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by

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Limited Cooperation in International Environmental Agreements

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Abstract
Governments' desire to ameliorate environmental problems may conflict with other goals. Policy levels which balance different objectives can be altered by policy changes in other countries. A decrease in the importance of the pollution problem, or an increase in its global extent, increase the likelihood that tighter environmental regulations in one region induce laxer policies elsewhere. The transboundary character and the importance of environmental externalities also affect the amount of cooperation needed to improve members' welfare in a coalition. More global pollution problems require a larger coalition. However, the critical coalition size may be larger or smaller for more severe problems.

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1 Introduction

There are over 140 international environmental agreements for the management of such resources as fisheries, river basins, Antarctica, and migratory and endangered wildlife (Barrett, 1991). Recent agreements to control pollution or reduce exploitation include the Montreal Protocol on Substances that Deplete the Ozone Layer and the Convention on Trade in Endangered Species (CITES). While most nations use the global resources and contribute to pollution problems, not all countries join environmental agreements. The Montreal Protocol, one of the more comprehensive agreements, has 100 members and CITES has 105 members (World Resources Institute, 1994-95). The 1985 Helsinki Protocol to reduce sulfur dioxide emissions, a major cause of acid rain, was signed by twenty-one European nations; thirteen nations, including Poland, Spain and the United Kingdom did not sign (The World Bank, 1992). We study the relation between international cooperation in pollution abatement and two characteristics of the pollution problem. The first of these is the importance of pollution control relative to other objectives. The second is the extent to which the problem is local or global.

Environmental externalities are significant reasons for government policies. However, non-environmental objectives may be affected by the same policies. For example, if there are labor market imperfections, the additional employment associated with increased production (and pollution) may be beneficial. Alternatively, increased domestic output may contribute to future productivity increases (through learning-by-doing) which are external to the firm, but captured largely by the domestic industry. A third example concerns imperfectly competitive world markets, where governments may want to promote the domestic industry in order to capture oligopoly rents. We refer to the non-environmental objective as “rent-shifting” because our model in the next section

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1See, Irwin and Klenow (1994) for empirical evidence on learning-by-doing in the semiconductor industry.
uses the example of imperfectly competitive world markets. However, we regard this as illustrative of a more general situation. The important point is that in addition to wanting to reduce environmental damages, governments have reasons for wanting to increase domestic production, and possibly to decrease foreign production (e.g., to raise world price). This seems like an accurate, if simplified, description of aspects of the current debate about environmental protection. When we speak of the importance of the environmental objective, we always mean “relative to the rent-shifting objective.” We assume that governments have a restricted policy menu, and attempt to meet more than one objective with each instrument.

We can think of environmental problems as existing on a continuum, from local to global. A problem (e.g. soil erosion) is local if damage generated within a country has no effect on the environment elsewhere. A problem (e.g. global warming) is global if the extent of environmental damage is independent of the source. In intermediate cases, a unit of pollution generated abroad causes less damage to a country than a unit of domestic pollution. When we speak of one problem as being “more global” than another, we mean that there is more spillover of damages, not that damages are more prevalent.

We study a non-cooperative Nash equilibrium in a policy-setting game played by a group of symmetric countries. The extent to which an environmental problem is global, and its (relative) importance, are the two key exogenous characteristics of this game. We use the model to study two types of questions. The first involves the comparative statics of the non-cooperative equilibrium, and the second, more important issue, involves the incentives for limited cooperation.

As an example of the first type of question, suppose that countries are at a non-cooperative equilibrium, and scientists discover that the environmental problem is more global than previously

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2In other situations, such as with acid rain, a unit of pollution may cause more damage in neighboring countries than the country that generates the pollution. We discuss how our model can be modified to deal with this situation.
thought. We explain why this parametric change can make the equilibrium policies either more or less environmentally friendly.

Our major interest, however, is in the effect of limited cooperation on policies. We examine the outcome when $S$ out of the total of $N$ countries join an environmental coalition. We consider two types of coalitions. The first, which we refer to as a "modest coalition," decides unilaterally to make its policies slightly more environmentally friendly; non-members make (Nash) equilibrium changes in their policies. An "ambitious coalition," on the other hand, adopts the Nash equilibrium policies in the game with non-members. The ambitious coalition model appears to be the obvious way to describe limited cooperation, since it involves an equilibrium outcome. The modest coalition model involves a non-equilibrium policy change by members. We concentrate on the modest coalition for two pragmatic reasons. First, we are able to obtain clear analytic results for that model, without resorting to numerical examples. Second, we think that model is a better description of reality. Most international environmental agreements appear to involve relatively modest change, and by definition these changes are environmentally friendly.

The formation of the coalition may induce the non-members to either tighten or weaken their environmental protection. In the latter case, there is "policy leakage." The usual leakage effect is that production and pollution migrate in response to stricter policies in one country. This effect is compounded or mitigated when other countries' policies change.

The induced changes in non-members' policies may lead to a decrease in members' welfare, relative to the initial (pre-coalition) equilibrium. We refer to the minimum coalition needed to insure that members are better-off as the "critical size.

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3One reader suggested that we rename the modest coalition a "stupid coalition." His reason is that since we do not require the coalition to choose a best response to the non-members, we are implicitly assuming that the coalition is a Stackelberg leader. Therefore we should require it to change its policies in the direction that improves its welfare. However, we are interested in describing an environmental coalition, and it seems reasonable to assume that such a coalition makes its policies more environmentally friendly. We want to know when such a coalition is consistent with an improvement in members' welfare.
We use our model to determine how the two exogenous characteristics of pollution (its global extent and relative importance) determine (i) whether policy leakage occurs, and (ii) the size of the critical coalition membership. We find that a decrease in the importance of the pollution problem, or an increase in its global extent, increase the likelihood that policy leakage occurs. Pollution problems which are more global require a larger coalition (more cooperation). However, the critical coalition size may be larger or smaller for more severe problems. These results, and their explanation, constitute the chief contribution of this paper.

The next section reviews related literature. We then describe the model and present the chief results for the case of a modest coalition in which governments use taxes. The following section considers the case where governments use quotas. We then consider an ambitious coalition. The conclusion summarizes the results and suggests directions for future work.

2 Literature Review

When governments use taxes, there is policy leakage if and only if taxes are strategic substitutes (Bulow, Geanakoplos and Klemperer, 1985). The question of whether policies are strategic substitutes or complements has been investigated in many contexts. In our model, the answer depends on the severity and global extent of the environmental problem.

The possibility that limited cooperation is disadvantageous (Salant, Switzer and Reynolds (1983), and Deneckere and Davidson (1985)), and the relation between this possibility and the strategic substitutability/complementarity of policies (Gaudet and Salant, 1991), is also widely understood. However, previous papers emphasized the importance of the policy menu (e.g., price or quantity) in determining whether limited cooperation improves members' welfare. Those papers examined the situation where there is a single externality. Where we have two externalities which pull in opposite directions (the environmental externality and that which creates the rent-shifting
incentive), the strategic characteristics of policies may change with parameter changes.\textsuperscript{4} Those parameter changes also have important effects on the benefits of cooperation.

A number of recent papers study the international implications of environmental policy, and of limited cooperation. There is a large literature on the strategic use of environmental policies under imperfect competition, including Barrett (1994), Conrad (1993), Kennedy (1994), Ulph (1994). For example, Ulph (1994) studies the equilibrium in a non-cooperative game between two countries with imperfectly competitive industries which create environmental damages. In that case, governments have both a rent-shifting and the usual Pigouvian motive for choosing taxes or quotas. We use the same framework with $N > 2$ countries; however, our objective is different, since we are interested in the welfare effects of agreements amongst a subset of countries.

The literature on limited cooperation in international environmental agreements is also substantial, including Black, Levi and deMeza (1993), Bohm (1993), Golombek, Hagem and Hoel (1994), Heal (1994) and Hoel (1994). For example, Bohm (1993), Golombek et al. (1994) and Hoel (1994) study the optimal design of a carbon tax when the policy choice by member governments may affect fuel demand in non-member countries which adversely affects the environmental quality in the member countries. We consider the relation between coalition size and welfare of members. Moreover, unlike the previous literature which addressed either local or global environmental problems separately (an exception is Kennedy (1994)), we consider both local and transnational problems in the same framework.

The problem of compliance or enforceability of international environmental agreements is an important concern in the absence of a super-national authority to check free-riding. Besides the vast game-theory literature on enforcement, Russell (1992) and Sandler and Sargent (1995) study

\textsuperscript{4}We recognize that parameter changes can lead to a change from strategic substitutability to complementarity even with a single externality. The obvious example is in a duopoly where a parameter determines whether goods are substitutes or complements in consumption.
the problem of compliance or enforceability of environmental agreements. We show that even if the agreement can be enforced at no cost, members’ welfare can still decline. Enforceability is clearly not sufficient for an agreement to be successful.

3 The Non-cooperative Equilibrium with Taxes

In this section we study the comparative statics of a partial-equilibrium, imperfect-competition model where \( N \) countries use an export tax/subsidy to increase their welfare. Welfare equals domestic industry profits (net of the transfer) minus pollution damages. (We ignore consumer welfare.) We study the symmetric subgame-perfect Nash equilibrium to a two-stage game. In the first stage, each country sets its tax/subsidy, and in the second stage industries choose output. The industry in each country is treated as a single firm which produces a homogeneous good; environmental damage depends on output.

The world price net of constant marginal production cost is \( p(Q) = a - b \sum q^i \) where \( q^i \) is the production of firm \( i \). Each firm maximizes profit, given the tax \( t^i \) and rivals’ aggregate output \( Q^{-i} \):

\[
\pi^i(q^i, Q^{-i}, t^i) = q^i p(Q) - t^i q^i, \quad \text{where} \quad Q = q^i + Q^{-i}.
\]

Firms’ decisions are strategic substitutes: each firm’s best-reply is decreasing in rivals’ output. Routine calculations show that the equilibrium level of output\(^5\) for firm \( i \) is:

\[
q^{i*} = \frac{a - N t^i + \sum_{j \neq i} t^j}{b(N+1)} \quad \text{and} \quad Q^* = \frac{Na - \sum t^i}{b(N+1)}.
\]

Firm \( i \)'s output is decreasing in own tax and increasing in rivals’ tax, and aggregate industry output decreases with any tax. This means that the absolute own-firm impact of a tax change is greater than aggregate change on the other firms. Thus, the diagonal dominance condition for stability in the quantity-setting game (Dixit, 1986) is satisfied.

Government \( i \) maximizes \( W^i \), welfare in country \( i \), taking as given rivals’ vector of taxes, \( t^{-i} \):

\[
W^i(t^1, \ldots, t^N) = q^i(a - bQ) - \frac{\gamma}{2}(q^i + \alpha Q^{-i})^2.
\]

The first term is firm \( i \)'s profit net of the transfer.

\(^5\)Quantities and price are always positive in this model.
and the second term is the damage due to emissions. The parameter \(0 \leq \alpha \leq 1\) describes the type of pollution problem. For local pollution, \(\alpha = 0\), and for global pollution, \(\alpha = 1\). As \(\alpha\) increases, the pollution becomes “more global.” (See Kennedy (1994), for a similar formulation). For some problems, such as acid rain, foreign pollution may cause more damage than domestic pollution, in which case \(\alpha > 1\). However, in that case a symmetric model is less plausible.

This model has two important features: (i) Domestic welfare depends on domestic and aggregate world production. (ii) Each government has a rent-shifting and an environmental objective. As we discussed in the Introduction, there are a number of situations that give rise to models with these characteristics.

The assumption of linearity is obviously restrictive. We studied a more general model, discussed in Appendix 2 and two footnotes, but this does not provide useful insights. Linearity allows a simple parameterization of the (relative) importance of the environmental problem and the extent to which it is global.

Solving for the symmetric Nash equilibrium in the game amongst governments, we obtain the tax:

\[
t^* = \frac{-F}{G + (N-1)H}
\]  

where \(F = -\alpha p(N-1) + \alpha(N - \alpha[N-1])(1 + \alpha[N-1])\); \(G = -2Np - (N - \alpha[N-1])^2\); and \(H = -\rho(N-1) + (N - \alpha[N-1])(1-2\alpha)\). The parameter \(\rho \equiv b/\gamma\) measures the relative importance of the rent-shifting opportunity and the pollution problem. When environmental damage is important relative to opportunities for rent-shifting, \(\rho\) is small. We obtain the well-known Brander and Spencer (1985) model by letting \(\rho \to \infty\), in which case the optimal policy subsidizes domestic production to capture oligopoly rents. As environmental damages become important, the optimal policy becomes
The equilibrium tax is 0 if and only if $\rho$ equals a critical value, which we denote $\hat{\rho} = [N - \alpha(N - 1)][1 + \alpha(N - 1)]/(N - 1)$; we obtain this expression by setting $F = 0$. Using the fact that the denominator in (1) is negative, and analyzing $\hat{\rho}$, we obtain

Remark 1. (i) For $\rho < \hat{\rho}$, $t^* > 0$. The equilibrium policy is a tax if and only if the rent-shifting opportunity is small relative to the environmental problem. (ii) The function $\hat{\rho}(\alpha, N)$ is increasing in $\alpha$ for $\alpha < 1/2$ and decreasing for $\alpha > 1/2$, and equals $N/(N-1)$ for $\alpha = 0$ or $= 1$. (iii) $\hat{\rho}$ is a decreasing function of $N$ for $\alpha$ close to 0 or 1; $\hat{\rho}$ is increasing in $N$ for $\alpha \approx 1/2$. □

Figure 1 graphs $\hat{\rho}$ for two values of $N$, with $N_2 > N_1$. (The graph of $\rho^*$ in Figure 1 is

\[ \rho \equiv \frac{b}{\gamma} \]
discussed later in the paper.) At points $A$ and $A'$, with $N = N_1$, $\rho$ is such that the rent-shifting and environmental incentives exactly balance, and the equilibrium policy is a zero tax. If the environment becomes more important (smaller $\rho$) the equilibrium policy becomes a tax. At $A$, the environmental problem is "fairly local" ($\alpha < 1/2$), i.e., the transboundary pollution is not severe; at $A'$ the transboundary problem is severe.

Suppose that we were in a world where the balance between environmental and rent-shifting incentives led to a zero equilibrium tax, and scientists suddenly announced that transboundary pollution was more of a problem than previously thought, so that $\alpha$ increases. Remark 1.ii says that if the problem was initially considered fairly local, governments would respond by imposing a tax. However, if the problem was initially considered fairly global, governments would respond by subsidizing the pollution-creating activity.

The first situation, at $A$, is what we expect. At the original equilibrium, an increase in $\alpha$ increases each country's marginal damages, which tends to make it want to reduce output, and thus to increase its tax. The second situation, at $A'$, is at first surprising. The explanation is that the equilibrium output of firms other than $i$, $Q^{-i}$, is an increasing function of $i$'s tax. For larger values of $\alpha$, country $i$ has a greater incentive to subsidize domestic production in order to reduce foreign pollution. Of course, when all countries behave in this way, we have the perverse effect that foreign-generated pollution increases for each country.

Now suppose that due to a resurgence in nationalism, $N$ increases. Country $i$ becomes smaller in relation to the rest of the world, $i$'s rent-shifting opportunities and its domestic output decrease, and aggregate output increases (for a given vector of taxes). Suppose that the combination of $\alpha$ and $\rho$ are at point $A$ in Figure 1, so that when $N = N_1$ the equilibrium policy is a 0 tax. An increase in $N$ causes the equilibrium policy to become a subsidy. Although the reduction in own-output is associated with a decrease in rent-shifting opportunities, it also means that marginal damages are
smaller. The increase in aggregate output has little effect on pollution damages, because pollution is primarily local for small $\alpha$. The net effect is to reduce the environmental incentives by more than the reduction in the rent-shifting incentives, causing the policy to become a subsidy. At $A'$, transnational pollution is important, and this can only be reduced by subsidizing domestic production. Again, an increase in $N$ causes the 0 tax to become a subsidy. At $A'' (\alpha \approx 1/2)$ foreign pollution is quite important, (so marginal damages are high) but not so important that country $i$ has an incentive to attempt to reduce foreign pollution by subsidizing domestic industry. These two effects therefore tend to offset each other. However, the loss in rent-shifting incentives associated with the increase in $N$ remains, so the equilibrium policy becomes a tax.

4 A Modest Coalition with Taxes

We now suppose that, beginning at the Nash equilibrium, a group of $S$ countries forms a "modest environmental coalition," which entails increasing their environmental tax by $dT$. The response of the remaining $N - S$ countries is endogenous. We want to know how the response of non-members to exogenous changes in coalition policies depends on the importance of the environmental damages, measured by $\rho$, and the type of pollution, measured by $\alpha$. The answer to this determines the severity of policy-leakage. If policies are strategic complements, environmentally friendly policies in one country encourage friendly policies elsewhere. However, if policies are strategic substitutes, friendly policies in one country encourage other countries to relax their environmental policies.

When coalition members use a tax of $\tau$, the equilibrium tax of each of the non-member governments ($t$) solves the following first order condition (after noting symmetry): $F + SH\tau + [G + (N -}
\[ S - 1)H|t = 0. \] This implies

\[
\frac{dt}{d\tau} = -\frac{SH}{G + (N - S - 1)H}. \tag{2}
\]

We previously noted that \( G < 0 \) because of \( i \)'s second order condition and \( G + (N - 1)H < 0 \), so the denominator in equation (2) is negative. Therefore, the sign of \( \frac{dt}{d\tau} \) equals the sign of \( H \): the policies are strategic substitutes if and only if \( H < 0 \). By setting \( H = 0 \) we obtain an implicit equation for a critical value of \( \rho \), which we denote \( \rho^*(\alpha, N) \). Analysis of \( H = 0 \) implies

Remark 2. (i) For \( \alpha < 1/2 \), \( \frac{dt}{d\tau} > 0 \) if and only if \( \rho < \rho^*(\alpha, N) \). For fairly local pollution problems, policies are strategic complements if and only if the environmental problem is sufficiently severe. (ii) For \( \alpha < 1/2 \), \( \partial \rho^*/\partial \alpha < 0 \) and \( \partial \rho^*/\partial N < 0 \). For fairly local pollution problems, an increase in the rate of spillovers or the size of the market makes it less likely that policies are strategic complements. (iii) For \( \alpha \geq 1/2 \), \( \rho^* = 0 \). For fairly global pollution problems, policies are always strategic substitutes. (iv) \( \rho^* < \rho \) for \( \alpha > 0 \). Policies are strategic complements only if the equilibrium policy is a tax. 

Figure 1 plots \( \rho^* \) for two values of \( N \), with \( N_2 > N_1 \). We first consider the case \( \alpha < 1/2 \). At point \( B \), non-members do not change their policies following a marginal change in the coalition’s tax. The rent-shifting and environmental incentives pull in opposite directions, and exactly balance. An increase in \( \rho \) means that the rent-shifting incentive increases relative to the environmental incentive. In this case, a coalition tax increase induces non-members to decrease their tax, so policies become substitutes. An increase in \( \alpha \) (making pollution more global), causes the non-members to gain more from the pollution reduction that results from the coalition’s tax increase. This decreases

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6In a general model, \( H \) is replaced by \( W_{m2} \), the change in the marginal welfare of a non-member country due to a change in the tax of a member country. \( G \) is replaced by \( W_{m1} \), the change in the marginal welfare of a non-member country due to a change in its own tax rate (see Appendix 2).
the marginal damage non-members face, causing them to relax their environmental policy. Thus, beginning at point $B$, an increase in $\alpha$ causes policies to become strategic substitutes. A necessary condition for policies to be strategic complements is that taxes are used in the original equilibrium.

An increase in $N$ causes $\rho^*$ to rotate down. A larger number of competitors/polluters decreases the range of $\rho$ for which policies are complements. As $N$ increases, the rent-shifting opportunities decrease, so it would appear that the environment would become relatively more important, and thus policies would be more likely to be complements. However, an increase in $N$ also means that production in each country is smaller, and since pollution is primarily local ($\alpha < 1/2$), this decrease means that marginal damages are much smaller. This effect is strong enough to outweigh the diminished rent-shifting opportunities.

When pollution is fairly global ($\alpha > 1/2$), policies are always strategic substitutes, regardless of the relative importance of rent-shifting and environmental concerns. This is because the two types of incentives pull in the same direction for large $\alpha$. For sufficiently global pollution problems, an increase in the coalition tax and the resulting decrease in pollution by firms in the coalition, decreases the marginal damage faced by non-members, even once their home firms have responded by increasing production. In that case, even if the rent-shifting opportunities, relative to the environmental costs, are negligible ($\rho \approx 0$), the non-members want to reduce their tax.

Our next question concerns the minimum coalition size necessary to insure that members benefit. For coalitions smaller than this critical size, cooperation is disadvantageous, because of the endogenous change in non-members' policies. We want to determine the relation between the type ($\alpha$) and severity ($\rho$) of pollution, and the critical coalition size. We take Country 1 as the representative country in the coalition, and totally differentiate its welfare, $W^1$. Using Country 1's first
order condition, and symmetry, gives

\[
\frac{dW_1^j}{dt} \bigg|_{t=t^*} < 0 \iff W_1^j((S-1) + (N-S)\frac{dt}{d\tau}) < 0
\]  

(3)

where \( W_1^j > 0 \) is the change in the welfare of country 1 due to a change in a rival’s tax, and \( dt/d\tau \) is the equilibrium response of a non-member to the change by the coalition.

Linearity of the model means that country 1’s equilibrium welfare depends on its own tax and the sum of rival’s tax. The term in brackets in (3) is the total policy response: \( S-1 \) coalition members increase their tax by \( d\tau \), and the \( N-S \) non-members change their tax by \( dt/d\tau \). From (3) we see that a sufficient condition for the coalition to be welfare improving, regardless of its size, is that the policies are strategic complements. A necessary and sufficient condition for a welfare improving coalition is that the fraction of rivals that join the coalition, \( (S-1)/(N-1) \), be sufficiently large. This fraction is obtained using (2) and (3):

\[
\frac{dW_1^j}{dt} \bigg|_{t=t^*} < 0 \iff \frac{S-1}{N-1} \gg \frac{H}{G}
\]  

(4)

Hereafter we refer to \( H/G \) as the “critical fraction.” This is the fraction of rivals that must join the coalition, in order for Country 1 to benefit from joining. We use the definition \( \alpha^* \equiv N(N + 1)/[4N + (N - 1)^2] > 1/2 \) to describe the properties of the critical fraction. We have

Remark 3. (i) \( \partial(H/G)/\partial \alpha > 0 \). More global pollution problems requires a greater critical fraction, i.e., more cooperation. (ii) For \( \alpha < \alpha^* \), \( \partial(H/G)/\partial \rho > 0 \). When pollution is “fairly local,” an increase in rent-shifting opportunity relative to the pollution cost increases the critical fraction. (iii) For \( \alpha > \alpha^* \), \( \partial(H/G)/\partial \rho < 0 \). For fairly global pollution, an increase in rent-shifting opportunity relative to the pollution cost decreases the critical fraction. (iv) \( H/G \leq 0 \) iff \( H \geq 0 \). A unilateral

\footnote{In the general case, \( H \) and \( G \) are as in footnote 6.}
tightening of environmental restrictions increases the country's welfare if and only if policies are strategic complements.

Figure 2: Minimum Critical Welfare Enhancing Coalitions. $\rho_2 > \rho_1$

Figure 2 plots the critical fraction as a function of $\alpha$, for two values of $\rho$, $\rho_2 > \rho_1$. We noted that a coalition always improves members' welfare when policies are strategic complements, as occurs for small $\alpha$ and small $\rho$ (Remark 2.i). Remark 3.iv says that policy complementarity is necessary as well as sufficient for unilateral action to be welfare improving. When $S > 1$, strategic complementarity is clearly not necessary, since the member countries internalize some of the market (pecuniary) externality. However, as pollution becomes more global, the policy leakage problem worsens, since non-members face lower marginal damages as a result of the coalition policy, and this causes them to relax their own policies. In order to offset this negative effect, the coalition must be larger if it is to remain welfare-improving for members (Remark 3.i.).

Remarks 3.ii and 3.iii are more subtle. For “quite global” problems, i.e. $\alpha > \alpha^* > 1/2$, policies
are strategic substitutes (Remark 2.iii), so when the coalition forms, outsiders respond by choosing lower taxes and increasing pollution. This has a large effect on coalition members for large $\alpha$; the effect is greater, the more important pollution is, i.e. the smaller is $\rho$. Therefore, for these values of $\alpha$, a smaller $\rho$ requires a larger minimum coalition size. When $\alpha < \alpha^*$ policies may be either substitutes or complements. We know that when they are complements, even a unilateral tax increase is welfare-increasing, so the only remaining case we need discuss is where $\alpha < \alpha^*$ and the policies are substitutes. Here, even though non-members respond to the coalition by relaxing their environmental taxes, the fact that pollution is fairly local means that the change in environmental damages within the coalition is small. However, the loss in oligopoly rent is still important. As the importance of this rent relative to the environment increases, the minimum coalition size must increase.

We close this section by considering the attitudes different groups have to the formation of a coalition. We view Country $i$ as consisting of “environmentalists,” who care only about environmental damages, $D_i(q_i + \alpha Q^{-i})$ and “republicans,” who care only about industry surplus net of transfers, $\pi^i(q^i, Q^{-i})$. Government $i$ chooses $t_i$ to maximize the weighted sum of welfare of these two groups, $W^i = \hat{\pi}^i - \eta D^i$, where $\eta$ is the weight given to environmentalist’s interests. Previously we had assumed that $\eta = 1$. Consider the incentives that the government, environmentalists, and republicans have for joining a coalition, given that we begin at the Nash equilibrium. Environmentalists would want to join any coalition. This result occurs because $d[\gamma(q^i + \alpha Q^{-i})^2]/d\tau < 0$ for all $\alpha$, $\rho$, $S$ and $N$. Country $i$'s environmental damage always decreases following an increase in the coalition tax, even if the coalition consists only of country $i$. This is a “diagonal dominance result.” The absolute value of the change in the sum of non-members’ taxes is always less than the exogenous change in the sum of members’ taxes, so country $i$’s level of pollution, $q^i + \alpha Q^{-i}$, always falls with an increase in the domestic tax. Since $dW^i/d\tau = d\pi^i/d\tau - \eta dD^i/d\tau$ may be positive or
negative, but $dD^i/d\tau < 0$, we see that there are coalitions which would increase country $i$'s welfare, but which domestic republicans would not want to join. Republicans might want to avoid forming a coalition which would increase domestic welfare, and environmentalists might want to form a coalition which would decrease domestic welfare. We restate this as

**Remark 4.** For a given initial equilibrium, increasing $\eta$, the social welfare weight on environmentalists' interests, reduces the critical fraction of coalition members. ■

This result is not surprising, since including environmentalists' concerns increases the benefits of a larger tax. Remark 4 should be contrasted to Remark 3.iii, which says that making the environment more important (decreasing $\rho$) can increase or decrease the critical coalition size. The experiment described in Remark 4 takes the initial equilibrium as given and considers how a marginal change in objectives affects incentives to form a coalition. The experiment in Remark 3 considers the effects of marginal changes in different equilibria caused by a change in the objective.

### 5 Environmental Agreements with Quotas

Some international environmental agreements, such as the Montreal Protocol, choose quantity restrictions, in which member states constrain their emissions to a specified level. We model this as a single stage game, where governments choose quotas. When the coalition is formed, members choose a quota $\bar{q}$, and the non-members' response is endogenous. It is no longer necessary to consider the firm's problem, provided that the quotas are binding, as we assume.

Welfare for the representative member, country 1, is $W^1(\bar{q}, Q^{-1}) = \bar{q}(a - b[\bar{q} + Q^{-1}]) - \frac{\gamma}{2}(\bar{q} + \alpha Q^{-1})^2$, where $Q^{-1} = (N - S)\bar{q} + (S - 1)\bar{q}$. Totally differentiating country 1's welfare, using its first order condition to evaluate the derivative at $\bar{q} = q$ (the symmetric Nash equilibrium), we obtain

$$\frac{dW^1}{dq} \bigg|_{\bar{q}=q} = \frac{\partial W^1(q, Q^{-1})}{\partial Q^{-1}} \frac{dQ^{-1}}{d\bar{q}} = -\gamma[\bar{q}\rho + \alpha(\bar{q} + \alpha Q^{-1})] \frac{dQ^{-1}}{d\bar{q}}.$$ (5)
A marginal decrease in $q$ in a neighborhood of the initial equilibrium raises a coalition member's welfare if and only if the equilibrium aggregate quota of the $N - 1$ other countries declines as a consequence. This implies

$$\frac{dW^1}{dq} \bigg|_{\bar{q}=q} \leq 0 \iff (N-S)\frac{dq}{dq} + (S-1) = 0. \tag{6}$$

Totally differentiating the equation which determines the symmetric equilibrium for non-members, gives

$$\frac{dq}{dq} = \frac{-S}{N-S+\beta}, \text{ where } \beta = \frac{\rho + (1-\alpha)}{\rho + \alpha} > 0 \tag{7}$$

Substituting equation (7) into (6), implies

$$\frac{dW^1}{dq} \bigg|_{\bar{q}=q} \leq 0 \iff \frac{S-1}{N-1} > \frac{\rho + \alpha}{2\rho + 1} \tag{8}$$

Coalition members gain from an exogenously induced marginal contraction of their quotas if and only if the fraction of rivals that join exceeds $(\rho + \alpha)/(2\rho + 1)$. We summarize the implications of equation (8) and compare the results to the tax setting case in

Remark 5. Under a quota, the critical fraction of members needed for a coalition to be welfare improving is (i) increasing in $\alpha$ (more global problems require more cooperation) (ii) increasing in $\rho$ for $\alpha < 1/2$ and decreasing in $\rho$ for $\alpha > 1/2$; (iii) strictly greater than 1 for all $\alpha$; and (iv) independent of $N$. (v) The critical fraction of coalition membership is always greater under quotas than under taxes. ■

The relation between $\alpha$, $\rho$, and the critical coalition size is qualitatively the same for quotas and taxes. With quotas, however, the critical fraction of members is independent of $N$. Remark 5.v
implies that a greater degree of cooperation is needed when governments choose quantities directly, rather than indirectly through tax policies. Moreover, a unilateral quota reduction by one country lowers their welfare even for purely local pollution problems (Remark 5.iii). This is in contrast to the case of taxes where unilateral actions for local pollution may be welfare increasing. The reason is that quotas are always strategic substitutes (equation 7) whereas taxes can be strategic complements or substitutes. Nevertheless, there always exists a critical coalition size less than \( N \) which is welfare enhancing.

6 The Ambitious Coalition: Equilibrium Response by the Members

In the previous sections we identified the formation of an environmental coalition with a marginal tightening of environmental policies. Here we consider the Nash equilibrium outcome when the coalition chooses a single tax to maximize members’ joint welfare. The previous scenario describes a “modest coalition,” and the scenario here, an “ambitious coalition.” In both cases, the non-members choose equilibrium policies under the Nash assumption. The important conclusion is that (relative to the original equilibrium with no coalition) members’ welfare in the ambitious coalition may decrease, even where their welfare in the modest coalition increases. This can occur because the members’ policy change is greater in the ambitious coalition, and therefore the response of non-members is also greater. If policies are strategic substitutes, so that non-members’ response harms the members, the latter are better-off when they are less ambitious.

The generality with which this possibility arises can be seen by considering the coalition’s “reduced form welfare function,” \( W(\tau, t) \), where \( \tau \) is the coalition tax, and \( t \) is the tax of non-members. Define \( W^*(\tau) \equiv W(\tau, t(\tau)) \) where \( t(\tau) \) is the equilibrium response of non-members. Denote the equilibrium symmetric tax before the coalition as \( t^n \) (\( n = \text{no coalition} \)) and the equilibrium taxes for members and non-members under the ambitious coalition as \( \tau^c, t^c \) (\( c = \text{coalition} \).
Suppose that policies are strategic substitutes, so that \( dt/dr < 0 \). The coalition's welfare gain resulting from a marginal decrease in their tax at the coalition-equilibrium is

\[
-dW^*(\tau^c)/d\tau = -W_1(\tau^c, t^c) - W_2(\tau^c, t^c)dt/dr = 0 - W_2(\tau^c, t^c)dt/dr > 0,
\]

where the second equality uses the coalition's first order condition, and the inequality uses the assumption of strategic substitutes and the fact that members always benefit when non-members raise their tax (\( W_2 > 0 \)). With strategic substitutes, the coalition would always be better-off by using a lower tax than its equilibrium level in the ambitious scenario. Previously we showed that their welfare gain in the modest scenario, \( dW^*(\tau^c)/d\tau = W_1(t^n, t^n) + W_2(t^n, t^n)dt/dr \), can be positive. (This is because \( W_1(t^n, t^n) = 0 \)) In that case, then, \( W^* \) is first increasing and then decreasing in \( \tau \). Consequently, it is possible that \( W^*(\tau^c) < W^*(t^n) \), even if \( dW^*(t^n)/d\tau > 0 \).

<table>
<thead>
<tr>
<th>( S )</th>
<th>( \tau^c )</th>
<th>( t^c )</th>
<th>Is Marginal Change Welfare Improving?</th>
<th>% Change in Amb. Coalition to Initial Welfare Level</th>
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</thead>
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<tr>
<td>1</td>
<td>4.55</td>
<td>4.55</td>
<td>no</td>
<td>0%</td>
</tr>
<tr>
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<td>6.59</td>
<td>4.17</td>
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</tr>
<tr>
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<td>no</td>
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</tr>
<tr>
<td>7</td>
<td>9.41</td>
<td>-1.13</td>
<td>yes</td>
<td>+5.7%</td>
</tr>
</tbody>
</table>

The symmetric Nash equilibrium tax, \( t^n = 4.55 \).

\( \tau^c \) is the member tax following an ambitious coalition formation.

\( t^c \) is the non-member tax following an ambitious coalition formation.

Table 1 provides an example of this possibility, for \( \alpha = .75, \rho = .25 \) and \( N = 10 = a \). Members' welfare is higher in a modest coalition for \( S \geq 4 \). In order for members to benefit from an ambitious coalition, however, we require \( S \geq 7 \). Since an ambitious coalition is never harmful when \( S = 1 \) (a "trivial coalition" which does not alter the original equilibrium), is always beneficial when \( S = N \), and may be harmful when \( 1 < S < N \), it is obvious that the members' benefits are not necessarily monotonic in \( S \). Table 1 also illustrates this possibility. For all of our numerical examples, we
found that the minimum critical coalition size for the ambitious coalition is no less than for the modest coalition. A sufficient condition for this comparison to hold is given by

**Remark 6.** Suppose that the function $W^*(\tau)$ is concave in $\tau$ and that $\tau^c > t^n$. (Formation of the coalition leads to a larger tax for members.) Then coalition members benefit from an ambitious coalition only if they benefit from a modest coalition. □

We showed above that the modest coalition always benefits members when policies are strategic complements, so Remark 6 is informative only for the case of strategic substitutes. If $W^*$ is concave in $\tau$, $dW^*/d\tau$ is decreasing in $\tau$. Given that $\tau^c > t^n$, in order for $W^*(\tau^c) > W^*(t^n)$ it must be the case that $dW^*/d\tau > 0$ over some interval of $\tau$. We know that $dW^*(t^n)/d\tau < 0$ in the case of strategic substitutes. Therefore, if $dW^*/d\tau > 0$ over any interval between $t^n$ and $\tau^c$, $dW^*(t^n)/d\tau > 0$. This inequality says that the modest coalition improves members’ welfare.

The hypotheses in Remark 6 hold for all of our numerical examples, and we expect them to hold quite generally, so it is somewhat surprising that we can not verify them analytically even in the linear model. We describe the reason for the difficulty in Appendix 3.

### 7 Policy Implications and Conclusion

Governments’ desire to ameliorate environmental problems may conflict with other goals. When their policy menu is limited, the stringency of environmental laws involves balancing different objectives. This balance can be altered by policies in other countries. We studied a model where imperfectly competitive markets give governments a rent-shifting objective, in addition to their environmental objective.

We used this model to investigate the relation between the amount of cooperation that is needed in order for an environmental agreement to improve members’ welfare, and the characteristics of the externalities. We focused on the extent to which the environmental problem is global, i.e., the
degree of spillovers, and the importance of the environmental objective relative to the rent-shifting objective. We considered two types of coalitions. A modest coalition is willing to consider only marginal policy changes from the initial non-cooperative equilibrium. An ambitious coalition is willing to make non-marginal changes. In the latter case, we studied the Nash equilibrium in the game between the coalition and non-members.

Modest coalitions require fewer members (less cooperation) in order to be welfare improving. There are well-known explanations, having to do with adjustment costs and uncertainty, why designers of international environmental agreements should have modest goals for policy reform. We identified an additional reason. The policy leakage problem is less severe for modest reform, so the danger that a coalition will back-fire and leave its members worse-off is correspondingly lower. The potential benefits of modest reform are lower, but the greater probability of such reforms being adopted is important.

For modest coalitions we showed that greater cooperation is needed, the more global is pollution. When spillovers are greater, coalition policy changes cause a larger change in non-member countries' incentives. Therefore, policy leakage is more of an issue with more global environmental problems. However, an increase in the severity of environmental problems can either increase or decrease the necessary amount of cooperation, depending on whether the environmental problem is quite global, or quite local. For example, for quite global environmental problems, an increase in the relative severity of the environmental problem makes it tempting for governments to try to reduce foreign-generated pollution. More cooperation is needed to counter this temptation.

We showed that, in the absence of a coalition, the equilibrium tax is always increasing in the relative severity of the environmental problem. However, increase in the degree to which environmental problems are global first increases and then reduces the equilibrium tax. For example, if the balance of incentives is such that the equilibrium policy is a zero tax, an increase in spillovers makes
the policy a positive tax when the degree of spillovers is initially small. When it is initially large, an increase in the degree of spillovers causes nations to subsidize production, thus increasing pollution. Again, the reason is that the policy leakage problem is severe for global environmental problems.

Problems of monitoring and enforcement are central to the design of successful international agreements. However, even when those problems can be solved, cooperation may not benefit members. In that case, we cannot expect an agreement to be signed.

Our results shed some light on the relation between the nature of environmental problems and the minimum size of a stable coalition. (We think it is reasonable to assert that a coalition is not stable if it leaves its members worse-off than at the non-cooperative Nash equilibrium.) A more challenging question concerns the relation between the nature of environmental problems and the maximum size of a stable coalition (i.e., the maximum amount of cooperation we can expect). The answer to this question will vary depending on the notion of stability that we adopt. The game-theoretic literature has a number of plausible definitions of stability. It will be interesting to determine the implications of these definitions for models of international pollution control.
References


Appendix 1: Derivations of Equations

For Referee's use (not intended for publication).

Diagonal dominance condition for stability in the quantity-setting (stage 2) game can be seen by taking the partial derivatives of \( q^i \), \( q^j = -N/b(N + 1) \) and \( q^j = 1/b(N + 1) \) where the subscript denotes the derivative with respect to \( t^i \). Since \( |q^i| > |\sum_{j \neq i} \frac{1}{b(N + 1)}| \), the diagonal dominance condition is satisfied in the quantity setting game.

**Derivation of Equation 1**

The \( N \) first order conditions for welfare maximization can be written as follows: \( F + Gt^i + H \sum_{j \neq i} t^j = 0 \) for all \( i = 1 \ldots N \) where \( F, G \) and \( H \) are given in the text. Using one of the first-order conditions and solving for the symmetric Nash equilibrium, we obtain equation (1). The denominator is negative because: \( G + (N - 1)H = -\rho(N^2 + 1) - (N - \alpha[N - 1])(\alpha N - \alpha + 1) < 0 \). Thus, the diagonal dominance condition for stability is satisfied for the tax-setting stage.

**Derivation for Footnote 3:**

At the symmetric Nash equilibrium: \( q^i = \frac{a-t}{b(N+1)} \) and \( Q^* = \frac{N(a-t)}{b(N+1)} \). For \( q^i \) and \( Q^* \) to be positive, \( a - t > 0 \), which is true because (using equation 1) \( a - t = \frac{aG + a(N-1)H + E}{G + (N-1)H} > 0 \). The denominator is negative and after algebraic manipulation the numerator can be shown to be negative. Also, \( p(Q) = a - bQ = (a + Nt)/(N + 1) \) which implies: \( \frac{aG + a(N-1)H - NE}{(N + 1)(G + (N-1)H)} > 0 \).

**Derivations for Remark 1:**

Using equation (1) and the diagonal dominance condition: \( t^* \leq \frac{N}{\alpha} \iff F \geq 0 \iff \rho \equiv \frac{N - \alpha(N - 1)(1 + \alpha [N - 1])}{N - 1} \leq 0 \). Therefore, for \( \rho \geq \rho \iff t^* \leq \frac{N}{\alpha} \). Also, \( \frac{\partial \rho}{\partial \alpha} = (N - 1)(1 - 2\alpha) \leq 0 \iff \alpha \leq \frac{1}{2} \) and \( \rho|_{\alpha=0} = \frac{N}{N-1} = \rho|_{\alpha=1} \). Therefore, as \( N \) increases the concavity of \( \rho \) increases but \( \frac{\partial \rho}{\partial N}|_{\alpha=0} \) or \( \alpha=1 < 0 \).

**Derivations for Remark 2:**

The first-order condition of government \( i \) is given by \( F + Gt^i + \sum_{j \neq i} Ht^j = 0 \). The \( S \) member countries set \( t^1 = \ldots = t^S = r \) and the \( N - S \) non-member countries set \( t^{S+1} = \ldots = t^N = t \).
The first-order condition for a non-member is given by: \( F + SH_r + [G + (N - S - 1)H]t = 0 \). Totally differentiating gives equation (2). The denominator of equation (2) is negative as noted previously because \( G + (N - 1)H < 0 \). Therefore, the sign of \( dt/d\tau \) depends on the sign of \( H \). Thus, \( \frac{dt}{d\tau} > 0 \iff H > 0 \) and \( H = 0 \to \rho = \frac{(1 - 2a)[(N - a)(N - 1)]}{N - 1} \equiv \rho^*(\alpha, N) \). Thus, \( \rho \geq \rho^*(\alpha, N) \iff H \leq 0 \iff \frac{dt}{d\tau} \leq 0 \). Moreover, \( \frac{\partial \rho^*}{\partial \alpha} = \frac{(N - 1)(\alpha - 1 - 2N)}{N - 1} < 0 \) for \( \alpha < \frac{1}{2} \) and \( \frac{\partial \rho^*}{\partial N} = \frac{-(1 - 2a)(1 - a)(N - 1) + \alpha}{(N - 1)^2} < 0 \) for \( \alpha < \frac{1}{2} \). For \( \alpha > 1/2 \), \( H < 0 \) for all \( \rho \geq 0 \), so \( dt/d\tau < 0 \).

Derivations for Coalition Formation with Taxes and Remark 3:

If \( \tau = t \), government 1's tax is a best-reply, so \( W^1_1 = 0 \). Totally differentiating, we obtain:
\[
\frac{dW^1}{dt} \bigg| _{\tau=t} = W^1_1 \frac{dt^1}{dt} + \cdots + W^1_N \frac{dt^N}{dt} \quad \text{and} \quad t^S+1 = \cdots = t^N = t \quad \text{and} \quad W^1_1 = W^1_k, j, k \neq 1, \text{ we get equation (3). After some algebraic manipulation we can show that } W^1_j = \frac{-(N^2 + 3N - 1) - p(1 - \alpha)(N - 1)](N^2 + N - 1)}{k(N + 1)^2(\alpha - (N - 1)H)} > 0. \text{ Therefore, } \frac{dW^1}{dt} \bigg| _{\tau=t} > 0 \iff (S - 1) + (N - 1) \frac{dt}{dt} > 0. \text{ After substituting in equation (2) from the text we get equation (4) where } \frac{H}{G} = \frac{p(1 - 2a)[(N - a)(N - 1)]}{2Np + (\alpha - (N - 1)H)^2} \geq 0 \text{ where } k \equiv (N - 1)(\alpha - 1) + \alpha + (1 - 2a)2N. \text{ The sign of } \frac{\partial (H/G)}{\partial \rho} \text{ depends on the sign of } K. \text{ Setting } k = 0 \text{ we obtain } 1 > \alpha^* = \frac{N(N + 1)}{4N(N - 1)} > \frac{1}{2}. \text{ Therefore, } \alpha < \alpha^* \iff \frac{\partial (H/G)}{\partial \rho} \geq 0 > 0. \text{ In addition, } \frac{\partial (H/G)}{\partial \alpha} = \frac{(N + 1)(N - 1)(N^2 - 2N + 1 - \alpha) + \alpha + a(\alpha + 2\rho)}{G^2} > 0.

Derivations for Remark 4:

Government i chooses \( t^* \) to maximize country i's total welfare: \( W^i = \pi^i - \eta D^i \), so: \( \frac{dW^i}{dt} = \frac{dt^i}{dt} - \eta \frac{dD^i}{dt} \). In order to show that including \( D^i \) in the welfare function decreases the critical coalition size we need to show that \( dD^i/d\tau < 0 \) for all \( \alpha, \rho, S \) and \( N \). This is trivially true by the diagonal dominance condition. Specifically, \( \frac{dD^j}{dt} = 2(\gamma q^i + \alpha Q^{-1})(\frac{dq^i}{dt} + \alpha \frac{dQ^{-1}}{dt}) < 0 \), because \( dq^i/d\tau < 0 \) and \( |dq^i/d\tau| > |dQ^{-1}|d\tau. \)

Derivations for Remark 5:

\[
\frac{\partial}{\partial \alpha} \left( \frac{t^{a+1}}{2\rho+1} \right) > 0 \quad \text{and} \quad \frac{\partial}{\partial \rho} \left( \frac{t^{a+1}}{2\rho+1} \right) = \frac{1 - 2a}{(2\rho+1)^2} \leq 0 \iff \alpha \leq \frac{1}{2}. \text{ Also, since } (\rho + \alpha)/(2\rho + 1) > 0 \text{ for }
all $\alpha$, $S > 1$. To compare the critical fractions for quotas and taxes we proceed in two steps. If $H > 0$, then $H/G < 0$ but $(\rho + \alpha)/(2\rho + 1) > 0$. If $H < 0$ then with some algebraic manipulation we can show that $H/G < (\rho + \alpha)/(2\rho + 1)$. Therefore, the critical coalition required for a quotas is always higher than taxes.
Appendix 2: The Nonlinear Model

Let $p(Q)$ be a general downward sloping demand function with $p'(Q) < 0$. We further assume
that $p'(Q) + q^i p'(Q) < 0$ for all $0 \leq q \leq \bar{Q}$. Thus, a given firm’s marginal profit must fall when
any rival firm increases its output. The goods are therefore strategic substitutes: each firm’s
best-reply function is downward sloping. The profit function is as in the text. The first and
second order conditions for profit maximization and an interior solution are: $p + q^i p' - t^i = 0$ and
$2p' + q^i p'' < 0$, respectively. The first-order condition defines firm $i$’s reaction function $r^i(Q^{-i}, t^i)$
such that $q^i = r^i(Q^{-i}, t^i)$ where
\begin{equation}
r^i_1 = -\frac{p' + q^i p''}{2p' + q^i p''}, -1 < r^i_1 < 0 \ \text{and} \ \ r^i_2 = \frac{1}{2p' + q^i p''} < 0 \tag{9}
\end{equation}
where subscripts denote a derivative with respect to the $i$th argument. Equation (9) shows that
if all the rivals jointly expand production, firm $i$ contracts, but by less than its rivals’ expansion.
In addition, equation (9) shows that reaction functions are downward sloping, an increase in the
emissions tax reduces the domestic firm’s output for any given output by the rivals, i.e. it shifts
downward the domestic firm’s reaction function.

Unlike the linear case, an explicit solution for the equilibrium level of output cannot be found.
However, the model assumptions insure a unique Nash equilibrium (for a proof see, Gaudet and
Salant, 1991) and is given by solving the reaction functions to yield the equilibrium output level:
$q^i = r^i(Q^{-i}, t^i) = x^i(t^1, \ldots, t^N)$. Note that $x^i$ is equivalent to $q^*^i$ in the text. The comparative
static properties of $x^i$ are given by:
\begin{equation}
dq^i = r^i_1 \sum_{j \neq i} dq^j + r^i_2 dt^i \tag{10}
\end{equation}
where

\[ dq^i = r_1^{-i}[dq^i + \sum_{k \neq i,j} dq^k] \]  \hspace{1cm} (11)

Assuming symmetry, \( j = k = -i \), equation (11) can be written as:

\[ dq^{-i} = r_1^{-i}[dq^i + (N - 2) dq^{-i}] = \frac{r_1^{-i}}{1 - (N - 2) r_1^{-i}} dq^i \]  \hspace{1cm} (12)

substituting (12) into (10)

\[ x^i \equiv \frac{dq^i}{dt} = \frac{r_2^{-i}[1 - (N - 2) r_1^{-i}]}{1 - (N - 2) r_1^{-i} - (N - 1)(r_1^{-i})^2} = \frac{N(p' + q_i p'') - q_i p''}{N p'[p' + q_i p''] + (p')^2} < 0, \text{ if } p'' \geq 0. \]  \hspace{1cm} (13)

where the first equality uses \( r_1^{-i} = r_1^i \) and the second equality uses equation (9). Also from (10) and (13)

\[ x_i^{-i} \equiv \frac{dq^{-i}}{dt} = \frac{r_1^{-i} r_2^{-i}}{1 - (N - 2) r_1^{-i} - (N - 1)(r_1^{-i})^2} = -\frac{p' + q_i p''}{N p'[p' + q_i p''] + (p')^2} > 0. \]  \hspace{1cm} (14)

Also by symmetry \( x_{-i}^i = x_i^{-i} \). We note that \( x_i^i < 0 \) and \( x_i^{-i} > 0 \) with \( |x_i^i| > |x_i^{-i}| \). That is, the absolute impact of a tax change on own firm is greater than that on the rival firm. Furthermore, \( |x_i^i| > |\sum_{j \neq i} x_j^i| \) which is the diagonal dominance condition.

The welfare of country \( i \) is given by: \( W^i(t^i, \cdots, t^N) = x^i p(Y) - D(x^i + \alpha[Y - x^i]) \) where \( Y = \sum_{i=1}^{N} x^i \) is the aggregate production and \( D(.) \) is the damage function with \( D' \geq 0 \) and \( D'' \geq 0 \). The first term is firm profits net of tax receipts and, the second term is damage due to emissions. The first and second order conditions for welfare maximization and an interior solution are, \( x_i^i p + x^i p' Y_i - D'(x_i^i + \alpha Y_i - x_i^i) = 0 \) and \( x_i^i p + 2 x_i^i p' Y_i + x^i p''(Y_i)^2 + x^i p' Y_i - D''(x_i^i + \alpha Y_i - \cdots \)
The change in the welfare of a member country due to a marginal tightening of the environmental policy is as given in the paper. However, the formula for \( \frac{dt}{d\tau} \) is more general now. To determine \( \frac{dt}{d\tau} \) as before we consider the maximization of the non-member country. The equilibrium tax of each non-member government solves the following first-order condition (\( o = \) non-member or outsider, \( i = \) member or insider, \( t = \) non-member's tax and \( \tau = \) member's tax): \( W_o^o = x_t^t \rho + x_o \rho' Y_t - D'(x_t^o + \alpha[Y_t - x_t^o]) = 0 \). Totally differentiating and noting symmetry we obtain

\[
\frac{dt}{d\tau} = \left\{ \frac{SW_{oi}}{W_o^o + (N - S - 1)W_{oi}}, \ i \neq o \right\}
\]

The sign of (15) is ambiguous and depends on the functional forms of demand and damage and also on the parameters like \( \alpha \), and the relative size of \( S \) compared to \( N \).

Using equation (15), the change in welfare of a member country due to a marginal in the environmental tax is given by

\[
\frac{dW_i^1}{d\tau} \bigg|_{\tau=t} \triangleleft 0 \iff \frac{S - 1}{N - 1} \triangleleft \frac{W_o^o}{W_{oi}^o}, \ i \neq o
\]

In other words, the welfare of member countries increases due to a marginal increase in taxes if and only if \( (S - 1)/(N - 1) > W_{oi}^o/W_{oo}^o \). The sign of the right hand side of equation (16) depends on the sign of \( W_{oi}^o \) which is the partial derivative with respect to the member tax of the first-order condition of a non-member country. The sign of \( W_{oo}^o < 0 \), as it is the second order condition of the
non-member country welfare maximization. The sign and the magnitude of the change in welfare of member countries depends in general on both demand and damage relations. As we saw in the linear case the sign of $W^0_\alpha$ can be anything. For the general case, the "critical fraction" is far too complicated to be analyzed without assuming functional forms.
Appendix 3: Comments on Remark 6

The problem is to show that $\tau^c > \tau^n$. It would be odd if formation of the coalition resulted in a smaller equilibrium tax. However, we cannot compare the expressions for $\tau^c$ and $\tau^n$ directly, because of their complexity. An alternative would be to show that $d\tau^c/dS > 0$. We can verify this inequality for $S = 1$ but not more generally.

**Derivations for $(d\tau^c/dS)|_{S=1} > 0$**

The coalition welfare is given by: $W^* = W^1 + (S-1)W^2$ where $W^1$ is the welfare of country 1 and $W^2$ is welfare of another representative member country. The first and second order conditions are given by: $W^1 = W^1 + (S-1)W^2 = 0$ and $W^*_1 = W^*_1 + (S-1)W^*_2 < 0$. The first order condition of a non-member, when non-members choose symmetric policies, is given by: $F + SH\tau + (G + [N-S-1]H)\tau = 0$. Totally differentiating both the member and non-member first order conditions and evaluating at $S = 1$ where $\tau = t$, we obtain (using Cramer's Rule): $\frac{d\tau}{dS} = \frac{-W^2(G+[N-2]H)}{W^*_1(G+[N-2]H) - SHW^*_1} > 0$. The denominator is positive because of stability and the numerator is positive because $G + [N-2]H < 0$ and $W^*_2 > 0$. Therefore, $\frac{d\tau}{dS}|_{S=1} > 0$.

The intuition for the inequality is that both the pecuniary and the environmental externality encourage members to cut their production. As $S$ increases, more of this externality is internalized, leading to a larger coalition tax. However, the inequality is not correct in general.

When we solve the model numerically, we find that it is possible that $d\tau^c/dS < 0$ when $\rho$ is small and $\alpha$ is large. However, in all of the examples where this occurs, the coalition tax exceeds the market price (net of production costs). In this case, it is optimal for firms in the coalition countries to "produce" negative quantities. This can be interpreted as saying that the coalition is a net importer of the product, so the tax is actually an import subsidy. The subsidy is used to discourage domestic production, and shift the pollution abroad. This makes sense, because when $\rho$ is small, the environmental damage is important. For large enough $S$, the coalition uses a smaller
tax, i.e., it decreases its import subsidy, as it internalizes more of the environmental damage. Aggregate production is always positive in this model, because at \( Q = 0 \) marginal environmental damage is 0, whereas marginal rents are positive. As \( S \) approaches \( N \), the tax falls below market price, and the coalition again becomes a net exporter. This explains the non-monotonicity of \( \tau^c(S) \).

In view of this non-monotonicity, lack of analytic results is not surprising.