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Identifying with Mathematics: the effects of conceptual understanding, motivation, and communication on the creation of a strong mathematical identity

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Identifying with Mathematics:
The effects of conceptual understanding, motivation, and communication on the creation of a strong mathematical identity

A Thesis submitted in partial satisfaction of the requirements for the degree Master of Arts
in
Teaching and Learning (Curriculum Design)

by

Veena Prakash Mansukhani

Committee in Charge:
Rusty Bresser, Chair
Cheryl Forbes
Libby Butler

2010
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Chair

University of California, San Diego

2010
DEDICATION

This work is dedicated to my sister, Asha, for her unconditional love and continuous support. This work is also dedicated to my parents for their guidance, support, and love throughout the years. Lastly, this work is dedicated to my students who live each day to the fullest, embrace the opportunities, and face the challenges presented in life.
EPIGRAPH

“You must be the change you want to see in the world.” – Mahatma Gandhi

“Small minds discuss persons.
Average minds discuss events.
Great minds discuss ideas.
Really great minds discuss Mathematics!”
- Anonymous

“Education is the most powerful weapon you have to change the world.” – Nelson Mandela
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Thank you Rusty for being my advisor. Your support, guidance, feedback, and positive attitude helped me tremendously during this process. I could not have done it without you. Thank you Cheryl for all of your help throughout the program. I appreciate your openness and willingness to meet me anywhere and anytime to discuss my project. Thank you to all of my students who worked so hard and made me proud. You all are the reason I have such a passion for teaching.

Thank you to all of my wonderful friends who understand my intense personality and strange obsession with perfection. Thank you for being sympathetic and supportive when I needed you.
ABSTRACT OF THE THESIS

Identifying with Mathematics:

The effects of conceptual understanding, motivation, and communication on the creation of a strong mathematical identity

by

Veena Prakash Mansukhani

Master of Arts in Teaching and Learning: Curriculum Design

University of California, San Diego 2010

Rusty Bresser, Chair

Many students struggle with understanding the importance of higher-level mathematics to their lives and rarely identify with the math they engage in. In order for students to identify with math, or see themselves as learners of mathematics, it is important to provide students with a curriculum that addresses conceptual understanding, motivation, and communication. Identifying with Mathematics was developed for educators to help students identify with math through investigation, communication, peer discussions, a meaningful project, and student reflections.

There were three goals of Identifying with Mathematics. The first goal was to help students see the relationship between mathematics and the real world while increasing their ability to solve problems. The second goal focused on creating a classroom
environment that facilitated communication and allowed students to effectively use academic language through meaningful peer discussions. The third goal was for students to create a strong mathematical identity by combining strategies that build conceptual understanding, motivation, and communication.

*Identifying with Mathematics* was implemented in a ninth-grade geometry classroom. Data from interviews, student surveys, student reflections, investigative lessons, a project, and field notes were analyzed to evaluate the effectiveness of the curriculum.

The results indicate that students’ perceptions of themselves as learners of mathematics became stronger. Students gained self-confidence while participation and motivation levels increased. In addition, students increased their use of academic language during classroom discussions and exhibited high-quality work in their projects. Overall, students’ mathematical identities became stronger as a result of a curriculum that empowered student to take charge of their education.
Chapter I.
Identifying with Mathematics

Each person’s identity defines who they are as a part of society or a specific community. As a whole, we usually think of identity as related to ethnicity, race, and culture. However, it can be beneficial to look at identity from a different perspective however. As young students mature into young adults, their connections with the material they learn in school contribute to the academic identities they form. In a math classroom, for example, there are certain students who are perceived by other students as well as their teachers to have strong connections with the content and high interest levels, and there are other students who are perceived to be “low level” math learners. Either way, these perceptions as well as the experiences students have in school help define their mathematical identities.

A math identity, as defined by scholars such as Sfard and Prusak (2005), Wegner (1998), and used in this paper, is the view students have of themselves with respect to mathematics and the way others view them as learners of mathematics. Martin (2009) states,

Mathematics identity refers to the dispositions and greatly held beliefs that individuals develop about their ability to participate and perform effectively in mathematical context and to use mathematics to change the conditions of their lives. A mathematics identity encompasses a person’s self understanding and how other see him or her in the context of doing mathematics.

This identity, whether strong or weak, can help determine the outcomes of decisions students make well into adulthood. The development of a strong math identity can help students select academic studies and employment that incorporate the use of mathematics.
which can be beneficial in allowing for a variety of career options in the future. A strong math identity can help support decisions to continue studies in mathematics.

I am currently teaching Geometry at a Title I charter school in southeast San Diego, CA. Title I is a federally funded program that provides schools with additional monies for students at risk of failure or living at or near poverty. The school is located in a low socio-economic area in which many of our students perform below grade level in mathematics and language arts. All ninth grade students at this school are required to take Geometry regardless of their past experiences in math. Through my interaction with my students, it has become apparent that very few relate mathematics to their lives.

Often, we see too many students disengage themselves from anything related to math and only take mathematics courses because they are required to. Students who do not see mathematics as an essential part of their lives may limit themselves to non-math related fields. Creating an identity centered on a mathematics community can improve students’ success while instilling intrinsic motivators that benefit their academic achievement. This paper explores the constructs of communication in the classroom, motivation, and conceptual understanding, while considering their contributions to the development of strong math identities among adolescents.
Chapter II.
Needs Assessment

Effective mathematics education at the high school level is exceedingly important in raising students’ abilities to reason, problem solve, and think abstractly. Our traditional techniques for teaching mathematics have inhibited some learners and their aptitude for conceptual understanding as well as motivation in the math classroom. Yet, up to this point, many teachers still use these approaches to teach mathematics. Traditional methods can be described as teacher-centered, lecture-based instruction with drill and practice worksheets. In a traditional classroom, student talk about content is often not required and there are few opportunities for students to be active learners. Currently, after reviewing test scores, speaking to teachers and administrators, and observing many classrooms, I believe teaching mathematics in traditional forms has not proven to be successful in the United States. The results of the data at my school show a low success rate suggestive of students who lack confidence and competence in mathematics, specifically Geometry.

Linking Mathematics and the Real World

Geometry is an extremely visual and unique subject that has many connections to life. Successful mathematics students must be able to investigate, discover, reason, problem solve, think abstractly, communicate about their experiences with their peers using academic language, and make connections between math and real life. Although many students can use formulas and procedures to find solutions, there is often a disconnect between what they learn in school and its relationship to practical applications. Mathematics is a subject that has a definite affiliation with the world, as
indeed, there is a specific union between any math concept and the world around us. For example, fields such as actuarial mathematics, biomathematics, biostatistics, computer science, finance and business, architecture, and teaching relate mathematics to its practical applications in the world. As teachers, it is our job to help students build those connections not only to achieve reasoning, problem solving and abstract thinking skills but to also motivate students to care about mathematics in the first place. Unfortunately, a learner’s frustration with conceptual understanding affects their engagement and motivation levels. Some students have a fear and even hatred toward mathematics because of the level of frustration they experience when they do not understand something. Conversely, students who understand the content presented are more likely to be engaged and motivated to go beyond what is expected. In order for students to be able to connect with mathematics, they must be given opportunities to talk in the classroom and discuss mathematical ideas. Vygotsky’s (1986) notion of sociocultural learning theory emphasizes the need for communication to take place in order for students to reach intersubjectivity, or negotiated meaning. Students need opportunities to talk and express their ideas with other students in order to reach higher levels of understanding. There is a link, then, between student engagement, motivation, and the use of talk in the classroom.

**Fear of Mathematics**

Given the relationship between mathematics and the world, it is important to develop curricula that help students realize the relevance of mathematics in their lives and find ways to engage students. Creating the right tools becomes a priority since they empower students to want to learn, which also supports conceptual understand. Another fundamental motivation to learning mathematics involves the development of students’
identities as members of the mathematics classroom (Anderson, 2007). Through such constructs as academic talk in the classroom, motivation, and conceptual understanding, students, in turn, develop mathematical identities that then mutually reinforce engagement and learning.

It is both evident and unfortunate that in the United States we find it more socially acceptable to struggle with mathematics than with any other subject. It is rare to hear a person say, “I can’t read,” but more widely accepted to hear someone say, “I am terrible at math.” However, the fallacy that learning mathematics requires special natural talents that only a few learners hold still exists among students (Anderson, 2007). There seems to be a link between these fears and struggles and the conceptual understanding of mathematics. A learner’s belief that they cannot “do math” unlocks an underlying fear of failure and a risk of appearing unintelligent. Unfortunately, this fear can even lead to hatred of the subject. From my experiences as a teacher, I believe the fear of failure and hatred of math is a major contributing factor to students’ inabilities to develop a strong math identity. It is easier at times to shut down instead of facing those fears and attempting to tackle them. In order to change this negative view, especially among students who see themselves as failures in the math classroom, teachers must set high expectations in their lessons and provide expert instruction. Unfortunately, students who start out with the greatest need for expert instruction are the least likely to get it (Resnick, 1995). The focus on achievement as the primary goal in teaching and learning can neglect the identities of students who have not been able to achieve high marks on testing in the math classroom (Martin, 2009). Instead, focusing on how students experience mathematics learning in relation to their life and their circumstances can help teachers
understand their lack of success in the classroom. It is essential that teachers have the tools to help students break through the anxiety and assist them in finding schematic connections between mathematics and the world.

**Mathematics on a Global Level**

The National Council of Teachers of Mathematics states that mathematics can and indeed must be learned by *all* students (NCTM, 2000). Although the methods used to teach mathematics in the past have allowed some students to thrive, it has also left many behind with a sense that it is sufficient enough to identify solely with basic arithmetic. Being able to execute and relate to higher-level mathematics in high school is an extremely effective tool in providing students with multiple opportunities and choices in their future.

National reports show that tests scores in mathematics have been on the decline compared to those of other countries. For years now, we have seen a definite need for change. The average SAT, Scholastic Aptitude Test, score in math in 2009 was 515, down five points from 2005 (College Board, 2009). This decline shows a need for change in our methods of teaching mathematics because scores should be moving in the positive direction. Students’ abilities to solve mathematical problems correctly have declined. The Trends in International Mathematics and Science Study (TIMSS, 2003) report demonstrated the average mathematics scores in the United States are lower than the scores in five other countries (all in Asia). In order for our students to compete for jobs in their future, they must be able to compete with students from other countries. In addition, the national average in the United States was 508, which is only eight points higher than the TIMSS scale average of 500. Because the United States is seen as a
country of economic power and democracy, one would expect that our scores on standardized tests would be well above the average. These statistics alone verify the need for a new and improved curriculum in high school.

There is a connection between test scores and comprehensibility in mathematics. Comprehensibility can be described as a student’s ability to understand and use mathematics in outside settings. Unfortunately, the current structures for teaching mathematics do not adequately address the need for comprehensibility in a math classroom. Instead of focusing on communication, conceptual understanding, and motivation, many traditional curricula focus on memorizing procedures, which can leave students at a disadvantage when it comes to the creation of a strong math identity. Relatively low test scores reveal that students in the United States struggle to compete with achievement on a global level. Looking at SAT scores in mathematics alone, there has been a small decline in average scores as each year passes.

I believe incorporating talk in the math classroom can help students internalize mathematical ideas and perform higher on standardized. Individuals who engage in communication help each other construct meaning of new content (Bayer, 1990). Therefore, students who are given opportunities to communicate about project based mathematics are probably likely to perform well on standardized tests while creating connections with mathematics and building an academic identity.

In my observation as a teacher, students do not see a correlation between the math they are learning in school and the world around them. Students who do not see themselves needing or using mathematics outside of the classroom may come to believe that basic arithmetic is sufficient enough to “get by” in life and fail to develop the identity
of a proficient mathematics learner (Anderson, 2007). Especially at the high school level when we begin to examine higher levels of math, students struggle to link math to real life applications. There are a few programs that help students build identities through the connections of math and the real world. For example, the Algebra Project, a program that is designed to teach Algebra through the use of real world artifacts, has resulted in significant gains in test scores and exemplifies the role of Algebra as a gatekeeper to higher education (Moses, 2001). However, these programs are few and far between. Because students’ dreams and aspirations may change throughout high school and into college, it is important for them to understand how and why mathematics plays an important role in determining and ensuring optimal career and educational choices in their futures.

**Testing Data**

The Principals and Standards for Teaching Mathematics require students to complete four years of math in high school (NCTM, 2000). During these four years, students are required to make connections between concepts learned in previous years to build upon their foundation of understanding. Failing to build a strong, layered foundation of learning can negatively affect students’ math performance and can inhibit students from relating and communicating about mathematics. According to the data from the 2009 released CST (California Standards Test) scores for Geometry alone, 53% of California students fall in the basic, below basic, and far below basic levels. This is an unfortunate number, and it shows how students taking higher-level mathematics are falling below proficient and advanced levels. Higher-level classes constitute high school classes from Algebra I through Calculus. Chart Academy (all names of people and
places used in this paper are pseudonyms) is a Title I school in southeast San Diego, CA. As a Title I school, students attending Chart Academy are living near or at poverty levels. Students are considered “at-risk” of dropping out of high school for a variety of reasons including low-income, high number of absences, low academic performance, single-parent homes, and residing in gang infused neighborhoods. However, the students at Chart Academy have much to offer and have not been given the correct tools with which to be successful. Traditional methods for teaching mathematics have not catered to their needs and it is imperative that teachers provide material that is most relevant for the students. Many of the students are far behind grade level expectations in both mathematics and language arts. It is important to note that Chart Academy requires every ninth-grade student to take Geometry regardless of past scores in math classes. The state comparisons in test scores include students of other grade levels who partake in Geometry.

As can be seen by Table 1 below, the CST released scores for Chart Academy show that 76% of students fall in the below to far below basic levels. Furthermore, 94% of Chart Academy students fall in the basic, below basic, and far below basic categories (see Table 1), which is 41% more than the California average. Seen from a different perspective, of the Chart Academy students who took the Geometry CST, only 6% were proficient and 0% were advanced whereas on the state level, 32% of students scored proficient and 15% advanced. There is a definite need for a curriculum that will not only help our students produce proficient to advanced results on tests but will also help them create meaning in their lives through mathematics. It is important that students are able
to generate a math identity in the classroom that can give them the confidence and competency levels to succeed academically and throughout their lifetimes.

**Table 1: Geometry CST 2009** Percentage of students at Chart Academy that received *Advanced, Proficient, Basic, Below Basic, and Far Below Basic* on the Geometry California Standardized Test (CST) in 2009.

<table>
<thead>
<tr>
<th>CST GEOMETRY 2009</th>
<th>9TH GRADE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chart Academy</td>
<td></td>
</tr>
<tr>
<td>STUDENTS TESTED</td>
<td>107</td>
</tr>
<tr>
<td>% ADVANCED</td>
<td>0%</td>
</tr>
<tr>
<td>% PROFICIENT</td>
<td>6%</td>
</tr>
<tr>
<td>% BASIC</td>
<td>18%</td>
</tr>
<tr>
<td>% BELOW BASIC</td>
<td>53%</td>
</tr>
<tr>
<td>% FAR BELOW BASIC</td>
<td>23%</td>
</tr>
</tbody>
</table>

Comparing 2009 data with scores from previous years show consistency in test results. Because Chart Academy did not offer ninth grade in 2008, Geometry was not a course option for students. However, Table 2 below shows the CST scores from 2007 when Chart Academy did offer ninth grade.

**Table 2: Geometry CST 2007** Percentage of students at Chart Academy that received *Advanced, Proficient, Basic, Below Basic, and Far Below Basic* on the Geometry California Standardized Test (CST) in 2007.

<table>
<thead>
<tr>
<th>CST GEOMETRY 2007</th>
<th>9TH GRADE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chart Academy</td>
<td></td>
</tr>
<tr>
<td>STUDENTS TESTED</td>
<td>89</td>
</tr>
<tr>
<td>% ADVANCED</td>
<td>0%</td>
</tr>
<tr>
<td>% PROFICIENT</td>
<td>15%</td>
</tr>
<tr>
<td>% BASIC</td>
<td>21%</td>
</tr>
<tr>
<td>% BELOW BASIC</td>
<td>47%</td>
</tr>
<tr>
<td>% FAR BELOW BASIC</td>
<td>17%</td>
</tr>
</tbody>
</table>

As can be seen in Table 2, 85% of students fell in the *basic, below basic, and far below basic* proficiency levels. Again, no students performed at the *advanced* level, and only
15% hit the proficiency mark. This is similar to the results from 2009 where the majority of the students did not reach proficiency. Instead of moving more students towards proficient and advanced scores, Tables 1 and 2 reveal consistently low scores from students at Chart Academy as well as a decline in proficient scores from 15% in 2007 to 6% in 2009.

Figure 1 shows a side by side comparison of the 2007 data and the 2009 data.

![Figure 1: CST Comparison 2007 and 2009](image)

**Figure 1: CST Comparison 2007 and 2009** California Standardized Test (CST) percentages for Geometry students at Chart Academy in 2007 and 2009.

Although there is a big drop in proficient levels, these data indicate some consistency among various years in which Chart Academy offered Geometry. There are too many students who are not reaching proficient levels of understanding in Geometry. These data suggest the need for a change in the current way we teach Geometry. Current pedagogical approaches to teaching mathematics are leaving many of our students behind in understanding as well as not providing the proper preparation for college.
Lastly, it is important to note that test results show that students are struggling not only in Geometry, but in Algebra as well. The CST scores from 2009 are relatively consistent with the 2008 released scores (see Table 3) which tested the same group of students in Algebra I. As can be seen in Table 3, when roughly the same group of students were tested in Algebra I in 2008, 89% of students fell in the *basic, below basic,* and *far below basic* band levels. Only 12% of students reached proficiency with 2% of the twelve reaching the *advanced* level. These results are a regular occurrence at Chart Academy.

**Table 3: Algebra I CST 2008** Percentage of students at Chart Academy that received *Advanced, Proficient, Basic, Below Basic,* and *Far Below Basic* on the Algebra I California Standardized Test (CST) in 2008.

<table>
<thead>
<tr>
<th>CST ALGEBRA I</th>
<th>8th GRADE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chart Academy</td>
<td>248</td>
</tr>
<tr>
<td>STUDENTS TESTED</td>
<td></td>
</tr>
<tr>
<td>% ADVANCED</td>
<td>2%</td>
</tr>
<tr>
<td>% PROFICIENT</td>
<td>10%</td>
</tr>
<tr>
<td>% BASIC</td>
<td>28%</td>
</tr>
<tr>
<td>% BELOW BASIC</td>
<td>44%</td>
</tr>
<tr>
<td>% FAR BELOW BASIC</td>
<td>17%</td>
</tr>
</tbody>
</table>

From these figures, it is evident that achievement in standardized testing is not simply a Geometry based problem, but a problem across different mathematical disciplines. There is a need for students to feel connected to the math they learn in order to increase their level of understanding and ability to perform on standardized tests. Test taking ability skills and lack of achievement may be linked to a lack in math identity.

Chart Academy is comprised of 71% Latino students and 20.5% African American students.
Table 4: Scores by Ethnicity  Differences in race/ethnicity test score data from the National Center for Education Statistics and the National Assessment of Educational Progress

<table>
<thead>
<tr>
<th>Race/Ethnicity</th>
<th>Test Scores</th>
<th>Percent at or above Basic</th>
<th>Percent at Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anglo Students</td>
<td>289</td>
<td>78</td>
<td>10</td>
</tr>
<tr>
<td>Latino Students</td>
<td>256</td>
<td>45</td>
<td>1</td>
</tr>
<tr>
<td>African American</td>
<td>250</td>
<td>40</td>
<td>1</td>
</tr>
</tbody>
</table>

Because over 80% of students at Chart Academy are of Latino or African American decent, it is important to look at the test scores in Table 4. Test results from NAEP (National Assessment of Educational Progress) also show students of Latino and African American decent underperforming in mathematics. For example, only 45 and 40 percent of Latino and African American students reached at least a basic level across the United States (NCES, 2009; NAEP 2009). In 2009 (see Table 4), African American students had an average score that was 39 points lower and Latino students had an average score that was 33 points lower than those of Anglo students. All students, including those who may not have succeeded in the past, are entitled to innovative ways of approaching mathematics education as a way to bridge this achievement gap. Furthermore, the need for a new approach can be beneficial to all students and can help them build levels of conceptual understanding and motivation around mathematical topics.

A Need for Change

Based on the findings presented above, it is evident that the need for a new approach to teaching mathematics is necessary. In order for students to succeed academically, they must be able to understand and use mathematics in their lives. In
order to create curricula that benefit all students, it is important to look at research that is relevant in the field of mathematics education. Building on and incorporating parts of the success cases in past studies can greatly enhance mathematics curricula.
Chapter III.
Review of Relevant Research

Teaching students in a low-socioeconomic area where many factors influence them outside of school leads to an even greater need for change in the way we have traditionally taught math because students are entitled to be presented with content in a manner that applies to their lives. When teachers design lessons that incorporate the lives of students, students are more likely to internalize the information and consider the math useful. This paper reviews the relationships and contributions of three different constructs in the creation of a strong math identity: peer communication in a classroom setting, motivation, and conceptual understanding.

Identity Formation

Identities are malleable, dynamic, and ongoing constructions of who we are as a result of our participation with others in the experience of life (Wenger, 1998). Children grow as mathematics students from elementary and middle school through high school and develop a sense of who they are as math learners based on their experiences in math classrooms. All of these experiences, as well as students’ interactions with their peers, teachers, and parents, influence the development of their math identities. Anderson (2007) found that “[e]ngaging in a particular mathematics learning environment aids students in their development of an identity as capable mathematics learners. Other students, however, may not identify with this environment and may come to see themselves as only marginally part of the mathematics learning community (p 8)”.

In order to create strong math identities in which students belong to and are accepted as participants of particular mathematical communities, learners need more than procedural
fluency. When teachers provide students with opportunities to feel successful and participate in school-based mathematical communities students can learn to use math to create meaning in their lives. Curricula must be developed to enhance that which leads to a strong math identity, specifically: a) communication b) motivation, and c) conceptual understanding. These curriculum constructs will lead to students’ success in mathematics and therefore prepare them for careers that require mathematics. The benefits of a student’s ability to create a math identity may be seen in their disposition to relate to and participate in math as well as acquire and retain a necessary set of skills that will eventually influence career choices.

**Identity and Math Experiences**

The ways students see mathematics in relation to the broader context of their lives can contribute either positively or negatively to their identity as mathematics learners (Anderson, 2007). Students who can both relate to and take ownership of their learning can create the factors such as conceptual understanding and motivation that influence their identities as mathematics learners. Identities are formed through peoples’ experiences and stories (Sfard & Prusak, 2005). Mathematical identities are, therefore, formed through experiences and stories in the math classroom. The daily, weekly, monthly, and yearly experiences that each student undergoes help form their overall identities as learners in a math classroom. Students who have positive experiences in mathematics tend to develop stronger identities while students who have more negative experiences have weaker identities. Black et al. (2009) offers a detailed description on identity:
We construct our identities as mathematics learners upon reflection on the subjectivities we have experienced when engaging in various forms of mathematical activity. Thus, our notion of identity (or identities) is historical in origin and emerges from the subjectivities (how one views oneself) we experience in the process of doing activities (p 58).

Therefore, students who experience success in the math classroom will have different identities from those who have typically experienced what they perceive as failure in the academic setting.

**Math Relevance Improves Conceptual Understanding**

Students must understand the relationship between mathematics and the real world in order to develop conceptual understandings and have a positive affiliation with the subject as a whole. In order for students to be able to acquire knowledge, they must be able to appropriate, or find a connection with, mathematical information (Boaler, 2002). Helping students build connections and relevance between what they learn in school and the world around them can help them achieve conceptual understanding in higher-level mathematics. Conceptual understanding is the "the comprehension of mathematical concepts, operations, and relations (Kilpatrick et al., 2001)" which requires students to go beyond computation and focus on big picture ideas in math. For example, students in Geometry must learn that three angles in a triangle always measure 180 degrees. In order for students to truly understand this concept, they must be able to relate triangles and their importance to the real world. Students who can investigate this concept through “hands-on” experiences that allow them to discover this relationship among all triangles will build stronger connections to this concept than students who are passively given the information.
Communication is foundational to motivation and conceptual understanding in forming the math identity. Communication is a key component to helping students construct meaning in mathematics (Bresser, Melanese, & Sphar, 2009). Students who are encouraged to talk in an academic setting about mathematical content are more likely to understand and be motivated in a classroom. In addition, language is an instrument for the negotiation of meaning (Bayer, 1990). If conversations do not take place in a classroom setting, students are less likely to find meaning in mathematical content. Building meaning of math in the lives of students can influence their motivation levels in the classroom. In turn, the constructs of motivation and engagement greatly influence students’ mathematical identities. If students participate in activities that appeal to them, their identity can be further developed through their personal motivation levels and engagement in such activities. Furthermore, providing students with engaging material stimulates a greater response to their levels of conceptual understanding and successes as students of mathematics.

Motivation

Students who possess a motivational orientation that focuses on learning and mastery of the subject matter are more apt to exhibit sustained engagement with schoolwork than students whose orientation is to merely perform satisfactorily or complete assigned work (Ames 1992; Thomas 2000). Thomas states that classrooms that promote student autonomy and cooperative goals as opposed to competitive goals allow students to focus on learning and mastery as the threat of competition is reduced. This research suggests that classroom teachers incorporate these constructs into their curriculum in order to help all children feel comfortable in their learning environment.
Learning is a natural part of evolving as human beings. Children want to learn and have a desire to make sense of the world (Oakes & Lipton, 1999). Therefore, intrinsic motivation, or motivation that comes from within the individual rather than from external factors, is more powerful in helping students develop as learners of knowledge. Providing students with opportunities to feel a sense of intrinsic motivation can influence their decisions to further their education in mathematics as well as in other disciplines throughout their future. In contrast to B. F. Skinners’ principles of behaviorism, motivation should come from within rather than external factors such as grades, gold stars, and rewards for stellar academic performance. Deci (1996) states that “children are not passively waiting to be drawn into learning by the offer of rewards but rather are actively engaged in the process of learning” (p. 20). It is evident that intrinsic motivation should be addressed in the design of content curriculum at all levels of mathematics.

Intrinsic motivation can come in many forms and it can be helpful in providing students with authentic learning experiences that a desire to learn in the student. For example, providing students with opportunities to have control and make decisions about their learning, creating meaningful and engaging curriculum, and holding students responsible for their learning can all lead to a student’s desire to learn more (Zemelman, Daniels, & Hyde 2005). According to Deci (1996), any act in which a person does not feel autonomous decreases their intrinsic motivation. In order to feel autonomous, a student strives to feel like a creator or designer of his or her own behavior and learning. When students feel as though their behavior is chosen by them and not by any form of external influence, they experience autonomy. Therefore, it is important that we create curriculum that allows students to control their own learning. Students need choices in
order to feel successful as it supports their independence. We need to actively encourage
self-initiation, experimentation, and responsibility. Deci (1996) stated that “motivation
requires that people see a relationship between their behavior and desired outcomes” (p. 59).

There are many reasons why students can lack the motivation they need to create
understanding in the classroom. Fear of failure, incuriosity, lack of ambition, lack of
responsibility, and combativeness all have the potential to contribute to low achievement
(Reiss, 2009). These factors can in turn lead to lack of conceptual understanding and low
achievement in and outside of the classroom. To perform well in a math environment,
students must not only be confident but have the confidence and motivational levels that
come from knowing that they know the content. Students’ success in the classroom
contributes to their motivational level and motivation does, in turn, affect a student’s
ability to achieve.

Chen and Stevenson (1995) conducted a comparative study incorporating
motivation and mathematics as part of the TIMSS study. The study compares the
motivation of culturally diverse groups of high school students. Chen and Stevenson also
look at various factors that influence student achievement such as effort, attitude,
interactions, and family lifestyles. The study is a quantitative effort to examine why
Asian-American and East Asian students tended to perform better than their Caucasian-
American peers on the standardized tests and math comprehension studied by the
researchers. The results of the study demonstrate that American students of all ethnic
backgrounds struggle with the ability to create a math identity. This comparative study
suggests the importance of creating a strong identity in math classrooms across the
United States in order for students to be able to keep up with students from other countries.

In order to increase students’ math identities and motivational level, they must be able to conceptually understand given tasks and content. Currently, the majority of American students distinguish having a good teacher as the number one factor contributing to their success rate. This is different from the majority of Chinese and Japanese students who feel that studying hard is the number one factor (Chen & Stevenson, 1995). From these results, it is apparent that students in the United States perceive their control of learning as a more marginal factor. There is a definite dependence of the role of the teacher in American society to accelerate academic achievement. Although these observations may be undesired, it is a reality of schooling in the United States, and teachers need to change the way they teach in order for students, especially those who fall behind national averages, to feel competent and triumphant in math. Because students in the United States highly depend on their teachers for guidance, providing curriculum that stimulates students to discover and communicate with one another can help build their conceptual understanding, motivation and in turn their identity.

It may be helpful to have a curriculum in which the control of learning is moved away from the teacher and towards to the student. Shifting this control of learning can help facilitate communication between students because students must rely on their prior knowledge, skills, and investigations in order to learn. In the TIMSS study, students were asked what motivates them to succeed in mathematics from a list of 9 choices: to gain more knowledge, to get good grades, to go to college, to please parents, to please
teachers, to get a better job in the future, and because they set high standards for themselves, had no other choice, or did not know what to do with their time. The top three choices in order of importance were to get a better job, to go to college, and to gain more knowledge. Those students who set high standards for themselves typically performed higher on the tests. From this study, it can be seen that high-achieving students are intrinsically motivated to succeed in math, and are therefore well prepared for success in the future. It is important that teachers use the intrinsic motivation already instilled in students to build curricula that appeals to students and incorporates communication in the classroom. From here, students can feel more successful and also more aware of mathematics in their lives. Although students have some choice in their own level of motivation, the way in which their material is presented to them can heavily sway their decisions and feelings about mathematics. This, in turn, will lead to a math identity in which students find meaningful connections with mathematics.

**Conceptual Understanding**

“The goal of teaching mathematics is to help all students understand concepts and use them powerfully” (Zemelman, Daniels and Hyde, 2005). The key to helping students build meaning and understand is for students to be able to take what they know and actually use it in the world around them. In order for students to be able to use mathematics in meaningful ways, they must be able to understand and apply their knowledge to different situations. True understanding of mathematics increases a student’s ability to comprehend and retain information which in turn increases a student’s aptitude to use this information (Bransford, Brown, and Cocking, 2000). Our society’s emphasis on standardized test performance has had a detrimental impact on math
instruction. Sadly, standardized tests have implications for our society, schools, and the future of our students. Although there is pressure for students to perform well on standardized tests, it is vital for students to be able to remember and use mathematics in different situations. Creating a learning environment that is both enjoyable and meaningful to students can help them conceptually understand mathematics. Students must be able to do well on standardized tests and learn mathematics in more robust ways that enhance their motivation and engagement levels in the classroom. It has become exceedingly imperative that students are able to draw upon learned concepts and connect them in new situations. Students must be able to construct their own meaning in what they see because, as can be seen by studies of early mathematical cognition, human beings never just take in and memorize material (Zemelman, Daniels, & Hyde 2005).

In order to achieve conceptual understanding, it is important to present content in a manner that allows students to truly become active in their own learning. Students who regularly partake in an active role in the classroom can build knowledge together. Engaging in conversations with classmates can help students build conceptual and shared knowledge through joint activities (Wells & Haneda, 1992). Traditionally, math is taught in a way that does not provide students with the tools to find connections between different levels of mathematics. In the past, most people have seen math as a collection of unrelated topics, theorems, procedures, and facts (Zemelman, Daniels and Hyde, 2005). In the United States, we teach and cover many different standards in one school year. Unfortunately, this does not allow for an in-depth investigation of each topic. Instead, we have focused on introducing many topics to students without creating opportunities for true mastery.
Coverage can be described as a means by which to attempt to relay all of the material and standards within a school year (Wiggins and McTighe, 2006). Because coverage does not allow students to study one particular concept in detail, students are unable to see relationships between different areas of mathematics (i.e. algebra, geometry, trigonometry, etc.) According to Zemelman (2005), there are five key processes that help build mathematical understanding: making connections, using reasoning and developing proofs, problem solving, creating relationships, and communicating ideas. Students must be able to connect newly learned information to prior experiences and knowledge. In order for students to start to see patterns and develop true conceptual understanding, they will need many more examples than what is provided for them in a textbook (Zemelman, Daniels, & Hyde 2005). It is important for students to be able to investigate and use their own reasoning skills to develop meaning and create rationale. Problem solving strategies allow for students to really think and maybe even struggle with a given task. Piaget’s (1970) idea of disequilibrium touches on the notion that learning is and should be unsettling at times. Learners should identify challenges and disequilibrium as something positive, and teachers should provide opportunities for students to experience disequilibrium in the cognitive process (Oakes & Lipton, 1999). “The best problems are authentic, challenging, intriguing, mathematically rich, and perhaps counterintuitive” (Zemelman, Daniels, & Hyde 2005).

*Principals and Standards for School Mathematics* (NCTM, 2000) emphasizes the importance of the communication of mathematical ideas as it helps students build conceptual understanding. The social aspect of learning is a huge contributor to students levels of conceptual understanding (Linn & Songer, 1991). Students construct
understanding of concepts by integrating their experiences, observations, activities, and ideas into their already existing schema. Conceptual understanding becomes extremely important in adolescent years due to the cognitive changes that take place during this time (Linn & Songer, 1991). In order for students to reach high levels of conceptual understanding, they must engage in learning that incorporates challenges and communication in the classroom.

**Communication**

The act of learning and acquiring knowledge is based on fundamental principles of social interactions. Taking classic learning theory into account, individuals have different schemata and have different ways of identifying knowledge (Oakes & Lipton, 1999). We use our past experiences and our past knowledge in order to build relationships in our minds with new information. Building on prior knowledge in the classroom can help students make connections with content material. Students need opportunities to discuss their prior knowledge with classmates in order to develop a stronger identity in math.

Vygotsky (1986) theorizes that thoughts take on new meanings when they are verbalized since word meanings have an association between the word’s sound and its content. Any opportunities students have to express their understanding or lack thereof is an important part of acquiring knowledge and constructing identity. Dialogue is essential to learning and should be embedded in each and every opportunity to gain knowledge. Because learning is social, conversations and discussions are pertinent to learning new information in the classroom as well as developing a sense of identity focused around math.
Semiotic mediation, or the role of talk, and the incorporation of real life activities can help students develop their identities as members of their communities as well as members of their classrooms (Wells & Haneda, 1992). A sociocultural approach to semiotic mediation emphasizes, 1) making connections from everyday life events, 2) mediated actions in social contexts, 3) developments of thinking through joint mediated activities, and 4) the idea that individuals are active agents in learning (Cole, 1996).

Providing students with a curriculum focused on communication and peer mediated discussions can help students reach a sense of understanding which can, in turn, affect their math identities.

Unfortunately, many teachers do not feel comfortable using classroom discussions in a mathematics class setting (Chapin, 2003). Providing students with opportunities to converse in a math classroom can help them think, express their ideas, reason, and problem solve. In order to provide meaningful experiences for students, teachers should be aware of the power of conversation in a classroom setting and strategically use discussions to help students learn. For example, in *Using Math Talk to Help Students Learn, Grades 1-6*, Chapin (2003) describes appropriate tools and strategies that can help make classroom discussions meaningful and purposeful. Revoicing, or repeating what students say to gain clarity, is an example of a tool teachers can use to facilitate discussion and understanding among students. Another strategy that students can use to reemphasize the importance of communication is to restate another students’ reasoning. This tool provides a way for students to clarify the claims of others. It also gives English Language Learners a chance to hear ideas and comments multiple times (Chapin 2003).
These are just some of the useful strategies that can be incorporated into mathematics curricula to build student confidence, communication skills, and understanding.

Cobb (1997) found that students’ reflective discourse may be a contributing factor to their mathematical learning. Reflective discourse can be a discussion among students in which mathematical topics are focused topics of conversation. Cobb’s study suggests that communication about mathematics within a classroom can be a significant tool in helping students reach higher levels of understanding.

As stated earlier, communication is a key component to providing opportunities for students to conceptually understand as well as motivating students to engage in mathematical content. Designing curriculum that promotes a sociocultural approach to learning can aid students in feeling autonomous which can lead to intrinsic motivation (Deci, 1996). Likewise, students who are motivated to learn strive to conceptually understand the concepts.

Within teaching students the importance of communication in the math classroom, it is important to teach students the academic language needed to have intellectual conversations about math. Academic language can be defined as the language needed by students to understand and communicate in an academic discipline. This can be especially difficult for English Language Learners (ELL) who are novices to the English language. Because Chart Academy is comprised of close to 45% of ELL students, this population of students is a concern for this school. Finding ways to help ELL students communicate effectively in an academic setting can raise achievement levels in mathematics. Conversational language, or language that students use in a non-academic setting, is different from academic language (Bresser, Melanese, & Sphar,
Students must know the difference between the two and be able to use academic language in an academic setting.

Students should be provided with necessary scaffolds and useful strategies in achieving the understanding and use of academic language in order to facilitate effective classroom discussions. For example, strategies such as activating prior knowledge, reducing the stress level in the classroom, creating vocabulary banks or word walls, practicing wait time, asking appropriate questions, and using manipulatives can all enhance the learning experience for English Language Learners (Bresser, Melanese, & Sphar, 2009). Although these strategies are effective for students learning English, they are also useful tools for the general education population as they promote a secure environment where students can feel safe to take risks.

**Supporting a New Curriculum**

When students are not able to make connections between the mathematics they learn in school and its perceived utility in their lives, they may construct an identity that does not include the need for advanced mathematics courses in high school (Anderson, 2007). Being unable to make connections, in turn, trickles down to students who fall behind in their attempts to understand mathematics and their ability to keep up with grade level content. By the time many students reach high school, especially those from low-income areas, their desire to learn mathematics and their capacity to achieve in mathematics is at great risk. Therefore, a curriculum that promotes motivation, conceptual understanding, and peer communication would help students identify with concepts that may seem foreign and unrelated to their lives.
Chapter IV.
Review of Existing Curricula

Throughout the years, there have been many curricula that present mathematics from a traditional approach, where tedious practice and rote memorization are the focus. To the contrary, the way in which children learn best involves constructing ideas and systems. As can be seen by supporting research, there is more to the learning process than simply memorizing material (Zemelman, Daniels, & Hyde, 2005). Because learning is a social process, communication and interactions with others must take place in order for material to be truly understood. In order to learn something new, individuals must first be able to activate their prior knowledge and form associations with new information. Based on findings by Hirst & Manier (1995), learning begins with an intrapersonal approach or inside ones’ head. Through interactions with others, an interpersonal exchange of information takes place which in turn leads to another intrapersonal level of understanding. These social interactions eventually lead to intersubjectivity or co-construction of meaning (Hirst & Manier, 1995). Curriculum should be developed to promote talk in the classroom and help students investigate concepts through “hands-on” and project based experiences. To that end, there are also some curricula that focus on the big picture and help students discover relationships. Project Based Learning is an approach to teaching that focuses on big picture ideas and helps students understand the importance of mathematics in the world. Project Based Learning helps students focus on big picture ideas and problem solving strategies through the use of major projects that depict the use of mathematics in the real world. In addition, students spend over half of their time in their homes and communities which makes it
even more important to be able to find these connections between school and the outside world (Bransford, 2000). Focusing on the big picture and providing links to the communities of the students allow learners to utilize the school experience outside the classroom.

Although both traditional approaches and approaches that incorporate big picture ideas through discovery learning have proven to be successful for some students, there are still many students who need a new approach in order to identify with higher-level math. Because all students need to see real world connections in order to bond with the math they learn, and because some also struggle with rote memorization, students need access to mathematics curriculum in innovative forms. Given the research reviewed in the section on motivation, traditional approaches to teaching mathematics can be problematic because students are not held responsible for their own learning. In addition, students are given mathematical facts instead of being required to discover these facts. Students learn by doing and should be active participants in their acquisition of knowledge (Van Tassell, 2001). The role of the teacher should shift from the traditional lecture-style approach to that of a facilitator, assisting students in understanding concepts through dialogue, investigations, and questioning. Project Based Learning may be superior in theory, but in its purest form does not allow students who are achieving below grade level standards sufficient opportunities to practice what they have discovered. Therefore, a balanced approach needs to be developed in which students can discover, investigate, think critically, problem solve, discuss mathematics, and have opportunities to practice.

In order for students to feel a connection with mathematics and develop an identity, they must be presented with material that is stimulating, engaging, and creates a
sense of autonomy. Anderson (2007) found that “[t]he types of mathematical tasks and teaching and learning structures used in the classroom contribute significantly to the development of students’ mathematical identities (p. 9).” If students are presented with material that can stimulate their interest and creativity, they are more likely to be engaged and develop a strong identity. When students are able to develop their own strategies and meanings for solving mathematics problems, they learn to view themselves as capable members of a community engaged in mathematical learning (Anderson, 2007). Students need opportunities to explore, discover, and communicate mathematical concepts.

In the elementary years, it is more common to see mathematics presented to students in an exploratory style. Unfortunately, as students transition into middle and high school, there is less discovery and more lecture and practice based instruction. This can lead students to detach themselves from mathematics in the upper grades. As a teacher in a school where students are “at-risk” of failure, it has become more evident and extremely important to help these students find a math identity to help them achieve both conceptually and on standardized tests. Given the modes of instruction at the high school level, students often lose their connection with their math content. Instead, they are given notes and facts proven to be true by mathematicians, and learners are asked to use these truths to solve problems from which they cannot create meanings. In order for a student to be able to create meaning, they must be able to see and discover the big picture while practicing the skills. For example, when a student is able to investigate a mathematical concept, communicate findings with classmates, and co-construct meaning from their investigation, they are more likely to see relationships between ideas and the real world.
Providing students with a curriculum that incorporates both conceptual understanding and procedural fluency is especially important for students who do not have the support from home and their parents. Parents may not understand the content that their students are required to know or parents may not be around to support their child in a manner that will help the child succeed in a math classroom. Students, therefore, need to feel autonomous in their decision to learn. It has been demonstrated that students whose motivation is autonomous rather than controlled seem to produce more successful results (Deci, 1996). Project Based Learning allocates time for students to investigate and discover the true meanings of what they are learning while providing learners with ownership and accountability. Rather than focusing only on direct instruction, Project Based Learning encourages individualized learning and student ownership over what they accomplish. A curriculum that incorporates Project Based Learning, together with a traditional practice, will allow students the opportunity to discover math while applying the skills necessary for superior performance on traditional exams. It may be beneficial for students to be provided with instruction that incorporates a balance between traditional and Project Based learning in order for students to benefit from both approaches.

*Discovering Geometry: An Investigative Approach* (Serra, 2008) and *College Preparatory Mathematics: Geometry* are curricula adopted for use by the state of California and many districts. In addition to these two textbooks, Project Based Learning will be defined and reviewed in this section. These curricula reflect a range of teaching modes and provide varying opportunities for students to communicate, practice, develop conceptual understanding, discover, and create meaning.
At Chart Academy, all ninth graders are enrolled in Geometry and use *Discovering Geometry: An Investigative Approach* published by Key Curriculum Press (Serra, 2008). While the curriculum contains a great amount of drill and practice, each lesson begins with an investigation. The book was designed to help students be actively engaged as they learn and gives them the tools to investigate theorems while learning by doing. *Discovering Geometry* has many “extras” including a quote at the beginning of each lesson, a challenge section with puzzles for improving skills, and exploration sections which include material such as geometric probability, building geometric solids, finding the height of any building, and many more. There are also suggestions for projects that relate to geometry including making kaleidoscopes, designing a racetrack, creating a mural, and opportunities for students to practice their skills using the latest educational technology.

Although there are attempts to help students create meaningful connections between a certain topic and the real world, there is a lack of unity and consistency in the way in which this curriculum is taught at Chart Academy. The curriculum has been implemented in a more traditional approach in the past, and some of the most relevant lessons that connect Geometry to the real world are not used.

Most of the sections in *Discovering Geometry* do not incorporate prior knowledge, and there is little connection to previous years of mathematics. The writing and introduction to new topics does not meet the needs of *English Language Learner* (ELL) students as there are few *Specifically Designed Academic Instruction in English* (SDAIE) strategies and limited picture examples. SDAIE strategies are designed with the
needs of ELL students in mind, but can benefit all students. Some strategies include use of manipulatives, visuals, and graphic organizers, providing opportunities for student interaction, embedding context in content, activating prior knowledge, incorporating a variety of cultures in lessons, reducing the amount of teacher talk, and multiple opportunities to check for understanding. Some students, especially those who are behind in grade level content, are at a disadvantage because their learning styles are not being met through the way in which the current curriculum is implemented. Students need a new approach to learning mathematics as solely relying on the way in which Discovering Geometry is implemented does not meet the needs of Chart Academy students. Unfortunately, Discovering Geometry does not meet the needs of all students due to the manner of implementation.

The curriculum superficially covers the California State Standards for Geometry by touching on each standard. Discovering Geometry (Serra, 2008) focuses on the breadth of each standard rather than the depth of each key concept. As can be seen by the reviewed research, a student’s ability to conceptually understand and draw connections between mathematics and the real world lies with deeply engaging and soaking in each concept. Coverage of content standards has proven to inhibit students from being able to find relationships between different areas of mathematics. It also does not allow students ample time to investigate topics that are imperative to student learning. This curriculum fails to significantly incorporate educational constructs critical to a Geometry curriculum. These constructs include conceptual understanding, motivation, creating an academic math identity, communication, and project based learning.
It is important that students create an identity around the idea of mathematics. They must see the relevance of what they are learning and how it applies to their own lives. In order for students to create this identity, they must believe that they are able to learn, retain, and use the information during other points in their lifetime. They must believe they are able to initiate and successfully implement specified tasks at various levels, and feel confident and competent to reproduce material at given times.

*Discovering Geometry* does not give students the opportunity to create meaningful connections and find a true sense of accomplishment through its curricular approach and the way in which it is implemented at Chart Academy. Instead, students are given information that they must memorize and retain without a sense of connection. Students are unable to identify with the material and struggle to create any sense of identity around Geometric concepts.

**Motivation**

*Discovering Geometry*, although attentive to covering and exposing students to all the necessary standards, fails to develop students’ motivation to learn and understand mathematics. The textbook is fairly traditional in the sense that each section begins with an introduction to the lesson which is either a picture related to the real world or a review of the previous section. Next, there is a section for investigation where students are to discover a certain theorem with prompts from the textbook and their teacher. Lastly, each section has an abundance of exercises (or drill and practice problems) where students are able to practice whatever it is that they discovered. Most sections also contain a review section where students can review skills they learned earlier in that chapter for testing. Students need to be able to find connections with the content in order
to be able to internalize new information and use it effectively. *Discovering Geometry* does not allow for students to create opportunities for engagement.

There are some attempts to engage students in projects related to the material, but the project instructions do not cater to English Language Learners and the directions are difficult to understand and ambiguous. *Discovering Geometry* focuses more on single lessons of investigation rather than bigger projects that incorporate multiple standards. The investigations ask students to follow step by step instructions to arrive at their discoveries. There is little room for student choice, and although there are some projects, the students do not get to choose a particular project that incorporates multiple standards. The projects are found in small boxes at the end of each section and are not considered a mandatory part of the curriculum. Given the time restraints of a class, there would be little time for students to actually arrive at the exploration sections as the exercises are long and tedious. If the explorations became the focus of the lesson and the exercises are secondary, students would be able to get more out of each lesson. Unfortunately, if students are to finish all the standards in one school year, the number of problems that *Discovering Geometry* incorporates per lesson exceeds the amount of time students spend on investigations.

Other sections, such as the exploration sections, attempt to create student engagement. Many of these sections incorporate Geometer’s Sketchpad which is a computer-based program. The explorations are well thought out and incorporate student discovery and exploration. Unfortunately, especially at schools in low-income areas, many students do not have access to a computer outside of school. Although some computers can be accessed, many students would not be able to explore computer-based
curricula on their own which is a key component to both motivation and conceptual understanding. The idea is critical and very important to the 21st century learner, but at this point it is not plausible to expect every student to have access to technology.

*Discovering Geometry* is a teacher-centered approach to instruction in the way that it is being implemented at Chart Academy, and although there is some investigation at the beginning of each section, students must rely on the teacher to guide them to correct answers to theorems. Because the textbook is already divided into sections with specific learning goals already in place, there is little room for student choice. There are no suggestions or approaches where students can actively choose what they want to learn and the order they want to learn it in. Providing choice to students is another way to motivate them (Deci, 1996; Zemelman, Daniels and Hyde, 2005) however *Discovering Geometry* does not cater to the incorporation of student choice in the classroom.

**Conceptual Understanding**

Because conceptual understanding requires students to go in depth when learning a new concept, *Discovering Geometry* fails to provide students with the time and dedication to truly investigating standards and topics. The curriculum scratches the surface of each key standard and provides students with many opportunities to answer and solve problems related to each standard, but it does not allow for much in-depth analysis. Most of the learning takes place in the classroom at a desk, and students are not asked to create connections to the world around them. In order to build conceptual understanding, students must be able to make the connections with the world, and experiencing mathematics in alternate settings may help students internalize the information and draw connections between math and the world around them. There is
also no distinct connection between the Algebra students learn in eighth grade and Geometry. This curriculum does not ask students to relate what they learn in ninth grade to what they learned in previous years. This again has an impact on students’ abilities to understand and apply what they learn.

**Communication**

*Discovery Geometry* (Serra, 2008) incorporates little to no peer discussions around mathematical concepts. Students are required to read through examples by experts, and solve problems individually during the practice section. There are no prompts for discussion and no open ended questions for students to discuss. The book provides a passive approach to learning where students are required to remain silent for each lesson. The book attempts to have students “discover” certain concepts, but eventually provides students with the necessary information they need in order to solve problems instead of allowing students time to discuss their findings with their classmates. Unfortunately, *Discovery Geometry* gives the impression that mathematics consists of being able to perform computations individually. There is no attempt to connect mathematics to the social world and provide opportunities for semiotic mediation in the classroom. However, although *Discovering Geometry* does not explicitly provide teachers with this opportunity, there are definitely opportunities for teachers to engage students in communication without the use of the text. It may be difficult for novice teachers to find entry questions for discussion and it would be helpful if *Discovering Geometry* embedded these questions into the curriculum.
College Preparatory Mathematics: Geometry

College Preparatory Mathematics (CPM, 2010) is a curricular approach to teaching mathematics that focuses on fostering long-term learning and retention. The core of the curriculum focuses on conceptual understanding with the development of rules and algorithms. It was adopted by the California State Board of Education in November 2007, and is focused on problem-based learning. CPM students are learning how to make mathematics useful in their daily lives and recognize the opportunities that a well-designed mathematics curriculum provides for their future. CPM fosters student study teams and encourages dialogue among students. There are some opportunities for practice within the curriculum, but most of the problems incorporate different standards and different techniques for solving problems. CPM is both discovery based learning and student centered.

Motivation

Because CPM is focused on problem-based instruction, there is a high level of motivational factors. The CPM textbook starts each and every chapter, or section, with an overlying problem for students to solve. Although this can be engaging and motivating to some students, there are other students who need variation. The problems incorporated in the CPM curriculum are definitely applicable to the real world and connect mathematics to many professions we see today. As a student though, it may be beneficial to see a variety of activities that surround different topics. For example, when students are asked to discover the trigonometric ratios, CPM requires students to go in detail about problems that incorporate the height of trees and the length of their shadows.
It would be helpful for students to be able to see the application of trigonometry in other forms as well that may be more relevant to students’ life experiences.

Students should also be asked to present their learning and demonstrate knowledge in a variety of ways. CPM does not incorporate many overarching projects and/or presentations which can promote student engagement and autonomy.

**Conceptual Understanding**

CPM does help students conceptually understand material through the use of real world connotations and problem-based instruction. Students are able to see how the mathematics relates to the world around them and actually discover solutions to a big problem faced by either different professions or different people in the world. However, conceptual understanding does not always show a relationship with the longevity of a student’s retention level. In other words, students may be able to conceptually understand a concept, but without ample practice may not be able to retain the information for a long period of time. Students need to be able to practice the overlying concept multiple times and see a variety of problems related to each topic. CPM gives students one big problem focused around a concept and students are asked to find a solution. However, CPM does not usually give students multiple opportunities to solve the same concept in different ways and apply their knowledge to problems that appear similar in nature.

**Communication**

College Preparatory Mathematics (2009) provides students with a variety of opportunities to discuss mathematics. During a particular lesson, students are asked to work through a problem in groups similar to a “math lab”. Students are encouraged to
work in study teams for the beginning of the lesson. Teachers are still required to provide direct instruction in an effort to offer explanation to students’ questions and confusions during the math lab. CPM offers opportunities for students to work in groups and solve problems and breaks down some of the problems into multiple scaffolded parts so students can discuss one piece at a time. However, the teacher acts as the provider of knowledge prior to the labs as well as post labs. Although the curriculum provides ample questions for students to discuss, the students are still quite reliant on the instructor to provide information.

**Project Based Learning**

Project Based Learning (PBL) can be described as a teaching practice that organizes standards and curriculum around in-depth projects (Thomas, 2000). There are many different forms of PBL, and different instructors use PBL to different extents in the classroom. PBL allows students opportunities investigate, discover, and take ownership of their own learning. It is a student-centered approach in which the teacher acts as a support rather than a focal point in the classroom. PBL provides learners with many opportunities to achieve conceptual understanding. The core of PBL focuses on creating an environment where students work autonomously over a long period of time on an intense problem or project, investigate overlying concepts in mathematics, discover meaningful results, apply their findings to various standards/branches of mathematics, and retain conceptual knowledge that can be accessed in another situation. The National Council of Teachers of Mathematics (NCTM) standard on data analysis and probability urges teachers to provide challenging tasks in which students formulate questions and
design their own studies that can be addressed with data they collect, organize, and analyze (Zemelman, Daniel, & Hyde 2005).

Especially in Geometry, concepts are best learned through real-world experiences which are fostered in the structure of project-based mathematics. Many of the recent applications of PBL can be seen in the math and science fields. Thomas (2000) describes some of the problem-based learning that has been implemented at the secondary level from the Center for Problem Based Learning at the Illinois Mathematics and Science Academy (IMSA). Although there are successful outcomes from research done on project based learning, there have originated some problems as well. Furthermore, there have been cognitive research studies that incorporate PBL such as motivation, expertise, contextual factors, and technology. As stated earlier in the section on motivation, students who are engaged in their work are more likely to have the intrinsic motivation to not only achieve, but master given content. Because PBL focuses on authentic learning experiences that integrate student-centered investigation and discovery, students have a greater ownership of their own learning and feel a sense of mastery after being able to conceptually understand their findings. As stated by Zemelman (2005), authentic lessons have three main components: (1) students construct knowledge, (2) students draw conclusions, and (3) students connect the topic to their own lives. It is in the best interest of the student that learning incorporates authenticity because this in turn facilitates conceptual understanding.

According to Thomas (2000), there are certain criteria that a project must have in order to be considered Project Based Learning. First, PBL is the center of the curriculum. Students are constantly engaged in an investigative project that leads to student-
discovered results. Second, all projects are created with the intention that students will stumble upon the essential concepts and standards of mathematics. Third, PBL focuses on student investigation. Students must be engaged in some sort of exploration that forces them to self-teach and struggle with desired discipline goals on their own. Fourth, the projects presented to the learner are student-driven. In other words, there is not one set of desired results that are expected to surface. Instead, students are responsible and expected to produce authentic results that have impact on and relate to the real world. Lastly, the projects are expected to have a real-life connection as opposed to operating solely within the confines of an artificial school domain. As stated earlier, these real world connections allow some students to master a discipline or concept through communication and understanding. However, although standardized testing may not be the most ideal solution to rating schools and students, it is an important part of public education. Therefore, students also need time to practice more traditional problems in order to have sufficient exposure to the standards and practice the expectations for high-stakes standardized tests.

Boaler, 1997, (in Thomas, 2000) found that students who attended a project-based school attained higher mathematical assessments results. This study was conducted on two British secondary schools, and was implemented over 3 years. The two schools were selected because of their differences in educational approaches: one focused on traditional methods while the other used a project-based approach to instruction. Students from both schools were selected based on similar backgrounds and abilities. The study showed that the students who were involved in the project-based methods school viewed mathematics as a “dynamic, flexible subject that involved exploration and
thought” (Boaler 1998). Involving students in exploration allows them to take ownership of their learning while creating meaning. Furthermore, the results from the mathematical assessments given to each focus group favored the students at the project-based school (Thomas 2000). These results indicate that students in the project-based school outperformed those students in the traditional school with similar abilities and had a higher passing rate on national exams. This study is important and illustrates the validity of PBL because it compares its effectiveness against more traditional teaching methods.

PBL is suitable for students with many different learning styles because it incorporates various student-learning characteristics. This approach is supported by Gardner’s theory of “multiple intelligences” in which he argues for a change in the current teaching methods (Gardner, 1983). PBL is a new and innovative way to teach students with different intellectual strengths, and it allows students to utilize different intelligences to explore and understanding mathematics.

By engaging in Project Based Learning, students are able to investigate, discover, and relate many different standards while achieving one overarching goal. Project Based Learning (PBL) allows students to see the bigger picture while identifying patterns and relationships between different branches of mathematics. It also allows students to develop metacognitive strategies while being able to represent their findings in their own creative way. Not only do they have to make connections, use reasoning, problem solve, investigate, discover, and represent, students who are engaged in PBL also need to continuously communicate their thinking and reasoning.
Summary

After reviewing current research and reflecting on the need for a new curriculum, I planned to implement a model of instruction in which students are motivated to learn and build conceptual understanding which in turn will lead to creation of a math identity. The curriculum builds on the strengths of existing curricula’s such as project based learning but incorporates peer-mediated discussions and a sufficient amount of more traditional practice. Because of the definite need for students to relate to and engage in mathematics, a curriculum in which students can take ownership of their learning in order to create connections is necessary. However, students must also be able to apply their knowledge to various settings whether it is real-world application or a standardized test. Traditional approaches to teaching mathematics have led students to feel disconnected from their material, and consequently many have trouble relating to math as an integral part of their lives. Students must be able to practice their skills in an academic setting and, although drill and practice is important, students need other ways to learn and engage themselves in the classroom that have practical applications. Likewise, if students are solely focused on project-based learning with no practice of learned skills, they are not given opportunities to succeed in an exclusively academic environment. As can be seen by the curriculum reviewed, there are no available learning materials that take both traditional and project based learning approaches into account.

A student’s peer communication, motivation, and conceptual understanding in the math classroom help construct a mathematical identity. As can be seen by the research presented, motivation can affect a student’s ability to succeed. Students who are not motivated are at risk of falling through the cracks. Students who have already fallen
behind grade level in terms of academic achievement are at risk of dropping out of high school. There are many factors that can inhibit a student’s achievement and motivation levels. Some of these may include but are not limited to home environment, social restraints, limited English skills, lack of understanding, and lack of parent involvement. Many students have expressed their discontent with mathematics because of the lack of connection with the subject. The curriculum I am proposing will expose students to mathematics in a way that is relevant and connected to their lives.

The proposed curricular approach will be examined through the implementation and evaluation of various activities and assessments. Students will take a pre- and post-evaluation to test for understanding. Also, students will be given a pre- and post-survey that will include questions about their feelings and connectedness to the material. From here, students will be asked to rate their sense of identity with the unit studied and compare it to past units learned. Throughout the implementation, students’ work will be observed for increased quality, understanding, and completeness.
Chapter V.

Identifying with Mathematics Curriculum

*Identifying with Mathematics* is a model of instruction that promotes student motivation and builds conceptual understanding as well as the development of a strong math identity by incorporating: conceptual understanding, motivation, and communication. A student’s conceptual understanding, motivation, and communication in the math classroom help construct a mathematical identity. A mathematical identity can be defined as the way in which students view themselves as learners of mathematics as well as the way in which others view students as learners of mathematics. Hoang (2007) claims that motivation can affect students’ abilities to conceptually understand. This, in turn, affects their success levels and inevitably their math identities. Students who are not motivated are at risk of disconnecting with mathematics in their high school and college years. This disconnect is detrimental to their abilities to understand, incorporate, and use math in their lives, and it can affect their ability to solve problems, reason, and use higher levels of logic. These skills are necessary and beneficial to have in today’s fast-paced and continuously changing world.

There are many factors that can inhibit a student’s conceptual understanding and motivation levels. Some of these marginalizing factors may include but are not limited to limited English skills, lack of understanding of the math content, and lack of parent involvement which lie outside of curricular control. However, lack of powerful instruction and engaging curriculum can affect the way students internalize information and levels of motivation and are greatly influenced by the way in which we teach.
mathematics. The *Identifying with Mathematics* approach is designed to be useful with students who have expressed their discontent with mathematics because of the lack of connection with the subject. The proposed curriculum addresses the language needs of ELL students as well as the general education students by embracing communication and academic vocabulary in the classroom through discovery activities and projects that promote motivation and conceptual understanding. *Identifying with Mathematics* helps bridge multiple needs in the classroom, and attends to the need for student centered instruction. *Identifying with Mathematics* exposes students to mathematics in a way that is relevant and connects to their lives.

**Goals and Purpose of the Curriculum Approach**

*Identifying with Mathematics* is a curriculum approach that increases a student’s identity levels with mathematics by incorporating various constructs important to raising student achievement. I designed a unit that incorporates both project-based and procedural fluency components. The goal is to raise a student’s conceptual understanding level through motivating and collaborative projects as well as procedural applications. The approach relies heavily on communication in the classroom through multiple opportunities for students to talk to one another through peer mediated discussions around mathematical concepts. As a result of participating in this approach, students should build engagement as well as understanding of the concept of area and its practical uses in society. Students use mathematical language and develop a strong sense of the bond between mathematics and the real world.
Identifying with Mathematics incorporates a project along with weekly real-life activities, daily preludes (or warm-ups), practice, and reflections. Students should be able to apply the concept of area to their lives by understanding its uses in the field of sports, design, fashion, and decorating. Students should also be able to put to memory and use the area formulas for rectangles, parallelograms, triangles, trapezoids, kites, and regular polygons. Students focus on relating area to the real world while continuing to practice their procedural fluency. In their lessons, activities, and projects, they express choice and creativity which are motivators for learning (Heyl, 2008).

The foci of the approach are conceptual understanding and motivation. Communication is a key component to helping students construct meaning in mathematics (Bresser, Melanese, & Sphar, 2009). Therefore, in order to achieve conceptual understanding as well as motivation, communication is taken into consideration as the foundation for the curriculum. Identifying with Mathematics looks at project based approaches to learning in accompaniment with procedural fluency. Table 5 below outlines the goals of Identifying with Mathematics.
Table 5: Curricular Goals The three goals of the *Identifying with Mathematics* curriculum.

<table>
<thead>
<tr>
<th>GOAL</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Help students create a strong relationship between mathematics and the real world while increasing their ability to solve procedural problems</td>
</tr>
<tr>
<td>2</td>
<td>Create a classroom environment that facilitates communication and allows students to effectively use academic language through meaningful peer-mediated discussions about mathematics</td>
</tr>
<tr>
<td>3</td>
<td>Incorporate both Goal One and Goal Two in developing a strong mathematical identity in and outside of the classroom by combining strategies that build both conceptual understanding and communication</td>
</tr>
</tbody>
</table>

Various evaluation strategies are employed to track students’ progress toward the goals. First, pre and post implementation tests, exit slips, and student reflections will be evaluated. Also, peer mediated discussions are assessed throughout the implementation process. Students keep a reflection journal to guide their thoughts about the material, and the journal serves as a place where students talk about their experiences in the lesson. This is important in forming a mathematical identity because experiences inside the classroom are contributors to each student’s identity development (Sfard & Prusak, 2005). They are asked to describe and discuss the relationship between the math they engage in and the world around them. Students are also asked to design a project that will relate to the real world. Students are observed by the teacher for meaningful discussions in mathematics. Daily, through teacher observations, a list of words is compiled that students use in their discussions with their classmates.
To measure identity, students take a survey that includes questions about their feelings regarding mathematics at the beginning of the unit, and then take another survey with the same questions after the area unit. In addition, student behaviors and engagement levels are observed through student conversations and completion of work before, during, and after implementation. The reflection journals also serve as a means to measure identity. Students are asked to reflect on various activities, lessons, and projects by writing about their thoughts, findings, peer discussions, experiment processes, real world connections, and what they learn.

**Breadth Versus Depth**

Some students need ample time to dive into the standards and dig deep into each one in order to achieve conceptual understanding. However, with the demands of high stakes testing and the lack of time in the classroom, many students are simply exposed to the standards. One way in which *Identifying with Mathematics* addresses the need for in-depth analysis of the standards while taking into account the time needed to address all standards is by combining multiple key standards into one project. Students are able to spend time developing and practicing multiple standards in one project. Also, at Chart Academy, math class periods are longer and the school day is extended to allow for more time in the classroom. The curriculum I developed was designed for 70 minute math periods. Going in depth of key standards may help students perform better on standardized tests. Spending more time on key standards and superficially covering those that are considered less important might still allow time for the state mandated breadth.
Curriculum Activities

As stated earlier in the needs assessment, traditional methods for teaching geometry focus on calculating and using formulas without much investigation or discovery, but students must be motivated and conceptually understand math in order to identify with it. In order for students to develop an identity with Geometry and math in general, they must be able to test, analyze, discuss, and understand the shapes with which they are presented. My review of research led me to design a curriculum that incorporates a combination of traditional and project based approaches to teaching and learning. This curriculum design emphasizes communication and focuses on the development of a math identity for the whole student. For example, teaching and learning should take into account the experiences and knowledge that students bring to the table and it should include relevance to the lives of the students. This curriculum incorporates daily, weekly, and biweekly skills, activities, and projects that will contribute to a student’s identity as a math learner. Pieces of various curricula including Discovering Geometry (Serra, 2008), College Preparatory Mathematics (2009), personally developed project based learning, Elementary and Middle School Mathematics (Van de Wall, 2001), and Math Matters (Chapin, 2006) are used to create an ideal unit in which students investigate, discover, practice, discuss, and understand area.

The need in mathematics to incorporate both project-based and traditional methods of instruction drives the proposed curriculum to include a combination of procedural fluency, discovery learning through kinesthetic activities, project-based learning, and peer-mediated discussions.
The entire unit takes about one month to implement. The first two weeks of implementation provide students with the opportunity to discover the formulas for areas of various polygons, practice using the formulas they derive, and form connections with the importance of area to the real world. During this time, students engage in hands-on kinesthetic activities that create meaning between area and the formulas which they will use to solve for area. Along with these activities, students are asked to practice their discoveries by solving generalized problems that integrate the area formulas. Students are also asked to engage in academic discussions with their peers through prompts and questions around concepts of area. The first part of the implementation focuses on student engagement and conceptual understanding. By the end of the first two weeks, students know, are able to derive, and are able to use the formulas in real world situations.

The last two weeks of instruction continue to incorporate procedural fluency in conjunction with a project which connects area to visual arts and architecture. Throughout the implementation students are required to build their mathematical communication skills as well as their autonomy through peer-mediated discussions surrounding their discoveries and projects.

Table 6 represents the curriculum features and their contribution to each of the constructs.
Table 6: Curriculum Constructs and Activities

Links each construct to each activity

<table>
<thead>
<tr>
<th>Curriculum Features</th>
<th>Conceptual Understanding</th>
<th>Motivation</th>
<th>Communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hands-on Discovery Activities</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Procedural Fluency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Project-Based Learning</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Mathematical Peer-discussions</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the table, it is evident that there is a great deal of overlap between the activities and the desired outcomes. For example, the hands-on discovery activities that are incorporated into *Identifying with Mathematics* contribute to a students’ level of conceptual understanding, motivation, and use of academic language. Procedural fluency is important in helping students understand how to solve a variety of problems within the same context, but does not increase motivation or communication levels. Project-Based Learning is both motivational and helps students draw connections between mathematics and the real world. Students are required to talk with group members about the specifics of their project as well as present their findings for a whole group discussion. Therefore, Project-Based Learning also addresses the use of academic language as well as conceptual understanding. Because communication is a key component in helping students construct meaning in mathematics (Bresser, Melanese, & Sphar, 2009), peer-mediated discussions can help students build understanding of mathematical concepts.
through collaboration and talk with their peers. Discussions are also extremely student centered which contributes to motivation and autonomy because students are asked to be active learners and take ownership of their learning. Lastly, peer-discussions require students to use academic language and expand their use of mathematical vocabulary. These factors contribute to the creation of a strong math identity among all students including English Language Learners.
Chapter VI.
Implementation and Revisions

_Identifying with Mathematics_ was designed to help students develop a strong math identity and see themselves as learners of mathematics. Traditionally, mathematics has been taught in such a way that is detrimental to a student’s understanding and acceptance of math in the real world. This traditional way was designed with direct instruction, rote memorization, and procedural practice as the focus. It is necessary that students see connections between what they are learning and their own lives. Teaching a student a formula and expecting him or her to use the formula to solve problems is simply not enough. Students must be able to discover these formulas on their own and distinguish the connection between the new concepts and past knowledge in order to build a schema that truly encompasses opportunities for conceptual understanding. When students can take ownership of their learning and are responsible for seeing mathematics as real mathematicians do, they can build a stronger identity around their own knowledge of mathematics. _Identifying with Mathematics_ takes into account a student’s conceptual understanding by introducing activities and projects that promote motivation, procedural fluency, communication and discussion, the use of academic language, and discovery.

**Environment**

The high school in which _Identifying with Mathematics_ was implemented is located in an urban area in southeast San Diego. The students at Chart Academy are predominately Latino and African American, and come from a very low socio-economic area. Chart Academy is a Title One school in which all students receive free lunch regardless of whether or not they qualify, and many families live at or below the poverty
Title one is a federally funded program that provides schools with additional monies for students at risk of failure or living at or near poverty.

**Table 7: Chart Academy Demographics** Percent of the student body at Chart Academy in each of the above categories

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>Percent of student body represented</th>
</tr>
</thead>
<tbody>
<tr>
<td>English Language Learners (EL)</td>
<td>44.8%</td>
</tr>
<tr>
<td>Fluent English Proficiency Students (FEP)</td>
<td>24.5%</td>
</tr>
<tr>
<td>Qualify for free/reduced price meals</td>
<td>86.5%</td>
</tr>
</tbody>
</table>

Close to 45% of students at Chart Academy are English Language Learners and close to 90% of students qualify for free or reduced price meals (see Table 7). This indicates the number of students at Chart Academy who are living near or at the poverty level.

**Table 8: Enrollment by Ethnicity** School and District Enrollment by Ethnicity

<table>
<thead>
<tr>
<th>Ethnicity</th>
<th>School Enrollment</th>
<th>School Percent of Total</th>
<th>District Percent of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latino</td>
<td>506</td>
<td>71.7</td>
<td>44.4</td>
</tr>
<tr>
<td>African American</td>
<td>144</td>
<td>20.5</td>
<td>13.2</td>
</tr>
<tr>
<td>Asian</td>
<td>31</td>
<td>4.4</td>
<td>8.9</td>
</tr>
<tr>
<td>Pacific Islander</td>
<td>18</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Anglo (White)</td>
<td>6</td>
<td>0.8</td>
<td>25.3</td>
</tr>
<tr>
<td>Filipino</td>
<td>1</td>
<td>0.1</td>
<td>6.6</td>
</tr>
<tr>
<td>American Indian</td>
<td>0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Multiple/No Response</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td>706</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

*Source: Ed-Data school reports*
Table 8 highlights the immense distinction between ethnicities at Chart Academy compared to the general district population in San Diego. As can be seen by the table, Chart Academy enrolls a much higher percentage of Latino and African American students and a much lower percentage of Asian and Anglo students compared to the average of other schools in the San Diego area.

The mission of Chart Academy is to accelerate academic achievement for all students through a college preparatory culture and curriculum. Because many of the students are behind grade level in competency, teachers at Chart Academy must seek out innovative ways to teach students new content while also teaching students past standards where proficient levels of understanding have not been met. This is a daunting task for many educators, but in order for students who are at risk of being credit deficient to compete with the other students in the United States, they must play “catch-up” and learn higher-level mathematics in addition to possibly basic arithmetic that has not yet been mastered.

This project was designed with the students of Chart Academy in mind, and there are many aspects that contributed to the design of this curriculum. For example, motivation among “at-risk” students of Chart Academy has been an ongoing issue that was addressed in Identifying with Mathematics. In addition, because of the high level of ELL students at Chart Academy, the use of academic language through communication in the math classroom was fundamental in the design.

The curriculum was implemented in four ninth-grade geometry classrooms. Two of the four classrooms were used to collect and analyze data. Classroom one consisted of 27 students ages 14 and 15. Most of the students at Chart Academy passed Algebra one.
Chart academy requires that all students in ninth grade take geometry regardless of their past experiences in the math classroom. Classroom two consists of 14 students within the ages of 14 and 15. Within classroom one, seven of the 27 students were English Language Learners and within classroom two, five were English Language Learners. Both classroom one and classroom two met every day for seventy minute periods with the exception of Monday which meet for 55 minutes. About half of my students in both classes struggled with basic arithmetic and had not been successful in math classes before. Based on the California Standardized Test (CST) results from last year, this entire class scored 1% advanced, and 11% proficient with the remainder of the students falling in the basic to far below basic band levels.

Through personal observation and field research, I noted that many students did not feel confident and successful in mathematics. Based on observations as well as some findings, there is a stigma that some people are born with a natural ability to learn math (Anderson, 2007). This implies that you either have it or you don’t. “It” refers to the ability to successfully engage, accomplish, contribute, and partake in mathematical concepts. Identifying with Mathematics was designed to help students discover geometry while engaging in projects that related geometry to the world around them. Students were also encouraged and expected to engage in mathematical discourse with one another to maximize motivation and understanding.

One factor that influenced the design of this curriculum was the lack of autonomy in the math classroom. In order for students to feel motivated, they must be active participants in their classroom. Many students were dependent on their teachers to provide them with the information they need in order to perform well on tests and solve
textbook math problems that may or may not explain the context in the real world. *Identifying with Mathematics* allowed students the freedom to discover relationships between shapes and their formulas for area without the need for large amounts of direct and traditional instruction by a teacher. The teacher was there to facilitate learning by asking clarifying questions, pushing student learning to the next level, and supporting students in their quest to connect concepts of area to the world around them.

I also observed that many of the students were passive learners in math classroom, which was another factor that influenced the design of this curriculum. Through implementation of *Identifying with Mathematics* students were asked to become active participants of the classroom and contributed to their own success and understanding in geometry. They were asked to engage in deep conversations brought about by guiding questions that prompted student findings and discoveries.

Finally, students needed to be able to see how the math they learn can contribute to society. Students engaged in a project that directly connected the mathematical concept of area to the world. They were asked to use the formulas which they derived to guide their exploration of area. The project linked standards based material with a creative and real-world, project based foundation.

The classroom environment and set-up allowed for students to work as a community of learners to achieve daily objectives. Students were arranged in groups of four which fostered student interaction and peer support. This set-up allowed for students to feel supported in their efforts to take responsibility for their learning and see themselves as well as others as true learners of mathematics. Students were encouraged to communicate and engage in mathematical discourse each and every class meeting.
Students were also encouraged to use the classroom as a means of support for their learning. The classroom was set up with a large number of charts, diagrams, problems, definitions, pictures, and more to help students feel comfortable in their classroom. The lay-out of the classroom allowed students to find multiple resources to use in solving new concepts or problems. The design of the classroom also fostered the appropriate scaffolds for English Language Learners by having charts and visuals accessible at all times.

**The Teacher**

Although at the time of implementation, I was only three years into my teaching career, I had worked with schools and students for many years. Throughout my college years, I took part in teaching fellowship in which I was placed in various high school math classrooms in low socio-economic areas in northern California to help build tutor relationships with students and teach one on one. Working with children in underserved communities who lack intrinsic motivation was my incentive to make a difference. After college, I decided to pursue my single subject credential in mathematics where I continued to work for agencies that seek positive change in our educational system.

My current position at the time of implementation allowed me to serve my students as well as my community through my commitment, dedication, and effort. As their teacher, I felt responsible to my students for providing the best possible instruction, and it is my duty to serve them as well as all children to my fullest potential. From my students, I had learned valuable lessons in making my content comprehensible, building relationships based on trust, scaffolding lessons for understanding, implementing positive reinforcement strategies, and committing to best practices for students. My passion for
students to not only understand higher-level mathematics but to use it to boost their lives led me to create authentic experiences for students. It is here that I realized students need to see the necessity for mathematics in order to feel confident using it in their lives. Designing a curriculum that is guided by research helped me to teach in a manner that was beneficial to all of my students.

**Background and Overview**


**Table 9: Math Content Standards in *Identifying with Mathematics*** California Mathematics Content standards that *Identifying with Mathematics* incorporates.

<table>
<thead>
<tr>
<th>STANDARD</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry 8.0</td>
<td>Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.</td>
</tr>
<tr>
<td>Geometry 10.0</td>
<td>Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.</td>
</tr>
<tr>
<td>Geometry 11.0</td>
<td>Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.</td>
</tr>
</tbody>
</table>

Table 9 indicates the three geometry standards that were covered in *Identifying with Mathematics*. Students needed to be able to know, derive, and solve problems involving perimeter and area as well as understand the relationship between the two. Also, students were required to compute the area for various polygons as mentioned in Geometry Standard 10.0 in Table 9. In addition to the California Mathematics Content Standards, *Identifying with Mathematics* also addressed multiple National Council of
Teachers of Mathematics (NCTM) process standards. *Identifying with Mathematics* addressed problem solving, communication, connections, reasoning, and proof.

*Identifying with Mathematics* required students to discover their own solutions to traditionally-provided problems, share their ideas with classmates, find connections between mathematics and the real world, and discover methods for proving mathematical conjectures through investigation. Students not only learned how to compute rigorous problems that contributed to their understanding, but they also derived and explained the relevance of what they learned through reasoning and proof development. *Identifying with Mathematics* went beyond the California State Standards by addressing other important factors critical to the development of a strong math identity. For example, students were encouraged to develop literacy skills through journal writing and they increased the use of academic vocabulary in the math classroom through peer-mediated discussions which incorporated NCTM’s communication standard. Students were also expected to reflect on their daily work through reflection journals and increased their ability to tackle rigorous problems through procedural fluency exercises. Finally, through motivating experiences that allowed students to take ownership of their learning such as discovery of formulas, students created a strong math identity around the necessity for mathematics in their lives while feeling successful and confident in the classroom.

Students were asked to push themselves to recognize how to learn math in innovative ways. *Identifying with Mathematics* took into account the various stages of Blooms Taxonomy (Bloom, 1956) in order to maximize student learning. Blooms Taxonomy was developed to categorize and classify levels of intellectual learning in a
classroom setting. Students were required to use various categories in *Identifying with Mathematics*. Table 10 describes the categories from Blooms Taxonomy and how they are incorporated into *Identifying with Mathematics*.

**Table 10: Blooms Taxonomy in *Identifying with Mathematics*** The incorporation of Blooms Taxonomy in *Identifying with Mathematics*

<table>
<thead>
<tr>
<th>BLOOMS CATEGORY</th>
<th>How is it used in <em>Identifying with Mathematics</em>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derive and Analyze</td>
<td>Students use manipulatives to derive and analyze necessary formulas for area from previously discovered formulas</td>
</tr>
<tr>
<td>Discuss</td>
<td>Students discuss key questions with their group members and push each other’s thinking through an increased use of academic language and conversation</td>
</tr>
<tr>
<td>Write</td>
<td>Students write synopses of their discussions and projects</td>
</tr>
<tr>
<td>Investigate</td>
<td>Students take ownership of their learning through investigations much like the mathematicians who originally discovered this area of mathematics</td>
</tr>
<tr>
<td>Practice and Apply</td>
<td>Students practice and apply their skills through procedural fluency exercises</td>
</tr>
<tr>
<td>Explore</td>
<td>Students explore connections between area and the world through a project</td>
</tr>
<tr>
<td>Create</td>
<td>Students create a design using geometric figures, combinations of polygons, and formulas as a foundation for the design</td>
</tr>
<tr>
<td>Reflect and Evaluate</td>
<td>Students reflect on their learning and evaluate their understanding through reflection journals</td>
</tr>
</tbody>
</table>

*Identifying with Mathematics* focused on creating those “Aha!” moments that contribute to the overall self-belief and achievement of the whole student.

This unit required almost no background knowledge of mathematics. Students who have very little experience with geometry had access to the curriculum as it truly focused on discovery from the beginning. Teaching prior to the implementation was limited to a basic understanding of what area represented through a class discussion.
Also, students had some experience naming basic shapes in geometry such as triangles, parallelograms, trapezoids, kites, and regular polygons. Students were familiar with using these names in a discussion around mathematics, and practiced having academic talks with their peers. Students engaged in topic discussions about mathematics weeks before implementation in order for them to be familiar with what was expected when having a group discussion.

In order to foster geometric thinking and conversation prior to implementation students filled out a graphic organizer (see Appendix) that focused on what it meant to use academic language and how academic language was used in a classroom setting. Next, students shared ideas from their graphic organizer with their group members as well as the rest of the class. From here, the group compiled a chart for the classroom centered on academic discussions and language. The chart contained starter sentences and tips for using academic vocabulary such as “highlight and define new or unfamiliar words to use in a discussion.” In addition to a group discussion about the importance of using academic language, it was important that the classroom was set up in a manner that fostered the use of new vocabulary words. As discussed in the review of research, word walls and vocabulary banks are effective ways for students to have access to new vocabulary they may need for their peer discussions. Figure 2 represents a word wall in the classroom.
Figure 2: Word Wall The word wall in our classroom. Each word is displayed on a separate index card with a definition and a picture to help students with communication in the classroom.

The word wall contained index cards that each had one new vocabulary word, its definition, and a picture describing the word. Students made use of the word wall multiple times per day during discussions, written reflections, and investigations.

Students were also asked to fill out a survey that helped the teacher decide how to design their final project components (see Appendix). It was important to give students an opportunity for choice in designing their project because research shows that choice can positively impact autonomy in the classroom (Deci, 1996). Students were asked to complete a survey (see Appendix) that helped the teacher determine how to design a project that would best meet the interests of the students.
Teaching and Learning Prior to Implementation

Before implementing *Identifying with Mathematics* it was important for students to know and understand the differences between their traditional views of learning mathematics and the more problem-based approach of the curriculum. Students began by experiencing a lesson grounded in discovery in a unit prior to the implementation unit. The unit required students to figure out the sum of the interior angles of a triangle using an investigation (See Appendix). First, students were asked to work in teams to take a triangle and arrange the angles in a way that would help them discover their sum.

Students cut the corners off of their triangles and arranged them in a way that formed a line. Since, students already knew that a straight line measures one hundred and eighty degrees, they were able to see that any triangle would have the same measurement as a line. Hence, they discovered that the triangle sum conjecture states that any three interior angles of a triangle have a sum of one hundred and eighty degrees. Figure 3 shows a before and after picture of the triangle investigation.
Figure 3: Triangle Sum Conjecture Activity A piece of an investigation that students completed. Students tore the edges (angles) off of the triangle and taped the three pieces (angles) together to discover that the three angles of any triangle form a straight line and therefore have a sum of 180 degrees.

After the investigation, students had a peer-mediated discussion. A peer-mediated discussion was a discussion among the four group members where students were given questions that promote collaboration and communication. The questions were open-ended and allowed for students to discuss their ideas surrounding a mathematical concept. Students had a few minutes to take notes on the questions individually. Next, students took turns discussing their ideas with their group while each group member jotted down notes. Finally, students came to a consensus or negotiated answer to each question and reported back to the entire class during a whole group discussion. Peer-mediated discussions created a safe risk-taking environment for students because they had time to discuss individually and in small groups instead of being directly singled out by a teacher. This was also an effective scaffold for English Language Learners as it
gave them time to decipher unfamiliar words with their group members and provided them an opportunity to talk in the classroom which was an important part of language development.

Since the peer-mediated group discussion process took time to learn, the strategies and etiquette of having an academic discussion needed to be addressed. In groups, students discussed the rules of a peer-mediated discussion and practiced a discussion around this exact topic. Students talked about what it meant to be a good listener and what a classroom should look and sound like during a discussion. After one minute of practice, the class charted ideas from each group. They compiled a list of procedures for discussions. Below, in Figure 4, is a picture of a figure that was made as a class around the topic of peer mediated discussions.

![Peer Discussion Chart](image)

**Figure 4: Peer Discussion Chart** This chart remained on the wall of the classroom. Students compiled the most important aspects of being able to use academic language during peer-mediated discussions.
After practicing with a non-math related topic, students continued this procedure for a month in their classes around mathematical topics two to three times each week in order to prepare for the *Identifying with Mathematics* unit.

**Implementation of Identifying with Mathematics**

**Pre-implementation Test, Reflection, and Project Survey**

Before introducing any of the lessons for this unit, the students completed a pre-implementation reflection that served as a basis for understanding their current knowledge level as well as motivation level. The reflection journal began with a letter written by me to the students expressing the endeavors in which they were about to take part. It discussed their upcoming roles as young mathematicians and their goals to discover the formulas for area. It also discussed their project and the role they would play in choosing their topics based on their interests. The reflection asked students to describe in words how they felt about partaking in this curriculum and their views of themselves as learners of mathematics up to this point in their lives. It also asked them to discuss their comfort levels, current academic grades, family support, and anything else they wanted to share about mathematics. While I read the introductory letter to the students, there was not a sound in the room. I realized that the students were ready for a change in pace, and many of them seemed intrigued with the assignment. All of the students took the survey and answered every question.

After the survey, students took a pre-implementation test in which they were asked to solve twelve area questions with regards to different shapes they would be learning about. Students were also asked to define the academic vocabulary that this unit entailed such as area, trapezoid, parallelogram, base and height. Next, students were
asked to write in the formulas for the area of basic geometric shapes. Finally, students were asked to answer questions relating area to the real world. They were asked to discuss the importance of understanding how to calculate area and where area is used in the world. The final questions on the pre-implementation test had students rate their interest level of this topic on a scale of one to five. The pre-implementation test served as a means to see what students already knew about area and its relationship to the real world as well as to examine their level of enthusiasm for the subject.

After the pre test, students were asked to complete a second journal reflection that helped them explain their thoughts about the pre-implementation test. Students were asked about their thoughts around their performance on the pre-implementation test. They were asked to describe how successful they thought they could be and how they felt they would perform on the post-implementation test. The idea was to see how students viewed themselves as learners of mathematics and determine their identity levels based on their attitudes about mathematics.

**Overview of Investigative Activities**

Students engaged in a five-lesson series of investigations that helped them discover the formulas for different geometric shapes. All of the activities had a similar structure in that they each included access to prior knowledge, an investigation of an area formula, peer-discussions, student reflections, group discussions, and procedural practice. The major difference between the activities was the shape which was being investigated. Table 11 represents an overview of the investigative activities.
Table 11: *Identifying with Mathematics* Investigation Activities Indicates the five investigatory activities in *Identifying with Mathematics*

<table>
<thead>
<tr>
<th>Activity Number</th>
<th>Title</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>Area of a Rectangle</td>
<td>Students were given the definition of area and used counting blocks, grid paper, and peer communication to determine the formula for the area of a rectangle</td>
</tr>
<tr>
<td>Two</td>
<td>From Rectangles to Parallelograms</td>
<td>Students used their discovery of the area formula for a rectangle to determine the area formula for a parallelogram</td>
</tr>
<tr>
<td>Three</td>
<td>From Parallelograms to Triangles</td>
<td>Students used their discovery of the area formula for a parallelogram to discover the area formula for a triangle</td>
</tr>
<tr>
<td>Four</td>
<td>From Parallelograms to Trapezoids</td>
<td>Students used the same discovery of the area formula for a parallelogram to discover the area formula of a trapezoid</td>
</tr>
<tr>
<td>Five</td>
<td>From Triangles to Regular Polygons</td>
<td>Students used their discovery of the area formula of a triangle to discover the area formula of a regular polygon.</td>
</tr>
</tbody>
</table>

All five activities required students to use prior knowledge of either shapes or area to help them determine new formulas. The scaffolds of each activity helped English Language Learners acquire access to the same material as all students. Again, the focus of each activity was for students to investigate, communicate, discover, and reflect. The activities were designed with motivation and conceptual understanding in mind. The activities were hands-on and collaborative which provided students engagement and promoted active learning. Also, because students were required to discover formulas with their group members instead of being provided with the information, more students were able to assimilate the information in their knowledge bank. Because the five activities were similar in structure, only activity one and activity five will be described in detail.
Activity One – Investigation: Area of a Rectangle

The first investigative activity in a series of five was titled “Investigation: Area of a Rectangle.” Upon entering the room, students were seated in their four-person groups and asked to begin the prelude. The prelude, a daily routine for the lessons, is a question that provokes thought and interest from the students much like a warm-up. Students were asked to respond to the following question: “Why is area important? Where can we use it in the real world?” After a class discussion of the prelude, students charted their answer with their group and their work was displayed on the walls. For example, students stated that we use area when we paint a room and it helps us figure out how much paint is needed for each wall. Following the prelude, I discussed the formalities of an investigative lesson with the students. During investigation lessons, students were asked to converse with their partners and seek out help from alternative resources such as charts, visual, and books before turning to me for questions.

To begin the first activity, students were seated in groups of four and asked to work with a partner to investigate the formula for the area of a rectangle. They were given the definition of area and asked to use that definition to find the formula. Students were given graph paper and counting blocks to help them investigate. First, students were asked to find the area of a rectangle in which the square units were drawn in. They were asked to find the area in any method they chose. Some students counted every single box while others counted one row and one column and multiplied them together. Next, students were given another rectangle that was represented in square units, but the actual squares or grid was not filled in for the students to be able to count squares. The rectangle had a base number and a height number. Students were asked to determine the
area of each rectangle based on these two numbers. They were asked to write out and explain in words how to determine the number of one by one squares inside a given rectangle based on the base and height numbers as seen in Figure 5 below.

<table>
<thead>
<tr>
<th>Explain how to determine the number of 1 x 1 squares that will fit in the below rectangle. Can you do it in 2 ways?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

**Figure 5: Example from Activity One** Taken from Activity One in the Appendix

Figure 6 is a dialogue between two students who were discussing the methods for determine the answer to the question stated in Figure 5.
Jesus: Well, I know it will be 18, right?

Christian: Ya, I think so, but how do we know that? We have to explain it in two ways.

Jesus: There are 3 columns and 6 rows, so we can fill in the lines.

Christian: No, aren’t there 3 rows and 6 columns because columns go up and down?

Jesus: Oh ya, you’re right! 3 rows and 6 columns. So, if we draw in the 3 lines like this (draws 3 lines) for 3 rows, then…

Christian: You just made 4 rows, we only need 2 lines in order to make 3 rows.

Jesus: Oh ya, ok, I’ll erase one (erasers one line)

Christian: Ok, so now we have 3 rows, we need to draw in the columns so that we can make our boxes. (draws in the 6 columns)

Jesus: So, if we count up all the boxes, we get 18 total

Christian: Yes, so one way you can say this is that there are 3 rows with 6 boxes each. See… (points to all 6 boxes across one row)

Jesus: But that is the only way I can see to do it. We have to explain it in two ways.

Christian: Well, we could go the other way.

Jesus: In each column there are 3 boxes. So, we have 6 columns with 3 boxes each. Does that work?

Jesus: Yes! It does. That is a different way of explaining the same thing. Either way, we end up with 18 total boxes for the area.

Figure 6: Student Dialogue 1 Represents a dialogue between two students around mathematical content

Finally, students were asked to come up with their own rectangle area conjecture.

Students determined the following: The area of a rectangle is given by the formula \( A = bh \) where \( A \) is the area, \( b \) is the length of the base, and \( h \) is the height of the rectangle. To close the lesson, students were asked share their experiences on their first investigative lesson in a three-minute quick-write journal reflection.
Activity Five – Investigation: From Triangles to Regular Polygons

Students entered the classroom and immediately got into their four-person groups. Because this was the final activity, students already knew what to expect and how the procedures work. It was important for students to feel some sense of self-direction by the end of the investigation lessons and routines were very important to uphold a good classroom culture at Chart Academy. The prelude, or warm-up, for this activity was “What is the formula for the area of a triangle? What is a regular polygon? How are triangles similar to regular polygons?” Students were asked to do a quick-write around these questions. All of the questions were a review from past lessons, and students should have some prior knowledge about at least one of the questions. After the quick-write students were asked to share their results with their group members. Each member passed their paper to the right so that each group member was reading another student’s answers. They were asked to comment on their classmates’ papers. Students responded with either comments or questions. Each group had one minute to spend on each paper before the papers were passed to the next group member. Since each group had four members, after three minutes, each group member had read the responses to all other students’ preludes. Next, students took another minute to read the responses that their classmates wrote on their papers. Students then took two minutes to have a peer-mediated discussion about the responses with their group members and came to a consensus about what they were going to report back to the class. Each group took turns giving one idea to add to the whole class discussion. Table 12 shows the responses in classroom one that were added to a group chart with the focus questions.
Table 12: Group Responses from each group in a classroom. The responses were used to create a classroom chart for student reference.

<table>
<thead>
<tr>
<th>Group Number</th>
<th>Contribution to Class Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The formula for the area of a triangle is $A = \frac{bh}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>A regular polygon has all equal angles and all equal sides</td>
</tr>
<tr>
<td>3</td>
<td>An equilateral triangle is a regular polygon</td>
</tr>
<tr>
<td>4</td>
<td>Triangles can make other regular polygons</td>
</tr>
<tr>
<td>5</td>
<td>You can break up a polygon into triangles</td>
</tr>
</tbody>
</table>

Next, from the response of Group 5, a very important vocabulary word needed to be introduced. The point in the middle from where each triangle was drawn has a name. The Apothem was introduced to the students. An apothem of a regular polygon is a perpendicular segment from the center of the polygon’s circumscribed circle to a side of the polygon. Since there were many geometry vocabulary words in the definition, students were asked to take a piece of the definition that contained a vocabulary word and describe the various words with their groups. They were asked to draw multiple pictures and point out the apothem.

Part three of this activity introduced the investigation to derive the formula for a regular polygon. Students were asked to work with their group partner to investigate. First, students were given a regular polygon and asked to divide it into isosceles triangles. Figure 7 represents the before and after of what students did with the polygon.
Students were asked to label each triangle with base $s$ and height $a$. Using what they knew about the area of a triangle, students then found the area in terms of $s$ and $a$ of one of the triangles. Students were able to see that the area of one of the triangles would be $s$ multiplied by $a$ and then divided by 2. Next, students were asked to determine the area of the entire polygon based on the number of triangles. Students discovered that since there were six total triangles that all had the same base and height, the total area would be $s$ multiplied by $a$ divided by 2 and then multiplied by six. So, the formula they discovered was $s$ (side length) times $a$ (apothem) times $n$ (# of sides) all divided by two.

Figure 8 is a conversation between two ninth graders in classroom two.
| Jenny: | So, we have to draw in the lines for the triangles like this, right? (drew in six lines from the apothem to make up six triangles) |
| Liz: | Now we have to label all the sides s. Why are they all s. |
| Jenny: | Because it’s a regular polygon, so all the sides are the same length |
| Liz: | Oh ya, that makes sense. And all the heights are a. |
| Jenny: | Ya. Ok, so we need to calculate the area of one triangle using the formula we discovered last week. |
| Liz: | So, the formula is $A=\frac{bh}{2}$ where $b$ is the base and $h$ is the height. So, we have to plug in $s$ and $a$ instead. |
| Jenny: | Ya, so we get the area of one triangle equals $sa/2$. |
| Liz: | Right, so since all of the triangles are equal, they will all have the same area. |
| Jenny: | Yup, next we have to figure out the area of the whole thing. Well, since we know that all the triangles are the same, can’t we just add them all together. |
| Liz: | Ya we can. So, the area would be $sa/2$ added together 6 times. |
| Jenny: | Or we could just multiply it by 6 to get the whole thing, right? |
| Liz: | Ya, that makes sense I think. |
| Jenny: | Oh! 6 is the number of sides of the whole polygon. I wonder if that has anything to do with the formula. |
| Liz: | Ya, it does. Because if we had a regular polygon that had 5 sides, it would be broken up into 5 triangles, so instead of multiplying by 6, we would multiply by 5, get it? |
| Jenny: | Ya, I get it. And since a triangle only has one triangle in it, we would just multiply by one. Ok, that makes sense now. So, you just multiply the answer by the number of sides it has. |
| Liz: | Ya, I think that’s exactly what you do. |

**Figure 8: Student Dialogue 2** A conversation between two students in the classroom who discuss mathematics at a higher-level than other students.

The discussion between Jenny and Liz demonstrated their ability to take one figure, namely a regular hexagon, and apply it to all other regular polygons. The activity (see appendix) scaffolds the discovery in a way that all students had access to the information they need in order to make the same discovery. Some students like Jenny and Liz were able to generalize the formula for all polygons while other students were simply able to find the area. Either way, students were able to access the material and discover the formula for the area on their own.
After the discovery, students were asked to write a brief reflection. They wrote about what they did to find their solution in part three (the discovery). Student responses to the lesson showed a great deal of understanding. Derrick, for example, said “We found the area of one triangle then, I multiplied the area by how many triangles there were in the polygon.” His explanation was accurate with his investigation. Evie, an advanced student, wrote “I cut the polygon and made triangles from the apothem. Then I found the area of that triangle. Then I added all the triangles together.” Other student responses included, “I got the answer of the triangle and then times by 6” and “I figured out what s and a were and then we multiplied them and divided by 2.” Students were beginning to be able to express their investigations in words and make meaning of the math they engaged in.

After the reflection, students were asked to solve procedural problems involving regular polygons. Students spent ten minutes solving problems independently. Figure 9 is a sample of student work.
Figure 9: Student Work Sample Represents a sample of student work. The procedural problems all come from the students investigation and discovery of the area formula.

Journal Reflection Three and Academic Language Quiz

After students completed the investigation lessons, they were asked to complete a reflection journal of their journey up to this point. Students were asked to reevaluate their feelings about area and its connection to the real world. They answered the same questions as their first journal such as whether or not they would be able to teach another student what they learned about area. They were also asked to answer questions about their excitement levels to start the project. The reflection took students about twenty minutes to complete. They also completed an academic language quiz (see Appendix) to test their knowledge of new vocabulary during the unit. Students were asked to describe and draw various words that were new to their peer-mediated discussions.
The Project: Overview

After the investigation lesson and for the remaining two weeks of implementation, students completed a project that joined the concepts of area to the world. All of the discovery work that students did was used in the project. The purpose of the project was for students to be able to make the connections between the academic and classroom use of area to the use of area in real life situations. I used the results from the survey that students took during pre-implementation to help determine the design. For example, there was a great deal of student interest in both sports and interior design. Keeping that in mind along with the importance for students to have choice in the classroom, I designed the project to include both areas of interest so that students could choose what appealed to them. The incorporation of student choice in the project was what set it apart from past projects the students have done.

Students were able to choose from three different topics: pattern design, interior design, and sports luxury box design. Although each topic was different, the contents of the projects were similar. All students used their knowledge of area to design a final product that was presented to their classmates. They all began with blueprints and rough drafts, and each project included a writing sample explaining their design. The project required students to calculate the area of their product, and each student displayed their work in a presentable manner. The project spanned over ten school days, and most of the students worked on their project as an in-class only assignment. There was little work that needed to be done outside of school. The beginning of the project brought about anticipation in the students because they knew it was going to be something new and different.
The Project: Launch Day

Students entered the classroom excited for the project details because they knew that their investigation activities were completed and the project was going to begin. The vibe on the launch day of the project was very different from the rest. Students entered the room extremely quietly and there was a sense of excitement in the room. Up to this point, the students did not know the details of what the project entailed, but they knew that the survey they took during the pre-implementation played a part in determining the logistics of the project. I began by passing out the overview of the project (see Appendix) and asked students to spend a few minutes reading it to themselves. Next, I asked students to conduct another peer-discussion to give them time to express any excitement, questions, or concerns they had up to this point. As a class, we charted ideas and comments that came from each groups’ discussions.

After the discussion, each group was responsible for reading and discussing one of the three project choices with the class. Instead of passively and quietly reading the guidelines for each project, the groups were responsible for a certain section, and each group shared the salient points of their section with the class. This way, students were actively participating in explaining the project and were able to answer each other’s questions about the project. After a question and answer session regarding the overview of the project, students were asked to look at the rubric. It was incredibly important that students knew exactly what was expected of them in order to produce high quality work.

In order for students to understand and use the rubric effectively, they did a gallery walk that was set up around the classroom. A gallery walk is a method used to have students out of their seats and walking around to acquire information. Stations are
set up around the room and students visit each station with their group to complete a task. For the project rubric, there were six categories: 1) accuracy of mathematical calculations, 2) correct and sufficient use of formulas, 3) project components, 4) communication of mathematical ideas, 5) neatness and effort, and 6) general directions. Each station housed one of the categories. Students with their group members spent two minutes at each station. They read the criteria to get an “A” in each of the categories. Students were asked to fill out a graphic organizer (see Appendix) in which they gave examples of what they thought was expected of them in each category.

After visiting all six stations, students returned to their seats with their groups to discuss the graphic organizer. Students were given five minutes to talk about the expectations of the project as well as any beginning ideas they had for their own project. The launch day ended with a closure activity called “popcorn”. Each student was asked to state which project they were leaning towards and one piece of the rubric that would represent “A” quality work. As the class went around the room, each student shared their answers before being dismissed from class.

**The Project: Student work**

Students used the majority of class time over the next two weeks to complete their projects. They began with an outline of their design using plain white paper. Next, they drew their design to scale using graph paper and made blueprints of their design.

For example, students who chose to design a luxury box were required to draw each piece of furniture to scale using the shapes indicated in the project overview (see Appendix). They needed to indicate each piece of furniture, determine the area of each piece, and determine the area of the entire box. If students chose to design the inside of a
one story house, they were required to use a certain square footage (1,600 square feet), and fit each room, bathroom, kitchen, and living room within those confines. Students used graph paper to draw the house to scale and create a blueprint. They were required to use at least four different shapes in their house, and were asked to calculate the area of each room. Lastly, if students chose to design a quilt, they used graph paper to help them with the calculations, and were required to use at least four different shapes. Students calculated the area of each shape in their quilt, the total area of the quilt, and the total area of all the shapes. This portion of the project took most students four days to complete.

After students finished their blueprints and area calculations, they were asked to write a one page essay on why they decided to design their project the way they did. They included the shapes, the formulas, their ideas, and their explanations for the design. Students spent one day writing out the essay by hand and completing a rough draft. The next day, students used school computers to type their essays and make any necessary corrections.

When students completed their essay, the next step was to work on their presentations. Students were asked to display their work in a presentation and prepare a two minute explanation for the class.

**The Project: Presentations**

For many of my students, this was the most challenging part of the project. Students at Chart Academy struggled with the idea that they would have to be presenting their math projects verbally in front of their peers. Presentations took place over two days. In order to help with both organization and calming of nerves, each student was given an outline of the order and day when he or she was scheduled to present. The
presentations were required to be no more than two minutes long due to time constraints. Students were also given a handout (see Appendix) with important points that needed to be included in the presentation. These included the project they chose, the shapes they used, the reasons for their design, the formulas they used, and one piece of information that made their design special. The presentation handout was a great way for me to double check that students were on the right track and that they were actively preparing for their presentation.

Because the projects were done on an individual level, each student presented his or her own project for the class. The students used various representations for presenting their project. Some students created a power-point while others used the document camera to display their work. There were also some students who decided to display their project in a booklet form. I used the project rubric as a guide for evaluating their work as well as their presentations. Although many of the students produced high quality projects, the presentations did not all exemplify high quality work. Some students were not prepared, and although they did perfect calculations and produced excellent representations of area and how it is used in the world, their presentations did not reflect the amount of work completed. I was disappointed to see students who worked so hard not be able to portray their efforts to the class. I was curious to see how students felt about the outcomes of the project as well as the presentation.

**Post Project Surveys**

After completing the project, a survey was given to the students to reflect on the overall process (see Appendix). Students spent twenty minutes answering questions regarding their project and their feelings towards taking an active role in mathematics.
This individual assignment was used to determine motivation levels of students after completion of the project as well as to see if student identities changed from journal reflection one. After completing all aspects of the curriculum, there were some parts to the project that need revisions before implementing Identifying with Mathematics a second time.

Revisions to Identifying with Mathematics

Although I was satisfied with the success of the implemented curriculum, there are definitely some changes that I would make should I implement Identifying with Mathematics again.

The first revision is the wording of the journal reflections. Because I included trigger words for students to use as a guide, I received many of the exact same words in their written responses. For example, in the first reflection journal, one of the questions is “How does this letter make you feel as a student? (excited, scared, confused, etc.)” Many of the students used excited, scared, and confused in their journal responses. There was little variation in word usage, and I regret including those trigger words in the questions. Also, students probably needed more time to practice the vocabulary words they learned before writing their reflections. When I implement Identifying with Mathematics again, I would eliminate the trigger words or compile a very large list of words for students to choose from.

The second revision comes from the project presentations. The students were not as prepared as I had hoped when it came to the presentations. If implementing the curriculum a second time, I would put a greater emphasis on end presentations as students should take pride in their work and want to discuss what they produced.
However, because of time constraints, the presentations seemed rushed and students did not have ample time to prepare. Students need more time to present and should practice at least three times before presenting to the class. I would also encourage the use of note cards which was not required during the first implementation. In order for students to build presentation skills, another aspect of this revision could come from the investigation lessons. Groups could practice presentation skills by sharing information in front of the class, and students could take turns presenting their findings each day. Also, I believe that I did not give students enough warning that they would be presenting their projects. It was not until the project launch day that students realized they would be presenting. If students are aware that they will be preparing a short presentation from the beginning of implementation, it may help them be more prepared for the presentation day.

The third revision I would make is to incorporate more technology into the project. Although not all of my students have access to computers, there are other resources they could use to seek out information they need. For example, towards the beginning of the project, students could use the internet to research examples for their chosen topic. Students could also use the internet to research the ties between area (or any branch of mathematics) and the real world, or they could research possible career choices that incorporate the use of area in daily tasks. I provided many examples for students to show them the connection between area and the real world, and I believe it would be beneficial for students to use technology to find the relevance on their own.

The fourth revision I would make to Identifying with Mathematics is to include more surveys, interviews, and reflections that helped me determine how students view
each other as learners of mathematics. A mathematical identity takes into account both how students see themselves as learners of mathematics as well as how students see each other as learners of mathematics. I found little evidence of how students view each other which lead me to believe that I needed to include more feedback of this sort in order to fully capture a student’s mathematical identity.

**Conclusion**

The entire implementation process took a total of four weeks. Throughout the process students had multiple opportunities to build conceptual understanding through discovery learning, and communicate by discussing their ideas with their classmates and teacher. The investigation activities as well as the project were student-centered and promoted active learning in the classroom. Students also had opportunities to reflect on their learning and answer questions regarding their view of mathematics. Lastly, students were able to draw connections between mathematics and the real world through the area project.

Creating an environment where students can take ownership and responsibility for their learning can lead to an increase in motivation and conceptual understanding levels. Incorporating communication in a math classroom can also contribute to students’ views of themselves as learners and contributor to the math classroom. Implementing *Identifying with Mathematics* yields some findings that are both informative and exciting, and can positively affect the way in which we approach mathematics education.
Chapter VII.
Evaluation and Assessment of *Indentifying with Mathematics*

**Curricular Goals**

I designed *Indentifying with Mathematics* to accomplish three main goals. Goal One sought to help students create a strong relationship between mathematics and the real world while increasing their ability to solve procedural problems by engaging in discovery lessons and a specially designed project. Goal Two focused on creating a classroom environment that facilitated communication and allowed students to effectively use academic language through meaningful peer discussions about mathematics. Goal Three incorporated both Goal One and Goal Two in developing a strong mathematical identity in and outside of the classroom by combining strategies that build both conceptual understanding and communication.

To evaluate the effectiveness of *Indentifying with Mathematics*, I used a variety of evaluation and data collection methods. Students engaged in quick writes, ongoing formative and summative assessments, analysis of peer-mediated discussions and reflections, and presentations to evaluate a student’s conceptual understanding. I conducted interviews with selected students regarding their math identities, academic understanding, and autonomy in the math classroom. The peer-mediated discussions, academic language quiz, student project essays, and student presentations were used to evaluate communication in the classroom. To measure identity, I looked at student surveys, student interviews, and journal reflections. Table 13 describes each goal in detail as well as the evaluation methods I used to assess *Indentifying with Mathematics*. 
Table 13: Evaluation Strategies of Goals States the goals and how each goal was evaluated post-implementation.

<table>
<thead>
<tr>
<th>GOAL</th>
<th>DESCRIPTION</th>
<th>EVALUATION METHOD</th>
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| 1    | Students create a strong relationship between mathematics and the real world while increasing their ability to solve procedural problems | • Student presentations  
      |                                                                                                | • Student projects  
      |                                                                                                | • Teacher questioning and observations  
      |                                                                                                | • Academic Language Quiz  
      |                                                                                                | • Quick writes |
| 2    | Students create a classroom environment that facilitated communication and allowed students to effectively use academic language | • Peer discussions  
      |                                                                                                | • Teacher observations  
      |                                                                                                | • Academic language quiz  
      |                                                                                                | • Student presentations  
      |                                                                                                | • Student project essay |
| 3    | Students develop a strong mathematical identity in and outside of the classroom                  | • Student interviews  
      |                                                                                                | • Student journal reflections  
      |                                                                                                | • Teacher observation and field notes  
      |                                                                                                | • Student surveys |

Data Collection Instruments and Strategies

Student Surveys

Throughout the curriculum, I gave students the opportunity to express their views about mathematics through the use of surveys. The surveys were used to help me guide the curriculum and understand the identities of my students. Student choice was a big component of my project, and I was able to capture this choice through the use of surveys. I used the surveys to help me develop the project. Students then were able to choose the topic that interested them the most from three different topics.
Reflection Journals

The reflection journals included a variety of questions that students were asked to answer in writing prior to and at the end of certain activities during implementation. The journals were organized into three different parts of the project to help determine a student’s thoughts throughout the implementation. Students were asked to write their answers to questions around the topic covered in this unit as well as their own views of themselves as learners of math. Over time, I was able to see changes in their answers as they continued to investigate and discover mathematics. One of the changes that I observed was that students become more confident and lengthy in their written answers. This change is discussed further below in the findings. Throughout the implementation, I modeled to students how to read, understand, and analyze the questions in the reflection journals.

The first reflection (see Appendix) was given to students prior to any activities. Students were asked to read a letter that I wrote to them discussing the change in what they perceived as “normal” classroom activities when it comes to mathematics. The letter addressed some of the goals I had set forth for my students including creating a strong identity with math through understanding, communication, investigation, discovery, and reflection. After the letter, students were asked to answer questions around their identity with mathematics. The questions revolved around their comfort level with math, their achievement with math in the past, and their motivation levels.

The second reflection (see Appendix) was given to students after they completed a pre-implementation test. The questions included in this reflection required students to write about their feelings about the pretest, their confidence levels, their degree of
independence when presented with unfamiliar material, and their views about whether or not they feel they would be successful in this unit.

The final reflection (see Appendix) was for students to think about what they had discovered in class over the past few weeks and talk about the impact on their own learning. Once again, students were asked to talk about their feelings about the unit and their view about mathematics after their discoveries. Students were also asked to discuss whether or not they felt as though they would be able to teach what they learned to another student or group of students. They were also asked to discuss their confidence levels with regards to geometry and math in general. The reflection journals were used to help students think about mathematics and discuss their understandings, concerns, and questions.

**Peer-mediated discussion write-ups**

Throughout the activities, students were asked to communicate with each other and discuss the mathematical ideas presented in the lesson. Students were prompted with questions related to the topic at hand and had time to discuss with a partner or in small groups as well as with the entire class. The peer-mediated discussions usually took place after a series of investigations in which students were asked to discover a formula relating to the area of a two-dimensional figure. Students expressed their thinking about the ways in which they arrived at their solutions.

**Pre-Implementation Test**

Before beginning the unit, students took a pre-implementation test to see what they already knew about area and polygons. On the test, students were asked to solve problems in which they were asked to calculate the area of various polygons. They were
also asked to define specific vocabulary words that were pertinent to the unit. Lastly, students were asked to answer questions in regards to their views of mathematics and the relationship between area and the real world. Because of time constraints, students did not take the same test post-implementation and I was not able to use this data in evaluating the curriculum.

**Investigation Activities and Project**

The investigation activities, which were discussed in detail in Chapter VI, served as a way for students to discover mathematical properties and take an active role in their learning. I used student responses and observations of the activities in evaluating *Identifying with Mathematics*. The project was graded according to a rubric that followed a similar format to the California Standardized Test format of Advanced through Far Below Basic. Students were evaluated in six different categories which are discussed in greater detail under examining the data.

**Observations and Field Notes**

Throughout implementation, I took copious notes during class time. These notes and observations were used and compared throughout the implementation process. During implementation, I would walk around the classroom with a clipboard and jot down notes during student investigations and discussions. I would also reflect on each days lesson and write down key points from each day. Many of the observations led to interesting findings that are discussed in detail below.

**Examining the Data**

The data were examined in multiple ways to evaluate the success of *Identifying with Mathematics*. I began by compiling all of my students’ work and my own field
notes and organizing them by activity. Next, I decided on an order in which to begin my evaluation. First, I looked at journals and surveys followed by peer-mediated discussions. After looking through student responses, I evaluated the investigation lessons and looked for various degrees of conceptual understanding. Lastly, I evaluated student projects using the project rubric (see Appendix) and read through my own field notes and observations.

**Student Journals and Surveys**

To begin analyzing the data, I read each student’s responses in their reflection journals and surveys from pre and post-implementation. After reading through all of the student responses, I decided to tally common response words for each question from every one of the students. It was not until after reading the responses that I realized many of the students had similar responses to certain questions. After tallying analogous feelings and responses, I compiled the information into a side by side comparison chart between pre and post-implementation responses. I categorized each word that students used into either a positive or negative affiliation with mathematics. In order words, each word was assigned either a plus sign for a positive view or a minus sign for a negative view. This helped me visualize and numerically evaluate the number of students whose responses to mathematics changed from a negative perspective to a positive one. Based on the responses, I made a bar graph (Figure 10) that represented pre and post-implementation responses to each of the different questions which will be discussed in further detail in the findings.
Peer-mediated Discussions

After reading through and analyzing journal and survey responses, I turned to the peer-mediated discussions. During implementation, I took copious notes on the use of academic vocabulary by the students as well as the engagement levels of each group. I noticed that the use of academic language as well as the sustainment of academic talk increased over time. To evaluate this, I took notes during student discussions and marked down every time a student used a new vocabulary word to discuss questions with their classmates. I also marked down the start and stop time of the discussions and noted when students began to waver into off-task conversations. I noted that at the beginning of implementation and during the first few peer-mediated discussions, students used, on average, three vocabulary words per discussion. I also noted that student academic conversations on average lasted no more than thirty seconds to one minute. The use of academic language and the time spent discussing mathematics increased over time as will be discussed further in the findings.

Investigation Activities

After reviewing the peer-mediated discussions, I decided to look through the investigation activities for evidence of conceptual understanding. During implementation, I was able to observe a change in pace during student investigations. There was a shift from students continuously being dependant on me for information to students being more dependent on their peers. Also, because of the discovery lessons, more students expressed understanding in their exit slips and closure activities. For example, I looked at samples from a previous unit and noted that 40% of my students were unable to explain the reasoning for their answers to a mathematical problem.
During this unit, I noted that 75% of my students were able to correctly justify their answers and write explanations that were accurate. This could show an increase in conceptual understanding which is discussed in detail below.

**Projects**

Finally, I evaluated student projects using the project rubric to check for conceptual understanding and academic language. I also took notes during the last two weeks of implementation to gauge motivation levels. I found some interesting results that could symbolize an increase in motivation and engagement. Although students produced high quality work on paper, their student presentations lacked practice and confidence. The work that students produced was not accurately presented in their presentations. Although student presentations did not exactly represent the amount of time and effort put forth into the project, students still showed high degrees of motivation and engagement during the project.

Based on the collected data, students’ showed an increase in both conceptual understanding and motivation. The time spent talking about mathematics and the use of academic language also increased as students gained confidence and comfort in their abilities. All of these finding will be analyzed in depth throughout the discussion of the findings as well as possible reasons for each of the findings and patterns.
Findings

Finding I. Students gained self-confidence in themselves as learners and teachers of mathematics.

In order to build a strong foundation for a math identity, students must feel confident in their abilities to perform mathematical procedures as well as discuss mathematical ideas with others. Through implementing Identifying with Mathematics, I found that the majority of my students gained self-confidence throughout the unit which greatly contributed to my third goal which was to build a stronger math identity.

To evaluate the rise in self-confidence I looked at student journal reflections, student surveys, and took field and observation notes. Looking at pre and post-implementation journal entries was extremely rewarding as I found multiple responses where students’ confidence in their knowledge increased. For example, one student who began the unit thinking she was not going to be able to teach another student about geometric area stated in her post-implementation survey, “I learned a lot in this unit and I feel like I can teach someone else how to calculate the area of different shapes because I know how to do it.” Students began to see themselves as learners and teachers of mathematics throughout the implementation.

As stated above, I tallied analogous responses by students and categorized them into positive or negative affiliations with mathematics. The graph in Figure 10 shows the percentage of positive, negative, and neutral responses pre and post-implementation.
Figure 10: Reflection Response Graph  The increase in positive responses from reflection one which was given pre-implementation and reflection three which was given post-implementation.

The data in Figure 10 show an increase in positive responses throughout the implementation process. In Reflection One, only 20% of students out of 41 students total indicated positive feelings about themselves as a learner of mathematics. On Reflection Three (post-implementation), 83% of students or 34 of 41 students indicated positive responses. This could indicate an increase in student confidence because the affiliation students have with mathematics became stronger and optimistic over time.

In addition, students were asked whether or not they believed they could teach students mathematics. On reflection one, 10% of students responded yes while 85% of students said they would not be able to teach another student math. The remaining 5% of students did not answer the question. On reflection three, 80% of students responded that they would be able to teach another student mathematics. The increase in students’ views
of themselves indicates an increase in their self-confidence levels and their abilities to teach already learned concepts to others. It is important to note that on reflection one, only 10% of students indicated that they may be able to teach an already learned concept to another student. After reviewing this in more detail and comparing pre and post-implementation reflections of this 10%, all but one of these students had originally noted that they would not be able to teach others. The switch from not being able to teach to maybe being able to teach may show an increase in their confidence levels as well.

Lastly, I took notes on student responses to questions and the number of times they contributed to whole group discussions about mathematics. What I noticed over time was the increase in the number of contributors as well as the number of ideas and opinions that surfaced during discussions. On the first day of implementation, I totaled seven students who contributed to class discussions. I decided to compare day one with a day in the middle of implementation as well as the last day of the investigation lessons. On day three of the investigation lessons (which was in the middle of implementation), thirteen students between the two classes contributed to whole group discussions. On the last day of investigations, twenty three students contributed to group discussions. The increase in the number of students who contributed to class discussions can signify an increase in self-confidence as a learner. It may also indicate that the environment that was created for the students was a safe place for students to take risks.

Overall, levels of self-confidence seemed to rise over time and more students felt that they would be able to teach other students. This rise in self-confidence may have strengthened their mathematical identities.
Finding II: Student discussions incorporated a greater use of academic language and the discussion write-ups became more detailed throughout the course of implementation.

Communication in a math classroom can create an active learning environment where students can discuss mathematical ideas rather than silently compute mathematics. As a teacher, this finding was extremely rewarding and exciting. Especially among “at-risk” students, the opportunities students have to talk and write in all classes can greatly contribute to their language and verbal skills. Throughout Identifying with Mathematics, the role of talk played a huge part in class lessons and students exhibited a greater use of vocabulary as well as sustained academic talk as each lesson passed.

Based on field notes, an average of three new vocabulary words were used per peer-mediated discussion during the first few days of implementation. In the discussion between a group of four below, I have italicized vocabulary words within their discussion. The discussion question was, “How are rectangles similar to parallelograms? Is there any way we can change a parallelogram into a rectangle?”
Annie: Um, ok, well let’s start with the first question I guess
Omar: Well, a *rectangle* looks like this (draws a rectangle), but I don’t remember what a *parallelogram* looks like.
Alexis: I think it looks like this (draws a trapezoid)
Natalie: No, that’s a *trapezoid* (group starts to laugh). This is a *parallelogram* (draws the correct figure)
Alexis: Oh ya, ok.
Annie: Ok, so we have to answer the question. How are they similar?
Natalie: Well, they both have four sides, but the *parallelogram* is slanted.
Natalie: How can we change a *parallelogram* into a *rectangle*?
Natalie: I don’t know, I guess just fold both ends in
Annie: Oh ok

**Figure 11: Student Dialogue 3** A conversation in the early stages of implementation between group members on the similarities and differences of rectangles and parallelograms.

At this point all group members stopped talking and waited for other groups to finish.

The conversation lasted about thirty seconds, and the majority of the other groups experienced the same time frame. Not only was the discussion lacking duration, but the lack of academic vocabulary and academic conversation indicated passive learning habits that needed to be broken.

On the final day of the investigation lessons, student discussions incorporated, on average, eight vocabulary words and students were able to sustain a longer period of academic talk. In Figure 12 are transcribed field notes I took during the last discussion of the same group as above. Annie, Omar, and Natalie are the three students who participated in this discussion as Alexis was absent for this lesson. The questions were, “What is a regular polygon? What is the formula for the area of a triangle? What do triangles and polygons have in common?”
Natalie: Well, I know that a *regular polygon* is a polygon with all *angles congruent* and all sides *congruent*.
Annie: Yes, it is *equiangular* and *equilateral*. So is an *equilateral triangle*, right?
Natalie: Ya, exactly, so that is something that *polygons* and *triangles* have in common. And we know the *formula* to find the *area* of a *triangle* is \( A = \frac{bh}{2} \).
Annie: Right, Omar, what do you think?
Omar: I agree with that you guys are saying. We also need to draw pictures of these figures. So, here is an example of a *regular polygon* (draws a regular hexagon). How do we mark it again?
Annie: Well, since it has six sides, we know it is a *hexagon*, and we mark all the *angles* and sides *congruent*.
Natalie: And we have to write that a *triangle* is a type of *polygon* to answer number three. What else?
Annie: I think that’s it. We just need to be ready to discuss our answer. I’ll answer number one.
Omar: I’ll say the *formula* for a *triangle*.
Natalie: Ok, I’ll answer how *triangles* and *polygons* are similar then.

**Figure 12: Student Dialogue 4** The same group of students as in Figure 11 discussing another mathematical idea at a later point in the implementation process.

The second discussion from this group shows greater insight into each question as well as a more organized compilation of ideas. Omar, who is typically less vocal and does not actively contribute to conversations, was prompted by Annie to give his ideas and opinions. The discussion showed a greater use of vocabulary and students incorporated past knowledge and vocabulary such as equiangular and equilateral into their discussion.

The increase in the use of academic vocabulary could be a result of the amount of practice students had over time. Because students knew what was expected of them and because students understood the importance of their discussions, they were able to incorporate more vocabulary and include all group members in each discussion. As time went on through the implementation process, I was able to see from my anecdotal notes during discussions that the majority of the groups were able to sustain academic talk...
around a given topic for a longer period of time and incorporate its relationship to the real world in each discussion. At the beginning, a typical peer discussion would last thirty seconds to one minute before students either began to veer into off-topic conversations or stop talk altogether. Towards the end, students were able to sustain academic talk for up to ten minutes.

Lastly, the discussion write up became more detailed over time. After analyzing the written responses to group discussions, I noticed that all of my students showed a greater use of detail in their discussion write ups as they were able to practice and gain more confidence in their discoveries. This could be due to a rise in conceptual understanding levels and confidence levels over time.

**Finding III: Students produced high-quality work that showed a great deal of effort, conceptual understanding, and motivation in their projects but presentations did not reflect the same high-quality work.**

To evaluate the projects, I used a rubric that contained six components which included accuracy of mathematical calculations, correct and sufficient use of formulas, project components, communication of mathematical ideas (the essay), neatness and effort, and general directions. Each component was critiqued on a proficiency scale where 90-100% represented “A” quality work, 80-89% represented “B” quality work, 70-79% represented “C” quality work, 60-69% represented “D” quality work, and 59% and below represented “F” quality work. Once each component was graded, the students overall scores were calculated by the average of all six components as well as the presentation grade. When evaluating the score for each component, I decided to focus
my attention on the components which most directly affect conceptual understanding, motivation, communication, and the presentations.

The graph in Figure 13 represents the breakdown of the grades of each of the components in which I focused my data collection.

![Graph](image)

**Figure 13: “High Quality Work” Graph** Represents the percentage of students who produced high-quality work in each of six categories as well as in their presentations. High-quality has been categorized by Advanced or Proficient marks or “A” and “B” marks on the project rubric.

As can be seen in Figure 13, 65% of students showed an advanced or proficient understanding and use of mathematical calculations. 70% of students correctly and sufficiently used the formulas at an advanced or proficient level. These percentages can be a result of an increase of conceptual understanding levels after the investigation lessons. The majority of the students were able to use the formulas they developed in meaningful ways to display accurate calculations in the area project.
Figure 13 also shows that 55% of students were able to communicate their mathematical ideas in their project at an advanced or proficient level. Given the high number of English Language Learner students, the scores on student essays reflected an increase in academic vocabulary usage. This could be due to the practice students had in peer-mediated discussions and journal reflections. It could also be that students felt more comfortable with their topic as they were able to choose what interested them the most. Therefore, interest level, comfort, practice, and active learning could have all contributed to students’ abilities to accurately and expressively write essays around a topic in math.

It is important to note that 25% of students displayed “C” quality work in their project essays. Only 20% of students did not demonstrate passing quality work. Therefore, 80% of students met the objectives they needed in order to receiving a passing grade on the project.

Only 15% of the students produced high-quality presentations that matched the amount of work and effort they put forth in their project. Although a large majority of the students fell into the “C” range, it was unfortunate to see such a low number of “A” and “B” presentations. There are many reasons why students could have performed poorly on their presentations. First, students were not given ample time to prepare and practice for their presentations. Because of time constraints and poor planning on my part, students only had one day to prepare for their presentations. This could have been a contributing factor to the low presentation scores. Second, students were not prepared for the requirement of presenting their work. It was not until the launch day of the project that students realized they would be required to present the project. Because my students were unfamiliar with presenting in a math classroom, it may be that many students need
more practice before perfecting the art of presenting mathematical work to a group of people.

Although the presentations did not display ample amounts of effort, overall the project was a success. Students displayed conceptual understanding in their calculations, explanations, and essays. The majority of the students also communicated their mathematical ideas effectively. Finally, students showed an increase in participation and motivation during the project which is described in detail in Finding IV.

Finding IV: There was an increase in student participation and motivation during the project.

After reviewing my observation notes, I did not notice any change in student motivation during the investigative lessons, but there was a definite change in student behavior after we moved from the investigative lessons to the “Bringing Area to Life” Project. Because of the project survey at the beginning of Identifying with Mathematics which helped me design the project according to student interests, it could be that students felt more motivated to undertake and complete a project focused around mathematics. Also, students were asked to choose one out of three topics that interested them. Providing students with choice may have enhanced their motivational levels.

In order to evaluate levels of motivation during the project it is important to note that 75% of my students produced neat work that showed sincere effort as can be seen in Figure 13. This indicates that students took pride in their projects and were engaged in creating work they could be proud of. This engagement level shows an increase in motivation levels over time.
Also, before beginning the project in Reflection Three students were asked about their interest level in starting the area project. 95% of students responded that they felt excited to start a project that was generated from their interests. On the question of whether or not the students were excited about way in which the project was designed, one student responded, “Yes, because that means the project is specifically for me.” Another student responded, “Yes, I have always liked to pick my work rather than to be told to do something.” The issue of choice likely had an effect on motivation levels as students were excited to embark upon a project that was created with their ideas in mind.

Next, student participation increased over time. As stated earlier, the number of different students who contributed to classroom discussions increased by sixteen students. 56% (23 of 41) of students in both classrooms contributed to classroom discussions at the end of implementations. This indicates that over half of the class was involved in classroom discussions around a mathematical topic. This percentage shows a great increase in the number of students who participated throughout implementing *Identifying with Mathematics* compared to the beginning stages in which only 17% participated. The number of students who received lunchtime help also increased over time. Figure 14 indicates the increase in participation levels from students in both classroom discussions and lunchtime help.
Figure 14: Participation Levels Percent of students who participated in classroom discussions and lunchtime help at early and late stages of implementation.

Finally, from my own teacher observations and notes, there were less classroom management issues during the time of the project. Because so many of my students arrive at school with outside factors that directly affect their moods and behaviors, some of my students struggle with behavior and effort inside the classroom. I noticed that there were few distractions during project time, and, on average, only one student had to leave the room during class time which was four times less than the average five students per period who asked to leave class. In addition, some students chose to work through their lunch period in order to complete the project on time. On average, during the two weeks of project time, I had about four students per lunch period in my classroom working on projects (see Figure 14). Prior to the project, I received no students during lunchtime
unless I was offering extra credit or extra help for a test. The increase in the amount of work time may indicate an increase in the level of intrinsic motivation.

**Finding V: Students’ mathematical identities became stronger over time.**

The last and most important finding shows that mathematical identities became stronger over time. Because of the data already presented, there is evidence that students’ communication, conceptual understanding, and motivation levels increased throughout implementation. As a result, students’ identities were supported by the curriculum.

Student interviews were evaluated to see how students’ views of themselves changed over time. In an interview with Andres, I was able to see the strength in his positive views of himself as a math learner. On the topic of his enjoyment level and mathematics, Andres informed me, “I do enjoy math because I find it interesting and it could be sometimes fun. So far, I like Algebra the best because it is not complicated. I would describe math as fun, exciting, but sometimes hard.”

In another interview with Tania, a student who has continuously struggled in her math classes, I saw an increase in her belief in herself as a learner of math. When asked about the area project and her confidence levels Tania responded, “The project was fun. I usually can’t do math and I stop trying, but I liked how I could choose my project, and I like making patterns, so it was fun and easy to finish the quilt. The math part was a little hard, but I am proud of what I did.” Tania’s view of herself and her ability to complete an assignment or project rose after her interest and motivation levels rose. It was exciting to see the changes occurring in my students as a result of a curriculum that they enjoyed and were motivated to finish.
William, another 9th grade student who is considered a behavior problem in most classes stated, “The project was cool. I liked being able to see how the math we learned is used in real life. I know we have a big project at the end of the year too and this project made me feel confident to do that one.” Williams attitude about the classroom changed over time, and his issues with behavior decreased because of his engagement level in the project.

Another example of an increase in identity can be seen in a response by Janelle who said, “The project was fun. I usually can’t do math and I stop trying but I liked how I could choose my project, and I like making patterns, so it was fun and easy to finish the project. The math part was a little hard but I am proud of what I did.”

Summary

The overall goal of Identifying with Mathematics was for students to develop a strong mathematical identity. Based on the data presented, students identities were strengthened due to an increase in conceptual understanding, use of academic language, discussions in the math classroom, and motivation levels. Students were able to experience investigations and take an active role in their learning as well as connect the math they used to the real world. It was amazing to see the difference in my students and their desire to learn mathematics. My hope is that they will continue to find joy in mathematics and continue to pursue math classes in the future.
Chapter VIII:
Summary and Conclusion

Reflecting upon my experiences of researching, developing, and evaluating curriculum has helped me come to several realizations and conclusions about my teaching as well as education in general. The overarching goal of helping students identify with mathematics on a level that is more rigorous than the average high school curriculum led me to understand the complexities and necessities of seeing oneself as a learner of mathematics. Throughout my research, I sought out methods, techniques, and opportunities that help students achieve greatness. As an instructor and an educator, I have also become an avid learner. My passion for learning has driven me to maximize my potential as a teacher-researcher by discovering ways to improve the curriculum for my students. A goal of mine during this experience was to learn, discover, and research techniques that help make academic content more comprehensible to students while contributing to curriculum reform.

This experience has helped me learn and understand a great deal about myself as a teacher as well as a learner. Primarily, I learned that in order to be a great teacher and effectively teach mathematics, it is important to be a researcher. I was able to find successful methods to reach my students through research on best practice in schools. With research to back up my curriculum, I learned how to effectively teach geometry to students who come from a world where a math class is not necessarily at the top of their priority list.
It was eye-opening to see how my views as a teacher changed after this experience. Initially, I was comfortable with my style of teaching and the way in which I conveyed the content to my students. After this experience, I became more aware of the needs of my students. In creating *Identifying with Mathematics* each piece of the curriculum was designed in an effort to support each student’s mathematical identity and help students make connections with the subject.

In order for students to understand the importance of mathematics in our society as well as incorporate mathematics in the real world, they must be able to identify with it and internalize it. Before implementing my curriculum, I noticed that many of my students disdained mathematics. I decided at that point that it was time to find innovative ways to change the lives of my students as well as their perceptions of mathematics. Through motivating and engaging investigative lessons, reflection journals, peer-mediated discussions, interviews, real world project based learning, and educational research, I created a curriculum that led to very interesting and exciting findings. First, students gained self confidence in themselves as learners and teachers of mathematics. Second, student discussions incorporated a greater use of academic language and student writings became more detailed and showed a greater sense of understanding over time. Third, there was also an increase in student motivation and participation during the project. And finally, students became more autonomous in finding solutions to problems throughout the implementation of the discovery activities.

Identity contributes to students’ perceptions about mathematics and their desire to practice mathematics (Boaler, 2002). A strong identity will influence a student’s decision to continue their studies in mathematics in a higher educational setting. Weak identities
turn students away from accepting mathematics in their lives and hinder their views of themselves as learners of mathematics, which in turn can hinder their conceptual understanding. Creating a strong identity can help students understand the relevance of higher-level mathematics in the world as well as help them gain self-confidence in a math classroom.

Motivation is a contributing factor in creating a strong mathematical identity. Students who are motivated have more positive views of themselves as learners of mathematics. I was able to observe this trait through the project-based learning that took place in my classroom. It was inspiring to see students who struggled with math and who had weak math identities thrive in a setting that incorporated choice and real-world connections. Many students were engaged in the project and were willing to ask clarifying questions as well as discuss their project with peers outside of the classroom. Students who embraced the project also received higher marks. Therefore, motivation can also contribute to success in the math classroom. Students who are motivated are more likely to understand the mathematics they engage in because the effort they put forth will help them accomplish their goals.

Conceptual understanding is another construct that could contribute to creating a strong math identity. As defined earlier, a math identity refers to the way in which students see themselves as learners of mathematics as well as how others view them as learners of mathematics. Students who understand the material they are presented with are more likely to see themselves as strong learners and contributors in the math classroom. I was able to observe this through the interviews I conducted with students. Those who had a strong math identity tended to contribute to classroom discussions more
than those who had a weak identity. Also, students with strong identities appeared to be the leaders of the classroom and often were the individual to whom students turned for help. During the implementation, I observed that the number of questions and comments from students increase over time. The love and enthusiasm I have for mathematics was finally starting to trickle down to my students as I observed them not only ask more questions but actually discuss mathematics without being prompted to do so. The level of understanding in the class led me to believe that the identities of my students became stronger.

In addition to conceptual understanding, I saw changes in my students abilities to communicate effectively and academically in the classroom. Being able to talk and discuss in a math classroom could not only increase conceptual understanding and motivation, but could also contribute to creating a strong identity with mathematics. From my observations I noticed that students who were actively engaged in learning and expressed their ideas and questions through peer communication were more likely to feel comfortable and confident as a student.

Although I found strong evidence that students’ views of themselves changed and strengthened over time, I was not able to evaluate data that suggest students’ views of each other changed. In implementing this curriculum again, I would be curious to see how students view other students as learners of mathematics, as this also contributes to a student’s mathematical identity. Further interviews and questioning would be needed to see this relationship. Although how students see each other is important in the creation of an identity, I believe students are greatly influenced with their own beliefs of themselves. For example, I believe that students who are confident in themselves as learners are more
likely to contribute to mathematical discussions, hold a higher degree of conceptual understanding, feel intrinsically motivated, and convince others of their ability to perform mathematics. This in itself likely affects the way students view each other as learners of mathematics.

In conclusion, students who have a strong math identity will likely understand math and use math in their futures. Strong identities also help students make decisions about continuing their education in math courses and going above what is required of them as students. Students can also begin to see themselves as a contributor to a class instead of bystanders or passive learners. Most importantly, I believe that creating a strong identity with math allows students to feel confident in their academic skills and succeed in the classroom.
Appendix

Identifying with Mathematics:
The effects of conceptual understanding, motivation, and communication on the creation of a strong mathematical identity

By: Veena Prakash Mansukhani
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Section I

Introduction:
For the Teacher
Tips for Implementation
For the Teacher

Identifying with Mathematics is a unit of study designed for a beginning geometry class. The activities and projects have been designed with existing research and best practice in mind. Identifying with Mathematics is an approach to building mathematical understanding and self-efficacy through a project- and activity-based curriculum. One of the goals of Identifying with Mathematics is to help students create a strong mathematical identity through communication, understanding, and bonds with the real world. A mathematical identity can be defined as the way in which students see themselves as learners of mathematics. Students are encouraged to investigate, communicate, derive, and discover, while also developing a greater sense of the necessity of mathematics in their lives.

Identifying with Mathematics incorporates various learning modalities through a variety of lessons that stimulate learning for the whole student. These include investigations through hands-on activity, communication through peer-mediated discussions, competency through procedural fluency, motivation strategies through reflection journals, and real-life applications through project based learning.

Students will be asked to discuss the activities in which they are engaged, which means that the students will be motivated and more likely to conceptually understand mathematics. Although Identify with Mathematics is designed with the mathematical concept of area as the unit of study, it can be modified to fit the needs of many different standards in mathematics as its foundation can be interpreted throughout different disciplines.
The curriculum is organized into five features: hands-on investigation activities, procedural fluency, project-based learning, mathematical peer discussions, and reflection journals. I decided to focus this curriculum on these features as each of them contributes to the constructs of motivation and conceptual understanding. Motivation and conceptual understanding include real world connections, math discourse (conversations, math vocabulary, persuasion, etc.), and procedural fluency. Each day students will engage in one or more of these features to build understanding of the concept of area and make connections to the world around them.

*Identifying with Mathematics* is broken down by daily activities which are designed to run 70 minutes long. Each lesson incorporates skills, prior knowledge, and concepts that have been mastered by the students from prior lessons. The days build on each other, and students can see a systematic and relevant connection between each activity. Since the students are asked to derive each area on their own, this curriculum allows students the ability to really understand and relate to the math because they are able to see the rationale for why each area is generalizable and accurate.

After the implementation of *Identifying with Mathematics* I was able to see positive growth in my students. Students’ identities grew stronger due to an increase in conceptual understanding, use of academic language, discussions in the math classroom, and motivation levels. Students were able to experience investigations and take an active role in their learning as well as connect the math they used to the real world. My hope is that your students will feel the same sense of accomplishment and enjoy *Identifying with Mathematics*!
Tips for Implementation

Identifying with Mathematics is an investigative approach to teaching Geometry in which students are working collaboratively to problem solve and recreate mathematics as mathematicians do. In order to promote a safe and comfortable learning environment for students, there are a few factors that should be addressed prior to implementation.

Grouping:

How do we choose and monitor groups? How do we help students “talk” about content?

In order to facilitate the best possible learning situations for students, it is important to consider the following when answering the questions above:

1. Know your students, their personalities, their language abilities, and their current content knowledge.
2. Use sentence starters to facilitate mathematical discussions.
3. Make sure vocabulary is visible to help English Language Learners.
4. Have students practice norms and procedures for group talk prior to implementation.

Safe Environment:

How do we create a safe environment where students feel comfortable taking risks?

In order to create a safe environment for students, consider the following possible tips:

1. Begin the year, the unit, or the lessons with an ice breaker that helps students feel comfortable around one another.
2. Discuss the severity of bullying and the importance of respect in and outside the classroom.
3. Try not to single out students and give students ample time to come up with responses before being asked to share in a whole group situation.
4. Allow students time to practice and try not to set expectations above what students are capable of handling.

Room Environment:

How can we create a room environment that fosters identity growth?

In order to create a room environment that fosters student learning consider the following tips:

1. Set up tables in collaborative groups of four to allow students immediate access to conversations with group members.
2. Make sure students have ample room in-between groups to minimize distractions during group discussions.
3. Make sure students understand the importance of being able to be actively involved in learning and allow time to discuss the expectations and procedures for talk in the classroom.
4.
Section II

List of Materials:
Letter of Consent
Overview of Curriculum
Letter of Consent

This letter is optional and up to the discretion of the teacher.

Dear Parents,

As your child’s math teacher, I continue to seek out and find innovative ways to improve teaching mathematics. I am currently participating in a masters program at the University of California, San Diego in which I am focusing on ways to improve teaching and learning in my field of study. I am working on a project developing a curriculum based on best educational practice which will include activities that we are currently doing along with activities that are designed to help foster and promote a strong identity with mathematics. My hope is that my students will see themselves as strong learners of mathematics. The curriculum will help students not only be successful, but feel successful in mathematics through hands-on activities, discussions with peers, projects and reflection journals.

When your child participates in this project he or she will be taking time in class to investigate, discover, communicate, and reflect on mathematical ideas. Through these activities, I hope to see students feel confident and really understand the standards-based math presented to them. If a student chooses not to participate in this project, he or she will still be required to complete all necessary school work, but I will not include any of his or her work/project/reflections in my study. A student’s decision to not participate in this study will not affect his or her grade or relationship with me or the school in any way.

All information gathered from this project will remain confidential. Your child’s identity as a participant will not be disclosed and pseudonyms will be used to protect your child’s identity. During this study students may be videotaped, tape recorded or interviewed. Students work will also be collected to evaluate the effectiveness of the curriculum. Images will only be used with your consent in this form. Please sign and return this form to me by February 22, 2010 only if you wish for your child to not participate in this project. If this form is not returned to me by February 22 2010, I will assume consent has been given.

If you have any questions or concerns about this activity, please feel free to contact me using the information below. Thank you for your help.

Sincerely,
Veena Mansukhani

If you agree to your student participating in this study, you do NOT need to return this form.

[ ] I give permission for my child to participate in this study but would not like my student to be videotaped or tape recorded.

[ ] I DO NOT give permission for my student to participate in this study.

Child’s Name (please print) ____________________________ Date: ________________

Parent Signature ______________________ Date: ______________

Again, if you are not opposed to your student’s participation, you do not need to return this form. If you would like for your child not to participate, please return this form by MONDAY FEBRUARY 22nd.
Overview of Curriculum

<table>
<thead>
<tr>
<th>Activities broken down by day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test and Reflection #1</td>
</tr>
<tr>
<td>Day 1: Investigation – Area of a Rectangle</td>
</tr>
<tr>
<td>Day 2: From Rectangles to Parallelograms</td>
</tr>
<tr>
<td>Day 3: From Parallelograms to Triangles</td>
</tr>
<tr>
<td>Day 4: From Parallelograms to Trapezoids</td>
</tr>
<tr>
<td>Reflection #2</td>
</tr>
<tr>
<td>Procedural Fluency Practice</td>
</tr>
<tr>
<td>Day 5: Area of a Regular Polygon</td>
</tr>
<tr>
<td>Reflection #3</td>
</tr>
<tr>
<td>Project Launch</td>
</tr>
<tr>
<td>Project – sketch</td>
</tr>
<tr>
<td>Project – blueprints</td>
</tr>
<tr>
<td>Project – area calculations</td>
</tr>
<tr>
<td>Project – area calculations</td>
</tr>
<tr>
<td>Project – final draft</td>
</tr>
<tr>
<td>Project – final draft</td>
</tr>
<tr>
<td>Project – essay (draft)</td>
</tr>
<tr>
<td>Project – type essay</td>
</tr>
<tr>
<td>Project – prepare for presentation</td>
</tr>
<tr>
<td>Project Presentations</td>
</tr>
<tr>
<td>Project Presentations</td>
</tr>
</tbody>
</table>
Section III

Prior Implementation Activities and Reflections:

Reflection Journal #1
Area Pre Test
Reflection Journal #2
Reflection Journal #1

Name: ________________________
Per: _____ Date: ________________

Reflection Journal #1: How do you currently feel about math?

Dear amazing student,

We are going to start a new unit of study about area. This unit is going to be focused around helping you create a strong identity with math through understanding how area relates to your life and why it is important. You are going to be asked to do a lot of investigation, communication, and discovery around this topic. Be prepared to talk and think a lot! One of the best ways for you to understand as a student is to communicate with your group members and discuss the math you are learning. You will be asked to discover the formulas for area on your own instead of being given all the information. You will also be asked to solve problems using the formulas you have developed but most of this unit will be focused on discovering. You will be asked to complete a project that relates area to the real-world. Area is used so much in the world around us, and it is important that you understand and accept why in order to truly connect with math. As you can see, most of the learning for this project will be taught by you! You will see what it is like to be a mathematician, and you will develop generalized formulas for the area of various shapes as mathematicians did many years ago. This will hopefully help you take ownership of your learning and help you realize that you know and can do more than you think you can! My hope is that you will find this unit interesting and motivating, and I hope you will have a desire to continue your journey in mathematics with confidence and enjoyment. I know you will thrive and do an amazing job on this unit because you will be responsible for teaching yourself as well as your classmates. I will be here along the way to facilitate your learning and ask questions that will lead to further analysis. Good luck, and I can’t wait to see what you are capable of doing!

Sincerely,
Ms. Mansukhani

Journal: Please answer the following questions in as much detail as possible. I want to see how you feel about math up to this point in your life.

1. How does this letter make you feel as a student? (excited, scared, confused, etc.)
2. Do you enjoy math in school? Do you believe it is important in life to know how to do math? Why/why not?

3. So far in your life, how do you view yourself when it comes to math? (strong/weak skills, strong/weak understanding, confident, insecure, successful, unsuccessful, etc.)

4. Do you feel comfortable in a math class? Do you like to raise your hand? How do you feel when you are called on to answer a question?

5. Do you think it takes a special talent to do well at math? Do you have this talent?
6. What motivates you to do well in a math class?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

On the questions below Please circle the number that most applies to you below

7. How important is it for you to get a good grade in math class?
   Not important  Somewhat important  Very important

8. How important is it to have a good teacher in order for you to understand?
   Not important  Somewhat important  Very important

9. How confident do you feel when it comes to being successful in math?
   Not confident  A little confident  Very confident

10. What would you say is your average grade in math?
    F  D  C  B  A

11. How much support do you get from your parents and people you live with at home?
    No support  Some support  A lot of support

12. Do your parents understand the math we are doing in Geometry?
    No, they don’t understand anything  They understand some  They know everything

13. Please write anything else you think I should know about you as a math learner:

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Area Pre Test

1. \[ \text{Area} = \text{?} \]
   \[ \text{7 cm} \]

2. \[ \text{Area} = \text{?} \]
   \[ \text{3 cm} \]

3. \[ \text{Area} = \text{?} \]
   \[ \text{square} \]

4. \[ \text{Area} = 100 \text{ cm}^2 \]

5. \[ \text{Area} = 65 \text{ m}^2 \]

6. \[ \text{Area} = \text{?} \]
   \[ \text{triangle} \]
7. Area = 60 m²

8. The figure below is a regular pentagon

9. The figure below is a regular hexagon.
   Area = 30 m²

10. 

11. 

12. Find the area of the shaded region
   
   Area of shaded region =
Vocabulary: Please define the following words as best you can and with as much detail as possible
1. Area: ____________________________________________
2. Trapezoid: _______________________________________
3. Parallelogram: ____________________________________
4. Regular Polygon: __________________________________
5. Base: ____________________________________________
6. Height: ___________________________________________

Formulas: Please write out the formulas for each of the shapes below
1. Rectangle: ______________
2. Square: ______________
3. Triangle: ______________
4. Parallelogram: __________
5. Trapezoid: ______________
6. Regular Polygon: __________
Please answer the questions below to the best of your ability:

1. Why is it important to know how to compute the area of geometric figures?

2. Where do we use area in the real world?

3. What types of jobs do you think there are that use area in their work? Make a list.

4. On a scale of 1 to 5 (1 being not interested at all and 5 being very interested) how interested are you in learning about area and doing a project around area? Circle one.

   1  2  3  4  5
Reflection Journal #2

Name: _______________________
Per: _____ Date: ______________

Reflection Journal #2 – Pre test

1. How do you feel after taking the pretest? Please explain in detail (at least 3 sentences)
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

2. How do you think you did? If I were to grade it, what grade do you think you received?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

3. When you arrived at a problem you did not know how to solve, what did you do and why?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
4. How do you think you will perform on the post test after we finish the unit and why? 
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

5. Did you realize that you know more about area than you thought you did? Why or why not?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

6. Tomorrow we are going to discover the area for rectangles and squares, do you think you already know what it is? If so, what is it? If not, how do you feel about being able to discover that?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

7. Do you think that you can be successful for this unit? Why or why not?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Section IV

Daily Investigations:
Overview of Investigations
Day 1: Investigation – Area of Rectangles
Day 2: From Rectangles to Parallelograms
Day 3: From Parallelograms to Triangles
Day 4: From Parallelograms to Trapezoids
Day 5: Procedural Fluency Practice
Day 6: Area of a Regular Polygon
Overview of Investigations

*Identifying with Mathematics* begins with five investigative activities that help students actively participate in mathematics. Each investigation requires students to collaborate, express their ideas and hypotheses, and draw conclusions about mathematical concepts. Below is a chart that describes the five activities.

<table>
<thead>
<tr>
<th>Activity Number</th>
<th>Title</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>Area of a Rectangle</td>
<td>Students were given the definition of area and used counting blocks, grid paper, and peer communication to determine the formula for the area of a rectangle</td>
</tr>
<tr>
<td>Two</td>
<td>From Rectangles to Parallelograms</td>
<td>Students used their discovery of the area formula for a rectangle to determine the area of a parallelogram</td>
</tr>
<tr>
<td>Three</td>
<td>From Parallelograms to Triangles</td>
<td>Students used their discovery of the area formula for a parallelogram to discover the area formula for a triangle</td>
</tr>
<tr>
<td>Four</td>
<td>From Parallelograms to Trapezoids</td>
<td>Students used the same discovery of the area formula for a parallelogram to discover the area formula of a trapezoid</td>
</tr>
<tr>
<td>Five</td>
<td>From Triangles to Regular Polygons</td>
<td>Students used their discovery of the area formula of a triangle to discover the area formula of a regular polygon.</td>
</tr>
</tbody>
</table>

Each of the activities are described in further detail on the following pages.
**Day 1: Investigation – Area of Rectangles**

Name: ______________________
Per: _____ Date: ______________

**DAY 1 – Investigation: Areas of Rectangles**

**Introduction:**

The *area* of a plane figure is the measure of the region enclosed by the figure. Today, using graph paper and counting blocks, we will discover the area of a rectangle and a parallelogram. You measure the area of a figure by counting the number of square units that you can arrange to fill the figure completely.

Length: 1 unit  
Area: 1 square unit (unit²)

---

**Part 1:** Find the area of the below figures using any method you choose.

**Figure 1:**

Area: __________________________
Part 2: The figures below are represented by square units. For example, Figure 3 has 5 square units that fit on the base and 6 square units that represent the height. Use the counting blocks to construct a rectangle of each figure below. Next, find the area of the below figures using any method you choose.

Figure 3:  \[
\begin{array}{c}
6 \\
5 
\end{array}
\]  Figure 4:  \[
\begin{array}{c}
4 \\
13 
\end{array}
\]

How many rows will fit? _____  How many rows will fit? _____
Area: ______________  Area: ______________

Part 3: Explain how to determine the number of 1 x 1 squares that will fit in the below rectangle. Can you do it in 2 ways?

Answer:

______________________________________________________________

______________________________________________________________

______________________________________________________________

Part 4: Using the below rectangle, select one side as the base.

How many unit squares will fit on the base? _______
How many rows will fill the rectangle? _______

\[
\begin{array}{c}
4 \frac{1}{3} \\
2 \frac{1}{2} 
\end{array}
\]
Rectangle Area Conjecture: The area of a rectangle is given by the formula $A = bh$, where $A$ is the area, $b$ is the length of the base, and $h$ is the height of the rectangle.

Practice:

1. Area = ________  

   \[
   \begin{array}{c}
   12 \text{m} \\
   19 \text{m}
   \end{array}
   \]

2. $A = ________$

   \[
   \begin{array}{c}
   4.5 \text{m} \\
   9.3 \text{m}
   \end{array}
   \]

3. Area = $96 \text{ m}^2$

   \[
   \begin{array}{c}
   12 \text{m} \\
   h
   \end{array}
   \]

   $h = ________$

4. Area = $273 \text{ cm}^2$

   \[
   \begin{array}{c}
   b
   \end{array}
   \]

   \[
   \begin{array}{c}
   21 \text{cm}
   \end{array}
   \]

   $b = ________$
Day 2: From Rectangles to Parallelograms

Introduction:
Today you will discover the formula for the area of a parallelogram using the area of a rectangle. In order to do this, let’s start by talking about the similarities and differences between a rectangle and a parallelogram.

Any side of a parallelogram can be called a base just like a triangle. The height is not necessarily the length of a side. An altitude is any segment from one side of a parallelogram perpendicular to a line through the opposite side. The length of the altitude is the height.

Example:

Part 1: Peer Mediated Discussion

Discuss the following questions with your group members.

1. How are rectangles similar to parallelograms?
2. Is there a way that we can change a parallelogram into a rectangle?
3. If we have a parallelogram with the same base and height as a rectangle, what do you think the relationship of the areas would be?

Write a brief summary of your discussion:

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Part 2: Hands-on Activity

**Step 1:** Using the parallelogram given to you, cut out the parallelogram along the altitude (height). You will end up with 2 pieces: a triangle and a trapezoid.

**Step 2:** Try arranging the two pieces into other shapes without overlapping them.

Can you form a rectangle? **YES**  **NO**

Calculate the area of this rectangle: Area = ________________

Do you think that the area changed since you cut and pasted the parallelogram into a rectangle? Why or why not?

Can you guess what the area of the parallelogram will be? Area = ______________

**Parallelogram Area Conjecture:** The area of a parallelogram is given by the formula________________, where $A$ is the area, $b$ is the length of the base, and $h$ is the height of the parallelogram.

Practice: Find the area of each parallelogram below

1. $A = \underline{\hspace{2cm}}$  
   ![Parallelogram 1](image1.png)

2. $A = 2508 \text{ cm}^2 \quad b = \underline{\hspace{1cm}}$ and Perimeter = __
   ![Parallelogram 2](image2.png)
**Day 3: From Parallelograms to Triangles**

Name: _________________________  
Per: ____ Date: ____________

**DAY 3 – From Parallelograms to Triangles**

Introduction:
Now that we know how to find the area of any rectangle and parallelogram, we will use that knowledge to investigate and discover the area of a triangle.

**Part 1: Investigation using paper, scissors and tape**

- Step 1: Take out a piece of paper and fold it in half
- Step 2: Draw a triangle on the folded paper
- Step 3: Keep the paper folded, and cut out the triangle making two identical copies
- Step 4: Try to construct a shape that you already know the area for (a rectangle or a parallelogram)
- Step 5: Tape the triangles together

Measure the base and the height of the figure you created to the nearest centimeter:

Base = _____________________  
Height = _____________________  

What is the area of the figure you created? Area = __________________________

Since your figure is made up of 2 equal triangles, what do you think the area of each triangle will be?

Area of each triangle = __________________________

How did you arrive at your answer?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Part 2: Peer Mediated Discussion
Discuss the following questions with your group:
1. What is the area of a parallelogram and a triangle?
2. What shape can you make with two congruent triangles?
3. How can you use what you know about the formula of a parallelogram to find the formula for a triangle?
4. Do you think this relationship true for all cases? Why or why not?

Write a brief summary of your discussion:
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Triangle Area Conjecture: The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where $A$ is the area, $b$ is the length of the base, and $h$ is the height of the triangle.
**Day 4: From Parallelograms to Trapezoids**

Name: _______________________
Per: ____ Date: ________________

**DAY 4 – From Parallelograms to Trapezoids**

Introduction:
By now, you are familiar with using formulas you know to derive other formulas. You are becoming young mathematicians! This same method can be used to find the area of any trapezoid. Let’s start with a peer discussion.

**Part 1: Peer discussion**

Discuss the following questions with your group members:
1. What is a trapezoid?
2. What is the formula for the area of a rectangle? A parallelogram? A triangle?

Write a brief description of your discussion:
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**Part 2: Constructing a Trapezoid and finding its area**

Step 1: Fold a piece of paper in half and draw any size trapezoid using a straight edge

What is the length of base₁? _______________________
What is the length of base₂? _______________________
Add your bases together: base₁ + base₂ = _______________________
What is the height of your trapezoid? ________________

Step 2: Keep the paper folded, and cut out the 2 equal trapezoids
Step 3: Construct a parallelogram out of the 2 trapezoids and tape them together
Measure the base and height of your parallelogram

Base = ____________________________

Height = __________________________

What is the area of your parallelogram? ________________________________

Since your parallelogram is made up of 2 congruent trapezoids, what do you think the area of each trapezoid will be?

Area of each trapezoid = _______________________

Part 3: Discovering the formula for the area

Answering the following questions will help you determine the formula for the area of a trapezoid:

1. What is the relationship of the length of the base of the parallelogram to the length of the sum of the bases of the trapezoid?
   a. Base of the parallelogram = ________________________
   b. Sum of the bases of the trapezoid = __________________

2. How many trapezoids did it take to make up the parallelogram?

What do you think the formula for the area of a trapezoid would be?

______________________________

Trapezoid Area Conjecture: The area of a trapezoid is given by the formula __________________, where A is the area, b1 and b2 are the lengths of the two bases, and h is the height of the trapezoid.
Day 5: Procedural Fluency Practice

Area Procedural Practice

1. \[ \text{13 cm} \quad 4 \text{ cm} \]
   \[ \text{Area} = \] ________

3. \[ \text{12 m} \quad 8 \text{ m} \]
   \[ \text{Area} = \] ________

2. \[ \text{15 in} \]
   \[ \text{Area} = 60 \text{ in}^2 \]
   \[ h = \] ________

4. \[ \text{4 m} \quad \text{b} \]
   \[ \text{Area} = 24 \text{ m}^2 \]
   \[ b = \] ________
5. \[
\text{Area} = \phantom{123}
\]
34 in
\[
\text{h} = 21 \text{ in}
\]

6. \[
\text{Area} = 360 \text{ cm}^2
\]
\[
h = \phantom{123}
\]
18 cm

7. \[
\text{Area} = \phantom{123}
\]
8 cm
\[
\text{h} = 4 \text{ cm}
\]
20 cm

8. \[
\text{Area} = 56 \text{ m}^2
\]
\[
h = \phantom{123}
\]
10 m

9. \[
\text{Area} = 108 \text{ m}^2
\]
\[
b = \phantom{123}
\]
6 m

9 m
Now that you have discovered how to find the formula's for the area of a rectangle, parallelogram, triangle, and trapezoid, please explain in detail how you did it! I will get you started on each one.

Parallelogram
First, we took out a piece of paper and folded it in half. Next, we drew a parallelogram on the folded paper. After that we ___________________________

__________________________________________

__________________________________________

__________________________________________

Triangle
First, we ____________________________

__________________________________________

__________________________________________

__________________________________________
Trapezoid

First we ____________________________________________
_________________________________________________
_________________________________________________
_________________________________________________
_________________________________________________
_________________________________________________

Directions: Please write down the formula for the area for each of the following:

Rectangle: \( A = \) ______________

Parallelogram: \( A = \) ______________

Triangle: \( A = \) ______________

Trapezoid: \( A = \) ______________
Day 6: Area of a Regular Polygon

DAY 4 - From Triangles to Regular Polygons
Name: ____________________
Per: _____ Date: __________

Introduction: Today you will be using the formula you developed for the area of a triangle to find the formula for the area of a regular polygon. We first will start with a few discussion questions.

Part 1: Peer Mediated Discussion

Discuss the following questions with your group members and write down your answers.

1. What is a regular polygon? Please describe it in words and draw an example.

2. What is the formula for the area of a triangle? ________________________________

3. What do triangles and regular polygons have in common?

__________________________________________________________________________

Pictures:

__________________________________________________________________________

__________________________________________________________________________
Part 2: Key Vocabulary

You can divide a regular polygon into congruent isosceles triangles by drawing segments from the center of the polygon to each vertex.

Ex:

Apothem: an apothem of a regular polygon is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon.

Part 3: Using triangles to derive the formula for a regular polygon

1) Draw isosceles triangles from the center (apothem) to the vertices of the hexagon.

2) Label each triangle with base s and height a

3) Using the formula for the area of a triangle, find the area of one of the triangles to the right.

Formula (using s and a): \( A = \) __________

Area of one triangle: __________

4) How many triangles does it take to make up the polygon? __________

5) Using this information, find the area of the entire polygon.

Area of polygon: _______________
Part 4: Reflection and deriving the formula

Please describe what you did to find your solution in part 3:

________________________________________________________________________

________________________________________________________________________

1) Using s and a from part 3, what formula did you use to find the area of the triangle? A = _______

2) How many sides did the polygon have? _____

3) What did you do to find the area of the entire polygon?
   (hint: multiply by the number of _________)

4) What is the formula to find the area of any regular polygon? _______________________

Part 5: Seeing the pattern

Fill in the following chart:

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>...</th>
<th>12</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of regular polygon</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Regular pentagon: $a = 3\text{cm}$ and $s = 5\text{cm}$, $A =$ 

5. Regular nonagon: $a = 9.6\text{cm}$ and $A = 304.4\text{ cm}^2$, $s =$ and $P =$ 

HW: Area of regular polygons

1. Find the area of a regular pentagon to the nearest tenth of a square centimeter if the apothem measures $8.9\text{ cm}$ and each side measures $10\text{ cm}$. 
Section V

Project:
- Project Choices and Overview
- Project Rubric
- Pattern Design Graphic Organizer
  - Pattern Design Examples
- Interior Design Graphic Organizer
  - Interior Design Examples
- Luxury Box Graphic Organizer: Baseball
- Luxury Box Graphic organizer: Football
- Luxury Box Graphic Organizer: Soccer
  - Luxury Box Examples
**Project Choices and Overview**

**Project: Bringing Area to Life**

Students! You are going to have the opportunity to engage in a project that takes the formulas/concepts of Area and relates it to the real world. Your task is to **pick one** of the below projects and create a presentation of learning that shows your understanding of area and its relation to life around you. The best advice for you is to be creative and display your BEST work. Each project will be graded on a 100 point scale (see rubric).

<table>
<thead>
<tr>
<th>Project #1: Pattern Design: Design a Quilt</th>
<th>Project #2: Interior Design: Design a House</th>
<th>Project #3: Design a Sports Luxury Box</th>
</tr>
</thead>
</table>
| Overview: You are asked to **design a quilt or blanket** that represents who you are as a person using the shapes that we have discussed in class on area. Your quilt should be well thought out, and designed with a plan in mind. Your final product should be colored, neat, and detailed. **Directions:**

**Step 1:** Design your quilt on a piece of graph paper (each square is 1cm²) that incorporates at least 4 of the different shapes we have learned thus far. Your quilt must have at least 8 total shapes on it. Your quilt should be well thought out and colored.

**Step 2:** Fill out the graphic organizer in which you calculate the area for each of your shapes, the total area for the entire quilt, the total area for all of the shapes, and the total area that will be blank (no shapes)

**Step 3:** Write neatly or type a one page explanation about the reason you chose to design |

| Overview: You are an interior designer and are asked to **design the layout for a one story house.** The house is a square and measures 40 feet by 40 feet which means its area is 1,600ft². The house has 3 bedrooms, 2 bathrooms, a living room, a kitchen, and a dining area. The couple that is moving in wants a design that incorporates different shaped rooms. Using at least 4 of the shapes that we have covered, design an apartment for this couple with their ideas in mind. **Directions:**

**Step 1:** Using a piece of graph paper to represent the house, design a floor plan for the house in which each square represents 1 square foot.

**Step 2:** Calculate the area of each room using the formulas you have developed in class. Make a table (graphic organizer) that incorporates the name of each room and its area.

**Step 3:** Write neatly or type a one page explanation as to why you chose to design your |

| Overview: In the world of professional sports, there are many people who like to watch the games from luxury boxes. You task is to choose one of the below sports and **create the ultimate luxury box** for the stadium. The sports stadiums you can choose from are: Soccer, Football, or Baseball. **Directions:**

**Step 1:** Choose a stadium. On the graphic organizer given to you, fill out the necessary information for the stadium you chose.

**Step 2:** Your design must include certain characteristics of a sports box (see graphic org.):

- 2 big screen TV’s (2ft by 3ft)
- A rectangular table with 4 triangular bar stools
- A couch facing the stadium (any shape)
- A kite shaped coffee table
- A fridge
- A bar in the |
<table>
<thead>
<tr>
<th>your quilt the way you did.</th>
<th>house the way you did.</th>
<th>shape of a trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 4</strong>: Display your work in any way you choose (poster board, booklet, tri-fold, PowerPoint, etc.)</td>
<td><strong>Step 4</strong>: Display your work in any way you choose (poster board, booklet, tri-fold, PowerPoint, etc.)</td>
<td>Using graph paper, design your luxury box from a bird’s eye view. Make sure you label each piece of furniture and include its area.</td>
</tr>
<tr>
<td><strong>Step 3</strong>: Write or neatly type a one page explanation as to why you chose to design your sports box the way you did.</td>
<td></td>
<td><strong>Step 4</strong>: Display your work in any way you choose (poster board, booklet, tri-fold, etc.)</td>
</tr>
</tbody>
</table>
### Project Rubric

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>“A” 90 – 100%</th>
<th>“B” 80 – 89%</th>
<th>“C” 70 – 79%</th>
<th>“D” 60 – 69%</th>
<th>“F” 50% and below</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accuracy of mathematical calculations</strong></td>
<td>□ Makes accurate calculations in <strong>all</strong> of the work</td>
<td>□ Makes accurate calculations in <strong>most</strong> of the work</td>
<td>□ Makes accurate calculations in <strong>some</strong> of the work</td>
<td>□ Makes accurate calculations in <strong>little</strong> of the work</td>
<td>□ Makes accurate calculations in <strong>none</strong> of the work</td>
</tr>
<tr>
<td><strong>Correct and sufficient use of formulas</strong></td>
<td>□ Uses all correct formulas and includes more than the minimum number of shapes required</td>
<td>□ Uses all correct formulas and includes the minimum number of shapes required</td>
<td>□ Uses most formulas correctly and includes the minimum number of shapes required</td>
<td>□ Uses some of the formulas correctly and does not include the minimum number of shapes required</td>
<td>□ Uses none of the formulas correctly and does not include the minimum number of shapes required</td>
</tr>
<tr>
<td><strong>Project Components</strong></td>
<td>□ Project contains steps 1, 2, 3, and 4 and each step is complete and accurate</td>
<td>□ Project contains steps 1, 2, 3, and 4 and each step is complete and <strong>mostly</strong> accurate</td>
<td>□ Project contains steps 1, 2, 3, and 4 and each step is <strong>mostly</strong> complete and accurate</td>
<td>□ Project is missing one of the steps and the steps are <strong>somewhat</strong> complete and accurate</td>
<td>□ Project is missing more than one of the steps and is <strong>not complete or accurate</strong></td>
</tr>
<tr>
<td><strong>Communication of mathematical ideas (essay)</strong></td>
<td>□ Uses correct geometric vocabulary <strong>frequently</strong> in the essay and visuals match mathematical thinking</td>
<td>□ Uses correct geometric vocabulary <strong>consistently</strong> in the essay and visuals match mathematical thinking</td>
<td>□ Uses correct geometric vocabulary <strong>occasionally</strong> in the essay and visuals mostly match mathematical thinking</td>
<td>□ Uses geometric vocabulary <strong>some of the time</strong> and visuals mostly match mathematical thinking</td>
<td>□ Does not use geometric vocabulary and has either no visuals or visuals do not match mathematical thinking</td>
</tr>
<tr>
<td><strong>Neatness and Effort</strong></td>
<td>□ Project is <strong>very</strong> neat with straight lines, color, and labeled parts</td>
<td>□ Project is <strong>mostly</strong> neat with straight lines, color, and labeled parts</td>
<td>□ Project is somewhat neat with straight lines, color, and labeled parts</td>
<td>□ Project is lacking color, straight lines, and/or labeled parts</td>
<td>□ Project is sloppy, not colored, and has nothing labeled</td>
</tr>
<tr>
<td><strong>General</strong></td>
<td>□ Follows all</td>
<td>□ Follows</td>
<td>□ Follows</td>
<td>□ Follows few</td>
<td>□ Follows no</td>
</tr>
<tr>
<td>Directions</td>
<td>directions given for each step of the project</td>
<td>most of the directions given for each step of the project</td>
<td>some of the directions given for each step of the project</td>
<td>directions given for each step of the project</td>
<td>directions given for each step of the project</td>
</tr>
</tbody>
</table>
Pattern Design Graphic Organizer

Pattern Design: Design a Quilt

Please fill out this graphic organizer based on the shapes you chose to incorporate in your design.

<table>
<thead>
<tr>
<th>SHAPE</th>
<th>AREA FORMULA</th>
<th>DIMENSIONS</th>
<th>AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td>A = bh</td>
<td>b = 13 cm, h = 10 cm</td>
<td>(13)(10) = 130 cm²</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Total area of quilt: ________________
Total area of all the shapes combined: ____________________
Total area of open (white) area: ________________________
Pattern Design Examples

Rectangles, Parallelograms, Triangles
Regular Polygons

Other examples of quilts inspired by Geometric Shapes
Please fill out this graphic organizer based on the shapes you chose to incorporate in your design.

<table>
<thead>
<tr>
<th>ROOM</th>
<th>SHAPE</th>
<th>AREA FORMULA</th>
<th>DIMENSIONS</th>
<th>AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex: Bedroom 1</td>
<td>Rectangle</td>
<td>A = bh</td>
<td>b = 13 ft, h = 10 ft</td>
<td>(13)(10) =130 ft^2</td>
</tr>
<tr>
<td>Bedroom 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bedroom 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bedroom 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bathroom 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bathroom 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Living Room</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kitchen</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dining Room</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total area of house: _________________________
Interior Design Examples

Total surface area = 144 m² (122 m²)
Total surface area: 140 m² (117 m²)
Design a Sports Luxury Box: Baseball Field

Find the area of the infield:

Shape of white area (diamond):

Formula to find area:

Diagonal$_1$ = 

Diagonal$_2$ = 

Area of field: 

<table>
<thead>
<tr>
<th>FURNITURE</th>
<th>SHAPE</th>
<th>AREA FORMULA</th>
<th>DIMENSIONS</th>
<th>AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex: TV #1</td>
<td>Rectangle</td>
<td>A = bh</td>
<td>b = 2 ft, h = 3 ft</td>
<td>(2)(3) = 6ft$^2$</td>
</tr>
<tr>
<td>TV #1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TV #2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Furniture</td>
<td>Shape</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>-------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Table</td>
<td>Rectangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Stools</td>
<td>Triangles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(don’t forget to multiply by 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Couch</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee Table</td>
<td>Kite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fridge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bar</td>
<td>Trapezoid</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total area of all the furniture: _________________________
Luxury Box Graphic Organizer: Football

Design a Sports Luxury Box: Soccer Field

Find the area of the field

Shape of the field: ___________

Formula to find area: ___________

Width = ___________

Length = ___________

Area of field: ___________

<table>
<thead>
<tr>
<th>FURNITURE</th>
<th>SHAPE</th>
<th>AREA FORMULA</th>
<th>DIMENSIONS</th>
<th>AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TV #1</td>
<td>Rectangle</td>
<td>A = bh</td>
<td>b = 2 ft, h = 3 ft</td>
<td>(2)(3) = 6 ft²</td>
</tr>
<tr>
<td>TV #2</td>
<td>Rectangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Table</td>
<td>Rectangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Stools</td>
<td>Triangles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(don't forget to)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
multiply by 4

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Couch</td>
<td></td>
</tr>
<tr>
<td>Coffee Table</td>
<td>Kite</td>
</tr>
<tr>
<td>Fridge</td>
<td></td>
</tr>
<tr>
<td>Bar</td>
<td>Trapezoid</td>
</tr>
</tbody>
</table>

Total area of all the furniture: _______________________

**Luxury Box Graphic Organizer: Soccer**

**Design a Sports Luxury Box: Soccer Field**

Find the area of the field

<table>
<thead>
<tr>
<th>Shape of the field:</th>
<th>Formula to find area:</th>
<th>Width =</th>
<th>Length =</th>
<th>Area of field:</th>
</tr>
</thead>
</table>

**EX:**

<table>
<thead>
<tr>
<th>Furniture</th>
<th>Shape</th>
<th>Area Formula</th>
<th>Dimensions</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV #1</td>
<td>Rectangle</td>
<td>A = bh</td>
<td>b = 2 ft, h = 3 ft</td>
<td>(2)(3) = 6ft²</td>
</tr>
</tbody>
</table>

TV #1

TV #2

Table

4 Stools (don’t forget to multiply by 4)

**FURNITURE** | **SHAPE** | **AREA FORMULA** | **DIMENSIONS** | **AREA** |
<table>
<thead>
<tr>
<th></th>
<th></th>
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<td></td>
<td>TV #2</td>
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<tr>
<td></td>
<td>Table</td>
<td>Rectangle</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td>4 Stools</td>
<td>Triangles</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Bar</td>
<td>Trapezoid</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Total area of all the furniture: _________________________
Luxury Box Examples:
Section VI

Post Assessments:

Reflection #3
Academic Language Quiz
Post Test
**Reflection #3**

**Reflection #3: Discovering Area**

Name: ________________________  
Date: _______________ Per: ______

Please respond to the following questions in complete sentences.

1. Now that you have discovered the formula for area for six different figures, how do you feel about yourself as a learner of geometry when it comes to area? (confident, scared, etc.)

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

2. Do you feel as though you really know the formula’s you are using? Why or why not?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

3. If you were to retake the pretest, how do you think you will do now that you have mastered the formulas for area? Explain.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

4. Do you think you would be able to teach another student how to find and use the formulas you have discovered? Explain.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
5. We are starting the project on area next week. You will be able to choose which project you would like to complete based on your interest. Does that make you feel more excited than if you were given a project and told to complete it? Why or why not?

6. Even though we have not yet discussed it in detail yet, do you think area is important to know in real life? Why or why not? Give examples of where you could use it.

7. Compare this unit (area) to another unit we have done in math. Do you feel more prepared and more confident with area than other units we have done? Why or why not?

8. Do you feel confident in your ability to solve for ANY missing variable in the area formulas? For example, if you are asked to find the height given the area, do you feel confident in your algebra skills to do that? Why or why not?

Please write out, from memory, the formulas for the area for each of the following shapes and what each of the parts stands for. Rectangle has been done for you:

Rectangle: $A = bh$ (Area equals base time height)  
Parallelogram:  
Triangle:  
Kite:  
Trapezoid:  
Regular Polygon:
Directions: Please match the following word with its definition.

1. Perimeter ______
2. Base ______
3. Altitude ______
4. Parallelogram ______
5. Formula ______
6. Trapezoid ______
7. Area ______
8. Height ______
9. Apothem ______
10. Base\(_1\) & Base\(_2\) ______
11. Kite ______

A. The two parallel sides of a trapezoid
B. A group of symbols that make a mathematical statement
C. A quadrilateral whose opposite sides are parallel and equal in length
D. A quadrilateral with two parallel sides that we call bases
E. The distance around a closed figure
F. The length of the altitude of a figure
G. The bottom side of a geometric figure from which the altitude can be constructed
H. Any segment from one side of a shape perpendicular to a line through the opposite side
I. The measure of the region enclosed by a plane figure
J. A perpendicular segment from the center of a regular polygon’s circumscribed circle to a side of the polygon.
K. A quadrilateral with two pairs of consecutive congruent sides and one pair of congruent angles

Directions: Please write the formula’s for the area of the following shapes:

Rectangle, Trapezoid, Kite, Parallelogram, Regular Polygon, Triangle
Please find the Area of the regular polygon below. Make sure you state your objective and explain how you found your answer. Also, use academic language when explaining your work.

Area = _____________________

Show your math work here:

Objective and Written Explanation:
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Section VII

Interviews:


**Interview**

**Mathematical Identity Survey: gauging the current identities of my students**
(some questions on this survey were taken from *Assessing Student Motivation in High School Mathematics*, Peter Kloosterman)

Focus students were interviewed through a face-to-face conversation on the following questions surrounding their math identity, autonomy as a math learner, motivation levels, and self-efficacy

1. Do you enjoy practicing mathematics? Why or why not? What types of math do you like best? What do you like the least?

2. What words would you use to describe school?

3. What words best describe mathematics in your opinion?

4. Over the years, how have you performed in mathematics academically? High/low grades?

5. How important is it for you to get a good grade in math?

6. Do you feel comfortable contributing to a classroom discussion about mathematics? Why or why not?

7. How has mathematics helped you in your life? Do you use it often?

8. Do you think that math is useful in your life? Why or why not?

9. Do you think math is useful to advance as a society? Why or why not?

10. When you learn a new topic in math, do you try to see how it relates to other math topics or topics you have learned before?

11. Are you confident in your skills as a math learner? What has helped or hindered this confidence? How good are you at math?

12. Are you better at some kinds of math than others?
13. Do you think that it takes a special talent to do math? Do you have this special talent?

14. Are you motivated to want to solve puzzles in mathematics?

15. Are there times when you struggle in math? How do you feel when you reach a point where you struggle?

16. In a given period, how often to you ask your teacher for help?

17. Do you think other students view you as a leader in the math classroom or someone they turn to if they have questions?

18. Is it important to you to look like a good math student or a bad math student in front of your friends and/or teacher?

19. How often do you do the least amount of work to get by in math?

20. Who do you turn to if you have questions?

21. On a scale of 1 to 10 (10 being the most), how much effort would you say you put into math during class? Do you always do everything your teacher assigns for you?

22. On a scale of 1 to 10 (10 being the most), how much effort would you say you put into practicing mathematics (geometry) outside of the math classroom?

23. In class, how does the math we engage in make you feel? Excited? Bored? Confident? Self-conscious?

24. How often in a given week do you feel frustrated because you don’t understand something in mathematics?

25. If you were not required to take math in high school, would you still take it as an elective? Why or why not?

26. What other math classes do you think you will take in High School?
27. What do you plan on doing after you graduate from High School (4-year college, 2-year college, army, job, etc.)?

28. How important is having a good teacher in your success as a math student?

29. What is your favorite subject?

30. How do you like math in comparison to other subjects?

31. Do you think that your parents use math a lot in their everyday lives? How?

32. Do your parents want you to do well in school? How much support do they give you? Do they help you with math homework? Do they understand your math homework?

33. What do your friends think about math? Do they like it? Are you influenced by what your friends think about math?
References


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talk in elementary classrooms. *Math Solutions*. Retrieved from


comparative study of Asian-American, Caucasian-American, and East Asian high

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and
258–277.


Penguins Books.

Basic Books.


backgrounds using project-based mathematics* (Masters thesis) Retrieved from
Education Resources Information Center. (No. ED501239).

Psychology: Theory and Practice of Doing and Knowing* (pp. 89–124).
Cambridge: Cambridge University Press.

Hoang, T. (2007). Creativity: A motivational tool for interest and conceptual
understanding in science education. *International Journal of Human and Social
Sciences, 1*(4), 215–221

collaborative approach to interdisciplinary learning.* Washington D.C: American
Psychological Association

Washington, DC: National Academy Press


