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EFFECT OF POLARIZATION OF SHELL-MODEL STATES IN REACTION CALCULATIONS FOR $^{40}$Ca($^{16}$O,$^{15}$N)$^{41}$Sc $^{1f_{7/2}}$.


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Abstract:

The reaction $^{40}$Ca($^{16}$O,$^{15}$N)$^{41}$Sc g.s. has been investigated in the context of a model where the final single-particle state in $^{41}$Sc was modified or polarized due to the proximity of the other nucleus. Calculations, performed with the finite-range distorted waves Born approximation code KUNDY, using two-centre shell model wave functions, demonstrate that this polarization produces effects on the cross section which indicate a resolution to some discrepancies between previous DWBA predictions and experiment.

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Experimental differential cross sections for heavy-ion induced nucleon transfer reactions often display a broad peak centred at an angle close to the classical grazing angle at energies for which this angle is removed from $0^\circ$. However, when distorted wave Born approximation (DWBA) predictions have been compared with experiment systematic discrepancies have become apparent\(^1\text{-}^{12}\). Empirically, the discrepancies are basically of two types: (1) the cross section is biased towards forward angles and exhibits little or no peaking at the grazing angle in contradiction to the calculation; (2) the cross section predicted by the DWBA shows the correct shape of the peak but consistently overestimates the peak angle, in some cases by as much as $30^\circ$\(^5\). Discrepancies of the second type have been observed in reactions involving transfer of protons\(^1\text{-}^6,^8,^9\) and neutrons\(^7,^{10,11}\) (largely on the former) and in two nucleon transfer between a light projectile and a heavier target\(^5\). In some cases the discrepancy is observed to decrease with increasing bombarding energy and to increase with decreasing binding energy of the final state, being sometimes absent for transfer to the most tightly bound states\(^2,^5,^7,^9,^{12}\) and appears to be larger when the $Q$-value and $l$ transfer of the reaction do not match semiclassical conditions\(^1,^4,^7,^{12}\). These discrepancies can be duplicated by DWBA calculations where optical model (OM) parameters have been arbitrarily varied away from those required to fit the appropriate elastic scattering data; typical adjustments are 10 to 15 percent for radii and 20 to 40 percent for diffuseness parameters\(^1,^2,^5\text{-}^{11}\). However, such discrepancies may also signify reaction mechanisms other than a direct transfer reaction. In particular, two distinct processes are possible. One is a two-step reaction mechanism involving an inelastic transition through a low-lying collective state. When this two-step process dominates, the cross section tends to be biased towards smaller angles rather than peaking at the grazing
angle. However, when this mechanism is not dominant over the direct transition and interferes destructively with it, the cross section may retain a peaked shape but is shifted forward in angle relative to the prediction for the direct transition. In those cases where the peak angle discrepancy appears as a systematic effect in a number of states in the same nucleus another reaction mechanism may be present. The reason for believing this to be the case is that the occurrence of such a systematic shift in angle is difficult to understand in terms of coupling of each of the final states to one or more intermediate states. The importance of the two-step process depends upon the strength of the inelastic transitions passing through the intermediate states as well as the parentage amplitudes involved in the direct and two-step processes. The relative strengths and phases of the parentage amplitudes are not expected to be similar for different final states. This suggests that coupling to low-lying collective states will not lead to a systematic effect in a group of states.

It is the purpose of the present note to present numerical results for a reaction mechanism that may lead to a systematic shift of the grazing peak angle relative to the predictions of a direct transfer mechanism. When two nuclei approach each other the individual asymptotic single-particle states may change under the influence of the neighbouring nuclear potentials in a reversible way because in an adiabatic process the single-particle motion is considered to be fast compared to the relative nucleus-nucleus motion. In such a model, nucleon transfer reactions occur between configurations which differ from the asymptotic ones, namely, between instantaneous, polarized, two-centre states. Thus, the approach of an attractive potential will increase the probability of transfer at larger impact parameters with a consequent shift of the cross section peak to an angle smaller than that predicted by the asymptotic states.
The effect is amplified for more weakly bound states. Furthermore, an increase in bombarding energy corresponds to less adiabaticity or less time for the instantaneous states to adjust to the presence of the reaction partner and consequently the effect of the polarization should diminish.

The present work aims at exploring observable effects of the polarization of single-particle states in the reaction $^{40}\text{Ca}(^{16}\text{O},^{15}\text{N})^{41}\text{Sc}$ through a model calculation using a simple schematic two-centre shell model \(^{16}\)) in the adiabatic (i.e. static nuclei) limit. The polarized single-particle wave functions thus produced are then used, with suitable approximations, in full-recoil finite-range DWBA calculations of the differential cross section for this reaction. A study based on more realistic, Woods-Saxon shaped, two-centre potentials is in progress and will be reported elsewhere. \(^{17}\) The two-centre shell model (TCSM) discussed here is that of Maruhn and Greiner\(^{16}\)). The potential is a modified asymmetric two-centre oscillator which includes the usual $l.s$ and $l^2$ forces. The only integrals of motion are energy $E$ and, for collisions of spherical nuclei, projection $\Omega$ of the angular momentum on the $z$-axis (intrinsic symmetry axis) which lies along the coordinate joining the two potential centres. The TCSM states may be specified either with the quantum numbers $E$ and $\Omega$, or with the quantum numbers $nlj\Omega$ of the eigenstates of the spherical harmonic oscillator (with $l.s$ force) which they approach asymptotically as the two-centre separation $R$ approaches infinity. Before TCSM states can be used in a reaction calculation they must first be decomposed into asymptotic states of good angular momentum:

\[ \phi_{E\Omega}(R,r) = \sum_{n'l'j'} A_{n'lj'j'}^{nlj\Omega}(R) \phi_{n'l'j'}^{\Omega}(r) \phi_{nlj\Omega}(r). \]  \((1)\)

A measure of the overall strength of the polarization is given by the total probability of the admixed states,
\[ p_{n'\ell'j'}(R) = \sum_{n'\ell'j'} |A_{n'\ell'j'}(R)|^2 = 1 - |A_{n'\ell'j'}(R)|^2 \sim 0 \quad (2) \]

where the prime in the summation indicates that \( n' \ell' j' \neq n\ell j \). The polarization amplitudes \( A_{n'\ell'j'}(R) \) are independent of the sign of \( \Omega \) and details of their calculation are discussed in ref. 19. For the case of the \(^{40}\text{Ca}(^{16}\text{O},^{15}\text{N})^{41}\text{Sc}\) g.s. reaction the polarization of the strongly bound \( \text{lp}_{1/2} \) proton state in \(^{16}\text{O}\) is weak, i.e. \(-1.1\) percent at grazing distances \( R_{\text{gr}} \sim 10 \text{ fm} \) and will be neglected in the following discussion. However, the \( \text{lf}_{7/2} \) proton state in \(^{41}\text{Sc}\) is strongly polarized, e.g. the polarization of the \(|\Omega| = 1/2\) component is 20.1 percent at \( R = 10 \text{ fm} \). The polarization decreases by approximately an order of magnitude if \(|\Omega|\) increases by unity since for increasing \(|\Omega|\) the states are located further away from the symmetry axis and are affected less by the approaching shell model potential. Therefore polarization of the higher \(|\Omega|\) components is neglected.

Fig. 1(a), (b) shows contour plots of the probability density of the asymptotic and polarized \( \text{lf}_{7/2} \) states. The polarization amplitudes for this state at grazing distances are depicted in Fig. 1(c) where the contours connect states of the same probability. A DWBA calculation of transfer to the polarized \( \text{lf}_{7/2} \) state as given by the expansion of eq.(1) would be very time consuming since more than 130 states \( \phi_{n'\ell'j'}(r) \) contribute with amplitudes whose absolute value is 0.02 or larger. However, since the reaction is localized to an annulus centred at \( R_{\text{gr}} = 10 \text{ fm} \), the calculation may be simplified. As shown in ref. 18 the radial dependence of the polarization amplitudes may be approximated by the same function for \( n' \ell' j' \neq n\ell j \):

\[ A_{n'\ell'j'}(R) \approx A_{n\ell j}(R) A_{n'\ell'j'}(R_{\text{gr}}), \quad (3) \]

this allows the radial dependence in eq. (1) to be taken outside the summation over \( n' \ell' j' \). The summation over the principle quantum number \( n' \) may then be performed thereby reducing the number of DWBA transition matrix amplitudes to
be calculated by approximately a factor of five. For R of the order of, or
greater than, the strong absorption radius \( 1.5(40^{1/3}+16^{1/3})^{-9} \text{fm} \), \( f^{1f7/2} \) may be well approximated by a Gaussian

\[
f^{1f7/2} 1/2 \approx 1.259 \exp[-(R-8.53)^2/9.37].
\]  

For the reaction \( A(a,b)B \) with \( a=b+1 \) and \( B=A+1 \), the usual DWBA transition matrix amplitude has the form \(^{15}\)

\[
T_{J_1,J_2}^{m_1,m_2} \propto \int \psi^-(k_B^b R_B^b) \phi^{*n_1^b J_2^b m_2^b} (r_B^b) V(r_B^a) \phi^{*n_1^a J_2^a m_2^a} (r_B^a) \psi^+(k_A^a R_A^a) \, dR_A^a \, dR_B^b
\]  

where \( r_a = R_{b1}^a, R = R_{A1}^b \) and \( R_B^b = R_{Bb}^b \). Incorporation of the TCSM states into the DWBA requires consideration of two distinct features: (1) each component of the expansion of eq. (1) carries a radial dependence on the separation between the nuclei in the respective channels (\( R_B^b \) or \( R_A^a \) in eq. (5)), and (2) the coordinate systems used in the DWBA and the TCSM are related through a rotation. The wave function of eq. (1) is expressed on a basis which has the symmetry axis connecting the line of centres of the two nuclei as the z-axis. To employ an amplitude of the type in eq. (5) it is necessary to relate this choice to the laboratory fixed axis which may be taken to be defined by \( k_A^a \). Denoting by \( D_{m_1 m_2}^{J_1 J_2} (\omega) \) the usual rotation matrix with Euler angles \( \omega \) it is possible to derive the expression for the DWBA transition matrix amplitude with the inclusion of polarization:\(^{15}\)

\[
T_{J_1 J_2}^{m_1 m_2} \propto \sum_{\Omega_2} \sum_{\Omega_1} \sum_{\mu_1 \mu_2} \int \frac{n_2^2 J_2^2 \Omega_2^2 (R_B^b)}{C_2^2} \psi^-(k_B^b R_B^b) \phi^{*n_1^b J_2^b m_2^b} (r_B^b) V(r_B^a) \phi^{*n_1^a J_2^a m_2^a} (r_B^a) \psi^+(k_A^a R_A^a) \, dR_A^a \, dR_B^b
\]  

where

\[
\begin{align*}
D_{m_2^2 \Omega_2}^{J_1^1 J_1^1} (\omega) & = D_{m_2^2 \Omega_2}^{J_1^1 J_1^1} (\omega) \\
& \times \phi^{*n_1^b J_2^b m_2^b} (r_B^b) V(r_B^a) \phi^{*n_1^a J_2^a m_2^a} (r_B^a) \psi^+(k_A^a R_A^a) \\
& \times dR_A^a \, dR_B^b
\end{align*}
\]  

(6)
In the present case polarization of the states in the incoming nucleus is neglected and therefore the asymptotic limit applies in eq. (6) \( A_{n_1 n_1' j_1 j_1'}(\omega) = n_{n_1 n_1' j_1 j_1'} \). The approximation of the radial dependence of the polarization amplitudes for the final channel together with the evaluation of the summation over \( n_2' \) leads to a tractable form of eq. (6).

The DWBA calculations were performed with the finite-range code KUNDY\(^{19}\) which includes recoil effects exactly. The complete calculation required 3.3 hours of CDC7600 computer time and called for the coherent summation of thirty sets of DWBA amplitudes, the first corresponding to the parent \( 1f_{7/2} 1/2 \) state and the remainder to the values of \( l_2' \) in the range 0 to 14 and \( j_2'=l_2+1/2 \). For values of \( l_2' \) beyond 8, double precision arithmetic (i.e. 29 vs 14 figures) and special numerical techniques were necessary to overcome severe losses of significance in the calculation. This problem was not eliminated by rearranging order of summations and is an attribute of the DWBA formalism on which the code is based\(^{20}\). The OM parameters used in the calculations were the unadjusted set of ref.\(^1\) which reproduces the elastic scattering of \(^{48}\)Ca\(^{16}\) at 40 MeV, namely, \( V=100 \) MeV, \( W=40 \) MeV, \( r_c=r_v=r_w=1.22 \) fm and \( a_v=a_w=0.49 \) fm with a volume type imaginary potential. The same parameters were used in both channels. For the \( 1p_{1/2} \) proton bound in \(^{16}\)O the wave function was obtained by adjusting the depth of a Woods-Saxon potential of radius \( r_o=1.2 \) fm, diffuseness \( a=0.65 \) fm and \( V_{so} = 12 \) MeV.fm\(^2\) to reproduce the separation energy. The Coulomb part of \( V_{bl} \) was included in the evaluation of the DWBA transition matrix amplitude.

Fig. 2 shows the DWBA predictions with and without polarization of the final state. For the latter the bound-state wave function of the proton in \(^{41}\)Sc was calculated by the same method as that for \(^{16}\)O with the same parameters. The very broad peak for the DWBA without polarization is largely due to poor matching between the two channels and because at this bombarding energy of 48 MeV the
exit channel is below the Coulomb barrier. The two curves differ substantially in shape, particularly for angles larger than the peak angle where the need for more data is imperative. At angles forward of the peak the DWBA with polarization is a distinct improvement and fits of this quality could only be obtained in the work of ref.1) by arbitrarily adjusting the OM parameters away from the values required to fit elastic scattering data. Both curves have been normalized to the data and as can be seen from Fig. 2 the peak of the differential cross section is shifted forward in angle (81 versus 63°) when polarization of the single-particle state in \( ^{41}\text{Sc} \) is included. Using a factor of 1.02 as the product of the spectroscopic factors\(^{1,21}\) the DWBA without polarization (dashed curve) has been multiplied by the significantly large number of 5.6 to obtain the normalization shown, while the DWBA with the inclusion of polarization (solid curve) has been multiplied by 1/43. This overestimate of the polarization calculation may be due to either model dependent features of the TCSM (e.g. assumption of oscillator potentials and parameterization of the barrier) or to the large polarization calculated in the adiabatic limit which may not be reached under the conditions of this experiment. For example, if polarization of the final state in \( ^{41}\text{Sc} \) was reduced to a few percent of its adiabatic limit the cross section would have the correct magnitude with the peak position essentially unchanged. This is so because the polarized components of the cross section (i.e. \( n_{2}^{l}l'j'_{2}n_{2}^{l}j_{2} \)) which peak at 63° contribute a factor \(-730\) more to the cross section than the asymptotic component \( n_{2}^{l}l'j'_{2}n_{2}^{l}j_{2} \) which peaks at 84°.

It was the aim of this study to estimate an upper limit (the adiabatic limit) on the increase in magnitude of the cross section and forward shift in angle produced by the inclusion of the polarization of shell-model states. It was found that, within this upper limit, the polarization process can account for the magnitudes and trends of the discrepancies that have been observed in comparing previous DWBA predictions with experiment in heavy-ion induced transfer reactions.
References

17) P. Lichtner and K. Pruess, to be submitted for publication.
18) K. Pruess, preprint LBL5066, accepted for publication in Nuclear Physics.


Captions

Fig. 1. Parts (a) and (b) show the probability density of the $1f^{7/2} 1/2^+$ proton state in $^{41}$Sc in a plane containing the symmetry axis for the asymptotic and polarized states respectively; the latter is for $^{15}$N at the grazing distance of $R=10$ fm. Units along both axes are in fm. Part (c) is a contour map of the polarization amplitudes of eq. (1) where the contours are in units of $10^{-2}$ and $j'=\ell'+1/2$ and $j'=\ell'-1/2$. The amplitudes are labelled by $n'$ and $\ell'$ and refer to an harmonic oscillator basis with $\hbar\omega=10.9$ MeV.

Fig. 2. Full finite-range DWBA predictions (including recoil effects) for $^{40}$Ca$(^{16}$O, $^{15}$N)$^{41}$Sc g.s. at 48 MeV (lab) as calculated by the code KUNDRY with (solid curve) and without (dashed curve) polarization of the final state in $^{41}$Sc with the same optical model parameters (see text). The data are those of ref. 1 (error bars are not shown); both curves have been normalized to the data (see text).
Fig. 1

(a) $|\psi|^2 \geq 3.16 \times 0.316 \times 10^{-3}$

(b)

(c)

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Fig. 2
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