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Linking snowpack microphysics and albedo evolution

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[1] Snow aging causes reflectance to vary significantly on timescales of days. This variability influences the strength of snow albedo feedback and can affect the timing of snowmelt. However, climate models have yet to incorporate important controls on snow aging and albedo evolution. We develop a physically based model that predicts evolution of dry, pure snow specific surface area, and apply aspherical ice particle theory to link these results with albedo evolution. This is the first theoretical study to quantify the relative roles of initial size distribution, vertical temperature gradient, and snow density in snow albedo evolution. Vapor diffusion caused by curvature differences causes rapid albedo decay in the first day following snowfall. Vertical temperature gradient generally dominates grain growth processes afterward but is modulated by snow density, irregularity in particle spacing, and temperature. These processes operate as a coupled system, which we uniquely represent without abrupt transitions between regimes. Model results agree very well with measurements of isothermal snow evolution and are within reasonable range of temperature gradient observations. We show that different snow state regimes cause albedo of nonmelting snow surfaces with identical initial albedo to vary by 0.12 or more after 14 days. Lack of quality observational data illuminates the need for well-controlled snow studies that simultaneously monitor specific surface area, temperature gradient, and albedo. Accounting for snow aging processes, especially temperature gradient, will improve understanding and assessment of snow albedo feedback and climate sensitivity. The modeling framework we develop will also be useful for air-snow chemistry studies that consider specific surface area.


1. Introduction

[2] The land surface plays an integral role in the planetary radiation budget. Snow is highly reflective and changes to its optical properties and spatial coverage modulate climate through snow albedo feedback [e.g., Budyko, 1969; Yang et al., 2001]. Slight changes in snow reflectance can double or halve the absorbed radiation, and many studies show snow to be a rapidly evolving medium [e.g., McGuffie and Henderson-Sellers, 1985; Aoki et al., 2003; Pirazzini, 2004]. This evolution is an important consideration in global climate models (GCMs), where energy estimation errors due to poor radiative representation can affect the timing of snowmelt and then amplify biases through snow-albedo feedback [Flanner and Zender, 2005].

[3] Previous studies account for the role of grain growth on albedo evolution only with empirical representations [e.g., Verseghy, 1991; Marshall and Oglesby, 1994; Douville et al., 1995; Loth and Graf, 1998]. Marshall [1989] parameterizes snow albedo for use in climate models, including a description of the evolution of snow grain size in dry and melting snow. The parameterization describes a constant growth rate for the first 2 weeks after snowfall, based on limited grain-size measurements in polar, surface snow [Stephenson, 1967; Warren et al., 1986]. Lack of observational data at the time prohibited her from deriving a temperature-dependence for grain growth during this initial growth phase. Using model results and recent observations, we will show that initial grain growth is nonlinear and depends on snowpack temperature, initial size distribution, vertical temperature gradient (TG), and snow density.

[4] Three recent studies demonstrate that representing ice media composed of nonspherical particles with a collection of spheres that conserves the total volume and total surface area (but not the total number of particles) yields predictions of hemispheric radiation fluxes typically within about 5% accuracy [Grenfell and Warren, 1999; Neshyba et al., 2003; Grenfell et al., 2005]. Suggested in earlier works [Bryant and Latimer, 1969; Wiscombe and Warren, 1980; Pollack and Cuzzi, 1980], this equal-V/S theory paves the way for an extremely powerful simplification that can be utilized when considering snow albedo evolution in GCMs, where generally only hemispheric fluxes are considered. It implies that if the specific surface area (SSA, \(S\), units of surface area per mass) of a snowpack is known, the snow can be represented optically with a collection of spheres, or effec-
tive radius ($r_v$), that conserves the snow’s volume to surface area ratio, regardless of the snow’s crystal habits. While this theory is of less use when directional reflectance is an important consideration [e.g., Dozier, 1989; Painter and Dozier, 2004], it can be utilized for estimation of the column energy budget in climate models. In support of this theory, we have found that snow reflectance predicted by different lognormal distributions of spheres which have the same volume to surface area ratio (but different mean radii, $r$) are nearly identical over the entire solar spectrum.

[5] Mean grain size of snowpacks generally increases with time, reducing albedo, especially in the near-infrared (near-IR) spectrum [e.g., Wiscombe and Warren, 1980]. Following snowfall and immediate mechanical deformation [Jordan, 1991], five primary processes govern the evolution of grain size. First, differences in curvature of the particles cause slight vapor density gradients via Kelvin’s Law [e.g., Colbeck, 1980; Arons and Colbeck, 1995]. This process operates in isothermal snow, and can dominate grain growth on short timescales in fresh snow. Second, macroscopic TG in the snow causes sharp intergranular vapor density gradients and bulk vapor diffusion through the ice matrix [e.g., Marbouty, 1980; Colbeck, 1983a; Gubler, 1985; Sturm and Benson, 1997], inducing temperature gradient growth. Third, snow subject to melting and refreezing experiences very dynamic growth as liquid H$_2$O is redistributed among the grains. Fourth, wind ventilation in surface snow also transports vapor. Finally, theory [Zhang and Seneibel, 1995; Colbeck, 2001] and recent observations using scanning electron microscopy [Rosenthal and Saleta, 2006] indicate that sintering may be an important mechanism for reducing snow SSA in low TG environments. We treat the first two effects in this study. We will utilize empirical representations of wet snow metamorphism [Brun, 1989; Marshall, 1989] for future model development.

[6] Snow albedo can also be strongly influenced by the accumulation of absorbing aerosols such as dust or soot [e.g., Warren and Wiscombe, 1980; Hansen and Nazarenko, 2004]. We neglect aerosols here, although the current study is a necessary precursor to understanding soot-albedo forcing because of the sensitivity of the forcing to snow grain size [Warren and Wiscombe, 1980]. In a coupled snow-aerosol model, aerosols will influence snow heating rates and affect grain growth through physically realistic means.

[7] The goal of this study is to apply basic microphysical principles to predict the evolution of dry snow SSA. Combined with equal-V/S theory, this will facilitate more realistic representation of snow albedo evolution. We prescribe snow temperature, temperature gradient, and density, which are all prognostic variables in many land surface models [e.g., Oleson et al., 2004]. Thus our microphysical module could be coupled to existing snow climate models [e.g., Jordan, 1991] without changing the bulk thermodynamics. Developing a full thermodynamic snow model is beyond the scope of this study. Our parameterization will be constrained by observation and be suitable for snowpack studies across a range of spatial scales.

2. Theory and Methods

[8] Vapor diffusion causes complex morphological changes to snow grains, forming intergranular bonds, faceted depth hoar crystals, and other complex shapes [e.g., Sturm and Benson, 1997]. Several studies have attempted to model dry snow metamorphism, accounting for some shape evolution in order to understand mechanical and thermal snow properties, with a motivation of understanding avalanche formation [Gubler, 1985; Brown et al., 2001; Lehning et al., 2002]. Because our goal is to predict evolution only of snow SSA and albedo, we adopt a more simplified approach to understanding grain growth, developing a one-dimensional representation of a collection of ice spheres.

[9] Snow aging enhances our Snow, Ice, and Aerosol Radiative (SNICAR) model [Flanner and Zender, 2005], which represents radiative transfer in the snowpack. SNICAR is a multilayer two-stream model based on Wiscombe and Warren [1980] and Toon et al. [1989] that treats snow as a collection of ice spheres. It obtains Mie parameters (single scattering albedo, extinction coefficient, and asymmetry parameter) for any lognormal size distribution from a lookup table computed offline. The model depends on vertically resolved effective radius ($r_v$), solar zenith angle, snow depth and density, direct and diffuse incident radiation, bare surface reflectance, and concentrations of absorbing impurities. We use 470 radiative bands in the solar spectrum (0.3–5.0 µm). In this study, we assume direct and diffuse incident fluxes that are typical of midlatitude winter.

2.1. Curvature Growth

[10] We begin with general theory of diffusional growth of spherical ice particles. All symbols discussed here are listed in the notation section. Fick’s Law, in the absence of any convection, describes diffusion of vapor through air in the presence of a vapor density gradient, $dp_v/dz$ as

$$J_v = -D_v \frac{dp_v}{dz} \quad (1)$$

where $D_v$ is the diffusivity of water vapor in air and is dependent on temperature [Pruppacher and Klett, 1998]. A convection term (simply wind vector multiplied by vapor density) is sometimes included in equation (1), but we neglect it in this study because of large uncertainty about circulation processes within the snowpack. We note, however, that wind has competing effects on albedo evolution. High sublimation rates and delayed settling of the finest suspended crystals from wind-entrained snow leave a surface composed of small crystals [Grenfell et al., 1994]. Conversely, wind accelerates grain growth by circulating vapor quickly through surface snow [Cabanes et al., 2003].

[11] Assuming an ambient vapor density, $\rho_{v,\text{amb}}$ and vapor density $\rho_{v,s}$ at the particle surface, the steady-state concentration profile at radial distance $x$, derived from the diffusion equation, is [e.g., Seinfeld and Pandis, 1998]:

$$\rho(x) = \rho_{v,\text{amb}} - \frac{r}{x} (\rho_{v,\text{amb}} - \rho_{v,s}) \quad (2)$$

where $r$ is the particle’s radius. The mass growth rate of a particle is

$$\frac{dm}{dT} = 4\pi r^2 D_v \frac{dp_v}{dx} \quad (3)$$
Combining equations (2) and (3), we get the general form of the steady-state growth equation for motionless aerosols employed in cloud and snow physics [e.g., Colbeck, 1983a; Pruppacher and Klett, 1998; Seinfeld and Pandis, 1998]:

\[
\frac{dm}{dT} = 4\pi D_v (\rho_{i,\text{amb}} - \rho_{s,\text{i}})
\]  

(4)

The difference between ambient vapor density and vapor density at the particle surface drives growth or sublimation of the ice particle. In the continuum regime, \(\rho_{s,\text{i}}\) is assumed to be in constant equilibrium with the particle surface during growth because growth progresses hundreds of times more slowly than diffusion to the particle surface [Seinfeld and Pandis, 1998]. Colbeck [1983b] also discusses why surface kinetic effects are small. Thus neglecting any solute effects, \(\rho_{s,\text{i}}\) is a function only of particle temperature and radius of curvature. For nonspherical ice shapes, the term \(4\pi r\) may be replaced with an equivalent “capacitance” for the shape, derived from electrostatic theory [e.g., Pruppacher and Klett, 1998], but these solutions are nontrivial [Chiruta and Wang, 2003].

[12] Kelvin's Law demonstrates that equilibrium vapor pressure over curved surfaces exceeds that over planar surfaces [e.g., Pruppacher and Klett, 1998]:

\[
\rho_i(r, T) = \rho_{eq} \exp \left( \frac{2\gamma}{R_i T \rho_i} \right)
\]  

(5)

where \(\rho_{eq}\) is the saturation vapor pressure over a planar surface, \(\gamma\) is the surface tension of ice against air, \(R_i\) is the specific gas constant for vapor, \(T\) is the system temperature, and \(\rho_i\) is the density of ice. We use \(\gamma = 0.109\) J m\(^{-2}\) from Pruppacher and Klett [1998]. Corresponding vapor density can be easily found with the Ideal Gas Law. The surface saturation ratio \((\rho_i/\rho_{eq})\) is only about 1.021 and 1.002 for \(r = 0.1\) \(\mu\)m and \(r = 1\) \(\mu\)m, respectively, and is very close to 1 for \(r > 10\) \(\mu\)m. While such small grain sizes are atypical of snow, fresh snow typically has branch dendrites with sharp curvature. Thus the Kelvin Effect is an important consideration in fresh snow [Colbeck, 1980, 1983a] but otherwise does not contribute to significant vapor density gradients.

[13] As sublimation or condensation occurs on a particle, latent heat is released or absorbed, altering the particle temperature. This temperature change has the effect of slowing both sublimation and condensation rates. An analytic approximation is derived for a particle’s mass rate of change which accounts for the latent heat effect [e.g., Rogers and Yau, 1994; Seinfeld and Pandis, 1998]. We define it here in terms of the environmental vapor pressure \(P_{amb}\):

\[
\frac{dn}{dT} = \frac{4\pi r \rho_{amb} \rho_i(r, T)}{\rho_{eq}} \left( \frac{1}{\rho_i T} - 1 \right) \rho_i \frac{\rho_i T}{\rho_{eq} D_v}
\]  

(6)

where \(K\) is the temperature-dependent thermal conductivity of air [Seinfeld and Pandis, 1998], and \(L\) is the latent heat of sublimation. Relative to equation (4), this approximation predicts differences in SSA of only about 4% after 14 days. [14] The key challenge, especially for TG conditions, is the determination of \(P_{amb}\). We do not know of any measurements of relative humidity inside the snowpack. However, air in surface snow is well-mixed with the lower atmosphere and thus likely has a similar vapor density. Indeed, seasonal sublimation totaling 15% of snowfall is observed in the Colorado Front Range [Hood et al., 1999]. During night, vapor saturation can induce frost deposition of small, ornate crystals, brightening the surface [Pirazzini, 2004]. In sub-surface snow, we expect the interstitial pore space to be consistently near saturation, given the high density of solid surface. In a coupled snow-atmosphere model, \(P_{amb}\) could be predicted for surface snow from atmospheric conditions. However, in this model we assume it is a volume-weighted mean of the equilibrium vapor pressures of all snow grains, as suggested by Adams and Brown [1982, 1983]:

\[
P_{amb} = \int_0^\infty p_i(r, T) r^2 P(r) \, dr
\]  

(7)

where \(P(r)\) is the probability density function of particles with radius \(r\). As we will see later, this formulation also facilitates a consistent representation of TG growth.

[15] For typical size distributions of snow grains, this weighted mean predicts mean pore vapor pressure slightly greater than equilibrium with respect to planar ice. Thus the smallest grains sublimate, while larger grains slowly grow. This formulation does not conserve mass (total ice mass only decreases with time, however), but as described earlier, the goal of this model is to predict SSA evolution using prescribed snow state variables. Furthermore, modeling the system as a closed box is made difficult by the fact that ice mass is about five orders of magnitude greater than vapor mass for typical snow density and temperature. We found that preventing numerical oscillations in pore vapor pressure requires model timestep on the order of \(10^{-3}\) s, starting from nonequilibrium conditions. In reality, however, sublimated vapor slightly raises local pore vapor pressure, inducing deposition on neighboring surfaces, including concave necks that bond sintered grains [Miller, 2002; Miller et al., 2003]. Incorporation of geometry with negative radius of curvature would enhance the Kelvin Effect. But the geometry suggested by Miller [2002] predicts concave ice volume that is a very small fraction of total ice volume and would hardly affect \(P_{amb}\) with our formulation.

[16] We assume a lognormal distribution of grain radii with initial geometric standard deviation \(\sigma_g\) and number-median radius \(r_n\):

\[
n(r) = \frac{1}{\sqrt{2\pi}r \ln(\sigma_g)} \exp \left[ -\frac{1}{2} \left( \frac{\ln(r/r_n)}{\ln(\sigma_g)} \right)^2 \right]
\]  

(8)

where \(n(r)\) is scaled to the probability density function \(P(r)\). Our parameter of interest is \(\tilde{S}\), which is simply total surface area of the particle ensemble divided by total mass:

\[
\tilde{S} = \frac{3}{\rho_i} \int_0^\infty r^2 P(r) \, dr
\]  

(9)
Similarly, effective radius, which drives the radiative transfer model, is also a surface area-weighted radius of the ensemble, and is directly related to $S$ for any collection of particles as

$$r_e = \frac{3}{\rho_s S}$$

Finally, $r_n$ is related to $r_e$ for a lognormal distribution as

$$r_n = r_e \exp \left[ -\frac{5}{2} \ln \left( \sigma^2 \right) \right]$$

[17] The initial size distribution determines the ensemble growth rate. Broad distributions with small median radii grow quickly as small particles completely sublimate, and monodisperse distributions do not evolve at all. Small size bins disappear permanently when all of their mass sublimes, and the distribution becomes nonlognormal. Assuming a broad distribution of $r$ for fresh snow hopefully captures realistic range of surface curvatures.

2.2. Temperature Gradient Growth

[18] Temperature gradient growth is a complex and poorly understood phenomenon. General observations of particle growth rates under TG are that they (1) increase with increasing TG [Marbouty, 1980; Fukuzawa and Akitaya, 1993], probably up to some limiting value, (2) increase with increasing temperature [Marbouty, 1980] and have little dependence on TG at low temperatures [Kamata et al., 1999], (3) increase with decreasing snow density [Marbouty, 1980; Sokratov, 2001; Schneebeeli and Sokratov, 2004], and (4) decrease with time and increasing particle size [Sturm and Benson, 1997; Baunach et al., 2001].

[19] Our approach captures these observations and represents curvature and TG growth in a unified manner. If we assume saturated pore vapor pressure along the temperature gradient axis, we can solve equation (1) for $dp_v/dz$ in terms of the temperature gradient $dT/dz$ to get the macroscopic vapor flux [Baunach et al., 2001]:

$$J_v \left( T, \frac{dT}{dz} \right) = -D_v \frac{p_{wv}(T)}{R_v} \left[ \frac{L}{R_v T} - 1 \right] \frac{dT}{dz}$$

$dT/dz$ is sign-dependent, but we always refer to it as positive in this study because of model symmetry along the TG axis. Conservation of mass requires that

$$\frac{dJ_v}{dz} = -\frac{dp_v}{dt}$$

Microphysical studies either assume $dJ_v/dz = dp_v/dt = 0$ [e.g., Adams and Brown, 1983; Gubler, 1985], or just $dp_v/dt = 0$ [Baunach et al., 2001; Lehning et al., 2002]. The latter studies predict a vertical flux divergence but conserve mass by depositing all excess vapor, equaling $dJ_v/dz \times \Delta z$, as ice. With this assumption, the densification of snow $(dp_v/dT)$ equals the divergence in vertical flux and is proportional to both $d^2T/\Delta z^2$ and $(dT/\Delta z)^2$ [Giddings and LaChapelle, 1962]. This approach was used by Sturm and Benson [1997] to calculate relative minima and maxima density positions in sub-Arctic snowpack, assuming measured temperature profiles.

[20] Applying this theory to grain growth, however, by distributing the excess vapor to available grains in any reasonable way, underpredicts grain growth by 1–2 orders of magnitude. Deficiency in this macroscopic approach suggests that vapor flux must occur on very small (i.e., interparticle) spatial scales. Evidence for this comes from measurements indicating that water molecules composing individual grains must sublimate and redeposit many times over during the course of a winter [Sturm and Benson, 1997]. Presumably, this deficiency is also why Baunach et al. [2001] and Lehning et al. [2002] add an intralatice vapor flux to their vertical flux divergence term in the Swiss SNOWPACK model. Realizing that interparticle vapor flux is required to achieve observed growth rates, early modeling studies have considered coupled source-sink particle configurations analogous to electrostatic capacitors [Colbeck, 1983a, 1983b; Sommerfeld, 1983; Gubler, 1985; Colbeck, 1993].

[21] Because ice conducts heat about 100 times more efficiently than air [Giddings and LaChapelle, 1962], we expect temperature gradient to be enhanced across the pore, relative to the macroscopic gradient. Therefore the top of a grain will tend to be warmer than its environment and the bottom colder, causing growth from the bottom and sublimation from the top. Observations of grains with rounded tops and faceted bottoms support this theory [Colbeck, 1983a; Sturm and Benson, 1997]. However, if we consider regular spacing between grains in a uniform vapor gradient field, all grains should have almost zero net growth resulting from TG (the only growth resulting from the slow, bulk vapor flux, equation (12)). The importance of irregular spacing for particle growth has been recognized [Colbeck, 1983a; Sommerfeld, 1983; Gubler, 1985]. Observations that only about 1 in 10 grains survive a season in a large temperature gradient [Sturm and Benson, 1997] offer strong evidence of preferential growth sites and competition for vapor. Observations of the largest crystals being surrounded by greater pore volumes [Akitaya, 1974; Colbeck, 1983a] imply greater vapor source for these particles and offer further evidence for the importance of particle spacing. Presumably, this is also why lower-density snow experiences more rapid growth [Marbouty, 1980; Fukuzawa and Akitaya, 1993]. Realizing the importance of irregularly spaced particles for growth, it is not surprising that growth occurs faster in greater temperature gradients [Marbouty, 1980; Fukuzawa and Akitaya, 1993], as enhanced vapor density gradients accentuate minute advantages in grain positioning. These realizations helped motivate the early capacitor models, but they have the burden of manually designating source and sink particles.

[22] In reality, the net growth or decay experienced by a particle depends on the sum contributions from all pore vapor sources/sinks. Our model assumes a single pore source/sink for each particle which accounts for all sources and sinks. To achieve this, we assign a single particle-pore distance vector, $\vec{h}$, to each particle, representing the vector sum of all particle-pore distances along the TG axis. Neglecting the Kelvin Effect, the sign of $\vec{h}$ determines growth or sublimation, and the magnitude determines mass rate of change, as greater spacings imply greater vapor
pressure differences. In a regular-packed lattice, \( \tilde{h} \) would be zero for every particle because each particle would have equally strengthened sources and sinks (again neglecting the small bulk flux from equation (12)), and only curvature growth would occur. To account for heterogeneous particle positioning, we synthesize Gaussian distributions of \( \tilde{h} \) for each particle size, with means equal to zero.

[23] What is the standard deviation of \( \tilde{h} \)? It is directly related to interparticle spacing variability, but lacking observations of such, we define a tunable parameter, \( \phi \), representing the degree of irregularity in particle packing, to scale the standard deviation of \( \tilde{h} \) to the mean particle spacing, \( \bar{s} \). The mean spacing between particle boundaries depends on snow density \( (\rho_s) \) and particle size as

\[
\bar{s}(r, \rho_s) = \left( \frac{4\pi^3 \rho_s}{3p_s} \right)^{1/3} - 2r
\]

These ideas conform with Colbeck [1993], who considers distributions of the normalized quantity \((a + 2r)/r - 2\). If we assume the same distribution of this quantity applies to all particle sizes, then mean spacing and standard deviation are related by the same scaler quantity for every particle size. With these arguments, we define a Gaussian probability density function of \( \tilde{h} \), given particle size and snow density, \( P(\tilde{h}|r, \rho_s) \), which has zero mean and standard deviation \( \phi \bar{s} \).

We can see that \( \bar{s} \to 0 \) as \( \rho_s/ \rho_0 = \pi/6 \). Therefore TG growth ceases at the limit \( \rho_s = 480 \text{ kg m}^{-3} \). Snow densities this high are rare in seasonal snowpack. Our limit is greater than the observed limit of 350 kg m\(^{-3}\) for TG growth forms [Marbouty, 1980], but our model predicts very slow growth at high densities.

[24] Having defined a representative particle-pore parameter \( \bar{h} \), we assume the pore vapor density is the mean of the equilibrium vapor densities at the top and bottom of the pore [Adams and Brown, 1982, 1983; Colbeck, 1983b]. This stems from the assumption that on small spatial scales, \( d_0/\bar{z}_2 = 0 \), and therefore, neglecting minuscule change in \( D_v \), \( dV_{\rho_s}/d\bar{z}_2 = 0 \) (equations (1) and (13)). Considering nonzero values of these terms, however, would alter our growth rates very little, as described above. Maintaining consistency with our curvature model, the equilibrium vapor densities at either pore boundary are also volume-weighted means of the ensemble of particle equilibrium vapor densities [Adams and Brown, 1982, 1983]. Then, the ambient pore vapor pressure, respective to each particle size and particle-pore spacing, is

\[
p_{amb}(r, \tilde{h}) = \frac{1}{2} \left( T - \tilde{h} \frac{dP}{\bar{z}} \right) + \frac{1}{T} \int_0^\infty \frac{p_0(\tilde{h}|r, T) r^3 P(r) \, dr \, T}{\int_0^\infty p_0(\tilde{h}|r, T) r^3 P(r) \, dr} - 2h \frac{dP}{\bar{z}}
\]

[25] Note that \( \tilde{h} \) designates vertical distance from pore center to particle center (rather than particle boundary) to account for the enhanced TG across the pore [Colbeck, 1983a], discussed above. Particle centers and pores at the same vertical level \( (\tilde{h} = 0) \) are at the same temperature, and no vapor diffuses between them. With a TG of zero, equation (15) reduces exactly to equation (7), irrespective of grain size. Thus we have a unified expression for ambient vapor pressure that includes the Kelvin Effect and TG effects. With \( p_{amb} \) determined, equation (6) drives the growth or sublimation of all particles. While the mean particle-pore spacing is zero for all particle sizes, appreciable growth of the ensemble occurs because the sublimating particles disappear completely, leaving behind only growing ones. In the studies described below, we use a timestep of 3600 s, 200 size bins, and 40 spacing bins per size bin.

3. Results and Discussion

[26] In this section we compare predictions by SNICAR with observations of isothermal snow SSA evolution and grain size evolution in snow with temperature gradient. Then, we show dependence of snow albedo evolution on snow properties, and compare SNICAR albedo with one 10-day observational time series. Finally, we discuss a simple and effective parameterization of SSA evolution suitable for climate models and air-snow chemistry studies [e.g., Domine and Shepson, 2002].

3.1. Isothermal SSA Evolution

[27] We first compare model predictions of isothermal growth with recent controlled laboratory experiments from Legagneux et al. [2004]. They gathered snow as it was falling and stored it at liquid nitrogen temperatures to prevent grain growth before measurement. During the experiment, they kept the snow uniformly at \(-15^\circ\text{C}\), and observed SSA evolution by measuring methane adsorption. They provide a physical basis for representing time-dependent SSA with an equation of the form:

\[
\tilde{S}(t) = \tilde{S}_0 \left( \frac{\tau}{t + \kappa} \right)^{(1/\kappa)}
\]

where \( \tilde{S}_0 \) is the initial SSA, and \( \tau \) and \( \kappa \) are empirical parameters. As we show later, this function also robustly fits model predictions over a range of temperature, TG, and density.

[28] We compare measurement and model results using different initial size distribution widths \( (\sigma_g) \). Legagneux et al. [2004] provide best-fit parameters of equation (16) for their measurements, which we reproduce in Table 1. We set \( \tilde{S}_0 \) and \( T \) to match the snow samples. Figure 1 shows model results against observation for their three fresh snow samples. SNICAR reproduces observed SSA decay from samples 1 and 2 quite well using \( \sigma_g = 2.3 \) but struggles to capture the long-term decay manifested in sample 3. Our choices of \( \sigma_g \) are within reasonable range of observed \( \sigma_g \). Using data provided by Teruo Aoki, we fit lognormal distributions to measurements of thousands of snow grains from four different snow samples [Aoki et al., 2000]. The best-fit values of \( \sigma_g \) for the four collections are 1.75, 1.80, 1.78, and 2.20. The snow studied by Aoki et al. [2000] was at least a day old, however, and we expect the size distribution to narrow with time, as small grains disappear. We also expect real variability in \( \sigma_g \) for fresh snow. Furthermore, we implicitly account for any sintering with our choice of \( \sigma_g \). Finally, we are likely accounting for the greater range of curvatures in real, aspherical grains by
assuming a broader distribution of spherical grains. The robustness of modeling SSA evolution with spheres must be tested against observations under variable snow temperatures though.

[29] Conditions which favor rapid curvature growth are wide size distributions of small particles. In Figure 1, SSA decreases rapidly during the initial day or two following snowfall, and subsequently tapers off as the distribution narrows and mass becomes concentrated with larger grains. Grain growth in the first two days has a strong dependence on $\sigma_g$, while growth after about day 3 has little dependence on $\sigma_g$. These model results are also supported by observations of temporal decrease in grain curvature of fresh snow [Fierz and Baunach, 2000].

### 3.2. Temperature Gradient Evolution

[30] Snow can be subject to TG well in excess of $100 \text{ K m}^{-1}$ [Fukuzawa and Akitaya, 1993; Sturm and Benson, 1997]. Cold, clear-sky nights favor large gradients, as strong radiative emission cools the snow surface more than the lower atmosphere, while snow at depth can remain near the melting temperature. With a goal of understanding avalanche formation, several studies have measured grain growth of high density, large-grained snow (characteristic of basal snow) subject to large TG over long time periods [Marbouty, 1980; Sturm and Benson, 1997; Baunach et al., 2001; Lehning et al., 2002]. These conditions induce depth-hoar formation, which is mechanically weak. Fukuzawa and Akitaya [1993], however, show that depth hoar can form very rapidly in surface snow.

[31] We compare model predictions with Fukuzawa and Akitaya [1993]. In laboratory studies, they induced temperature gradients from 150 to 300 K m$^{-1}$ in low-density snow (80–100 kg m$^{-3}$) made with an ice-slicer. They maintained a mean temperature of $-16^\circ\text{C}$ at the sampling depth (1 cm). They report mean diameter, $d$, as that of spheres with equal cross-sectional area as the photographed crystals, and note that this method can lead to high estimation biases. Experiments were conducted for up to 50 hours. We replicated these experimental conditions for all temperature gradients with SNICAR, using different values of $\phi$, and present a scatterplot of modeled versus observed mean radius in Figure 2.

[32] Fukuzawa and Akitaya [1993] observe highly linear growth rates, whereas SNICAR predicts more rapid initial growth that tapers off. On the basis of our isothermal snow analysis, we used $\sigma_g = 2.3$, while snow produced by an ice-slicer may be more homogeneously sized. However, we do not attribute the nonlinear growth evolution to curvature effects, as a sensitivity study with monodisperse grain size showed only slightly more linear growth. The large TG of these studies overshadows any curvature effects, except in the first couple of hours. Interestingly, similar nonlinear growth functions have been observed in long-term, high TG studies [Sturm and Benson, 1997; Baunach et al., 2001], as mentioned above. Nonetheless, model-measurement agreement is quite good when we assume $\phi = 5$. Also, while we must use mean radius, $r$, for comparison with Fukuzawa and Akitaya [1993], we emphasize that it is not the parameter of interest, having little bearing on snow radiative properties. In fact, the time-progression of $r$ and $r_e$ can be inversely related if mass transfer is skewed towards one end of a broad size distribution. Hence model-measurement agreement of $r$ is no guarantee that SNICAR predicts realistic albedo evolution. Fukuzawa and Akitaya [1993] is, however, the most relevant and comprehensive observational study on TG growth that we are aware of.

[33] We also compare model predictions with two long-term laboratory observations, presented in Table 2. These studies examine growth in denser, larger-grained snow. They have less relevance to surface snow but nonetheless offer some insight into SNICAR’s performance. Baunach et al. [2001] use the same equal-area method for determining grain size as Fukuzawa and Akitaya [1993] and Lehning et al. [2002] publish grain size referring to the greatest extension of the grain. In the long term, $\phi = 7$ provides better agreement with these data, but the measurement

### Table 1. Parameters $\hat{S}_{0b}$, $\kappa$, and $\tau$ for Observations of Fresh Snow Evolution From Table III of Legagneux et al. [2004]

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\hat{S}_{0b}$ m$^2$ kg$^{-1}$</th>
<th>$\tau$, hours</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>87</td>
<td>7.1</td>
<td>4.6</td>
</tr>
<tr>
<td>2</td>
<td>100.7</td>
<td>10.2</td>
<td>3.6</td>
</tr>
<tr>
<td>3</td>
<td>59.2</td>
<td>12.5</td>
<td>4.1</td>
</tr>
</tbody>
</table>

![Figure 1](https://example.com/image1.png) **Figure 1.** Comparison of model predictions of isothermal specific surface area evolution with measurements from Legagneux et al. [2004]. The three panels each show one observed time series and three modeled time series, assuming different initial size distribution widths. Model initial effective radius is chosen to match initial observed SSA.
technique of Lehning et al. [2002] gives greater radius that the mean radius that we model.

[34] Finally, we compared SNICAR predictions with recent observations of SSA evolution under TG conditions [Schneebeli and Sokratov, 2004]. They use X-ray computed microtomography (XMT) to observe three-dimensional snow microstructure undergoing TG metamorphism. While this technique holds excellent promise for understanding the physics of crystal metamorphism, as sublimation and deposition on individual crystals are observed real-time, SSA evolution was not the focus of this study. Unfortunately, SSA deduced from XMT depends on scan resolution, so results from this method are inconsistent with the gas adsorption technique [Legagneux et al., 2004]. We found best agreement with their results using $3 < \phi < 5$, but hesitate to place much emphasis on XMT observations until more controlled experiments of fresh snow SSA evolution are needed to realistically assess SNICAR’s predictions of TG growth in the context of albedo evolution. Placing the heaviest emphasis on Fukuzawa and Akitaya [1993], and considering a mean value of the other studies, $\phi = 5$ is a reasonable assumption. We assume this value for the rest of the study, but should reassess it as future observations becomes available.

3.3. Snow Albedo Evolution: Model Sensitivity to Physical Parameters

[36] In this section we use SNICAR to examine the influence of $\sigma_g^*$, temperature, TG, and snow density on snow albedo evolution. Isolating these parameters also helps us assess if SNICAR captures the basic observations of TG growth listed in methods. Although $r_e$ is most influential on near-IR albedo (0.7–5.0 $\mu$m), we only examine broadband albedo (0.3–5.0 $\mu$m). Grain size varies with snow depth, influencing bulk snow albedo [e.g., Grenfell et al., 1994], but here we assume an optically thick snowpack of uniform time-evolving effective grain size. SNICAR predicts broadband albedo variation of only 0.0075 when $r_e$ varies from 50–500 $\mu$m beneath a 5 mm LWE layer with $r_e = 50 \mu$m. Thus assuming a homogeneous, optically thick snowpack is reasonable for fresh snowfall on top of existing snow. However, we expect $r_e$ time evolution to vary within a fresh snow layer in a strong surface TG. We assume direct incident flux with a zenith angle of 60$^\circ$. Model snowpack configurations for our four experiments are summarized in Table 3. Also listed are the initial snow albedos, corresponding to initial effective radii, $r_{e0}$. Equation (10) relates $S$ to $r_{e0}$ but we use $r_e$ in these discussions because of its common use by the radiative transfer community.

[37] Figure 3 shows the temporal evolution of $r_e$ and albedo (plotted on different axes) for these configurations. Model Experiment A depicts isothermal snow evolution with four different initial size distributions. We see that large $\sigma_g$ drives rapid initial albedo decay. However, comparison of the two $\sigma_g = 2.3$ simulations shows that larger initial effective radii mitigate the effect that large $\sigma_g$ can
have by reducing the Kelvin Effect. Only the combination of small \( r_{e0} \) and large \( \sigma_g \) drives rapid initial albedo decay. After 14 days, however, the albedo range is only 0.04 for the given range of initial conditions.

[38] Model Experiment B demonstrates the effect of temperature on albedo evolution while holding \( \sigma_g \) and \( r_{e0} \) fixed with a modest (also fixed) TG. In contrast to the effects of \( \sigma_g \) and \( r_{e0} \), temperature differences produce widespread albedo differences with time. For this configuration and these three temperatures, the albedos after 14 days are 0.79, 0.81, and 0.85.

[39] Model Experiment C isolates the influence of TG with all other initial parameters fixed. We see that, given realistic ranges of the physical parameters, TG can be the most influential on albedo. For this range of TG, albedo and \( r_e \) range by 0.09 and 530 mm, respectively, after 14 days. In a sensitivity test with \( \Delta T = 50{\circ}C \), albedo varied by only 0.017 after 14 days under the same range of TG. Thus our model conforms with observation that TG becomes unimportant in colder snow [Kamata et al., 1999]. We attribute this behavior to the nonlinear dependence of saturation vapor pressure on temperature. Vertical vapor density gradients drive TG growth, and \( d\rho_a/dz \) decreases with decreasing temperature in near-saturation conditions because of the Clausius-Clapeyron relationship.

[40] Finally, model Experiment D shows that snow density also modulates the importance of TG. All albedo change with \( \rho_s = 480 \text{ kg m}^{-3} \) is from curvature growth, since \( a = 0 \) (equation (14)). The range of albedo after 14 days for \( 50 < \rho_s < 350 \text{ kg m}^{-3} \) is about 0.05. While Marbouty [1980] suggests that variable snow densities less than 150 kg m\(^{-3}\) do not affect TG growth, SNICAR predicts continual increasing influence as \( \rho_s \to 0 \). It may be reasonable to cap the effect of \( \rho_s \) at some low value, but given observational uncertainties and realistic snow densities, we refrain from doing so here.

### 3.4. Observed Albedo Evolution

[41] At this time we cannot conduct a meaningful comparison of model and observed albedo evolution because (1) we know of no observational studies simultaneously measuring albedo, temperature gradient, and size distribution, (2) magnitudes of the competing wind effects (ventilation and fine crystal deposition) are unknown and not included in SNICAR, (3) we do not know the importance of, or consider, nighttime frost formation of fine, “bright” crystals.

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**Figure 3.** Model parameter study illustrating the evolution of snow effective radius \( (r_e) \) and albedo evolution, isolating dependence of (a) initial size distribution, (b) temperature, (c) temperature gradient, and (d) snow density. Time evolution of effective radius is plotted in black against the left axis and broadband albedo in green against the right axis.
Figure 4. Observed and modeled albedo decay at Niwot Ridge following the 2 January 2001 snowfall event. Error bars represent one standard deviation of all measurements composing each day’s albedo change.

consistent zenith angle. We also include snow aging parameterization from the NCAR Community Land Model 3 (CLM) [Oleson et al., 2004], and NASA GISS GCM ModelE [Schmidt et al., 2006], which is based on Loth and Graf [1998], who, in turn, use albedo decay from Verseghy [1991] for dry, deep snow. CLM dry snow aging depends on snow temperature, while the nonmelting relationships described by Verseghy [1991] and Loth and Graf [1998] do not. We have included the albedo increase that CLM would prescribe for the 2 mm LWE snowfall on day 3. To reduce zenith-angle dependence, all curves depict albedo change, rather than absolute albedo. The error bars represent one standard deviation of measured albedo reduction, centered about each day’s mean albedo change. The 10-min measurements are normalized to their base albedo at time zero, and the standard deviation is derived from all 24 daily measurements. The three SNICAR predictions are of direct-radiation albedo evolution with \( \frac{dT}{dz} = 20, 40, \) and 80 K m\(^{-1}\), assuming \( \sigma_g = 2.3, \rho_s = 100 \text{ kg m}^{-3} \), and vertically homogeneous grain size (\( r_{so} = 50 \mu m \)), which is justified in this case because the snowfall event was large and rapid. SNICAR and CLM models are both driven with hourly mean air temperature, which we use as a rough surrogate for snow temperature. This assumption should cause little error for these conditions, as driving SNICAR with the mean (constant) temperature alters 10-day albedo change by \( \sim \)1%.

We make several observations here. First, the large 1-day albedo change (\( -0.03 \)) is characteristic of rapid curvature growth. We can replicate this with small TG and \( \sigma_g > 2.3 \), or with large TG. Second, \( \frac{dT}{dz} = 80 \text{ K m}^{-1} \) reproduces observed albedo decay during the first 4 days very well. Third, there is an albedo rise on Day 5 that could be explained by atmospheric- or frost- deposition of fine crystals, or noise. If deposition is the cause, grain growth of the underlying snow may proceed at a similar rate as predicted with \( \frac{dT}{dz} = 80 \text{ K m}^{-1} \). Fourth, SNICAR captures this observational trend better than the GCM parameterizations, which predict excessive albedo decay after day 3. CLM implicitly accounts for globally uniform accumulation of impurities, which is one reason for its greater predicted albedo reduction. In future GCM studies, we will account for time-dependent accumulation of impurities with online atmospheric transport and deposition.

3.5. Empirical Parameterization

Legagneux et al. [2004] propose equation (16) as an empirical representation for observed isothermal SSA evolution. We show that equation (16) robustly fits predictions of SSA evolution over a wide range of temperature, TG, and snow density. The simplicity of this equation is attractive because of the numerous size bins that SNICAR requires to capture curvature growth. Resulting computational savings open the door for its use in climate models and snow chemistry studies which utilize SSA.

We compute best-fit parameters \( \tau \) and \( \kappa \) for equation (16) to match 14-day simulated SSA over the domain \( 210 \leq T \leq 273 \text{ K}, 0 \leq \frac{dT}{dz} \leq 300 \text{ K m}^{-1} \), and \( 50 \leq \rho_s \leq 400 \text{ kg m}^{-3} \). Figure 5 depicts time evolution of SSA predicted by SNICAR and equation (16) with best-fit parameters over some of this domain. Agreement is exceptionally good, even with large TG and range of \( \rho_s \). Best-fit parameters for
the curves shown in this figure are listed in Table 4. Implementation of this method simply requires the time-derivative of equation (16) and an online lookup table retrieving best-fit parameters as a function of $T$, $dT/dz$, and $\rho_s$. The authors can be contacted for a comprehensive table.

4. Conclusions

We have developed a new, physically based model which predicts the evolution of dry snow specific surface area (SSA) and is suitable for coupling to full snow thermodynamic and air-snow chemistry models. Recent studies [Grenfell and Warren, 1999; Neshyba et al., 2003; Grenfell et al., 2005] justify use of snow SSA to obtain accurate hemispheric radiative fluxes, even for aspherical particles, thus linking our results to albedo evolution. Our model suggests that curvature-driven vapor diffusion dominates mass transfer of fresh snow under low temperature gradient. Vertical temperature gradients exceeding 20 K m$^{-1}$, however, induce vapor density gradients which otherwise dominate grain growth and albedo decay. The influence of temperature gradient is controlled by temperature, snow density, and variance of interparticle spacing.

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Model results track laboratory observations of isothermal SSA evolution very well. Predictions of temperature gradient growth compare favorably with observed mean radius evolution, but simultaneous measurements of

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$dT/dz$, K m$^{-1}$ & $\rho_s$, kg m$^{-3}$ & Snow Temperature, °C & & & \\
\hline
& & -50 & -20 & -10 & 0 \\
\hline
0 & 150 & $\tau$ & 43.6 & 7.1 & 4.5 & 3.2 \\
& & $\kappa$ & 11.4 & 6.7 & 6.1 & 5.8 \\
50 & 150 & $\tau$ & 27.5 & 47.1 & 21.0 & 11.9 \\
& & $\kappa$ & 15.3 & 1.7 & 1.8 & 1.9 \\
200 & 50 & $\tau$ & 370.6 & 5.2 & 2.5 & 1.5 \\
& & $\kappa$ & 0.9 & 1.9 & 1.9 & 1.9 \\
200 & 300 & $\tau$ & 47.2 & 35.0 & 15.5 & 8.8 \\
& & $\kappa$ & 11.7 & 1.8 & 1.9 & 1.9 \\
\hline
\end{tabular}
\caption{Best-Fit Parameters of Equation (16) for the Range of Temperatures, Temperature Gradient, and Snow Density Shown in Figure 5}
\end{table}
SSA and temperature gradient are needed for thorough model evaluation. The 14-day albedo change of dry snow with identical initial effective radii varies from $-0.01$ to $-0.13$, depending on snow conditions. Model predictions track one 10-day time series of clear-sky albedo measurements from Niwot Ridge better than two GCM parameterizations, but too little is known about the snowpack conditions to draw any definitive conclusions. Last, we show that a simple representation of SSA evolution robustly describes our model over a wide range of parameters. Its simplicity and effectiveness suggest that it could be a valuable addition to climate and snow chemistry models.

[51] Existing GCM representations of snow aging do not consider temperature gradient in albedo evolution, although this and several other studies [Marbouty, 1980; Fukuzawa and Akitaya, 1993; Sturm and Benson, 1997] show it to be very important. Investigations into the effects of blowing snow, wind ventilation, and frost formation are also needed for a thorough understanding of snow albedo evolution. This study also highlights the need for high-resolution experimental studies that simultaneously observe snow temperature gradient, SSA, accumulation of soot and dust, and albedo. Such data would provide stronger basis for defining model parameters describing snow SSA and albedo evolution. If models are to accurately predict climate changes due to greenhouse and other forcings, they must capture influences of all important processes involved in snowpack evolution.

Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Mean particle boundary-boundary spacing, m</td>
</tr>
<tr>
<td>$\tilde{a}$</td>
<td>Mean particle boundary-boundary spacing, m</td>
</tr>
<tr>
<td>$D_v$</td>
<td>Diffusivity of vapor in air, m$^2$s$^{-1}$</td>
</tr>
<tr>
<td>$d$</td>
<td>Mean particle diameter, m</td>
</tr>
<tr>
<td>$h$</td>
<td>Vertical distance from particle center to pore center, m</td>
</tr>
<tr>
<td>$J_v$</td>
<td>Vapor flux, kg m$^{-2}$s$^{-1}$</td>
</tr>
<tr>
<td>$L$</td>
<td>Latent heat of fusion, J kg$^{-1}$</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Thermal conductivity of air, J cm$^{-1}$s$^{-1}$K$^{-1}$</td>
</tr>
<tr>
<td>$m$</td>
<td>Particle mass, kg</td>
</tr>
<tr>
<td>$P$</td>
<td>Probability</td>
</tr>
<tr>
<td>$p_e$</td>
<td>Equilibrium vapor pressure at particle surface, Pa</td>
</tr>
<tr>
<td>$P_{eq}$</td>
<td>Equilibrium vapor pressure over planar surface, Pa</td>
</tr>
<tr>
<td>$P_{amb}$</td>
<td>Ambient (environmental) vapor pressure, Pa</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Specific gas constant for vapor, J kg$^{-1}$K$^{-1}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Particle radius, m</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>Mean particle radius, m</td>
</tr>
<tr>
<td>$r_e$</td>
<td>Effective radius, m</td>
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<tr>
<td>$r_{e0}$</td>
<td>Initial effective radius, m</td>
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<tr>
<td>$r_n$</td>
<td>Number-median radius, m</td>
</tr>
<tr>
<td>$S$</td>
<td>Specific surface area, m$^2$kg$^{-1}$</td>
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<tr>
<td>$S_0$</td>
<td>Initial specific surface area, m$^2$kg$^{-1}$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature, K</td>
</tr>
<tr>
<td>$z$</td>
<td>Distance along temperature gradient axis, m</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Surface tension of ice against air, J m$^{-2}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Empirical parameter for SSA evolution</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Density of ice, kg m$^{-3}$</td>
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<td>$\rho_s$</td>
<td>Density of snow, kg m$^{-3}$</td>
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<tr>
<td>$\rho_v$</td>
<td>Density of water vapor, kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_v,s$</td>
<td>Equilibrium vapor density at particle surface, kg m$^{-3}$</td>
</tr>
</tbody>
</table>

Acknowledgments. We dedicate this paper to the fond memory of Walter Rosenthal, whose enthusiastic conversations on sintering will be missed. We thank three anonymous reviewers for critical analysis and helpful comments about our methods. We thank Tetsuo Aoki for providing snow grain size measurements, Anne-Sophie Taillandier and Florent Domine for providing advice and references about SSA evolution, Michael Lehning and Charles Fierz for providing snow grain size observations, Jeff Dozier and Walter Rosenthal for comments on sintering, and Steve Warren for advice on relationships between size distributions and albedo. Data were obtained from the NSF supported Niwot Ridge Long-Term Ecological Research project and the University of Colorado Mountain Research Station. Funding for this work was provided by NSF/NCAR SGER ATM-0503148 and NASA Earth System Science Fellowship NNG05GP30H. Computations supported by Earth System Modeling Facility NSF ATM-0321380.

References


