as expected: consumption, employment and investment are all procyclical and investment is more volatile than GDP while consumption and employment are less volatile. Looking at international correlation, new data confirms that consumption, output, investment and employment are generally positively correlated across countries. Although output is more correlated across countries than consumption for the period prior to 1985 (the empirical support of the C-O anomaly), consumption is more correlated across countries than output for the modern period. On average, output and consumption correlations are "close" and positive. I also add statistics of the international market, which are commonly overlooked. Imports, exports, the terms of trade and the real exchange rate are all highly volatile relative to GDP. Imports and exports are both procyclical, but imports are more closely related to output than exports. The trade balance and the real exchange rate are countercyclical while the terms of trade is acyclical. The analyses presented in Chapter 3 and 4 utilize this updated set of statistics in evaluating the fit of the models.

By assuming labor is supplied to the market inelastically, the model cannot account for international employment dynamics. In chapter 3, endogenous labor supply was incorporated into the Ghironi and Melitz model. Adding a labor-leisure decision to the household’s problem resulted in increased output volatility. Procyclical employment and positive international correlation of employment are successfully reproduced by the model. Further, changes in labor supply improve the international correlations of output, consumption and investment. Increasing the elasticity of labor supply can result in improved output volatility without worsening any of the other results. Past researchers suggest that the cor-
relation of business cycles produced by the model can be further improved by slowing the rate of investment exodus from one country to the other through asset market frictions. By increasing costs associated with purchasing foreign bonds, the model can in fact generate stronger correlations of output, consumption and new firm construction. However, this is achieved at the expense of the international market and the labor market. Increased asset market frictions boost the volatility of exports and reduce the volatility of the trade balance as countries try to maintain more balanced trade. Changes in labor supply when strong bond market frictions are present induce a reduction in employment in the foreign country, worsening the correlation of employment across countries.

Without any notion of capital, the definition of "investment" in the original Ghironi and Melitz (2005) is inconsistent with the real business cycle literature. Chapter 4 adds a more formal capital market and allows for capital in the production of final consumption goods. At first, the inclusion of capital into the model improves the correlation between imports, exports and GDP when foreign bonds purchases are costly and economic fluctuations are driven by productivity shocks to intermediate goods production. The presence of capital, however, results in negative international correlations of output, investment and consumption and inadequately captures the volatility of the international market. Increases in the price of capital when there are positive productivity shocks leads to decreases in the real interest rate which induces statistical declines in foreign variables after transforming them into comparable terms. When productivity shocks to final goods are incorporated into the model, dramatic improvements in international correlations along with higher volatilities of international market variables are achieved. Positive shocks to final goods production
directly increase the exchange rate, making positive international correlations of output, consumption and investment statistically achievable. Since shocks to final goods production also acts as demand shocks for intermediate goods, there is increased volatility of trade measures and international prices. When endogenous labor supply is incorporated into the model with capital accumulation, increased output volatility along with procyclical employment and positive correlation of employment across countries are achieved when business cycles are driven by shocks to intermediate goods production. When shocks to final goods production are added, the model can generate positive international correlation of employment with low labor supply elasticities. This is achieved, however, at the expense of output volatility.

Future work includes incorporating unemployment into the model. Concerns over the transmission of job-loss across countries are of concern to policy makers which suggests an adequate concept of involuntary unemployment needs to be added to the framework. Other potential extensions to the model include adding multiple countries and "common shocks", which would likely result in stronger international correlations without having to sacrifice international market volatility. Not only can the inclusion of multiple countries explain changes in the distribution of global wealth over the business cycle, it can contribute to current debates on multilateral trade agreements. Modern concerns about financial collapse (such as the Asian Financial Crisis or the current housing market crisis) and the transmission of recessions across countries require more refined financial markets be built into the model. These are left for future research.
Bibliography


Appendix A

Appendix

A.1 Appendix to Chapter 3

A.1.1 Details of the Model with Endogenous Labor Supply

The Balance of Payments

I construct the balance of payments equation by first imposing stock market clearing and bond adjustment costs transfers on the consumer’s budget constraint. The budget constraint for the home country becomes:

\[
C_t + B_{t+1} + Q_t B_{st+1} + (N_{ht} \tilde{v}_t) = w_t l_t + (1 + r_t) B_t + Q_t (1 + r^*_t) B^*_t + N_{dt} \tilde{v}_t + N_{dt} \tilde{d}_t
\]

I generate the budget constraint for the foreign agent:

\[
C^*_t + B^*_{t+1} + Q^{-1}_t B^*_{st+1} + (N^*_{ht} \tilde{v}^*_t) = w^*_t l^*_t + (1 + r_t) B^*_t + Q^{-1}_t (1 + r^*_t) B^*_{st} + N^*_{dt} \tilde{v}^*_t + N^*_{dt} \tilde{d}^*_t
\]
I multiply the foreign budget constraint by $Q_t$ to transform it into home consumption terms and subtract it from the home budget constraint. Imposing the bond market clearing conditions suggest the following equation for net foreign assets:

$$2(1 + r_{t+1}) B_t + 2(1 + r^*_t) Q_t B^{st} = \left[ C_t - Q_tC^*_t \right] + \left[ N_{dt} \tilde{v}_t - Q_t N^*_t \tilde{v}^*_t \right] + 2[B_{t+1} + Q_t B^{st+1}] - [w_t l_t - Q_t w^*_t l^*_t] - [N_{dt} \tilde{d}_t - Q_t N^*_t \tilde{d}^*_t]$$

$$- [N_{xt} \tilde{d}_t - Q_t N^*_t \tilde{d}^*_t]$$

(A.1)

**Equilibrium Conditions**

The model is characterized by the following nonlinear system of equations:

1. **Price and Profit definitions:**

   $$\tilde{\rho}_{dt} = \left( \frac{a}{a - 1} \right) \frac{w_t}{Z_t \tilde{z}_d}$$

   (A.2)

   $$\tilde{\rho}^*_{dt} = \left( \frac{a}{a - 1} \right) \frac{w^*_t}{Z_t \tilde{z}_d^*}$$

   (A.3)

   $$\tilde{\rho}_{xt} = \left( \frac{a}{a - 1} \right) \frac{w_t}{Z_t \tilde{z}_d^*}$$

   (A.4)

   $$\tilde{\rho}^*_{xt} = \left( \frac{a}{a - 1} \right) \frac{w^*_t}{Z_t \tilde{z}_d^*}$$

   (A.5)

   $$\tilde{d}_{dt} = \frac{1}{a} C_t \tilde{\rho}_{dt}^{-1-a}$$

   (A.6)

   $$\tilde{d}^*_{dt} = \frac{1}{a} C^*_t \tilde{\rho}^*_{dt}^{-1-a}$$

   (A.7)

   $$\tilde{d}_{xt} = \frac{1}{a} C_t Q_t \tilde{\rho}_{xt}^{-1-a} - \frac{w_t F_{xt}}{Z_t}$$

   (A.8)

   $$\tilde{d}^*_{xt} = \frac{1}{a} C^*_t Q^*_t \tilde{\rho}^*_{xt}^{-1-a} - \frac{w^*_t F_{xt}}{Z^*_t}$$

   (A.9)

2. **Price Index**:\(^1\)

   \(^1\)Using the equation for the price index of consumption, we substitute in average prices.
\[ 1 = (N_{dt}\tilde{\rho}_{dt}^{1-a} + N_{xt}\tilde{\rho}_{xt}^{1-a}) \quad (A.10) \]
\[ 1 = (N_{dt}\tilde{\rho}_{dt}^{1-a} + N_{xt}\tilde{\rho}_{xt}^{1-a}) \quad (A.11) \]

3. Expected Profit:
\[ \tilde{d}_t = \tilde{d}_{dt} + (\frac{N_{xt}}{N_{dt}})\tilde{d}_{xt} \quad (A.12) \]
\[ \tilde{d}_t^* = \tilde{d}_{dt}^* + (\frac{N_{xt}^*}{N_{dt}^*})\tilde{d}_{xt}^* \quad (A.13) \]

4. Free Entry:
\[ \tilde{v}_t = w_tF_{Et}/Z_t \quad (A.14) \]
\[ \tilde{v}_t^* = w_t^*F_{Et}/Z_t^* \quad (A.15) \]

5. Zero-Profit Intermediate Exporter\(^2\):
\[ \tilde{d}_{xt} = \frac{a - 1}{k + 1 - a} \frac{w_tF_{Xt}}{Z_t} \quad (A.16) \]
\[ \tilde{d}_{xt}^* = \frac{a - 1}{k + 1 - a} \frac{w_t^*F_{Xt}}{Z_t^*} \quad (A.17) \]

6. Share Exporting Firms:
\[ (1 - G(z_{xt})) = \frac{N_{xt}}{N_{dt}} = z_{\text{min}}(\frac{k}{k + 1 - a})^{k/(a-1)}z_{xt}^{-k} \quad (A.18) \]
\[ (1 - G(z_{xt}^*)) = \frac{N_{xt}^*}{N_{dt}^*} = z_{\text{min}}(\frac{k}{k + 1 - a})^{k/(a-1)}z_{xt}^*{-k} \quad (A.19) \]

\(^2\)To generate this condition, we utilize the cutoff condition, \( \tilde{d}_t^* = \frac{w_{Et}F_{Xt}}{Z_t} \), the useful transform, \( z_{xt} = (\frac{k}{k + 1 - a})^{1/(1-a)}z_{xt} \), and the definition for exporter profit, \( \tilde{d}_{xt} \).
7. Number of Firms:

\[ N_{dt} = (1 - \delta)(N_{dt-1} + N_{et-1}) \]  
(A.20)

\[ N_{dt}^* = (1 - \delta)(N_{dt-1}^* + N_{et-1}^*) \]  
(A.21)

8. Euler Equation for Domestic Bonds:

\[ C_t^{-1}(1 + nB_{t+1}) = (1 + r_{t+1})\beta E_t C_{t+1}^{-1} \]  
(A.22)

\[ C_t^{*-1}(1 - nB_{st+1}) = (1 + r_{t+1}^*)\beta E_t C_{t+1}^{*-1} \]  
(A.23)

9. Euler Equation for Foreign Bonds:

\[ Q_tC_t^{-1}(1 + nB_{st+1}) = (1 + r_{t+1}^*)\beta E_t Q_{t+1}C_{t+1}^{-1} \]  
(A.24)

\[ Q_tC_t^{*-1}(1 - nB_{st+1}) = (1 + r_{t+1})\beta E_t^* Q_{t+1}C_{t+1}^{*-1} \]  
(A.25)

10. Euler Equation for Mutual Fund Shares:

\[ \tilde{\upsilon}_t = (1 - \delta)E_t(C_{t+1}C_t^{-1})^{-1}(\tilde{d}_{t+1} + \tilde{\upsilon}_{t+1}) \]  
(A.26)

\[ \tilde{\upsilon}_t^* = (1 - \delta)E_t^*(C_{t+1}C_t^{-1})^{-1}(\tilde{d}_{t+1}^* + \tilde{\upsilon}_{t+1}^*) \]  
(A.27)

11. Labor Market Clearing\(^3\):

\[ C_t^{-1/\lambda} w_t^{1/\lambda} = \frac{a - 1}{w_t} (N_{Dt} \tilde{d}_{Dt} + N_{xt} \tilde{d}_{xt}) + \frac{(aN_{xt} F_{Xt} + N_{Et} F_{Et})/Z_t}{Z_t} \]  
(A.28)

\[ C_t^{*-1/\lambda} w_t^{*1/\lambda} = \frac{a - 1}{w_t^*} (N_{Dt}^* \tilde{d}_{Dt}^* + N_{xt}^* \tilde{d}_{xt}^*) + \frac{(aN_{xt}^* F_{Xt} + N_{Et}^* F_{Et})/Z_t^*}{Z_t^*} \]  
(A.29)

\(^3\text{When labor is supplied inelastically, we set the left-hand side of the equations to 1.}\)
12. *Net Foreign Assets:*

\[ 2(1 + r_{t+1})B_t + 2(1 + r^*_t)Q_tB_{st} = [C_t - Q_tC^*_t] + [N_{et}v_t - Q_tN_{et}^*\tilde{v}_t^*] + 2[B_{t+1} + Q_tB_{st+1}] \]

\[-[w_t\ell_t - Q_tw^*_t\ell^*_t] - [N_{dt}\tilde{d}_{dt} - Q_tN_{dt}^*\tilde{d}_{dt}^*] \]

\[-[N_{zt}\tilde{d}_{zt} - Q_tN_{zt}^*\tilde{d}_{zt}^*] \]  \hspace{1cm} (A.30)

Note that the mutual fund market clearing conditions in addition to the zero net supply of bonds conditions have already been included in the above equilibrium equations. The 29 equations are log-linearized around the symmetric steady state to form 29 linear equations in terms of percentages (denoted by ^)\(^4\).

In addition to these equations, the stochastic processes for the technology innovations must be specified:

\[
\begin{bmatrix}
\hat{Z}_t \\
\hat{Z}^*_t
\end{bmatrix}
= \begin{bmatrix}
s_{11} & s_{12} \\
ns_{21} & s_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{Z}_{t-1} \\
\hat{Z}^*_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\xi_{Zt} \\
\xi^*_{Zt}
\end{bmatrix}
\]  \hspace{1cm} (A.31)

\[E_t(Z_t) = E_t^*(Z^*_t) = \bar{Z} \]

\[\sigma_{\xi_{Zt}} = \sigma_{\xi^*_{Zt}} = \sigma_{\xi} \]

\[\rho_{\xi_{Zt}\xi^*_{Zt}} = \rho_{\xi Z \xi^* Z} \]

\(^4\text{Since the variable, } B_t, \text{ has an expected value of zero in equilibrium, it is linearized with respect to steady state consumption: } \hat{B}_t = \frac{B_t - \bar{B}}{\bar{B}} = \frac{B_t}{\bar{B}}.\]
A.1.2 Divisible versus Indivisible Labor

Although endogenizing the labor supply decision allows the model to capture employment dynamics, the formulation employed in the previous sections does not allow the model to analyze changes in unemployment. By construction, any unemployment in the model is completely voluntary. Past research by Hansen (1985) and Rogerson (1988) suggests a simple modification can be made to allow for endogenous involuntary unemployment ("indivisible labor"). In addition to improving the realism of the model, indivisible labor can also increase the volatility of labor hours relative to the real wage; a feature of the data that divisible labor models have difficulty matching. Results from incorporating indivisible labor into the framework developed so far suggest this feature can distort the pattern of trade in a way that causes excessively volatile output. In the model with indivisible labor, positive productivity shocks lead to huge gains in output which induces massive investment exodus and negatively correlated business cycles across countries.

The consumer’s problem is reformulated as follows. When considering divisible labor, I will follow Rogerson (1988). Workers have 1 unit time to spend either working ($l_t$) or in leisure. The consumer’s instantaneous utility function takes the form:

$$U_t(C_t, l_t) = \log C_t + H \log(1 - l_t)$$

where the parameter, $H$, is calibrated such that the steady state fraction of time working
is 1/3. The agent’s maximization problem is still written as:

\[
\max_{\{C_s, l_s, B_{s+1}, B_{s+1}, x_{s+1}\}} \sum_{s=t}^{\infty} \beta^{s-t} [U_s(C_s, l_s)] \quad \text{s.t.} \quad C_s + B_{s+1} + Q_s B_{s+1} + \frac{n_s}{2} (B_{s+1}^2) + \frac{n_s}{2} (Q_s B_{s+1})^2 + (N_{hs} \tilde{v}_s) x_{s+1} =
\]

\[
w_s l_s + (1 + r_s) B_s + Q_s (1 + r_s^*) B_{s+1} + \Gamma_s + x_s (N_{ds} \tilde{v}_s + N_{ds} \tilde{d}_s)
\]

The labor supply equation is:

\[
l_t = 1 - H \frac{C_t}{w_t} \quad (A.32)
\]

This equation is linearized and then included in the equation system that is used to solve the model.

For indivisible labor, the representative agent only has two choices of labor supply: to work a fixed number of hours or no hours at all. The agent contracts with firms to work \(l_0\) hours with probability \(\phi_t\). The expected amount of labor that the household will supply is \(l_t = \phi_t l_0\).

The consumer’s expected instantaneous utility function takes the form:

\[
U_t(C_t, l_t) = \phi_t (\log C_{ct} + H \log (1 - l_0)) + (1 - \phi_t) (\log C_{ut} + H \log (1))
\]

where \(C_{ct}\) is consumption when the worker is employed and \(C_{ut}\) is consumption when the worker is unemployed. It is assumed that the consumer is fully insured against becoming unemployed so that consumption in the unemployed state is the same as consumption in the employed state \((C_{ct} = C_{ut} = C_t)\). The expected instantaneous utility function is then:

\footnote{Instead of choosing the number of labor hours to supply, workers are choosing a probability of becoming employed each period, \(\phi_t\).}
\[ U_t(C_t, l_t) = \log C_t + \phi_t H \log(1 - l_0) \]

which can be rewritten using the definition of \( l_t \) as:

\[ U_t(C_t, l_t) = \log C_t + l_t \frac{H \log(1 - l_0)}{l_0} = \log C_t + Gl_t \]

where \( G = \frac{H \log(1 - l_0)}{l_0} \). The first order condition governing labor supply becomes:

\[ w_t = -GC_t \quad \text{(A.33)} \]

The parameters \( G \) and \( l_0 \) are calibrated so that the steady state fraction of time spent working (\( l_t \)) is 1/3 and the parameter \( H \) matches the case of divisible labor. This equation is linearized and then included in the equation system that is used to solve the model.

The simulation results for the models with divisible labor and indivisible labor that follow Hansen’s (1985) exercise are reported in table A.1. The model with divisible labor generates qualitatively similar results to the model described in the previous section. The model with indivisible labor actually worsens the model’s fit by exacerbating the volatility of output and investment, generating countercyclical exports, and inducing negative correlation of output, investment and labor hours across countries. Inspection of the impulse response functions for both the divisible labor and indivisible labor models suggest the worsened fit is due to a mixture of open capital markets and strong responses of labor demand.

In the model with divisible labor, a positive productivity shock in the home country leads to an overall expansion at home. Consumption increases, the profitability of
Table A.1: Simulation Results: Divisible Versus Indivisible Labor.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Statistics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_Y$ (%)</td>
<td>1.54</td>
<td>1.38</td>
<td>9.67</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.75</td>
<td>0.52</td>
<td>0.10</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>3.41</td>
<td>4.51</td>
<td>11.20</td>
</tr>
<tr>
<td>$\sigma_L/\sigma_Y$</td>
<td>0.61</td>
<td>0.40</td>
<td>1.16</td>
</tr>
<tr>
<td>Correlation with GDP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{C,Y}$</td>
<td>0.88</td>
<td>0.98</td>
<td>0.48</td>
</tr>
<tr>
<td>$\rho_{I,Y}$</td>
<td>0.90</td>
<td>0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_{L,Y}$</td>
<td>0.81</td>
<td>0.98</td>
<td>0.99</td>
</tr>
</tbody>
</table>

| International Comovement  |      |      |      |
| Correlation               |      |      |      |
| $\rho_{Y,Y}$              | 0.56 | 0.20 | -0.12|
| $\rho_{C,C}$              | 0.45 | 0.17 | 0.21 |
| $\rho_{I,I}$              | 0.37 | -0.26| -0.39|
| $\rho_{L,L}$              | 0.39 | 0.41 | -0.19|

| International Market Statistics |      |      |      |
| Volatility                     |      |      |      |
| $\sigma_{IM}/\sigma_Y$        | 3.31 | 0.73 | 0.21 |
| $\sigma_{EX}/\sigma_Y$        | 3.74 | 0.77 | 0.20 |
| $\sigma_{NX}/\gamma$         | 0.40 | 0.37 | 0.88 |
| $\sigma_{TOT}/\sigma_Y$       | 1.69 | 0.27 | 0.04 |
| $\sigma_{Q}/\sigma_Y$         | 3.37 | 0.08 | 0.03 |
| Correlation with GDP          |      |      |      |
| $\rho_{IM,Y}$                 | 0.60 | 0.84 | 0.54 |
| $\rho_{EX,Y}$                 | 0.28 | 0.09 | -0.28|
| $\rho_{NX/Y,Y}$               | -0.36| -0.46| -0.45|
| $\rho_{TOT,Y}$                | 0.07 | 0.57 | 0.76 |
| $\rho_{Q,Y}$                  | -0.18| -0.55| -0.03|

(1) Data  
(8) Divisible Labor  
(9) Indivisible Labor
firms rises, there’s increased firm construction and higher demand for imported goods as aforementioned. In the labor market, there is increased demand for labor to spend on production and new firm construction but reduced supply of labor due to the expansion of consumption. The model predicts an increase in the real wage and in increase in employment, which boosts output. In the foreign country, less is spent on firm construction and investment dollars are redirected abroad. There is downward pressure on labor demand due to the fall in firm construction but upward pressure on labor demand due to the expansion of the export sector. Labor supply increases due to a reduction in the variety of goods which resulted from firm attrition. The model predicts a fall in the real wage, but an increase in employment and a boost in output.

When labor is indivisible, inspection of the impulse response functions for the endogenous variables of the model suggest a strong increase in employment in the home country in response to a positive productivity shock. Holding labor supply and new firm construction constant, a positive productivity shock leads to an increase in labor demand and higher employment in equilibrium. In the model with indivisible labor, the increase in labor demand leads to a larger increase in employment than in the model with divisible labor. Firms in the home country become more profitable under the indivisible labor specification than they do under divisible labor. Since firms are more profitable under the indivisible labor specification, there is a larger boost in new firm construction than in the divisible labor framework. Finally, allowing for changes in labor supply, the labor supply curve in the indivisible labor model rises quite dramatically since the positive productivity shock boosts consumption relatively more than in the model with divisible labor.
Employment is more volatile as is the real wage, leading to a dramatic increase in income.

In the foreign country, there is a large exodus of investment in the indivisible labor framework due to the dramatic increase in the profitability of home firms. Firm attrition leads to a large decrease in labor demand. Although labor supply increases in the foreign country due to a fall in consumption, the model predicts a lower real wage as well as lower employment in equilibrium. National income falls as does the demand for home’s exports. The model predicts negatively correlated output and employment across countries and countercyclical exports, none of which match the empirical facts.
A.2 Appendix to Chapter 4

A.2.1 Details of the Model with Capital Accumulation

The theoretical model is heavily influenced by Ghironi and Melitz (2005) and is quite similar to the model developed in Chapter 3. Details of the model follow. Note that the model described here considers two countries, home and foreign (which is denoted by *). Both countries are large and assumed to be structurally identical. That in mind, I construct the framework for one country (the home country) knowing that a symmetric framework exists for the foreign counterpart.

The Consumer’s Problem

Denote the universe of intermediate input varieties as \( \Omega \). At any time, \( t \), a subset \( \Omega_t \subseteq \Omega \) are actually produced by firms. \( \Omega_t \) contains both domestically produced and imported goods. Further, it need not be the case that \( \Omega_t = \Omega_s \) for \( t \neq s \) nor \( \Omega_t = \Omega^*_t \). A composite intermediate good, \( M_t \), is produced using Dixit-Stiglitz technology:

\[
M_t = \left( \int_{\omega \in \Omega_t} m_t(\omega)^{(a-1)/a} \, d\omega \right)^{a/(a-1)} \tag{A.34}
\]

where \( m_t(\omega) \) denotes the quantity of variety \( \omega \) used in the production of the composite, and \( a \) denotes the elasticity of substitution across varieties. The composite intermediate good is then combined with capital, \( K_t \), to produce consumption using a simple Cobb-Douglass production process:
\[ C_t = A_t M_t^b K_t^{1-b} \]  \quad (A.35)

\( A_t \) denotes total factor productivity at time, \( t \). At \( A_t = b = 1 \), the model reverts to the structure described by Ghironi and Melitz (2005).

It is straightforward to construct demand equations and price indices for \( m_t, M_t, K_t \) and \( C_t \). Denote \( P_{Xt} \) as the price index for good \( X \). I construct the following conditional demand equations by solving a straight-forward cost-minimization problem\(^6\):

\[
\begin{align*}
    m_t(\omega) &= M_t(P_t(\omega)/P_{Mt})^{-a} \quad \text{(A.36)} \\
    M_t &= \frac{C_t}{A_t} \left( \frac{P_{Mt}}{P_{Kt}} \right)^{1-b} \quad \text{(A.37)} \\
    K_t &= \frac{C_t}{A_t} \left( \frac{P_{Mt}}{P_{Kt}} \right)^{b} \quad \text{(A.38)}
\end{align*}
\]

I also construct the associated price indices:

\[
P_{Mt} = \left( \int_{\omega \in \Omega_{Dt}} P_t(\omega)^{1-a} d\omega \right)^{1/(1-a)} \quad \text{(A.39)}
\]

\[
P_{Ct} = \frac{P_{Mt}^{b} P_{Kt}^{1-b}}{A_t^{b} (1-b)^{1-b} b} \quad \text{(A.40)}
\]

Next, I turn to the household’s problem faced by the representative agent. It is assumed that there exists a representative household that chooses to work, consume and save. The household is endowed with 1 unit of time that it can divide between labor, \( l_t \), and leisure. Labor earns the real wage, \( w_t \). The household can choose to save by purchasing

\(^6\)The production function for \( C_t \) is essentially a multi-step CES production function. First, I find the lowest-cost way to produce one unit of the intermediate index, \( M_t \). Then, I solve a cost-minimization problem for \( C_t \) over \( M_t^b \) and \( K_t \).
bonds, \(B_{t+1}\), which each cost one unit of consumption, but yield \((1 + r_{t+1})\) units in the next period. Both domestic and foreign bonds (\(B_\ast\)) are available to the household and are subject to a quadratic transaction cost which is rebated, lump-sum (\(\Gamma_t\)), to the household. The household can also purchase shares of a mutual fund, \(x_{t+1}\), which entitle the owner to a fraction of the profits of the producing firms. Arbitrage prices the shares at the firms expected discounted value (\(N_{ht}\hat{v}_t\) for new shares, \(N_{dt}\hat{v}_t\) for old shares). Households are obligated to hold bonds and stocks for only one period before they are resold in asset markets. Finally, the households have the option of saving real wealth in the form of savings accounts, \(S_{t+1}\). Savings are used to construct capital in the next period and earn a real return of \(R_t\). The consumer’s real period budget constraint is then given by:

\[
C_t + B_{t+1} + Q_t B_{st+1} + \frac{n_2}{2} (B_{t+1}^2) + \frac{n_2}{2} (Q_t B_{st+1}^2) + (N_{ht}\hat{v}_t)x_{t+1} + S_{t+1} = \]

\[
w_t l_t + (1 + r_t)B_t + Q_t (1 + r_\ast)B_{st} + \Gamma_t + x_t (N_{dt}\hat{v}_t + N_{dt}\hat{d}_t) + R_t S_t
\]

where \(n_2 \geq n_1 > 0\) are scale parameters on foreign and domestic bond adjustment costs, \(\Gamma_t = \frac{n_1}{2} (B_{t+1}^2) + \frac{n_2}{2} (Q_t B_{st+1}^2)\) in equilibrium and \(Q_t\) is the real exchange rate.

The instantaneous utility function for the agent is assumed to have the form:

\[
U_t(C_t, l_t) = \log C_t - \frac{l_t^{1+\lambda}}{1 + \lambda}
\]

\(^7\)All savings instruments are denoted with the time subscript in which they yield a return.
The agent’s maximization problem can then be written as:

\[
\max_{C_s, l_s, B_s, x_s, F_t} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log C_s - \frac{1}{1+\lambda} \right] \quad \text{s.t.} \\
C_s + B_{s+1} + Q_s B_{ss+1} + \frac{n_1}{2}(B^2_{s+1}) + \frac{n_2}{2}(Q_s B^2_{ss+1}) + (N_{hs} \tilde{v}_s)x_{s+1} + S_{s+1} = w_s l_s + (1 + r_s)B_s + Q_s(1 + r_s^*B_s) + \Gamma_s + x_s(N_{ds} \tilde{v}_s + N_{ds} \tilde{d}_s) + R_s S_s
\]  

(A.42)

The first-order conditions for the consumer’s problem generate an equation that guides labor supply in addition to three Euler equations. The labor supply equation is:

\[l_t = C_t^{-1/\lambda} w_t^{1/\lambda}\]  

(A.43)

which suggests that $1/\lambda$ represents elasticity of labor supply. In the original Ghironi and Melitz (2005) framework, labor was supplied inelastically ($l_t = 1$). The Euler equation for domestic bonds is:

\[C_t^{-1}(1 + n_1 B_{t+1}) = (1 + r_{t+1}) \beta E_t C_{t+1}^{-1}\]  

(A.44)

Similarly, the Euler equation for foreign bonds is:

\[Q_t C_t^{-1}(1 + n_2 B_{st+1}) = (1 + r_{t+1}^*) \beta E_t Q_{t+1} C_{t+1}^{-1}\]  

(A.45)

The Euler equation for stocks is:

\[\tilde{v}_t = \beta(1 - \delta) E_t (C_{t+1}/C_t)^{-1}(\tilde{d}_{t+1} + \tilde{v}_{t+1})\]  

(A.46)

Notice that repetitive forward substitution of the Euler equation for stocks generates the expected value of the firm’s lifetime profit stream. Finally, the Euler equation for savings
\[ C_t^{-1} = \beta E_t R_{t+1} C_{t+1}^{-1} \]  

(A.47)

**The Firm’s Problem**

I now consider the problem faced by a typical intermediate good producer in the home country. It is assumed that there exists an unbounded mass of firms in the economy that may begin production at any time. These firms are monopolistically competitive producers of unique varieties. Further, it is assumed that firms produce for domestic and foreign markets separately\(^8\). Given the demand for their products derived in the previous section, the firm’s problem is as follows:

**Step 1:** Decide whether or not to enter.

**Step 2:** Upon entry, choose how much output to produce for each market and what prices to set.

**Step 3:** Death or exit.

In practice, firms will first derive the solutions to step 2 and step 3 and form estimates on their future profit streams before choosing whether or not it’s worth it to enter the industry.

Prior to entry, the firm calculates average (expected) profit. Each firm \(j\) in the home country produces output according to the production function:

\[ y_{jt} = Z_t z_j L_{jt} \]  

(A.48)

\(^8\)Markets are segmented – firm’s practice third-degree price discrimination.
where \( y_{jt} \) denotes the quantity of output firm \( j \) produces, \( Z_t \) denotes an economy-wide technology variable, \( z_j \) is a productivity draw specific to firm \( j \) and \( L_{jt} \) is the quantity of labor firm \( j \) hires in the production of output. The firm will choose how much to produce for the domestic market \((m_{djt})\) and how much to produce for export \((m_{xjt} = m^*_{m,jt})\). Exports are subject to an iceberg cost, \( \tau \geq 1 \). Thus, total output for the firm is given by \( y_{jt} = m_{djt} + \tau m_{xjt} \).

The firm’s expenses include a wage bill, a start-up cost paid in terms of labor, and an exporter fee (also paid in terms of labor). The wage bill is simply \( w_tL_{jt} \). Start-up costs are a fixed cost paid once during the period the firm begins production. The "production" of entry is given by \( F_{Et} = Z_tL_{Ejt} \) which suggests a total fixed cost of \( w_tF_{Et}/Z_t \). Exporter fees are paid each period the firm chooses to export \( (m_{xjt} > 0) \). The production of the exporter fee is given as \( F_{Xt} = Z_tL_{Xjt} \). Thus, the per-period exporter cost is \( w_tF_{Xt}/Z_t \). Dropping the start-up cost, we form the firm’s real per-period profit maximization problem:

\[
\max_{c_{xjt}, c_{djt}} \Pi_{jt} = \frac{P_{djt}}{P_{Ct}}m_{djt} + \epsilon_t\frac{P_{xjt}}{P_{Ct}}m_{xjt} - \frac{w_t}{Z_tz_j}(m_{djt} + \tau m_{xjt}) - I_{xt}\frac{w_tF_{Xt}}{Z_t} \tag{A.49}
\]

where \( I_{xt} = 1 \) if \( c_{xjt} > 0 \), else \( I_{xt} = 0 \). \( \epsilon_t \) denotes the nominal exchange rate. It is important to note that \( P_{xjt} \) is measured in terms of foreign currency. For any variable, \( X \), we denote real prices as \( \rho_X = P_X/P_C \) and substitute the demand functions from the previous section into the firm’s problem (as is standard with monopolistic competition) to generate the first order conditions for the firm\(^9\):

\(^9\)It is assumed that firms do not observe the impact of price setting on the aggregate price level when solving the firm’s maximization problem. Also, note that the definition of \( P_{Ct} \) implies \( M_t = bC_t\rho_{M_t}^{-1} \)

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where \( Q_t = \epsilon_t P_{Ct}^* / P_{Ct} \) is the real exchange rate. For simplicity, it is assumed that \( \epsilon_t = 1 \).

We then use the demand equations to find the optimal real prices charged by the firm:

\[
m_{djt} = bC_t \rho_{Mt}^{a-1} \left( \frac{a}{a - 1} \frac{w_t}{Z_t z_j} \right)^{-a} \\
m_{xjt} = bC_t^* \rho_{Mt}^{\ast a-1} \left( \frac{a}{a - 1} \frac{w_t \tau}{Z_t z_j Q_t} \right)^{-a}
\]

The prices suggest a constant markup over marginal cost \((w_t/Z_t z_j)\).

Profits for the firm can be divided into "domestic production profits" and "export profits". They are given as:

\[
ds_{djt} = \left( \frac{b}{a} \right) C_t \rho_{Mt}^{a-1} \rho_{djt}^{1-a} \\
ds_{xjt} = \begin{cases} \left( \frac{b}{a} \right) C_t^* \rho_{Mt}^{\ast a-1} \rho_{xjt}^{1-a} - \frac{w_t F_{Xt}}{Z_t} & \text{if } c_{xjt} > 0 \\ 0 & \text{if } c_{xjt} = 0 \end{cases}
\]

For a continuum of potential firms, there exists some productivity draw, \( z_{xt} \), such that the firm which receives that draw upon entering at time \( t \) earns zero profits whether it exports or not (the exporter cutoff). For this firm, \( \left( \frac{b}{a} \right) C_t^* \rho_{Mt}^{\ast a-1} \rho_{xjt}^{1-a} = \frac{w_t F_{Xt}}{Z_t} \).

Knowing the profits for any given productivity draw, potential firms decide whether or not to enter. Their productivity draw is not assured prior to entry, so firms form an
expectation of their profits (they calculate an "average" profit from entry). To do this, we employ the "special averages" developed by Melitz (2003). First, we suppose that the productivity draws follow a Pareto distribution with CDF $G(z) = 1 - (z_{\min}/z)^k$ and PDF $g(z) = k z_{\min}^{-k} z^{-k-1}$ where k denotes the shape parameter of the distribution. Next, define:

$$\tilde{z}_{1-a}^d = \int_{z_{\min}}^{\infty} z^{1-a} g(z) dz$$  \hspace{1cm} (A.56)$$

$$\tilde{z}_{1-a}^{xt} = \int_{z_{xt}}^{\infty} z^{1-a} \frac{g(z)}{1 - G(z_{xt})} dz$$  \hspace{1cm} (A.57)$$

Finally, substitute these definitions in constructing average prices:

$$\tilde{\rho}_{1-a}^d = \int_{z_{\min}}^{\infty} \rho_{rj}^{1-a} g(z) dz = (\frac{a}{a-1} \frac{w_t}{Z_t z_d})^{1-a}$$ and $$\tilde{\rho}_{1-a}^{xt} = \int_{z_{xt}}^{\infty} \rho_{rj}^{1-a} \frac{g(z)}{1 - G(z_{xt})} dz = (\frac{a}{a-1} \frac{w_t}{Z_t z_{xt} Q_t})^{1-a}$$. We define $N_{dt}$ as the number of firms that produce for the domestic market and $N_{xt}$ as the number of firms that produce for export. The model is recast in terms of average prices:

$$\tilde{\rho}_{dt} = (\frac{a}{a-1} \frac{w_t}{Z_t z_d})$$  \hspace{1cm} (A.58)$$

$$\tilde{\rho}_{xt} = (\frac{a}{a-1} \frac{w_t}{Z_t z_{xt} Q_t})$$  \hspace{1cm} (A.59)$$

$$\tilde{\rho}_{Mt} = (N_{dt} \tilde{\rho}_{1-b}^d + N_{xt}^{\tau} \tilde{\rho}_{1-b}^{xt})^{1/(1-b)}$$  \hspace{1cm} (A.60)$$

$$\tilde{d}_{dt} = b \frac{\gamma a^{-1}}{a C_t \tilde{\rho}_{Mt}^{1-a} \tilde{\rho}_{dt}^{1-a}}$$  \hspace{1cm} (A.61)$$

$$\tilde{d}_{xt} = b \frac{\gamma a^{-1}}{a C_t \tilde{\rho}_{Mt}^{1-a} Q_t \tilde{\rho}_{2xt}^{1-a} - \frac{w_t F_{xt}}{Z_t}}$$  \hspace{1cm} (A.62)$$

Completing the integral for the special productivity averages suggests that $\tilde{z}_D = (\frac{k}{k+1-a})^{1/(a-1)} z_{\min}$ and $\tilde{z}_{xt} = (\frac{k}{k+1-a})^{1/(a-1)} z_{xt}$ where $k > a - 1$ for boundedness. Knowing that all existing firms produce for the domestic market, and a fraction of those become exporters, the expected per-period profit for a potential firm, on average, is:
\[
\tilde{d}_t = \tilde{d}_{tt} + (1 - G(z_{xt}))\tilde{d}_{xt}
\]

(A.63)

There are two important features guiding firm creation and destruction. The first is a lag in production. A firm that enters in period \(t\) starts producing at period \(t + 1\). The entering firm, however, is still counted as a firm in period \(t\). The total number of firms that exist at period \(t\), \(N_{ht}\), is given by the number of producing firms that already exist plus the number of new firms, \(N_{et}\).

\[
N_{ht} = N_{dt} + N_{et}
\]

Second, firms are subject to an exogenous exit shock. The number of firms that "survive" to produce in period \(t + 1\) is given by:

\[
N_{dt+1} = (1 - \delta)N_{ht} = (1 - \delta)(N_{dt} + N_{et})
\]

(A.64)

Therefore, the expected value of the firm’s lifetime profit stream, \(\tilde{v}_t\), is given by:

\[
\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [1 - \delta]^{s-t} \left[ \beta^{s-t} \left( \frac{C_s}{C_t} \right)^{-1} \right] \tilde{d}_s
\]

(A.65)

where \(\left[ \beta^{s-t} \left( \frac{C_s}{C_t} \right)^{-1} \right]\) is the stochastic discount factor to be discussed shortly and \((1 - \delta)\) is the firm’s survival probability. Firms continue to enter as long as the discounted value of their profit stream exceeds the cost of entry. Therefore, the entry cutoff is determined by:

\[
\tilde{v}_t = w_t F_{Et}/Z_t
\]

(A.66)
Market Clearing and Equilibrium

I now turn to important market clearing conditions implied by the model. The most obvious of these conditions is labor market clearing. As determined in the consumer’s problem for the home (foreign) country, labor supply is given as

\[ l_t = C_t^{-1/\lambda} w_t^{1/\lambda} \]

when labor supply is endogenous and \( l_t = 1 \) when labor is supplied inelastically. Labor demand comes from three sources:

1. **Production of intermediate inputs.** Each firm that produces for the domestic market requires \( L_{djt} = \frac{m_{djt}}{Z_t z_j} \) units of labor. From demand, \( m_{djt} = C_t (\varepsilon_H^e \alpha_D^b) \rho_{Ht}^b \rho_{Dt}^{a-b} (\rho_{djt})^{-a} \rightarrow \)

\[ \frac{m_{djt}}{Z_t z_j} = C_t (\varepsilon_H^e \alpha_D^b) \rho_{Ht}^b \rho_{Dt}^{a-b} (\rho_{djt})^{-a} \frac{1}{Z_t z_j w_t^a} \frac{a-1}{a} \]

\[ = C_t (\varepsilon_H^e \alpha_D^b) \rho_{Ht}^b \rho_{Dt}^{a-b} (\rho_{djt})^{-a} \frac{a-1}{a w_t^a} = a d_{djt} \frac{a-1}{a w_t^a} = d_{djt} \frac{a-1}{w_t^a}. \]

On average, \( N_{dt} \) firms each demand \( \tilde{d}_{dt} \frac{a-1}{w_t^a} \) units of labor. Similarly, each firm that produces for export requires \( L_{xjt} = \frac{m_{xjt}}{Z_t z_j} \) units of labor. From demand, \( m_{xjt} = C_t (\varepsilon_H^e \alpha_M^b) \rho_{Ht}^b \rho_{Mt}^{a-b} (\rho_{xjt})^{-a} \rightarrow \)

\[ \frac{m_{xjt}}{Z_t z_j} = C_t (\varepsilon_H^e \alpha_M^b) \rho_{Ht}^b \rho_{Mt}^{a-b} (\rho_{xjt})^{-a} \frac{1}{Z_t z_j w_t^a} \frac{a-1}{a} \frac{a}{Q_t} \]

\[ = C_t (\varepsilon_H^e \alpha_M^b) \rho_{Ht}^b \rho_{Mt}^{a-b} (\rho_{xjt})^{-a} \frac{a-1}{aw_t^a} \frac{1}{Q_t} = \]

\[ \frac{a-1}{w_t} d_{xjt} + (a-1) F_{Xt}/Z_t. \]

On average, \( N_{xt} \) firms each demand \( \tilde{d}_{xt} \frac{a-1}{w_t} \) units of labor. Similarly, each firm that produces for export requires \( L_{xjt} = \frac{m_{xjt}}{Z_t z_j} \) units of labor. From demand, \( m_{xjt} = C_t (\varepsilon_H^e \alpha_M^b) \rho_{Ht}^b \rho_{Mt}^{a-b} (\rho_{xjt})^{-a} \rightarrow \)

\[ \frac{m_{xjt}}{Z_t z_j} = C_t (\varepsilon_H^e \alpha_M^b) \rho_{Ht}^b \rho_{Mt}^{a-b} (\rho_{xjt})^{-a} \frac{1}{Z_t z_j w_t^a} \frac{a-1}{a} \frac{a}{Q_t} \]

\[ = C_t (\varepsilon_H^e \alpha_M^b) \rho_{Ht}^b \rho_{Mt}^{a-b} (\rho_{xjt})^{-a} \frac{a-1}{aw_t^a} \frac{1}{Q_t} = \]

\[ \frac{a-1}{w_t} d_{xjt} + (a-1) F_{Xt}/Z_t. \]

2. **Exporter costs.** \( N_{xt} \) firms must hire labor to produce the exporter fee of \( F_{Xt}/Z_t \).

Demand for labor to produce the exporter fee is thus \( N_{xt} F_{Xt}/Z_t \).

3. **Start-up costs.** \( N_{Et} \) entering firms must hire labor to produce the entry fee of \( F_{Et}/Z_t \).
Demand for labor to produce the entry fee is thus $N_{Et} F_{Et} / Z_t$. Labor demand in the model is given as

$$L_t = \frac{a - 1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et}) / Z_t$$

The equation that governs equilibrium in the labor market when labor supply is endogenously determined is:

$$C_t^{-1/\lambda} w_t^{1/\lambda} = \frac{a - 1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et}) / Z_t \quad (A.67)$$

When labor is supplied inelastically, the labor market clearing condition is:

$$1 = \frac{a - 1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et}) / Z_t \quad (A.68)$$

The second market clearing condition ensures market clearing in the mutual fund market (shares of the portfolio sum to unity):

$$x_{t+1} = x_t = 1 \quad (A.69)$$

The third market clearing condition ensures zero net supply of bonds. The representative agent in each country is either a borrower or a lender, but not both. If the agent produces bonds to sell, they must be purchased by agent in the foreign country:

$$B_{t+1} = -B^*_t$$

Thus, we generate the zero net supply conditions:

\[B_{t+1} = -B^*_t\]  

\[A.69\]  

As shown in the appendix to this chapter.
\[ B_{t+1} + B^*_t = 0 \] (A.70)

\[ B_{st+1} + B^*_st+1 = 0 \]

The presence of adjustment costs ensure that the unique steady state bond issuance is zero \((B = 0)^{11}\).

The next market clearing condition applies to the market for capital (when capital is relevant in production). In equilibrium, savings today become the foundation for tomorrow’s capital stock \((S_{t+1} = K_{t+1})\). In equilibrium, the gross return to savings \((R_tF_t = R_tK_t)\) should be equal to total payments to capital plus the return of non-depreciated capital \((\rho K_t + (1 - \delta K)K_t)\). This suggests the equilibrium price of capital is given by:

\[ \rho_{Kt} = R_t - 1 + \delta_K \] (A.71)

Capital demand for the production of consumption in period \(t\) is given by:

\[ K_t = \frac{C_t}{\rho_{Mt} \left( \frac{1 - b}{\rho_{Kt}} \right)^b} \] (A.72)

while supply of capital in the next period is determined by the Euler equation:

\[ C_t^{-1} = \beta E_t R_{t+1} C_{t+1}^{-1} \] (A.73)

\(^{11}\)Again, this can be seen by comparing the home agent’s Euler equation for home bonds and foreign agent’s demand for home bonds at the steady state then applying the zero net supply condition. For the home agent, \(\beta(1 + r) = (1 + nB)\). For the foreign agent, \(\beta(1 + r) = (1 + nB^*)\). The zero net supply condition implies \(B = -B^*\). Thus, \((1 + nB) = (1 - nB) \rightarrow B = 0\).
The final market clearing condition ensures balance of payments. To generate this condition, we consider the consumer’s budget constraint. Note that capital market clearing suggests:

\[ S_{t+1} - R_t S_t = K_{t+1} - (\rho K_t K_t + (1 - \delta_K) K_t) = K_{t+1} - (1 - \delta_K) K_t - \rho K_t K_t. \]

I impose mutual fund market clearing and lump-sum transfers of bond adjustment costs:

\[ C_t + B_{t+1} + Q_t B_{t+1} + (N_{t+1} \tilde{v}_t) + K_{t+1} = w_t l_t + (1 + r_t) B_t + Q_t (1 + r_t^* B_{t+1} + N_{dt} \tilde{d}_t + N_{dt} \tilde{d}_t + \rho K_t K_t + (1 - \delta_K) K_t. \]

We generate the budget constraint for the foreign agent:

\[ C_t^* + B_{t+1}^* + Q_t^* B_{t+1}^* + (N_{t+1}^* \tilde{v}_t^*) + K_{t+1}^* = w_t^* l_t^* + (1 + r_t^*) B_t + Q_t^* (1 + r_t^* B_{t+1}^* + N_{dt}^* \tilde{d}_t^* + N_{dt}^* \tilde{d}_t^* + \rho K_t^* K_t^* + (1 - \delta_K) K_t^*. \]

We multiply the foreign budget constraint by \( Q_t \) to transform it into home consumption terms and subtract it from the home budget constraint. Imposing the bond market clearing conditions suggest the following equation for net foreign assets:

\[
2(1 + r_{t+1}) B_t + 2(1 + r_{t+1}^*) Q_t B_{t+1} = [C_t - Q_t C_t^*] + [N_{ex} \tilde{v}_t - Q_t N_{ex} \tilde{v}_t^*] + 2[B_{t+1} + Q_t B_{t+1}]
- [w_t l_t - Q_t w_t^* l_t^*] - [N_{dt} \tilde{d}_t - Q_t N_{dt} \tilde{d}_t]
- [N_{st} \tilde{d}_{st} - Q_t N_{st} \tilde{d}_{st} + \tilde{K}_{t+1} - Q_t \tilde{K}_{t+1}^*]
- (1 - \delta_K) [K_t - Q_t K_t^*] - [\rho K_t K_t - Q_t \rho K_t K_t^*]
\]

Equilibrium in this model is characterized by the 35 equations described below. These equations are log-linearized around the symmetric steady state to form a log-linear linear equation system (denoted by \( \hat{\cdot} \))\(^{12}\). In addition to these equations, the stochastic

\[^{12}\text{Since the variable, } B_t, \text{ has an expected value of zero in equilibrium, it is linearized with respect to steady state consumption: } \hat{B}_t = \frac{B_t - 0}{\mu} = \frac{B_t}{\mu}.\]
processes for the technology innovations are specified:

\[
\begin{bmatrix}
\hat{Z}_t \\
\hat{Z}_t^* \\
E_t(Z_t)
\end{bmatrix} = \begin{bmatrix}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{Z}_{t-1} \\
\hat{Z}_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
\xi_{Zt} \\
\xi_{Zt}^*
\end{bmatrix} \tag{A.75}
\]

\[
\begin{bmatrix}
\hat{A}_t \\
\hat{A}_t^* \\
E_t(A_t)
\end{bmatrix} = \begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{A}_{t-1} \\
\hat{A}_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
\xi_{At} \\
\xi_{At}^*
\end{bmatrix} \tag{A.79}
\]

Additional definitions are required. Following, income (GDP) is defined as:

\[
Y_t = w_t l_t + \rho_K K_t + N_{dt} \tilde{d}_{dt} \quad (A.83)
\]

Further, total imports is constructed using the average revenue foreign exporters earn from their sales abroad:

\[
IM_t = b N_{xt}^* C_t \tilde{p}_{Mt}^{a-1} \tilde{r}_{xt}^{1-a} \quad (A.84)
\]
Total exports is constructed using the average revenue domestic exporters earn from their sales abroad:

\[ EX_t = bQ_tN_xtC_t^{\hat{\rho}_t\rho_{Mt}^{1-a}} \]  
(A.85)

Terms of trade is defines as the ratio of import prices to export prices:

\[ TOT_t = \frac{\hat{\rho}_t^*}{Q_t\hat{\rho}_t} \]  
(A.86)

Investment is defined as expenditures on new firm entry plus the purchases of new capital:

\[ I_t = N_{et}\hat{v}_t + K_{t+1} - (1 - \delta_K)K_t \]  
(A.87)

In the model, the price of consumption \((P_{Ct})\) is measured as a welfare-based price index following Feenstra (2003). It is thus important to transform this welfare-based index into one which closer matches the price index calculated in the data. To do so, recall:

\[ 1 = \frac{[N_{dt}\hat{\rho}_{dt}^{1-a} + N_{xt}\hat{\rho}_t^{1-a}]^{b/(1-a)}\rho_{Kt}^{1-b}}{A_t(1-b)^{1-bb}} \]

If we assume that all prices are, on average, \(\hat{\rho} = \hat{\rho}_t/P_{Ct}\), we construct:

\[ \frac{P_{Ct}}{\hat{\rho}_t} = \frac{[N_{dt} + N_{xt}^{*}]^{b/(1-a)}}{A_t(1-b)^{1-bb}} \]  
(A.88)

Any variable measured in terms of real consumption, \(X_t\), is adjusted to this index: \(\hat{X}_t = \frac{P_{Ct}}{\hat{\rho}} X_t\). Further, since the real exchange rate is constructed using the welfare-based price indices \((Q_t = P_{Ct}^*/P_{Ct})\), we construct an adjusted real exchange rate:
\[
\tilde{Q} = \tilde{P}_t^* / \tilde{P}_t = Q_t \left[ \frac{A_t^*}{N_{dt} + N_{xt}^*} \right] \left[ \frac{N_{dt} + N_{xt}^*}{N_{dt}^* + N_{xt}} \right]^{b/(1-a)} \quad (A.89)
\]

Equilibrium in this model is characterized by the following equation system:

1. **Price and Profit definitions:**

   \[
   \begin{align*}
   \tilde{\rho}_{dt} & = \left( \frac{a}{a-1} \frac{w_t}{Z_t z_d} \right) \\
   \tilde{\rho}_{dt}^* & = \left( \frac{a}{a-1} \frac{w_t^*}{Z_t^* z_d^*} \right) \\
   \tilde{\rho}_{xt} & = \left( \frac{a}{a-1} \frac{w_t}{Z_t z_{xt}} \right) \frac{Q_t}{Q_t} \\
   \tilde{\rho}_{xt}^* & = \left( \frac{a}{a-1} \frac{w_t^*}{Z_t^* z_{xt}^*} \right) \frac{Q_t}{Q_t} \\
   \tilde{\rho}_{Mt} & = (N_{dt} \tilde{P}_dt^{1-a} + N_{xt} \tilde{P}_xt^{1-b})^{1/(1-b)} \\
   \tilde{\rho}_{Mt}^* & = (N_{dt} \tilde{P}_dt^{1-a} + N_{xt} \tilde{P}_xt^{1-b})^{1/(1-b)} \\
   \tilde{d}_{dt} & = \left( \frac{b}{a} \right) C_t \tilde{P}_{Mt}^{1-a} \tilde{\rho}_{dt}^{1-a} \\
   \tilde{d}_{dt}^* & = \left( \frac{b}{a} \right) C_t \tilde{P}_{Mt}^{1-a} \tilde{\rho}_{dt}^{1-a} \\
   \tilde{d}_{xt} & = \left( \frac{b}{a} \right) C_t \tilde{P}_{Mt}^{1-a} \tilde{P}_{xt}^{1-a} - \frac{w_t F_{xt}^*}{Z_t^*} \\
   \tilde{d}_{xt}^* & = \left( \frac{b}{a} \right) C_t \tilde{P}_{Mt}^{1-a} \tilde{P}_{xt}^{1-a} - \frac{w_t^* F_{xt}}{Z_t} \\
   \end{align*}
   \]

2. **Price Index:**

   \[
   1 = \frac{\rho_{Mt}^{b} \rho_{Kt}^{1-b}}{A_t (1 - b)^{1-b} b^b} \quad (A.100)
   \]

   \[
   1 = \frac{\rho_{Mt}^{sb} \rho_{Kt}^{1-b}}{A_t^* (1 - b)^{1-b} b^b} \quad (A.101)
   \]
3. **Expected Profit:**

\[
\tilde{d}_t = \tilde{d}_{dt} + \left( \frac{N_{xt}}{N_{dt}} \right) \tilde{d}_{xt} \\
\tilde{d}_t^* = \tilde{d}_{dt}^* + \left( \frac{N_{xt}^*}{N_{dt}^*} \right) \tilde{d}_{xt}^* 
\]

(A.102)  

(A.103)

4. **Free Entry:**

\[
\tilde{v}_t = w_t F_{Et}/Z_t \\
\tilde{v}_t^* = w_t^* F_{Et}/Z_t^* 
\]

(A.104)  

(A.105)

5. **Zero-Profit Intermediate Exporter** \(^{13}\):

\[
\tilde{d}_{xt} = \left[ a - 1 \right] w_t F_{Xt}/Z_t \\
\tilde{d}_{xt}^* = \left[ a - 1 \right] w_t^* F_{Xt}/Z_t^* 
\]

(A.106)  

(A.107)

6. **Share Exporting Firms:**

\[
(1 - G(z_{xt})) = \frac{N_{xt}}{N_{dt}} = z_{min}^k \left( \frac{k}{k + 1 - a} \right)^{k/(a-1)} z_{xt}^{-k} \\
(1 - G(z_{xt}^*)) = \frac{N_{xt}^*}{N_{dt}^*} = z_{min}^k \left( \frac{k}{k + 1 - a} \right)^{k/(a-1)} z_{xt}^*_{-k} 
\]

(A.108)  

(A.109)

7. **Number of Firms:**

\[
N_{dt} = (1 - \delta)(N_{dt-1} + N_{et-1}) \\
N_{dt}^* = (1 - \delta)(N_{dt-1}^* + N_{et-1}^*) 
\]

(A.110)  

(A.111)

8. **Euler Equation for Domestic Bonds:**

\[
C_t^{-\gamma}(1 + nB_{t+1}) = (1 + r_{t+1})^\beta E_t C_{t+1}^{-\gamma} \\
C_t^{*-\gamma}(1 - nB_{st+1}) = (1 + r_{t+1}^*)^\beta E_t C_{t+1}^{*-\gamma} 
\]

(A.112)  

(A.113)

\(^{13}\)To generate this condition, we utilize the cutoff condition, \(C_t(\varepsilon_{ft} \alpha_{kt})^\rho_{ftk}^\beta \rho_{ftk}^{\gamma - b} Q_t \rho_{ftk}^{1-a} = \frac{w_t F_{xt}}{Z_t} \), the useful transform, \(z_{xt} = \left( \frac{k}{k+1-a} \right)^{1/(1-a)} z_{xt} \), and the definition for exporter profit, \(\tilde{d}_{xt} \).
9. Euler Equation for Foreign Bonds:

\[ Q_tC_t^{-\gamma}(1 + nB_{t+1}) = (1 + r_{t+1}^*)\beta E_t Q_{t+1}C_{t+1}^{-\gamma} \]  
(A.114)

\[ Q_t^{-1}C_t^{s-\gamma}(1 - nB_{t+1}) = (1 + r_{t+1})\beta E_t^* Q_{t+1}^{-1}C_t^{s-\gamma} \]  
(A.115)

10. Euler Equation for Mutual Fund Shares:

\[ \tilde{v}_t = \beta(1 - \delta)E_t(C_{t+1}/C_t)^{-\gamma}(\tilde{d}_{t+1} + \tilde{v}_{t+1}) \]  
(A.116)

\[ \tilde{v}_t^* = \beta(1 - \delta)E_t^*(C_{t+1}^*/C_t^*)^{-\gamma}(\tilde{d}_{t+1}^* + \tilde{v}_{t+1}^*) \]  
(A.117)

11. Labor Market Clearing\(^{14}\):

\[ C_t^{-\gamma/\lambda}w_t^{1/\lambda} = \frac{a - 1}{w_t}(N_{Dt}\tilde{d}_{dt} + N_{xt}\tilde{d}_{xt}) + \frac{(aN_{xt}F_{Xt} + N_{Et}F_{Et})/Z_t}{aN_{xt}^*F_{Xt}^* + N_{Et}^*F_{Et}^*/Z_t^*} \]  
(A.118)

\[ C_t^{s-\gamma/\lambda}w_t^{s1/\lambda} = \frac{a - 1}{w_t^*}(N_{Dt}^*\tilde{d}_{dt}^* + N_{xt}^*\tilde{d}_{xt}^*) + \frac{(aN_{xt}^*F_{Xt}^* + N_{Et}^*F_{Et}^*)/Z_t^*}{aN_{xt}F_{Xt} + N_{Et}F_{Et}/Z_t} \]  
(A.119)

12. Net Foreign Assets:

\[ 2(1 + r_{t+1})B_t + 2(1 + r_{t+1}^*)Q_tB_{st} = [C_t - Q_tC_t^*] + [N_{et}\tilde{v}_t - Q_t N_{et}^*\tilde{v}_t^*] + 2[B_{t+1} + Q_t B_{st+1}] \]

\[ -[w_t l_t - Q_t w_t^{l*}] - [N_{dt}\tilde{d}_{dt} - Q_t N_{dt}^*\tilde{d}_{dt}^*] \]

\[ -[N_{xt}\tilde{d}_{xt} - Q_t N_{xt}^*\tilde{d}_{xt}^*] + [K_{t+1} - Q_t K_{t+1}^*] \]  
(A.122)

\[ -(1 - \delta_K)[K_t - Q_t K_t^*] - [\rho_{Kt} K_t - Q_t \rho_{Kt}^* K_t^*] \]

\(^{14}\)When labor is supplied inelastically, we set the left-hand side of the equations to 1.
13. **Demand for Capital in Period t:**

\[
K_t = \frac{C_t}{A_t} \left( \frac{\rho_{Mt}}{\rho_{Kt}} \right)^b \tag{A.123}
\]

\[
K_t^* = \frac{C_t^*}{A_t} \left( \frac{\rho_{Mt}^*}{\rho_{Kt}^*} \right)^b \tag{A.124}
\]

14. **Supply of Capital in Period t+1:**

\[
C_{t-1} = \beta E_t (\rho_{Kt+1} + (1 - \delta_K)) C_{t+1}^{-1} \tag{A.125}
\]

\[
C_{t}^{* -1} = \beta E_t (\rho_{Kt+1}^* + (1 - \delta_K)) C_{t+1}^{*-1} \tag{A.126}
\]

**A.2.2 Investment Volatility and Shocks to the Production of Final Goods.**

To see that there is an increase in investment volatility when there are shocks to the production of final goods, we return to the equation governing conditional demand for capital: 

\[
K_t = \frac{C_t}{A_t} \left( \frac{\rho_{Mt}^1}{\rho_{Kt}^1} \right)^b \tag{A.123}
\]

Recall the the price index for consumption is given as 

\[
P_{Ct} = \frac{P_{Mt}^b P_{Kt}^{1-b}}{A_t (1-b)^b} \tag{A.123}
\]

We can rewrite this equation as 

\[
A_t (1-b) P_{Ct} = P_{Mt}^b P_{Kt}^{1-b} (1-b)^b b^{-b} \tag{A.123}
\]

In its nominal form, capital demand is 

\[
K_t = C_t A_t^{-1} P_{Mt}^b P_{Kt}^{1-b} (1-b)^b b^{-b} \tag{A.123}
\]

Multiplying and dividing by \(P_{Kt}\) and substituting in the expression for the price index yields 

\[
K_t = C_t (1-b) \rho_{Kt}^{-1} \tag{A.123}
\]

In this form, the demand for capital is expressed as a function of aggregate consumption and the price of capital only. A positive productivity shock leads directly to an increase in consumption, resulting in a rise in capital demand. Since \(K_t\) is a state variable, at time \(t\) the amount of capital supplied is fixed, therefore a positive productivity shock results directly in an increase in capital prices.

Since the shock, \(A_t\), is persistent, households expect there to be higher capital prices in the future. An increase in \(E_t \rho_{Kt+1}\) results in an increase in the amount consumers
are willing to save, resulting in higher $K_{t+1}$. Investment spending on capital, $K_{t+1} - (1 - \delta_K)K_t$, thus increases.