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Abstract
The paper explores how the coordination of vehicle schedules in a public transit system affects generalized costs. We consider an idealized system that delivers its users to a common destination by requiring each to transfer from a feeder- to a trunk-line vehicle. Continuum models are used first to analyze cases in which the trunk-line vehicle schedule is given exogenously. We find that when feeder vehicles are dispatched in coordination with this exogenous trunk-line schedule, the reduction in user cost often outweighs the added cost to the feeder operation. In cases when the frequencies of trunk and feeder services can be established jointly, the models show that coordination can be Pareto improving, meaning that operator and user costs both diminish. Conditions that give rise to these cost savings are specified. Practical implications are discussed.

Keywords: schedule coordination; feeder transit

1. Introduction
By reducing the times spent in transferring between vehicles, schedule coordination can diminish the costs that a transit system imparts to its users. In the long run, this can bring broader benefits to both the transit agency and society at large by inducing greater transit ridership. These matters have been studied extensively in the literature (Chien and Schonfeld, 1998; Chowdhury and Chien, 2002; Li et al., 2009).

What seem to have garnered less attention are questions on how coordination can affect aspects of transit cost beyond just the user cost. Yet, a decision on whether to deploy some proposed coordination scheme will often depend upon these other costs; e.g. a scheme that saves user transfer time is more likely to be adopted if it also reduces, or at least does not significantly increase, transit operating costs (Hickey, 1992; Schumann, 1997).

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The present paper therefore explores how schedule coordination affects some of the costs that are imparted to transit operators as well as its users. This is done by applying continuum approximations of generalized cost to a simple transit network of many parallel feeder lines that connect to a single trunk.

Background is furnished in the following section. We describe: existing models for designing public transit systems, the scope of the present study, and the hypothetical system upon which the study is based. A continuum model is used in section 3 to explore impacts of coordination when the schedule of trunk-line service is given exogenously. We find that by dispatching feeder vehicles in coordination with the given trunk schedule, total user cost can significantly diminish while little or no extra cost is imparted to the operator of the feeder service. The continuum model is expanded and used in section 4 to explore cases in which the trunk and feeder service frequencies can be optimized jointly. Here we find that coordination can be Pareto improving, such that costs diminish for all parties. Practical implications are discussed in section 5.

2. Background

This section: reviews relevant literature (sec. 2.1); describes our general approach to the present analyses (sec 2.2); and presents the trunk-feeder network to be used in these analyses (sec. 2.3).

2.1 Literature Review

Numerous models have been developed for designing transit systems. Most furnish values for decision variables (e.g. the distances between stops and between routes, vehicle headways, etc.) that minimize some generalized cost. Roughly speaking, these models fall into two categories, as described below.

In the first category of models, input parameters and decision variables take values that are discrete in space. Thus for example, travel demands are specified as origin-destination matrices, and the locations available for stops and routes may be constrained by details of the local topography. Models of this discrete type can generate design solutions that incorporate many realistic details. This feature has made these models popular in the literature: they have been used to design systems that deliver users from many origins to many destinations (Chien and Schonfeld, 1998; Kuah and Perl, 1989; Kuan et al., 2006; Martins and Pato, 1998; Shrivastav and Dhingra, 2001; Shrivastava and O'Mahony, 2006; Verma and Dhingra, 2006); as well as systems in which the distribution of trips is many-to-one (Chien and Yang, 2000; Chowdhury and Chien, 2001).

The realism imbedded in these discrete models adds complexity, however. As a result, solutions are typically obtained through heuristic methods that do not always guarantee global optimality. And these solutions often do not unveil relations between the input parameters and the optimal values of the decision variables. Thus it can be difficult to glean general insights from these models to inform high-level system design.

In the second category of models, input parameters and decision variables are approximated as smooth, continuous functions, such that travel demands, routes and stops are expressed per unit length or unit area (Daganzo, 1999; Newell, 1973). Continuum Approximation (CA) models of this type have primarily been used to design many-to-one systems (Byrne and Vuchic, 1972; Chien, et al., 2002; Clarens and Hurdle, 1976; Hurdle, 1973; Kuah and Perl, 1988; Wirasinghe, 1977; Wirasinghe, 1980). In essence, CAs omit much of the real-world detail of discrete models in favor of simplicity, and therefore tend to generate solutions that take relatively simple, closed forms. These can often unveil relations between the input parameters and the optimal values of the decision variables. The resulting insights therefore tend to be more general than those obtained from a discrete model.

2.2 Study Scope

In the pursuit of general insights, we will analyze the costs of a simple trunk-feeder transit network using CAs. Previous research involving models of this kind typically ignored trunk-line costs, and assumed that trunk-vehicle headways were sufficiently small as to render user transfer costs negligible as well. Our models will differ from these earlier CAs in that we will initially add a term to describe a user's cost in transferring from a feeder- to a trunk-line vehicle, and will later add another term to describe trunk-line operating cost (Clarens and Hurdle, 1975; Hurdle, 1973; Kuah and Perl, 1988; Wirasinghe, 1980). We will thus consider a user's costs incurred as she: accesses a feeder line by travelling toward it; waits for a feeder vehicle; and eventually transfers to a trunk vehicle. These will be estimated for the so-called average-case user during her one-way trip. The costs of accessing a feeder-line stop while travelling parallel to the line, and of travelling aboard vehicles are both ignored, since these are invariant to schedule coordination.

The costs for the system operator(s) will depend upon the vehicle-hours of service that are to be provided. These will depend on factors that include: service frequencies, the density of feeder lines, the physical lengths of those lines and vehicle travel speeds. We will assume that any cost of controlling vehicles to maintain a schedule is the same, whether or not the schedule is aimed at coordinating trunk and feeder services; and that all transit vehicles have sufficient capacities to accommodate boarding demands. [1]

2.3 Case Study

The idealized network on which we will base our study is shown in Figure 1. It consists of a trunk line, operating at a headway H, that runs in the y-direction to a Central Business District (CBD) at location $(0, R)$; and parallel feeder lines. The latter are each of length L, run perpendicular and connect to the trunk, and collectively span distance L_r along the trunk. The service region is thus a rectangle of dimension L_r x L, as shown in the figure.

[1] The presumed invariant cost of control is therefore ignored in our analyses. Moreover, new control methods for maintaining a transit schedule can be deployed quite inexpensively; see Daganzo and Pilachowski, 2009 and Pilachowski, 2009. Further discussion on the costs of control, and on issues regarding vehicle capacity, are offered in section 5.

We will assume that L_r is sufficiently large that the service region holds many feeder lines; and that these lines can be located anywhere throughout the region, as might occur when the feeder service is provided by buses on a dense network of streets. Trunk-line stops will not necessarily be placed at every junction with the feeder lines, meaning that a feeder vehicle may be required to travel in the y -direction to reach a trunk-line stop. However, this travel distance will be assumed negligible as compared with L, a feeder vehicle's one-way travel distance in the x -direction.

Travel demand in the service region is expressed as a continuous, time-independent density function. We will assume that this density varies gradually along x and is independent of y, as might occur, for example, if development arose along the trunk line and gradually diminished at greater distances from it. We therefore denote this density function $\delta(x)$. By assuming that demand is independent of y, both the spacing between neighboring feeder lines, r, and the feeder-vehicle headway, h, will be fixed throughout the service region. This will simplify our analysis for the case when trunk and feeder schedules are established jointly (in Section 4). This assumption of uniform demand also means that all feeder lines will have the same number of stops, since we will take stop locations to be unconstrained by topography and would be selected instead to minimize costs (see Kuah and Perl, 1988). We can therefore assume that feeder-vehicle speeds are the same on all lines. We select a feeder-vehicle commercial speed, v_f, that is slower than the cruise speed to roughly account for the time lost in serving passengers. (Crude estimates for commercial speed suffice, as we shall see in sec. 3.)

Finally, we will assume that the distribution of trips is many-to-one, with all users bound for the CBD. A case like this might arise (approximately) during the morning rush in a mono-centric city.

3. Exogenous Trunk-Line Schedules

We first consider the case in which trunk-line vehicles operate with a headway that is given and that cannot be altered to accommodate feeder operation. This case can arise when, for example, the trunk line is long and connects both to feeder lines inside our service region, and to additional lines that reside outside it. We present CAs for estimating generalized costs when trunk and feeder services are uncoordinated (sec. 3.1); reformulate the models to estimate costs when feeder service is operated in coordination with the exogenously-specified trunk-line schedule (sec. 3.2); and make comparisons (sec 3.3).

3.1 Cost Models for Uncoordinated Service

The CAs presented below are comparable to those derived in earlier work (Hurdle, 1973; Wirasinghe, 1980; Kuah and Perl, 1988). We will merely add a term to estimate the user cost of transfers. All parameters and decision variables to be used in our models are defined in Table 1.

We define Z to be the generalized cost per unit time for the service region. It is the sum of four cost components: the users' access to feeder lines (along the y direction), their wait times at feeder stops, their wait (i.e. transfer) at the trunk stop, and the feeder operating cost. Thus,

$$Z = \alpha_1\, r + \alpha_2\, h + \alpha_2\, H + \alpha_3 \frac{1}{h}\frac{1}{r} \quad (1)$$

Where

$\alpha_1 = \frac{C_a}{4v_a} L_r \int_0^L \delta(x)dx$

$\alpha_2 = \frac{C_w}{2} L_r \int_0^L \delta(x)dx$

$\alpha_3 = C_f \frac{2L}{v_f} L_r$.

We minimize Z, taking r and h as decision variables. The optimal spacing for the feeder lines, as a function of h, is therefore

$$r^*(h) = \alpha_1^{-\frac{1}{2}} \alpha_3^{\frac{1}{2}} h^{-1/2} \; . \quad (2)$$

Substituting (2) into (1), we minimize with respect to h and find the optimal feeder-vehicle headway to be

$$h^* = (\alpha_1 \alpha_2^{-2} \alpha_3)^{\frac{1}{3}} = \left(\frac{2C_a C_f L}{v_a v_f C_w^2 \int_0^L \delta(x)dx}\right)^{\frac{1}{3}} \; , \quad (3)$$

which leads to an optimal feeder line spacing of

$$r^* = (\alpha_1^{-2} \alpha_2 \alpha_3)^{\frac{1}{3}} = \left(\frac{16 v_a^2 C_w C_f L}{v_f C_a^2 \int_0^L \delta(x)dx}\right)^{1/3} . \quad (4)$$

Substituting (3) and (4) into (1), we find that the minimum generalized cost for the service region is

$$Z^* = 3(\alpha_1 \alpha_2 \alpha_3)^{\frac{1}{3}} + \alpha_2\, H = 3\left(\frac{C_a C_w C_f}{4 v_a v_f} L \left(\int_0^L \delta(x)dx\right)^2\right)^{\frac{1}{3}} + \frac{C_w}{2} H \int_0^L \delta(x)dx)\, L_r \; . \quad (5)$$

Note from (5) how this minimum cost (excluding the waiting cost at the trunk stop) is robust to variations in the input parameters, meaning that coarse estimates for their values will yield near-optimal designs for the feeder system.[2]

Note too that (5) furnishes a cost in the absence of coordination between the trunk and feeder schedules; this is evident from the third term in (1) which takes the average transit time to be half the trunk-vehicle headway. Schedule coordination is examined next.

3.2 Coordinating Feeder Service with the Trunk

Consider a simple scheme in which we coordinate schedules within the service region by choosing a common feeder headway, h_c, that is some integer multiple of the trunk line's exogenous headway; i.e.

$h = h_c = kH$

where k is any positive integer.

One could select the value of k that makes h_c closest to the optimal feeder line headway, h^*, obtained from (3). Referencing (2), one would then obtain:

$$r_c^* = \alpha_1^{-\frac{1}{2}} \alpha_3^{\frac{1}{2}}\, h_c^{-\frac{1}{2}} \quad (6)$$

[2] Methods for transforming idealized CA design variables to real-world environments can be found in Kuah and Perl (1988) and Wirasinghe (1980).

and

$$Z_c^* = 2\alpha_1^{\frac{1}{2}}\alpha_3^{\frac{1}{2}}\, h_c^{-\frac{1}{2}} + \alpha_2 h_c \ . \qquad (7)$$

3.3 Cost Comparisons

Selecting a line spacing as in (6) can, to a large degree, compensate for the added cost of choosing an $h_c \neq h^*$, such that the user and operating cost incurred within the feeder subsystem changes very little. To illustrate this, we note first that $h_c = kH$ can be rewritten as $h_c = mh^*$, where m is some non-negative constant that should be close to 1 (by virtue of having chosen suitable k). In the uncoordinated case, the costs within the service region for user access, for the wait for feeder vehicles and for feeder operation are each $(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$. In the coordinated case, the access and feeder operating costs are each $m^{-\frac{1}{2}}(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$, and the user wait cost for feeder vehicles is $m(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$. The percent difference in the sum of access, waiting, and feeder operating cost between coordinated and uncoordinated operation is therefore $\frac{2m^{-\frac{1}{2}}+m-3}{3}$. This difference is small for m close to 1, as shown in Figure 2.

Since schedule coordination eliminates the user transfer cost at the trunk stop, the difference between Z_c^* and Z^* is $\left(2m^{-\frac{1}{2}} + m - 3 - \frac{m}{k}\right)(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$. And since $h_c = kH$, coordination yields a savings in total cost only when k is small. This makes sense: when H is small compared to h^*, there is no need for schedule coordination.

4. Endogenous Trunk-Line Schedules

Suppose now that the trunk-line component of the system serves only the feeder lines that reside within the service region of Figure 1. For this case, we will compare the costs of uncoordinated services against those that occur when the trunk and feeder lines operate with the same headway and in coordinated fashion. We will also demonstrate how this simple coordination scheme can be Pareto improving. CAs are presented for the uncoordinated and coordinated cases (secs. 4.1 and 4.2, respectively). Estimated costs are compared to identify the conditions needed to achieve Pareto improvements (sec. 4.3); and the cost savings are quantified for a range of operating environments on our hypothetical network (sec. 4.4).

4.1 Uncoordinated Trunk-Feeder Service

The trunk and feeder headways, H and h respectively, are treated for now as separate decision variables, such that Z, the sum of relevant generalized costs for the entire service region is given by

$$Z = \beta_1 H^{-1} + \alpha_2 H + 3(\alpha_1\alpha_2\alpha_3)^{1/3} \qquad (8)$$

Where

$$\beta_1 = C_t \frac{2R}{v_t} \ .$$

The first term in (8) is the trunk operating cost, obtained by multiplying the operating cost rate, C_t, with the needed number of trunk-line vehicles ($\frac{2R}{v_t}\ H^{-1}$). The second term is the average transfer cost, where in the absence of coordination, the average wait time is again assumed to be $H/2$. The third term describes the remaining two user-cost components and the feeder operating cost, given in (5).

We find that

$$H^* = \sqrt{\frac{4C_tR}{C_wv_tD}}$$

and that

$$Z^* = 2\sqrt{\frac{C_wC_tRD}{v_t}} + 3\left(\frac{C_aC_wC_fL}{4v_av_f}\right)^{\frac{1}{3}}\left(\int_0^L \delta(x)\,dx\right)^{\frac{2}{3}}L_r \tag{9}$$

where $\mathrm{D} = L_r\int_0^L \delta(x)dx$ is the service region's total hourly travel demand.

Note that the minimum cost, Z^*, is expressed purely as a function of its input parameters.

4.2 Coordinated Service

When trunk and feeder services are coordinated and operate at a common headway, H_c, the sum of relevant generalized costs for the service region, Z_c, is given by

$$Z_c = \beta_1{H_c}^{-1} + \alpha_2H_c + \beta_2{H_c}^{-\frac{1}{2}} \tag{10}$$

where

$$\beta_2 = 2(\alpha_1\alpha_3)^{\frac{1}{2}}\ .$$

The second term in (10) is the user waiting cost for a feeder vehicle (only), since we assume zero user cost in transferring to the trunk. The third term, consisting of the access to feeder lines and the feeder operating cost, comes from (7). Note that (10) is convex in H_c, since the sum of convex functions is itself convex. Thus, the minimum cost, Z_c^*, can be obtained numerically from (10).

4.3 Conditions for Pareto Improvement

We now compare the cost models of secs. 4.1 and 4.2 to determine the bounds for which schedule coordination is Pareto improving. The user costs will be examined first. These diminish with coordination if the sum of the user-cost components of (9) is less than that of (8); i.e., if

$$C_wD\,\frac{1}{2}H_c^* + {H_c^*}^{-\frac{1}{2}}(\alpha_1\alpha_3)^{\frac{1}{2}} < \sqrt{\frac{C_wC_tDR}{v_t}} + 2(\alpha_1\alpha_2\alpha_3)^{1/3}. \tag{11}$$

Next we determine the conditions in which coordination reduces the operating cost for trunk and feeder services combined. To this end, we compare the operating-cost components of (9) and (8). If

$$C_t\frac{2R}{v_t}\ {H_c^*}^{-1} + (\alpha_1\alpha_3)^{\frac{1}{2}}{H_c^*}^{-\frac{1}{2}} < \sqrt{\frac{C_wC_tDR}{v_t}} + (\alpha_1\alpha_2\alpha_3)^{1/3}, \tag{12}$$

the trunk and feeder operating cost is reduced.

If both the trunk and feeder services are furnished by a single entity, coordination will be Pareto improving whenever (11) and (12) both hold. Of course, trunk and feeder services are often provided separately by distinct agencies. In this latter case, coordination can still reduce the costs for every party involved.

To see why this is true, note first from (12) that the operating cost for feeder service (alone) is reduced if $H_c^{*-\frac{1}{2}}(\alpha_1\alpha_3)^{\frac{1}{2}} < (\alpha_1\alpha_2\alpha_3)^{1/3}$, which can also be expressed as

$$H_c^* > \left(\frac{(\alpha_1\alpha_3)^{\frac{1}{2}}}{(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}}\right)^2. \tag{13}$$

Referring to (3), we can see that (13) reduces to the inequality $H_c^* > h^*$.

As regards the operating cost of trunk service (alone), inspection of (7) and (9) reveals that the optimal trunk-vehicle headway is guaranteed to be larger with coordination. This is because i) the coordinated model, given by (9), shifts the waiting cost ($\alpha_2 H$) at trunk stops to feeder stops, and ii) this waiting cost is now weighed against both the trunk and feeder operating costs (rather than solely against the trunk operating cost). These two factors combined assure that coordination results in a larger trunk headway. The larger headway leads to a lower trunk operating cost.

4.4 Illustrations of Pareto Improvement

To illustrate the benefits of coordination, we present two scenarios. The first corresponds to a region where the average wage rate is low, as typically occurs in a developing country; and the second to a region with a high average wage rate, as in a more industrialized country. We assume in both scenarios that the user value of time, for both access and waiting, is equivalent to the user wage rate. Additionally, while some components of the hourly cost rates for both the feeder and trunk systems will remain roughly the same across the two scenarios (fuel, depreciation, etc.), the overall hourly operating cost will differ due to differences in labor costs. Accordingly, the input cost parameters for the developing country ("low") and the industrialized country ("high") are shown in Table 2.

We assume that both scenarios are governed by the same system characteristics: v_a= 5 km/hr, L_r= 10 km, L= 5 km, and R= 20 km (refer again to Table 1 for the definitions of these parameters). Feeder- and trunk-vehicle speeds are chosen to reflect typical urban bus and light rail speeds: 15 and 30 km/hr, respectively.

Both scenarios are analyzed under a range of demand densities for the trip origins. For simplicity, uniform densities are used. (Similar results were found for demand densities that varied with y and caused uncoordinated feeder-vehicle headways to vary across feeder lines.) These uniform densities ranged from 10- to 200 pax/ km^2-hr, in increments of 10 pax/ km^2-hr, such that the total demand in the service region ranged from 500- to 10,000 pax/hr.

Costs saved through coordination are shown as functions of these demands in Figures 3 and 4. Figure 3 displays savings for the low-cost scenario, and Figure 4 for the high-cost one. Both figures present curves for the savings in total user cost, total (trunk and feeder) operating cost and feeder operating cost alone.

5. Conclusions

By applying continuum approximations to a simple trunk-feeder transit network, we find that schedule coordination can save more than just user costs. In cases when the vehicle headway on the trunk is given exogenously, service can be coordinated by operating the feeder vehicles at headways that are integer multiples of those on the trunk. By suitably adjusting the feeder-line spacing, this simple coordination scheme can eliminate user waiting cost at the trunk station, while often adding little or no cost elsewhere in the system. Better still, if the headways for trunk and feeder vehicles are both decision variables, schedule coordination can often be Pareto improving and benefit all parties.

We acknowledge that these findings came by analyzing an idealized network, and by adopting a number of (often simplifying) assumptions. And we concede that there are limits on the extent to which these findings can be exploited. For example, when a trunk line's schedule is exogenous, an operator does not always have free reign to locate the feeder lines in response to that schedule. Even when this freedom exists, moreover, feeder-line spacing would usually have to be optimized with respect to a limited portion of the day (e.g. the morning rush), since the headways scheduled for a trunk tend to change over the day. Furthermore, we did not account for any real-world "control" costs of coordination, which might include transit signal priority, vehicle tracking, etc. If this is the case, the cost of control may override coordination benefits, particularly when the optimal coordinated headway takes a low value. Finally, the passenger-carrying capacity of trunk-line vehicles can also limit possibilities, since the joint selection of a feeder- and trunk-vehicle headway tends to expand the latter. This limitation can be remedied at relatively low cost when trunk vehicles can be enlarged; e.g. by using articulated buses or by adding cars to trains. Or, an operator might increase capacity by dispatching trunk vehicles in small platoons, though the resulting increase in trunk operating cost might then become large (Sivakumaran et al., 2010).

All this notwithstanding, the present findings can inform transit system design. In those instances when trunk and feeder services are provided by distinct agencies, the findings speak to the benefits that might come via institutional cooperation. They may also motivate transit agencies to explore alternative schemes for delivering service. Consider, for example, a case in which many-to-one service is provided on a network with a long trunk line that spans a long service region. Pareto improvements might come by partitioning the network into narrower sub-regions and assigning trunk-vehicles to serve sub-regions in dedicated fashion. A better understanding of the cost-saving potential of schedule control might ultimately give rise to any number of innovations in transit service. The present paper represents a step forward in this regard.

Acknowledgement

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Figure 1

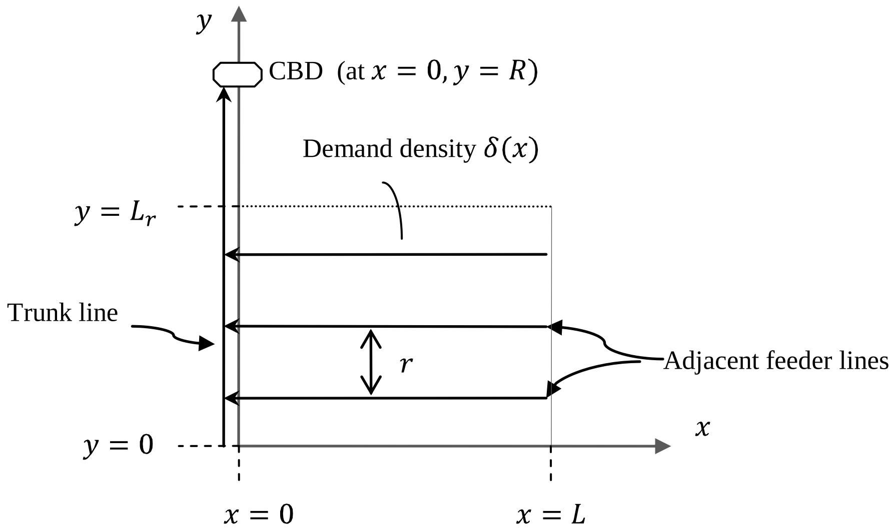

Fig. 1. Hypothetical trunk-and-feeder operating environment.

Figure 2

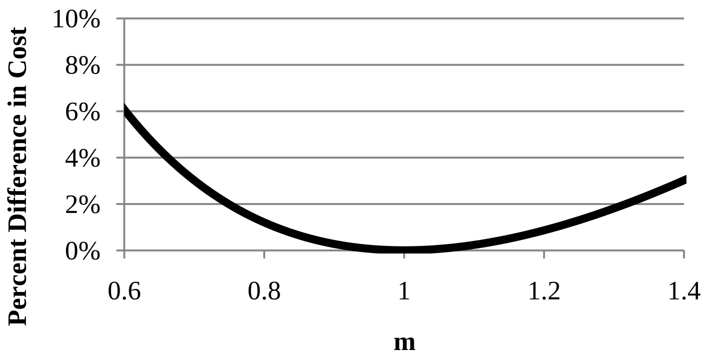

Fig. 2. Percent change in the sum of access, waiting, and feeder operating costs through coordination.

Figure 3

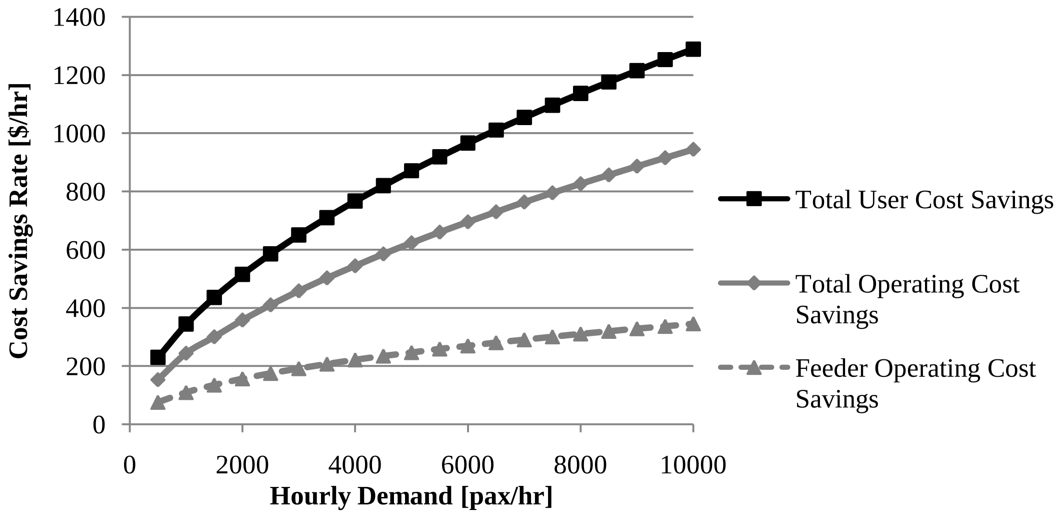

Fig. 3. Cost Savings from Coordination, for "Low" Cost Parameter Values

Figure 4

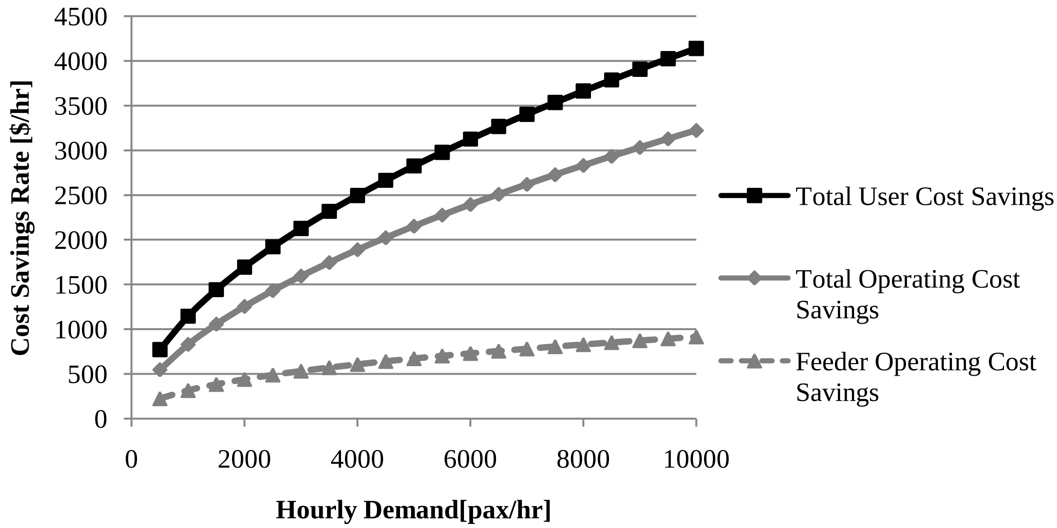

Fig. 4. Cost Savings from Coordination, for "High" Cost Parameter Values

Table 1

Table 1
Description of input parameters and decision variables

Term	Description	Units
$\delta(x)$	Demand density	pax/km^2-hr
v_a	User speed in accessing the transit system	km/hr
v_f	Feeder vehicle speed	km/hr
v_t	Trunk vehicle speed	km/hr
C_a	User value of access time	$/hr
C_w	User value of waiting time	$/hr
C_f	Feeder vehicle operating cost rate	$/hr
C_t	Trunk vehicle operating cost rate	$/hr
L	Width of the service region	km
L_r	Length of the service region	km
R	Distance to CBD from y = 0	km
r	Feeder line spacing	km
h	Feeder headway	hrs
H	Trunk headway	hrs

Table 2

Table 2
Cost parameter values for two scenarios

Term	Low Value [$/hr]	High Value [$/hr]
C_a	3	15
C_w	3	15
C_f	40	100
C_t	180	300