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ON THE THEORY OF NON-LEPTONIC HYPERON DECAYS

A. Pais

October 27, 1960
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ON THE THEORY OF NON-LEPTONIC HYPERON DECAYS

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ABSTRACT

Recent experimental results on non-leptonic hyperon decays are taken to suggest that there exists a doublet approximation for strong and weak interactions and that this higher symmetry is useful at least for some reactions in which hyperons do, K-particles do not occur explicitly. The doublet approximation is characterized by a doublet spin $I$ which $= 1/2, 1, 0$ for baryons, $\pi, K$ and by a K-spin. It is not necessary to assume that the strong K-interactions are weak compared to the strong $\pi$ interactions. For the mentioned reactions it is necessary to assume that the strong interactions which violate $I$ play a minor role compared to those which conserve $I$.

The following refinement of the non-leptonic $\Delta T = 1/2$ rule is proposed. ($T =$ isotopic spin.) The weak non-leptonic interactions consist of two parts $H^{(0)}, H^{(1)}$ with $\Delta I = 0, 1$ respectively. In the doublet approximation $H^{(0)}$ and $H^{(1)}$ separately conserve parity in the presence of all strong $\pi$- and K-interactions. $H^{(0)}$ and $H^{(1)}$ together violate parity however. In addition to $\Delta I = 1$, $H^{(1)}$ should in general satisfy a further constraint, but there are classes of graphs for which $\Delta I = 1$ is sufficient.

Current x current structures for $H^{(0)}$ and $H^{(1)}$ are examined. Results of a foregoing paper can be viewed as a special case of the $\Delta I = 0, 1$ rule. The same is true for results obtained by Feldman, Matthews and Salam and by Wolfenstein. The considerations of these authors can be extended to wider classes of graphs.
Odd relative helicity for $\Lambda \rightarrow p + \pi^-$, $\Sigma^+ \rightarrow p + \pi^0$ is a consequence of the $\Delta I = 0, 1$ rule only. So is the prediction that $\Xi$-decay is strongly $P$-violating.

The parity properties of $H^0, H^1$ are sufficient conditions. It is a delicate question whether they are necessary. For a subset of graphs they are not necessary, but this set seems arbitrary. Assuming the parity conditions to be necessary, the schizon scheme is ruled out.

It is suggested that the non-leptonic weak interactions are generated by the strong interactions. It is observed that an $H^{(1)}$ is generated by assuming that the $\pi(K)$ fields have small $K(\pi)$ components. An $H^{(0)}$ is generated by assuming that the doublets $N_1(N_2)$ have small $N_2(N_1)$ components; likewise for $N_2$ and $N_1$. This procedure also generates a non-electromagnetic $\Delta T = 3/2$ interaction. This last coupling is small in the sense that it only contributes to $K^{+}_{\pi2}$ to the extent that the doublet approximation is not valid.
ON THE THEORY OF NON-LEPTONIC HYPERON DECAYS*

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I. INTRODUCTION

Beyond the demonstration of the existence of isotopic spin (T) and strangeness (S) rules, the study of strong reactions have so far taught us little about more intimate connections between the varieties of strongly interacting particles. Attempts to consider some of the new particles as composites in terms of others have till now not produced any insight which cannot as well be reached by assuming that any one baryon, say, is neither less nor more elementary than any other. In this paper we continue to adopt this last view. From this standpoint one may try to further interconnect particles and interactions by asking for stronger symmetries than those which yield T and S conservation. It is known that such symmetries cannot exist rigorously. Nor is there thus far any indication that some of the strong interactions are relatively weak compared to others so that expansions in the former might be a useful procedure. Conjectures that one part of the strong interactions possesses symmetries stronger than another therefore have had as yet to remain in a speculative stage. Unless we find some qualitative clues, the strong interaction problems appear to be in somewhat of a deadlock. It is the purpose of this paper to discuss certain weak decay

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reactions which, in this author's view, provide us with such a clue about the strong interactions.

The reactions we have in mind are

\[ \Sigma^+ \rightarrow n + \pi^+ , \quad (A^+, \alpha^+), \]  
\[ \rightarrow p + \pi^0 , \quad (A^0, \alpha^0), \]  
\[ \Sigma^- \rightarrow n + \pi^- , \quad (A^-, \alpha^-). \]  

We shall often refer to these reactions as \( \Sigma_+^+ \), \( \Sigma_0^+ \), \( \Sigma^- \), respectively. Their amplitudes and asymmetry parameters will be denoted by \( A, \alpha \), labeled as indicated. Experimental results \( ^3 \) are compatible with the requirement of the \( \Delta T = 1/2 \) rule that \( A^+, A^-, A^0 \sqrt{2} \) shall form a triangle. If we neglect final state interactions this triangle can conveniently be drawn in the so-called \( (s, p) \) plane. \( ^4 \)

Concerning the \( \Delta T = 1/2 \) rule (which to this author seems neither less nor more mysterious than the \( \Delta T = 0 \) rule of the strong interactions) we shall adopt the same view as in a previous paper. \( ^5 \) For the purpose of this study, the rule will be supposed to be rigorous for non-leptonic decays. At the same time we do not wish to prejudge the question whether deviations from \( \Delta T = 1/2 \) are electromagnetic only.

Experiment further indicates \( ^4 \) that the \( \Sigma \)-triangle is oriented in a rather special way which can be expressed by

\[ \alpha^+ \approx 0 , \quad \alpha^- \approx 0. \]  

\( \Delta T = 1/2 \) implies that if \( \Sigma_+^+ \) is nearly pure \( s(p) \) wave, then \( \Sigma^- \) is nearly pure \( p(s) \) wave. It is presently not known which is which. This can be decided experimentally. \( ^6 \)
Equation (1.4) is rather remarkable. It shows that insofar as $\Sigma^+$ and $\Sigma^-$ are concerned it may be a good approximation to say that $\Sigma^+$ and $\Sigma^-$ each do have a well-defined parity relative to the $\pi$-nucleon system; and that the parity of $\Sigma^+$, whatever it is, is opposite to that of $\Sigma^-$. Now either a system has a well defined parity relative to another or it hasn't, so what does "a good approximation" mean? To see this, note the following. It is easy to give examples of a weak interaction $H_1$ which leads to a $\Sigma$-triangle so oriented that $\alpha^+ = \alpha^- = 0$. But this by no means solves the problem.

Consider $\Sigma^-$-decay as a first example. Let $H_1$ be such that $\Sigma^-$ is pure $p$ wave, say. However the strong interactions generally allow $\Sigma^-$ to be part of the time a $\Sigma^+$, for example $\Sigma^+ + 2\pi^-$. During this time $H_1$ can induce s-wave decay $\Sigma_+^+$. The final $n\pi^-$ state can then be reached by strong reabsorption of a $\pi^+\pi^-$ pair. The net result is an s-wave contribution to $\Sigma^-$. Similarly, virtual $\Sigma_0^+$ decay would give a mixed $(s, p)$ contribution to $\Sigma^-$. In other words even if the weak interactions properly orient the triangle, the strong interactions in general do not respect this orientation. That is, to stick with the example, unless we could provide a reason which would inhibit virtual $\Sigma^- \rightarrow \Sigma^+$ transitions. This is possible, though not rigorously.

As a next example consider the sequence

$$\Sigma^- \rightarrow \Lambda + \pi^- \rightarrow (p + \pi^-) + \pi^- \rightarrow n + \pi^-$$

where the strongly P-violating $\Lambda$-decay is involved and for the rest only P-conserving strong interactions. Why is the $\Sigma^-$ nearly impervious to this violation?

We would like to point out that the so-called doublet approximation (DA; also known as restricted symmetry) provides a natural though not rigorous answer to these questions. Here one assumes even $\Sigma\Lambda$-parity, neglects the
\[ \Lambda = \frac{\Lambda^0 + Z^0}{\sqrt{2}} , \quad \Sigma^0 = \frac{-\Lambda^0 + Z^0}{\sqrt{2}} . \]  

The baryons then regroup in terms of four doublets, see Eq. (2.2) below. The reason that \( \Sigma^- \rightarrow \Sigma^+ \) is inhibited is (Section 2(a)) that they belong to different doublets which in the DA do not intercombine. The reason that \( \Sigma^- \) is not affected by P-violation in A-decay is that \( \Sigma^- \) cannot combine with \( Y^0 \) for much the same reason. Thus if the \( Y^0, Z^0 \)"parts" of \( \Lambda \) separately are P-conserving in decay, then \( \Sigma^- \) will stay P-conserving. While if \( Y^0, Z^0 \) have opposite parity in decay, the physical particle \( \Lambda \) will be strongly parity violating.

These remarks may serve to indicate the general approach planned in this paper. (On purpose we have not included \( \Sigma^+ \)-decay in these few examples, as there a more delicate problem arises, see Section 2(b).) We shall endeavor to arrange things so that we get exact P-conservation for \( \Sigma_+^+ \) and \( \Sigma_-^- \) in the approximation where a stronger symmetry than charge independence is supposed to hold. As has been shown, \(^2\) the DA is the weakest symmetry stronger than charge independence. Thus the DA is the natural starting point. If we succeed we shall be able to assign to \( \Sigma^- \) for example a parity relative to the \( \pi \)-nucleon system, but only to the extent that the DA holds. When we talk of P-conservation in certain decays we shall always refer to a situation where a symmetry higher than charge independence is assumed. Thus we do not at all anticipate that P-conservation in \( \Sigma_+^+ \) and \( \Sigma_-^- \) would be as good as it is in atomic physics for example.

The following must be strongly emphasized. The quest for stronger symmetries has so far most often seemed a self-inflicted agony. A symmetry
first set up has subsequently to be broken. The difference in the present case is that the assumed symmetry leads to interesting physical conclusions which at least so far are not in qualitative disagreement with experiment. Even so, the question always remains, what is the influence of those strong interactions which do not respect the DA? We shall not face this question in this paper. As long as it is not answered the work does not represent a theory but a program. However, this time it would appear to be perhaps a quite promising program.

Remark. For reasons given in I we lay the emphasis on the near parity conservation of $\Sigma^+$ and $\Sigma^-$ and consider the near equality of their rates as more of an accident. No doubt this equality will eventually be a vital clue as well. However it does not seem to raise such a qualitative puzzle as the parity aspect does.

In this paper we shall apply the DA both to $\pi$ and K-couplings. We do this mainly to emphasize that on the whole it is not the essential point which kinds of fields and interactions follow the DA. It is not relevant therefore whether K- and $\pi$-couplings are of the same order of strength or not. What is relevant on the other hand is of course the role of the interaction which breaks down the doublet symmetry. In Section 2(a) we treat the $\pi$-couplings in the DA and define the relevant quantum numbers.

In Section 2(b) a proposed refinement of the $\Delta \pi = 1/2$ rule is stated. It is suggested that the non-leptonic interactions consist of two parts $H^{(0)}$ and $H^{(1)}$ which separately conserve parity but which clash when taken together. To separate any parity violating interaction in two separately P-conserving parts is of course totally trivial. One could do the same for $\beta$-decay. What is not trivial in the present case is that these separate
parts are simultaneously subject to a condition in terms of a quantum number other than parity, namely the doublet spin. We therefore suggest that doublet spin and parity properties are correlated in a definite way.

The main theorems on $\Lambda$ decay are given in Section 2(d), on $\Xi$ decay in Section 4.

In certain instances the DA will be insufficient for the purpose of obtaining the desired parity properties. We shall then consider two (not mutually exclusive) approaches a) the use of further invariance arguments, see Section 2(c), b) the investigation of special types of virtual transitions, see Section 6. In this second approach we follow ideas due to Feldman, Matthews and Salam$^9$ and to Wolfenstein$^{10}$ and try to generalize their results.

In the work of FMS$^9$ some emphasis is laid on the differences of the dispersion approach as compared to the Lagrangian methods used by others. Rather than to underline such differences, the present work aims to emphasize above all what such varied techniques actually have in common. As has been stated in I, the essence of the problem seems to be the establishment of shared symmetries of weak and strong interactions. In Section 6(c) the connection between the FMS results and the present argument will in fact be established through the analysis in terms of symmetry arguments of the weak vertices used by these authors.

In Section 5 we discuss $K$-particle effects in the DA. In particular we show in Section 5(d) that such reactions as $K \rightarrow \pi$, $K^0 \rightarrow 2\pi$, $K^+ \rightarrow 3\pi$ can be described in the DA in terms of the weak interactions $H^{(0)}$, $H^{(1)}$ introduced in Section 2(b) even though these interactions separately conserve parity. Here we meet with a very essential point that has been brought out by the work of Wolfenstein.$^{10}$ There are in fact specific graphs, see
Section 6(a), which give pure opposite parity contributions to $\Sigma^+$ and $\Sigma^-$. These graphs follow the doublet rule $\Delta I = 0$ or 1. But for these Wolfenstein graphs the condition that $H^{(0)}$, $H^{(1)}$ are $P$-conserving (see Section 2(b)), though sufficient, is not necessary. On the other hand we shall also see in Section 6(a) that we can retain Wolfenstein's results but extend them to a larger class of graphs if indeed, as proposed in Section 2(b), $H^{(0)}$ and $H^{(1)}$ separately conserve parity.

The great importance of this question lies in the following. By an argument given in I, if $H^{(0)}$ and $H^{(1)}$ separately do conserve parity then an incompatibility exists between the rule proposed in Section 2(b) and the schizon scheme; see also Section 3(f).

While most of the arguments summarized before do not have reference to specific structures of the weak interactions beyond their doublet spin properties, a certain interest attaches to the question, how can these interactions be brought in current $\times$ current form. This problem is dealt with in some detail in Section 3 where it is shown that the present work goes beyond I in two respects a) in I we used global symmetry from the start, in this paper the weaker DA is sufficient in many instances, b) the $(js, jt)$ coupling used in I represents a special choice for $H^{(0)}$, $H^{(1)}$. Other possibilities are noted.

The concluding Section 7 is mainly devoted to a few general remarks on a) the possible generic connection between weak and strong interactions, b) possible non-electromagnetic deviations from $\Delta T = 1/2$, c) the question of the leptonic decays. We shall state which remarks made in I apply to the particular $(js, jt)$ coupling scheme used there; and which remarks have a wider validity.
If the present approach is correct, a new question arises. Why should the DA manifest itself as a useful symmetry in non-leptonic hyperon decay but not in the reactions studied previously?\textsuperscript{2} Those reactions all involve real K's, the decays virtual K's only. We are therefore led to surmise that where K-particles appear only as a virtual cloud, the DA is more easily discernable. Results on \(\pi\)-hyperon scattering\textsuperscript{12} may perhaps shed light on this point.
2. THE DOUBLET APPROXIMATION

(a) Strong \( \rho \)-Interactions

The DA for this coupling has been discussed elsewhere. We briefly state the main points. It is necessary for the existence of this approximation that the \((\Sigma, \Lambda)\)-parity be even. The \( \rho \)-couplings are considered under the neglect of the \((\Sigma, \Lambda)\)-mass difference and are of the typical form

\[
H_\rho = \left[ G_1 \bar{N}_1 \gamma_5 N_1 + G(\bar{N}_2 \gamma_5 N_2 + \bar{N}_3 \gamma_5 N_3) + G_4 \bar{N}_4 \gamma_5 N_4 \right] \rho
\]

(2.1)

where

\[
N_1 = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_2 = \begin{pmatrix} \Sigma^+ \\ \rho^0 \end{pmatrix}, \quad N_3 = \begin{pmatrix} \Sigma^- \\ \rho^- \end{pmatrix}, \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}
\]

(2.2)

and where \( \rho^0, \Xi^0 \) are given in Eq. (1.5). The space time structure of the couplings is immaterial for the argument, as long as the pseudoscalar nature of \( \rho \) is guaranteed. In fact we are not even committed to the form \((2.1)\). What then is the essence of the DA?

First of all Eq. \((2.1)\) implies that we are free to rotate the \( \gamma \)-spin together with \( \rho \). \( \gamma \) is the isotopic spin for \( N_1, N_4 \) but not for \( \Sigma, \Lambda \). We call it the doublet spin, to which we refer in general as \( I \). For \( \rho \) we have \( T = I = 1 \). Secondly we are free to rotate in the \( (N_2, N_3) \)-plane'. We can unite \( N_2 \) and \( N_3 \) to

\[
N = \begin{pmatrix} N_2 \\ N_3 \end{pmatrix}
\]

(2.3)

and assign to this "doublet" (each component of which is multi component
itself) a spin \( K = 1/2 \), with \( K_3 = +1/2 \) \((-1/2)\) for \( N_2(N_3) \). The relation between \( T \), \( I \) and \( K \) is

\[
T = I + K. \tag{2.4}
\]

We shall see later (Section 5) that if \( K \)-particles participate in the DA we have for them \( I = 0, \ T = K = 1/2 \). We call \( K \) the \( K \)-spin. The DA for strong \( \pi \)-coupling is now defined generally by the statement that \((I, K, I_3, K_3)\) are good quantum numbers. One may simultaneously and independently apply to \( I \) and \( K \) the usual rules of the vector addition model.

Observe that \( H_\pi \) not only conserves baryons, but also conserves individual doublets. Thus the DA guarantees that virtual transitions \( \Sigma^- \rightarrow \Sigma^+ + \pi \)-mesons are forbidden. This is just the inhibition we are after.

(b) \( \Delta T = 1/2 \) and \( \Delta I = 0, 1 \)

We shall now suppose that the non-leptonic decay interactions do not only satisfy \( \Delta T = 1/2 \) but more specifically that they also have \( \Delta I \) properties. To see what such a statement means it is instructive to reason by analogy with the \( \Delta T = 1/2 \) rule itself.

If we say that non-leptonic decay reactions satisfy \( \Delta T = 1/2 \), we may also say that the decay interaction satisfies \( \Delta T = 1/2 \). This trivial statement is of course due to the fact that the strong interactions satisfy \( \Delta T = 0 \), so that they cannot modify the \( \Delta T \) properties of the weak interaction. Electromagnetic corrections are of the \( \Delta T = 1 \) type and make the \( \Delta T = 1/2 \) rule impure. Likewise, if we say that a weak interaction has a certain \( \Delta I \) this is only meaningful to the extent that the strong interactions have \( \Delta I = 0 \)--that is the DA. "Corrections" to the DA will make the \( \Delta I \) rules for weak interactions impure. (As we have stated in the Introduction, we
shall not discuss here the influence of such distortions on the decay processes.

If we assign a $\Delta I$ to a weak interaction, the latter should be expressable in terms of doublets, insofar as baryons are concerned. We are therefore working in an approximation in which the weak and strong interactions share the doublet symmetry.

All nonleptonic, $\Lambda$, $\Sigma$, $\Xi$-decays have $|\Delta K_3| = 1/2$. It is easily seen that actually $\Delta K = 1/2$. As $\Delta T = 1/2$, it follows that $\Delta I = 0$ or 1, according to the vector relation

$$\Delta T = \Delta I + \Delta K.$$ (2.5)

The most general decay interaction $H$ therefore is of the form

$$H = H^{(0)} + H^{(1)},$$ (2.6)

where $H^{(0)}$, $H^{(1)}$ are characterized by $\Delta I = 0$, 1 respectively.

For $\Sigma$-decays the $\Delta I_3$-assignments are as follows:

$$\Sigma^- \rightarrow n + \pi^- : |\Delta I_3| = 1.$$ (2.7)

$$\Sigma^+ \rightarrow n + \pi^+$$

$$\rightarrow p + \pi^0 : \Delta I_3 = 0.$$ (2.7)

We note that according to Eq. (2.7) $H^{(0)}$ allows $\Sigma^+$-decays but forbids $\Sigma^-$. $H^{(1)}$ allows in general all three decays, because $\Delta I = 1$ implies $\Delta I_3 = \pm 1, 0$. Thus we can generally write $H^{(1)}$ as

$$H^{(1)} = H^{(1)}_1 + H^{(1)}_{-1} + H^{(1)}_0$$ (2.8)

where the subscript refers to the $\Delta I_3$ value.
As \( H^{(1)} \) allows all \( \Sigma \)-modes the first question is whether we could a) restrict ourselves to the \( H^{(1)} \) term in Eq. (2.6), b) choose the parity structure of \( H^0_0^{(1)} \) to differ from \( H^{\pm 1}_1 \) in such a way that \( \Sigma_+^+ \) (which proceeds via \( H^0_0^{(1)} \)) has opposite parity compared to \( \Sigma_-^- \) (which proceeds via \( H^{1-1}_1 \)). This in itself is indeed feasible but it would be in violent contradiction with \( \Delta T = 1/2 \). For clearly \( \Sigma_0^+ \) would now also be parity conserving. Instead of a \( \Sigma \)-triangle we would therefore have two amplitudes aligned along the \( s \) (or \( p \)) axis, the third aligned along the \( p \) (or \( s \)) axis in the \( (s,p) \) plane.

Let us digress for a moment from the main program which is to understand the parity properties of \( \Sigma \)-decays if \( \Delta T = 1/2 \) is assumed to be strictly valid. It may be worth while to note that the discussion of Eq. (2.8) shows that one can conceive of (non-electromagnetic) violations of the \( \Delta T = 1/2 \) rule of such a nature that the \( \Sigma \)-triangle does not longer exist rigorously, while yet the parity-conservation in the \( n \) \( \mp \) channels remains intact. Such violations should be relatively small however, as deviations from \( \Delta T = 1/2 \) do not seem to be large.

We now return to the main program and try a different tack. Suppose that we could find an argument additional to the \( \Delta I = 1 \) specification of \( H^{(1)} \), in such a way that \( H^{(1)} \) would contribute to \( \Sigma_0^+ \) but not to \( \Sigma_+^+ \). Then \( \Sigma_+^+ \) would go via \( H^{(0)} \) only; \( \Sigma_-^- \) goes anyway via \( H^{(1)} \) only; while \( \Sigma_0^+ \) would go via both \( H^{(0)} \) and \( H^{(1)} \). We shall come back at length to the construction of this additional argument. Accepting for the moment that this can be found, we can clearly achieve the parity properties of \( \Sigma \)-decays by the following hypothesis.

\[ \Delta I = 0, 1 \] rule. The weak non-leptonic interactions consist of two parts \( H^{(0)} \), \( H^{(1)} \) with \( \Delta I = 0, 1 \), respectively. \( H^{(0)} \) and
$H(1)$ separately conserve parity, but clash when taken together. That is to say, $H(0) + H(1)$ violates parity. For $H(1)$, $\Sigma^+$ is to be inhibited by an argument additional to $\Delta I = 1$.

This rule interlocks doublet spin and parity. As we shall see in Section 3, the $\eta_s, \eta_t$ couplings of I are special examples of $H(0), H(1)$. The parity condition on $H(0), H(1)$ is a sufficient condition. There may perhaps be accidents where this condition would not be necessary, for examples see Section 6.

The next task is to find the additional argument concerning $H(1)$. This is a more subtle problem and there are at least two avenues of approach which are by no means mutually exclusive. 1) While $\Delta I = 1$ specifies $H(1)$ insufficiently, nevertheless $\Delta I = 1$ is adequate by itself if in the calculation of $\Sigma^+$-decay probabilities there are specific virtual transitions which are strongly predominant. This is conceivable, see Section 6. 2) For $H(1)$ we need a stronger symmetry than the DA to reach our goal. On the one hand it is distinctly unsatisfactory to employ strong symmetries. On the other hand the particular symmetry we shall invoke in the next subsection will allow us at once to tie parity conservation in $\Sigma^+$ and $\Sigma^-$ to parity violation in $\Lambda$-decay.

(c) Further discussion of $H(1)$

It does not affect $H(0)$ if we subject $H(1)$ to an additional symmetry argument. Indeed it is typical for the weak processes which concern us here that without loss of rigor we may subject $H(1)$ and $H(0)$ to different invariance requirements as long as $H(1)$ (or $H(0)$) shares that invariance with the strong interactions. This is true as $H(1)$ and $H(0)$ can never interfere because weak interactions are considered to first order only.
We can even go further. By the same token $H_{\pm l}^{(1)}$ and $H_0^{(1)}$ do not interfere. We seek for an argument which inhibits $\Sigma_+^+$, a reaction which can proceed via $H_0^{(1)}$. We shall state the argument in terms of a symmetry shared by $H_0^{(1)}$ and the strong interactions. By our reasoning it is entirely immaterial whether $H_{\pm l}^{(1)}$ shares this additional symmetry or not. (As it happens, it does not.) Now $\Sigma_-$ proceeds via $H_{-l}^{(1)}$ only. Therefore it remains true that this reaction is P-conserving in the DA without any additional argument.

In general we may say that the shared invariance of strong interactions and a partial weak interaction is a legitimate tool because we deal with problems linear only in the weak interactions.

The additional argument on $H_0^{(1)}$ is now that it shares with the strong interactions invariance for

$$N_1 \rightarrow i \epsilon \tau_2 N_2, \quad \pi^\pm \rightarrow -\epsilon \pi^\mp,$$

$$N_2 \rightarrow -i \epsilon \tau_2 N_1, \quad \pi^0 \rightarrow -\epsilon \pi^0,$$

where the $\epsilon$'s are phase factors equal to $\pm 1$. Equation (2.9) leaves Eq. (2.1) invariant provided we neglect the $N_1 - N_2$ mass difference while

$$\epsilon = \pm 1 \quad \text{for} \quad G_{\pm l} = \pm G. \quad (2.10)$$

This alternative for $\epsilon$ corresponds in essence to $G^\pm$-symmetry defined and discussed in I. In addition we must rotate $N_3$ and $N_4$ appropriately and, for $\epsilon = -1$ but not for $\epsilon = +1$ we must apply $N_3 \leftrightarrow N_4$. For what follows in this section we need not say more about the doublets 3 and 4.
If \( H_0^{(1)} \) shares the invariance under Eq. (2.9) then

\[
(\Sigma^+ | n \pi^+)^{(1)} = \epsilon_1 \epsilon_2 \epsilon (n | \Sigma^+ \pi^-)^{(1)}
\]  (2.11)

where the superscript \((1)\) indicates that we refer to the transition brought about by \( H_0^{(1)} \) (in the presence of \( H_\pi \)). We require

\[
\epsilon_1 \epsilon_2 \epsilon = -1
\]  (2.12)

and now apply an argument (already used in I) which was first employed by Treiman\(^{14}\) in a similar context. Namely, under the neglect of final state interactions

\[
(n | \Sigma^+ \pi^-)^{(1)} = (\Sigma^+ | n \pi^+)^{(1)}
\]  (2.13)

Hence it follows from Eqs. (2.11-13) that \((\Sigma^+ | n \pi^+)^{(1)}\) vanishes under the stated conditions. The argument thus amounts to the following. The amplitude in question is a function of \( m_p \), \( m_\Sigma \) and the momentum transfer \( \Delta = (q_\Sigma - q_p)^2 \). Under the conditions stated \((m_p = m_\Sigma)\) this function is equal to minus itself for all values of \( \Delta \).

It is by no means obvious that \( H^{(1)} \) can be constructed so as to satisfy Eqs. (2.9) and (2.12). In fact we shall see in Section 3 that several expressions for \( H^{(1)} \), specified by \( \Delta \Gamma = 1 \) only, will have to be discarded if Eqs. (2.9) and (2.12) hold true. Thus the argument restricts the dynamical form of the weak interactions. There are however several possibilities for \( H^{(1)} \) which do satisfy the requirements. From now on I call these the allowed forms of \( H^{(1)} \).
(d) Two theorems on P-violation in Λ-decay

(A) It follows from the $\Delta I = 0, 1$ rule that parity is violated in Λ-decay in the same approximation that parity conserved in $\Sigma^+$ and $\Sigma^-$, provided $H^{(1)}$ is of the allowed form. Proof. Equation (2.9) also implies

$$ (Y^0 \mid p\pi^-)^{(1)} = \varepsilon_1 \varepsilon_2 \varepsilon(p \mid Y^0 \pi^+)^{(1)} \ . \quad (2.14) $$

Hence the argument which led to $(\Sigma^+ \mid n\pi^+)^{(1)} = 0$ also gives $(Y^0 \mid p\pi^-)^{(1)} = 0$. Thus $Y^0 \to p + \pi^-$ proceeds only via $H^{(0)}$. On the other hand $Z^0 \to p + \pi^-$ is a $\Delta I_3 = -1$ transition and can therefore proceed only via $H^{(1)}$. But $H^{(0)}$ and $H^{(1)}$ clash in parity. Therefore from Eq. (1.5) it follows that $P$ is violated in Λ-decay.

Actually this result can be sharpened considerably.

(B) It follows from the $\Delta I = 0, 1$ rule and for allowed $H^{(1)}$ that

$$ \alpha^0 = -\alpha^\Lambda \quad \quad \quad (2.15) $$

where the left (right) side of this equation refers to the helicity of the proton in $\Sigma^+ \to p\pi^0$, ($\Lambda \to p\pi^-$). Here the experimental information $R_+^+ \approx R_-^-$ has been used.\(^3\)

To prove this statement we note that by (A) the transition $Y^0 \to p\pi^-$ proceeds via $H^{(0)}$ only. But $H^{(0)}$ has $\Delta I = 0$ and therefore satisfies doublet charge symmetry, as a result of which

$$ (Y^0 \mid p\pi^-) = (\Sigma^+ \mid n\pi^+) \ . \quad (2.16) $$

This result is a consequence of the DA only. Equation (2.16) was also obtained in I but under much more restrictive conditions.\(^{15}\)
The second part of the proof consists in showing that

\[(\Sigma_0^0 \mid p \pi^-) = (\Sigma^- \mid n \pi^-)\]  

(2.17)

provided \(H^{(1)}\) is of the allowed form. Equation (2.15) follows from Eqs. (2.16) and (2.17) by an argument given in I.

The verification of Eq. (2.17) has to wait till Section 3(d). We have now in fact pushed the argument as far as is feasible independently of the structure of \(H^{(0)}\) and \(H^{(1)}\). The next task is to consider these dynamical structures more closely.\(^{16}\) Concerning the symmetries used we shall arrive at the following conclusions. (a) For \(\Sigma^-\) to be \(P\)-conserving the DA is sufficient. (b) The same is true for Eq. (2.16) which makes up half of the relation (2.15). (c) For \(\Sigma_+^+\) to be \(P\)-conserving, the additional argument (see Eqs. (2.11-13)) goes beyond the DA but the \(\Xi\)-nucleon mass difference may retain its actual value. (d) The same is true for Eq. (2.17). Not until Section 6 shall we see that the conditions mentioned under (c), (d) may also be weakened to the DA symmetry if certain virtual transitions predominate.
3. DOUBLET SPIN STRUCTURES FOR WEAK INTERACTIONS

(a) Baryon currents. Examples

To begin with we consider $\Delta T = 1/2$, $|\Delta S| = 1$ interactions of the form (baryon current, $\Delta S = 0$) x (baryon current, $|\Delta S| = 1$). The results so obtained are immediately applicable to more general situations. In this section we shall have no need to specify the space time structure of currents.

Consider first $\Delta S = 0$, $T = 1$ currents without any DA assumption. These are bilinear in $(N_1, N_1)$; $(\Sigma, \Sigma)$; $(\Sigma, \Lambda)$ or $(N_4, N_4)$ and are easy to write down. We shall now use the following device. Even in the presence of the $\Sigma, \Lambda$ mass difference we shall express the currents in terms of $N_1, \ldots, N_4$ of Eq. (2.2). We consider $Y^0, Z^0$ as mathematical constructs defined by Eq. (1.5) in terms of the real particles $\Lambda, \Sigma^0$.

Of course we shall use expressions in terms of doublets with the ulterior motive to go to the DA. It is however quite essential to realize the following. If we assume the $\Delta T = 1/2$ rule to be rigorous (barring electromagnetism) then any weak interaction should satisfy $\Delta T = 1/2$ not only in the DA but also in the actual split $(\Sigma, \Lambda)$ situation. The device just mentioned guarantees from the outset that this requirement is met. The need to bear this point in mind was first emphasized by Treiman.\textsuperscript{14}

With the exclusion of a remark to be made in Section 7 we shall ignore in this paper the question whether the procedure just mentioned is strictly necessary. This question is tied to whether or not deviations from $\Delta T = 1/2$ are indeed purely electromagnetic.

The most general form for $\Delta S = 0$, $T = 1$ baryon currents is

\[ T = 1: \quad \alpha_1 \bar{N}_1 \tau N_1 + \alpha(\bar{N}_2 \tau N_2 + \bar{N}_3 \tau N_3) + \alpha_4 \bar{N}_4 \tau N_4 + \alpha' j' \]  

(3.1)
where \( \tilde{\tau} \) is again the doublet spin. We shall use the notation
\[
\tilde{\tau}^\pm = \frac{1}{\sqrt{2}} (\tau_1 \pm i \tau_2) ; \quad \tilde{J}^\pm = \frac{1}{\sqrt{2}} (J_1 \pm i J_2).
\]

(3.2)

\( \tilde{J}' \) is given by
\[
\tilde{J}'^+ = N_2 S_3 \sqrt{2} ; \quad \tilde{J}'^- = N_3 S_2 \sqrt{2} ; \quad \tilde{J}_3' = -(N_2 S_2 - N_3 S_3).
\]

(3.3)

Consider next the \( |\Delta S| = 1, \, T = 1/2 \) current. Write it first in terms of the physical baryons, then transcribe to the doublet language. The most general result is
\[
|\Delta S| = 1, \, T = 1/2: \quad \beta_1 s_1 + \beta_2 s_2 + \beta_3 s_3 + \beta_4 s_4 ,
\]

(3.4)

with
\[
s_1 = \begin{pmatrix} N_3 S_1 \\ N_2 S_1 \end{pmatrix} ; \quad s_2 = \begin{pmatrix} N_4 S_2 \\ -N_4 S_3 \end{pmatrix}.
\]

(3.5)

\[
\begin{align*}
s_3 &= \begin{pmatrix} N_3 \tilde{\tau}_3 S_1 + N_2 \tilde{\tau}^- S_1 \sqrt{2} \\ -N_2 \tilde{\tau}_3 S_1 + N_3 \tilde{\tau}^+ S_1 \sqrt{2} \end{pmatrix} ; \\
s_4 &= \begin{pmatrix} -N_4 \tilde{\tau}_3 S_2 + N_4 \tilde{\tau}^- S_3 \sqrt{2} \\ -N_4 \tilde{\tau}_3 S_3 - N_4 \tilde{\tau}^+ S_2 \sqrt{2} \end{pmatrix}.
\end{align*}
\]

For any \( s \) we shall often use the notation
\[
s = \begin{pmatrix} s^- \\ s^0 \\ s^+ \end{pmatrix} ; \quad \bar{s} = \begin{pmatrix} \bar{s}^+ \\ \bar{s}^0 \\ \bar{s}^- \end{pmatrix}.
\]

(3.6)
The method of first writing down currents in terms of the usual baryons and then transcribing to doublets is too roundabout. A simpler procedure is the following.

(b) Baryon currents, General method

Just as $\tau$ acts on the doublet spin components we introduce $\rho$ which acts on the K-spin components of N, see Eq. (2.3):

$$\rho_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \rho_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho_3 = -\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

(3.7)

$$\rho^\pm = \frac{1}{\sqrt{2}} (\rho_1 \pm i \rho_2).$$

Note the minus sign in the definition of $\rho_3$. We can now write Eq. (3.3) in the compact form $j' = \bar{N} \rho N$.

Thus we can look upon the $T = 1$ current of Eq. (3.1) as follows. The first three terms correspond to $I = 1, K = 0$; we can in fact write the $\alpha$ term of Eq. (3.1) as $\alpha \bar{N} \tau N$. The $j'$-term has $I = 0, K = 1$. This is an example of the general rule that we will get all currents by a vector addition procedure of all possible $I$ and $K$ to the desired $T$. This formal procedure is independent of the $(\Sigma, \Lambda)$ mass difference. (Of course only then can $I$ and $K$ serve as good quantum numbers if this difference is neglected.)

The following lemmas will be obvious.

1) Any bilinear baryon current; whether $\Delta S = 0$ or 1 can only have $I = 0$ or 1.

2) S-conserving currents have $K = 0$ or 1.

3) $|\Delta S| = 1$ currents have $K = 1/2$. 

Thus one can write down the following complete list of currents.

\[ \Delta S = 0. \]

\[ T = 0; \quad I = K = 0 : \rho = \eta_1 \bar{N}_1 N_1 + \eta \bar{N} N + \eta_4 \bar{N}_4 N_4 \quad (3.8) \]

\[ I = K = 1 : \rho' = \bar{N} \rho \tau N \quad (3.9) \]

\[ T = 1; \quad I = 1, K = 0 : j = \alpha_1 \bar{N}_1 \tau N_1 + \alpha \bar{N} \tau N + \alpha_4 \bar{N}_4 \tau N_4 \quad (3.10) \]

\[ I = 0, K = 1 : j' = \bar{N} \rho \tau N \quad (3.11) \]

\[ T = 2 ; \quad I = K = 1. \quad \text{Current is } v^3 \text{ where} \]

\[ v^2 = \bar{N} \rho^{-} \tau^{-} N \]

\[ v^1 = - \frac{1}{\sqrt{2}} \bar{N} (\rho^{-} \tau_3 + \rho_3 \tau^{-}) N \]

\[ v^0 = \sqrt{\frac{2}{3}} \bar{N} [\rho_3 \tau_3 - \frac{1}{2} (\rho^{-} \tau^+ + \rho^+ \tau^{-}) ] N \quad (3.12) \]

\[ v^{-1} = \frac{1}{\sqrt{2}} \bar{N} (\rho^+ \tau_3 + \rho_3 \tau^+) N \]

\[ v^{-2} = \bar{N} \rho^+ \tau^+ N. \]

\[ |\Delta S| = 1. \]

\[ T = 1/2 ; \quad I = 0, \quad K = 1/2 : \quad s = \beta_1 s_1 + \beta_2 s_2 , \quad (3.13) \]

\[ I = 1, \quad K = 1/2 : \quad s' = \beta_3 s_3 + \beta_4 s_4 , \quad (3.14) \]

(See Eq. (3.5).)
\( T = \frac{3}{2}; I = 1, K = 1/2. \) Current is \( u^T = \xi_1 u_1^T + \xi_4 u_4^T; \)

\begin{align*}
  u_1^{3/2} &= -\bar{N}_3 \tau^- N_1, & u_4^{3/2} &= -\bar{N}_4 \tau^- N_2 \\
  u_1^{1/2} &= \sqrt{\frac{2}{3}} [\bar{N}_3 \tau_3 N_1 - \frac{1}{\sqrt{2}} \bar{N}_2 \tau^- N_1], & u_4^{1/2} &= \sqrt{\frac{2}{3}} [\bar{N}_4 \tau_3 N_2 + \frac{1}{\sqrt{2}} \bar{N}_4 \tau^- N_3] \\
  u_1^{-1/2} &= \sqrt{\frac{2}{3}} [\bar{N}_2 \tau_3 N_1 + \frac{1}{\sqrt{2}} \bar{N}_3 \tau^+ N_1], & u_4^{-1/2} &= \sqrt{\frac{2}{3}} [-\bar{N}_4 \tau_3 N_3 + \frac{1}{\sqrt{2}} \bar{N}_4 \tau^+ N_2] \\
  u_1^{-3/2} &= \bar{N}_2 \tau^+ N_1, & u_4^{-3/2} &= -\bar{N}_4 \tau^+ N_3.
\end{align*}

All currents have the indicated T-properties for the actual values of the \( \Sigma, \Lambda \) masses. We next discuss the possible structures of \( H^{(0)} \) and \( H^{(1)} \) in terms of these currents. As \( H^{(0)}, H^{(1)} \) have definite I-properties we shall now have to use the currents in their true doublet form.

(c) Structure of \( H^{(0)} \)

Let \((I_0, K_0)\) denote the \((I, K)\) values of \( \Delta S = 0 \) current; likewise \((I_1, K_1)\) refers to \( \Delta S = 1 \). To construct an \( H^{(0)} \) we need \( I_0 = I_1 = 0 \) or 1. In either case \( K_0 \) can be equal to zero or one. Thus there are four possibilities.
\[ I_0 = I_1 = K_0 = 0 : \quad \rho(s^0 + s^0), \quad (3.16) \]

(See Eqs. (3.6), (3.8), (3.13).)

\[ I_0 = I_1 = 0 ; \quad K_0 = 1 \quad \overset{\text{\(N\)}}{\rho^+ N s^-} - \frac{1}{\sqrt{2}} \overset{\text{\(N\)}}{\rho_3 N s^0} + \text{h.c.} \quad (3.17) \]

(See Eqs. (3.7), (3.11), (3.13).)

\[ I_0 = I_1 = 1 ; \quad K_0 = 0 \quad : \quad j t \quad (3.18) \]

\[ t = \xi_1(t_{12} + t_{21}) + \xi_4(t_{34} + t_{43}) \quad (3.19) \]

\[ t_{ij} = \overset{\text{\(N\)}}{N_i} \tau N_j. \]

Here we meet a typical recoupling problem. Couple the currents of Eqs. (3.10) and (3.15) together to \( \Delta T = 1/2 \). The answer is

\[ j t \sim - \frac{\sqrt{2}}{3} \left( j^+ s^- - \frac{1}{\sqrt{2}} j^0 s^0 + \text{h.c.} \right) \]

where the term in brackets itself has \( \Delta T = 1/2 \); see Eq. (3.14). Equation (3.18) is the \( jt \) coupling discussed in I.

\[ I_0 = I_1 = 1 , \quad K_0 = 1 \quad : \quad \overset{\text{\(N\)}}{N \rho^+ \tau N(\beta_1 \overset{\text{\(N\)}}{N_3} \tau N_1 + \beta_2 \overset{\text{\(N\)}}{N_4} \tau N_2)} - \frac{1}{\sqrt{2}} \overset{\text{\(N\)}}{\rho_3 \tau N(\beta_1 \overset{\text{\(N\)}}{N_2} \tau N_1 - \beta_2 \overset{\text{\(N\)}}{N_4} \tau N_2)} + \text{h.c.} \quad (3.20) \]

There is an obvious structural connection between Eqs. (3.17) and (3.20).
(d) Allowed structure of $H^{(1)}$

As was stated in Section 2(c) we mean by this an interaction with $\Delta I = 1$ and which shares with the strong interactions the invariance under Eq. (2.9) with the condition (2.12).

The transformation (2.9) is the product of an $I$-spin rotation and a $1 \leftrightarrow 2$ substitution. The currents of Eqs. (3.9), (3.11), (3.12) all contain $N_2 N_3$ and $N_3 N_2$. Such terms cannot possibly respect $1 \leftrightarrow 2$. Hence $\rho'$, $j'$ and $v$ cannot appear in the allowed structures of $H^{(1)}$.

There remain three possibilities, all of which with $K_0 = 0$.

\[
I_0 = 1, \quad I_1 = 0'
\]

\[
j^+ s^- - \frac{1}{\sqrt{2}} j^0 s^0 + h.c.
\]

(3.21)

This is the interaction called $(j s)$ in $I$.

\[
I_0 = 1, \quad I_1 = 1
\]

\[
j^+ s'^- - \frac{1}{\sqrt{2}} j^0 s'^0 + h.c.
\]

(3.22)

We shall call this interaction $(j s')$.

\[
I_0 = 0, \quad I_1 = 1
\]

\[
\rho(s^0 + s'^0).
\]

(3.23)

We denote this coupling by $(\rho s')$. It is easily shown that the interactions (3.21-23) do share the invariance for the transformation (2.9) and do obey the condition (2.12) provided that

\[
(j s) : \quad \alpha_1 = \dagger \alpha
\]

\[
(j s') : \quad \alpha_1 = \dagger \alpha
\]

for $\epsilon = \dagger 1$. (3.24)

\[
(\rho s') : \quad \eta_1 = \dagger \eta
\]
Thus, see Eq. (2.10), allowed structures for \( H^{(1)} \) exist for both \( G^+ \)- and \( G^- \)-invariance.

We are now ready to derive Eq. (2.17) and thus complete the proof of Theorem (B), Section 2(d). One shows in fact that, under the conditions (3.24), \( H_{-1}^{(1)} \) shares with \( H_\pi \) the invariance under

\[
N_3 \rightarrow i \epsilon_1 \tau_2 N_1, \quad \pi^+ \rightarrow -\epsilon \pi^+,
\]
\[
N_1 \rightarrow i \epsilon_3 \tau_2 N_3, \quad \pi^0 \rightarrow -\epsilon \pi^0,
\]

provided the phases satisfy

\[
\epsilon_1 \epsilon_3 \epsilon = -1.
\]

Equation (2.17) follows from Eqs. (3.25-26) by the same argument used in connection with Eqs. (2.11-13). Hence Eq. (2.17) has been derived both for \( G^+ \)- and \( G^- \)-invariance.\(^{16}\)

It may be useful to state the conditions under which the helicity relation (2.15) holds. For Eq. (2.16) the DA is sufficient. For Eq. (2.17) it is insufficient. Following the various transformations one concludes that Eq. (2.17) nevertheless holds in the presence of the true \( \Xi \)-nucleon mass difference for \( G^+ \)-symmetry. \( G^- \) necessitates full global symmetry.

(e) \( \pi \)-currents. Space parity and \( G^+ \)-symmetry

The baryon currents of the foregoing sections may be completed with meson currents. Here we consider the \( \pi \)-field only. (Currents involving \( K \)'s occur in Sec. 5.) For \( \pi \) we have \( I = 1 \), \( K = 0 \). The only baryon current to which \( \pi \)-terms may be added is \( j \) of Eq. (3.10). Representatives are
\( \partial \) denotes a spatial derivative

\[
J(\pi)_A = \partial \pi \quad ; \quad J(\pi)_V = \pi \times \partial \pi .
\]  

(3.27)

The \( \Delta \mathcal{I} = 0, 1 \) rule and all that follows hold true for

\[
J_A \rightarrow J_A + J(\pi)_A \quad ; \quad J_V \rightarrow J_V + J(\pi)_V
\]

as long as the DA only is invoked. But for the stronger \( G^\pm \)- symmetries used in Eq. (3.24) something new happens.

According to Eq. (3.24) the \( j \) current which enters in the \( G^\pm \) case is

\[
J_{\pm} = \overline{N}_1 \uparrow N_1 \pm \overline{N} \uparrow N + \overline{N}_4 \uparrow N_4 .
\]  

(3.28)

Under the transformations \( (N_1 \leftrightarrow N_2 \quad ; \quad N_3 \leftrightarrow N_4) \quad ; \quad (N_1 \leftrightarrow N_2 \quad ; \quad N_2 \leftrightarrow N_4) \) we have \( J_{\pm} \rightarrow \pm J_{\pm} \) while for \( G^\pm \) we have \( J(\pi)_A \rightarrow \pm J(\pi)_A \); \( J(\pi)_V \rightarrow J(\pi)_V \).

Hence for \( G^\pm \) we may add \( J(\pi)_A \) to \( J_A \) and still all arguments of Section 3(a) hold true. But while we can add \( J(\pi)_V \) to \( J_V^+ \) (for \( G^+ \)), we cannot add \( J(\pi)_V \) to \( J_V^- \) (for \( G^- \)). This indicates that we can further restrict the allowed structure of \( H^{(1)} \) by arguments concerning conserved currents.

The phenomenon just described is due to the fact that with respect to the group \( G^- \) (not \( G^+ \)) a non-trivial parity is introduced in isotopic space. For \( \pi \)-currents this isotopic parity is linked to the space parity.

(f) Structure of the non-leptonic decay interaction

This completes the survey of the doublet spin structure of \( H^{(0)} \) and \( H^{(1)} \). The next question is how one guarantees that these two interactions separately are P-conserving but clash when taken together. In I this question has been discussed for the particular choice \( H^{(0)} = j_t, \quad H^{(1)} = j_s \) (see I Eq. (48)) but all arguments apply equally well to any allowed \( H^{(0)}, H^{(1)} \).
The same is true for the possibility mentioned in I that all $S$-violating currents are either all pure $V$ or all pure $A$. Furthermore a general argument was given in I that showed the parity clash idea and the universal Fermi interaction (total current) x (total current) to be incompatible. In particular the $\Delta I = 0$, 1 rule and the schizon scheme are mutually exclusive. See however the remarks on this question in Section 6(a).

The particular coupling scheme discussed in I is clearly not unique. It is not the purpose of the present paper to express preferences for one or another form of $H^{(0)}$, $H^{(1)}$. It is remarkable however to note the following. The $\Delta I = 0$, 1 rule and a coupling of $(\Delta S = 0, T = 1) \times (|\Delta S| = 1, T = 1/2)$ currents are actually compatible provided the $S$-conserving current does not only contain $I = 1$, $K = 0$ but also $I = 0$, $K = 1$. In fact from Eq. (3.17) and (3.21) we derive the following. If we couple the $T = 1$ current

$$\bar{N}_1 \gamma_\lambda \tau N_1 \pm \bar{N} \gamma_\lambda (\tau + \text{const}) \gamma_5 N + \bar{N}_4 \gamma_\lambda \tau N_4$$  \hspace{1cm} (3.29)

to the $|\Delta S| = 1$, $T = 1/2$ current $s$ given by Eq. (3.13) then all requirements of the $\Delta I = 0$, 1 rule can be fulfilled. In Eq. (3.29) we have exemplified a space time structure to which corresponds a pure ($V$ or $A$) $S$-violating current. It is interesting to note that in such a coupling scheme (unlike the one discussed in I) the $S$-violating current is purely of the $T = 1/2$ kind.

Finally we note that the relation I Eq. (4) between the rates of $\Sigma^+$- and $\Lambda$-decay is a consequence of the $\Delta I = 0$, 1 rule rather than of the more specific form of coupling used in that paper.
4. THEOREM ON P-VIOLATION IN $\Xi$-DECAY

In accordance with the DA we consider the amplitude for $\Xi^- \rightarrow \Lambda + \pi^-$ as the sum of the amplitudes for

$$\Xi^- \rightarrow Y^0 + \pi^-, \quad \Delta \Lambda_3 = -1, \quad \Delta \Lambda = 1 \quad (4.1)$$

$$\Xi^- \rightarrow Z^0 + \pi^-, \quad \Delta \Lambda_3 = 0 \quad (4.2)$$

The $\Delta \Lambda_3$, $\Delta \Lambda$ as far as specified are also obvious. But the $\Delta \Lambda$ for reaction (4.2) cannot be fixed by an argument similar to the one given for $Y^0 \rightarrow p + \pi^-$ in Section 2(d).

However, let us ask if it is possible to relate $\Xi$-decay to the $\Sigma$-decays. For this we should relate $(\Xi^- | Y^0 \pi^-)$, $(\Xi^- | Z^0 \pi^-)$ to $(\Sigma^+ | n \pi^+)$, $(\Sigma^- | n \pi^-)$. Such relations are possible if and only if we consider situations more degenerate than the DA. The general method for judging what the possibilities are is the following. The DA implies that the $I$-group may be used. Additional degeneracy implies additional invariance for substitutions between such doublets as are taken degenerate. We ask for the possible and minimal degeneracies which relates $\Xi$ to $\Sigma$ decay. These are the following.

a) Let the strong interactions share with $H^{(1)}$ the invariance for

$$N_4 \rightarrow \xi_3 N_3, \quad N_2 = \xi_1 N_1, \quad \pi \rightarrow \epsilon \pi \quad (4.3)$$

It follows that

$$(\Xi^- | Y^0 \pi^-) = \epsilon \xi_1 \xi_3 (\Sigma^- | n \pi^-) \quad (4.4)$$

b) Let the strong interactions share with $H^{(0)}$ the invariance for

$$N_4 \rightarrow i \xi_2 \tau_2 N_2, \quad N_3 \rightarrow i \xi_1 \tau_2 N_1,$$

$$\pi^+ \rightarrow -\epsilon \pi^+, \quad \pi^0 \rightarrow -\epsilon \pi^0 \quad (4.5)$$
then

\[(\Xi^- | Z^0 \pi^-) = \epsilon \xi_1' \xi_2' (\Sigma^+ | n \pi^+) \quad (4.6)\]

As \((\Sigma^+ | n \pi^+)\) obeys \(\Delta I = 0\), it follows from Eq. \((4.6)\) that the reaction \((4.2)\) proceeds via \(\Delta I = 0\) under the stated conditions. Thus according to Eq. \((1.6)\) the minimal conditions which relate \(\Xi\) to \(\Sigma\) decay imply that parity is violated in \(\Xi^- \rightarrow \Lambda \pi^-\) in the same approximation that parity is conserved in \(\Sigma^+\) and \(\Sigma^-\).

This is the counterpart of theorem \((A)\) for \(\Lambda\)-decay, see Section 2(d). Theorem \((B)\) has no analog. That is, unlike the phases in Eqs. \((2.16), (2.17)\), those in Eqs. \((4.4), (4.6)\) are not uniquely fixed.

We show this by one counter example. Let \(H^{(0)}\) be of the type \(j t\). Then one easily shows: \(\epsilon \xi_1' \xi_2' = \pm 1\) for \(\xi_1 = \pm \xi_4\) (see the definitions in Eq. \((3.19)\)). Let \(H^{(1)}\) be of the type \(j s\). One proves: \(\epsilon \xi_1 \xi_3 = \pm 1\) for \(\beta_1 = \pm \beta_2\). Apparently one needs not only an argument about the structure of the interaction, but an even more detailed argument about the structure of the \(S\)-violating current. At any rate we have \(|\alpha_\Xi| = \alpha_\Lambda\) for full global symmetry.
5. INFLUENCE OF K-PARTICLE EFFECTS

(a) Strong K-couplings

It is the purpose of this section to study the extent to which the \( \Delta I = 0, 1 \) rule and all subsequent statements can be upheld in the presence of strong K-interactions, but only in as far as the latter respect the DA. Thus we need in particular those strong K-couplings which satisfy \( \Delta I = 0 \), just like \( H_\pi \) of Eq. (2.1). For the present we do not speculate on whether these specific K-couplings do or do not form a major part of all K-interactions. Rather do we ask, if they exist what is their influence.

To start with we follow a procedure similar to the one of Section 3(a). Let \( H_K \) be the most general K-coupling bilinear in baryons, \(^1\) linear in K. Transcribe this general coupling in terms of doublets, without implying any mass degeneracy. Write K as a spinor,

\[
\mathbf{K} = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}.
\]

Then

\[
H_K = H_K^{(0)} + H_K^{(1)},
\]

\[
H_K^{(0)} = \bar{s} K + s \bar{K} \quad , \quad (\Delta I = 0)
\]

\[
H_K^{(1)} = \bar{s}' K + s' \bar{K} \quad , \quad (\Delta I = 1).
\]

Here \( s \) and \( s' \) are the same structures as already introduced in Eqs. (3.13-14). (Of course the constants \( \beta \) in those equations have a different magnitude here.) The products in Eqs. (5.2-3) are in the usual sense of spinor multiplication, \( s \bar{K} = s^- K^- + s^0 K^0 \), etc.
If we now go to the DA then these two couplings have distinct $\Delta I$ properties, as indicated. To verify this, remember the assignments of Eqs. (3.13-14) for $s, s'$ and use $I = 0; T = K = 1/2$ for the K-particles.

Thus $H_K^{(0)}$ respects the DA and is an interaction discussed previously.19 (The $S_1, S_2$ of previous work2 are equal to $\frac{1}{2} S + K_3$; $\frac{1}{2} S - K_3$ respectively.) $H_K^{(1)}$ breaks the DA through interference with itself and with $H_K^{(0)}$. These are consequences of the $\Delta I$ assignment, not of the particular trilinear structure which has only been mentioned to exemplify the argument.

We now ask if the previous results hold true if we include $H_K^{(0)}$ in the strong interactions and to what extent the answers depend on the characteristic relative parities of K-particle physics. These are 1) the parity $P(K^+)$ of charged K's relative to Λ-nucleon, 2) the parity $P(\Xi)$ of cascade relative to nucleon, 3) the parity $p(K)$ of charged relative to neutral K-particles.

It is therefore necessary to write out the interaction (5.2) in some more detail. We have

$$H_K^{(0)} = [F_1 \bar{N}_1 \ 0_{12} \ N_2 + F_2 \bar{N}_3 \ 0_{34} \ N_4]K_0$$

$$+ [F_1 \bar{N}_1 \ 0_{13} \ N_3 - F_2 \bar{N}_2 \ 0_{24} \ N_4]K^+ + \text{h.c.}$$

(5.4)

where all 0-operators in essence represent either 1 or $i \gamma_5$. The parity possibilities are completely specified as follows.
\[
0_{13} = i \gamma_5 (1) \quad \text{for} \quad \text{P}(K^+) \text{ odd (even)}
\]

\[
P(\Xi)\text{even} : \quad 0_{12} = 0_{34}, \quad 0_{13} = 0_{24}
\]

\[
P(\Xi)\text{odd} : \quad 0_{12} \neq 0_{34}, \quad 0_{13} \neq 0_{24}
\]

\[
p(K)\text{even} : \quad 0_{12} = 0_{13}, \quad 0_{24} = 0_{34}
\]

\[
p(K)\text{odd} : \quad 0_{12} \neq 0_{13}, \quad 0_{24} \neq 0_{34}
\]

(5.5)

Note that in principle one can dispose independently over P(\Xi) and p(K).

We now observe that the assignment \( \Delta I = 0 \) to \( H_K^{(0)} \) is independent of \( P(K^+) \), \( P(\Xi) \), \( p(K) \). This is true because each of the four terms of Eq. (5.4) satisfy \( \Delta I = 0 \) individually. Hence, as explained in Section 2(b), it remains a meaningful procedure to assign a \( \Delta I \) to a weak interaction, whatever the parities are which enter in \( H_K^{(0)} \).

It follows therefore that the \( \Delta I = 0 \), 1 rule of Section 2(b) can be maintained as long as the additional argument for \( H_K^{(1)} \) can be upheld in the presence of \( H_K^{(0)} \). We shall see presently that this argument can be fully maintained if \( P(\Xi) \) is even, but that some \( P \)-violation may occur if \( P(\Xi) \) is odd. All arguments will turn out to be independent of \( p(K) \), however.

A remark on this latter parity is in order here. If \( p(K) \) is odd then \( I \) is still a good quantum number, but \( K \) is not. As has been noted elsewhere, odd \( p(K) \) implies deviations from \( \Delta T = 0 \) in strong interactions, hence from \( \Delta T = 1/2 \) in weak interactions. Thus odd \( p(K) \) could only then be a possibility if the virtual \( K \)-interactions would play only a minor role in non-leptonic hyperon decays.
(b) Non-leptonic hyperon decays

In this section we discuss the additional argument for $H^{(1)}$ in the presence of $H^{(0)}_K$ and also the extension of the argument for $\Lambda$- and $\Xi$-decays in the presence of this $I$-conserving strong $K$-interaction. In all these instances the reasoning follows the same pattern. Wherever we have used symmetries stronger than the DA we ask if these symmetries can be extended to include shared invariance for $H^{(0)}_K$.

a) Eq. (2.11). Complete Eq. (2.9) as follows: $N_3 \rightarrow i \tau_2 N_4$, $N_4 \rightarrow i \tau_2 N_3$, $K^0 \rightarrow K^0$, $K^+ \rightarrow -K^-$. Equation (2.11) remains true in the presence of $H^{(0)}_K$ if $F_1 = F_2$ and $P(\Xi)$ is even. For odd $P(\Xi)$ charged $K$-couplings cause a deviation. The same applies to Eq. (2.14).

b) Eq. (2.16) remains valid.

c) Eq. (2.17). Complete Eq. (3.25) with $N_2 \rightarrow i \tau_2 N_4$, $N_4 \rightarrow i \tau_2 N_2$, $K^0 \rightarrow K^0$, $K^+ \rightarrow K^-$. With $F_1 = F_2$, $P(\Xi)$ even, Eq. (2.17) remains valid. $K^0$-couplings cause a deviation for odd $P(\Xi)$.

d) Eq. (4.4). Complete Eq. (4.3) with $N_3 \rightarrow N_4$, $N_1 \rightarrow N_2$, $K^+ \rightarrow -K^-$, $K^0 \rightarrow K^0$. Eq. (4.3) remains valid for $F_1 = F_2$, even $P(\Xi)$. For odd $P(\Xi)$ the $K^+$-couplings cause a deviation.

e) Eq. (4.6). Complete Eq. (4.5) with $N_2 \rightarrow i \tau_2 N_4$, $N_1 \rightarrow i \tau_2 N_3$, $K^0 \rightarrow K^0$, $K^+ \rightarrow K^-$. Eq. (4.6) remains valid for $F_1 = F_2$, even $P(\Xi)$. For odd $P(\Xi)$ the $K^0$-couplings cause a deviation.

All results stated in this section are independent of $p(K)$.

(c) $K\pi$-currents

Direct $K\pi$-transitions can be brought about through $K\pi$-terms in the currents which enter the weak interactions. This mechanism is additional to the one already met in the foregoing. Their inclusion does not change
the essence of the previous argument.

We have specified \( I = 1, \ K = 0 \) for \( \pi \), \( I = 0, \ K = 1/2 \) for \( K \).

Hence the only baryon currents to which \( K \pi \)-terms can be added are \( t \) and \( s' \), see Eqs. (3.14) and (3.19):

\[
t ightarrow t + t_K
\]

\[
s' \rightarrow s' + s_K'
\]

where (up to a constant)

\[
t_K = \pi(K^0 + \bar{K}^0)
\]

\[
s_K' = \left( \begin{array}{c} K^+ \pi^0 + K^0 \pi^+ \sqrt{2} \\ K^+ \pi^- \sqrt{2} - K^0 \pi^0 \end{array} \right).
\]

Note that \( t_K \) involves neutral K-particles only. The discussion of \( K^2 \)-currents follows similar lines.

\( \text{(d) Weak non-leptonic K-transitions} \)

Now that strong \( K \) and \( \pi \)-interactions as well as decay couplings have been defined with respect to the DA, non-leptonic K-decays can be discussed in this approximation. Once again, we do not insist that such a description of the reactions represents the complete picture. Rather do we pose the following conditional problem. What can be said about \( H^{(0)} \) and \( H^{(1)} \) if it were true that deviations from the DA modify only slightly the description of non-leptonic K-decay.

Note that a transition \( K^+ \rightarrow \text{system} \) with \( S = 0 \) is necessarily of the type \( \Delta I = 1 \). For if it were a \( \Delta I = 0 \) transition, then the final state would have \( Q = 0 \). In particular
\[ K^+ \rightarrow 2\pi^+ + \pi^- \quad \text{or} \quad \pi^+ + 2\pi^0 \quad \text{have} \; \Delta I = 1 \quad (5.8) \]

\[ K^- \rightarrow \pi^- , \quad \bar{K}^0 \rightarrow \pi^0 \quad \text{have} \; \Delta I = 1 . \quad (5.9) \]

On the other hand

\[ K_1^0 \rightarrow \pi^+ + \pi^- \quad \text{or} \quad 2\pi^0 \quad \text{have} \; \Delta I = 0 . \quad (5.10) \]

Thus the \( \theta^0 \) and \( \tau^\pm \) modes proceed via \( H^{(0)} \) and \( H^{(1)} \) respectively. \( K_2^0 \) goes via \( H^{(1)} \). Representative graphs are shown in Fig. 1. For each of the two reactions one graph goes via nucleons, the other via cascades.

Let us now continue to assume that \( H^{(0)} \) and \( H^{(1)} \) conserve parity. Then in general the \( \Delta I \) properties of these couplings are no sufficient guarantee for actually allowing the reactions (5.8) and (5.10) to occur.

Using the C-operators of Eq. (5.4) we find

\[ K^+_{\pi^3} \quad \text{and} \quad K \rightarrow \pi \quad \text{allowed via} \]

\[
\begin{cases}
\text{nucleons if } & 0_{13} = 1(i \gamma_5) \quad \text{for } H^{(1)} = 1(i \gamma_5) \\
\text{cascades if } & 0_{24} = 1(i \gamma_5) \quad \text{for } H^{(1)} = 1(i \gamma_5) .
\end{cases}
\]

These conditions are evidently general and do not depend on the particular graphs shown in Fig. 1. Likewise for \( K_{\pi^2}^0 \) we have

\[ K_{\pi^2}^0 \quad \text{allowed via} \]

\[
\begin{cases}
\text{nucleons if } & 0_{12} = 1(i \gamma_5) \quad \text{for } H^{(0)} = 1 \gamma_5(1) \\
\text{cascades if } & 0_{34} = 1(i \gamma_5) \quad \text{for } H^{(0)} = 1 \gamma_5(1) .
\end{cases}
\]
To see when $K^+_2$ and $K^+_3$ are both allowed in the DA we must distinguish two cases.

a) $P(\Xi)$ even. The conditions are

\begin{align*}
\text{if } 0_{12} = 0_{13} &= 1 \quad \text{then } H^{(0)} = i \gamma_5, \quad H^{(1)} = 1, \quad (5.13) \\
\text{if } 0_{12} = 0_{13} &= 1 \gamma_5 \quad \text{then } H^{(0)} = 1, \quad H^{(1)} = 1 \gamma_5, \quad (5.14) \\
\text{if } 0_{12} &\neq 0_{13} \quad \text{then } H^{(0)} = H^{(1)}. \quad (5.15)
\end{align*}

Equation (5.15) implies that if $p(K)$ were odd it would be impossible to assume that all four decays: $K^0_2$, $K^+_3$, $\Sigma^+_2$ and $\Sigma^-_3$ are well approximated by the DA. While Eqs. (5.13), (5.14) show that for even $p(K)$ the assumption of $P$-conserving but clashing $H^{(0)}$, $H^{(1)}$ is adequate to describe all these decays in the DA.

b) $P(\Xi)$ odd. The transitions (5.8-10) are always allowed. Thus even $P(\Xi)$ excludes odd $p(K)$; odd $P(\Xi)$ does not exclude odd $p(K)$.

We conclude the following. It is possible to endow $H^{(0)}$, $H^{(1)}$ with all the properties stated in the $\Delta I = 0, 1$ rule of Section 2(b) and have both $K^0_2$ and $K^+_3$ allowed in the DA. For this last purpose it is of course totally unnecessary to assume that these two separate weak interactions do conserve parity. The parity properties of $H^{(0)}$, $H^{(1)}$ become only manifest in the general discussion of $\Sigma$-decays given in the foregoing.

But even in this last respect we must make a proviso. It is conceivable that also for $\Sigma$-decays we could allow $H^{(0)}$ and $H^{(1)}$ to be parity violating provided that certain dynamical accidents happen.

It cannot be anybody's purpose to give a complete theory of accidents. Let us nevertheless consider some specific examples in the next section.
6. QUESTIONS OF DOMINANT VIRTUAL TRANSITIONS

(a) Dominance of $K_N$ and $K_{N^2}$

Consider the contribution to $\Sigma^-$ and $\Sigma^+$ from the graphs drawn in Fig. 2. The $\Sigma^-$ graph was first considered by FMS. The possibility to have P-conservation if $K_N$ and $K_{N^2}$ dominate was first noted by Wolfenstein who also stated the relevance of the DA with regard to the $\Sigma^+ \to Y^0$ graph. We add the following comments.

1) From the present point of view we deal here with special $H(0)$ and $H(1)$ transitions as indicated in the Figure, see Eqs. (5.9-10).

2) These graphs provide examples of P-conserving contributions even if $H(0)$ and $H(1)$ do not separately conserve parity, see the discussion of Section 5(d). Thus if such graphs would entirely dominate $\Sigma^+$ and $\Sigma^-$, the argument given in I concerning the incompatibility of the schizon scheme with the parity structure of the non-leptonic weak interactions would not apply.

3) We consider next examples of graphs which enter in the same order as those of Fig. 2 but which give P-violation if $H(0)$, $H(1)$ are P-violating. Consider the weak $K_N$ and $K_{N^2}$ transitions of Eqs. (5.9-10) as brought about by

$$K^- \to n + \bar{p} \text{ (weak)} ; \quad n + \bar{p} \to \pi^- \text{ (strong)}$$

$$K^0 \to n + \bar{n} \text{ (weak)} ; \quad n + \bar{n} \to \pi^+ + \pi^- \text{ (strong)}.$$  \hspace{1cm} (6.1)

This is only one of many ways in which $K_N$ and $K_{N^2}$ vertices can be generated but Eq. (6.1) will suffice to illustrate the point. The weak links of Eq. (6.1) are drawn in Fig. 3. These links do not only suffice to generate the graphs of Fig. 2 but also those of Fig. 4. It is obvious that
these last graphs give in general (that is, barring still further accidents) P-violating contributions to the decays if \( H^0 \), \( H^1 \) violate parity.

4) However, it follows from the argument given in Section 5(d) that it means no restriction to the Wolfenstein argument to let \( H^0 \) and \( H^1 \) be P-conserving. But in turn, if this parity condition is satisfied we are also guaranteed that the graphs of Fig. 4 conserve parity.

5) Consider the graphs of Figs. 2 and 4 from the point of view of the \( \Delta I = 0, 1 \) rule. As explained in Section 2(b) we need in general an additional argument concerning \( H^1 \). But for the graphs under consideration no argument beyond the DA is necessary.

6) We conclude the following. To achieve P-conservation in \( \Sigma^+ \) and \( \Sigma^- \) the Wolfenstein graphs are acceptable whether or not \( H^0 \) and \( H^1 \) separately conserve parity. If they do conserve parity we can extend without further ado the Wolfenstein argument so as to include the graphs of Fig. 4. The DA is not only necessary but also sufficient for this subset of transitions.

The connection between the Wolfenstein model and the \( \Delta I = 0, 1 \) rule has thus been established by focusing the attention on the properties of the weak vertices. A similar argument will make clear the connection between this rule and the results of FMS.

(b) Single baryon dominance

The considerations of this section were entirely stimulated by the results of FMS. Following these authors we consider the particular chains
of weak and strong interactions drawn in Fig. 5. The strengths of the strong vertices are expressed in terms of $G_1$ and $G$, see Eq. (2.1). In the spirit of FMS one may consider $G_1$ and $G$ as renormalized constants however. As is evident from the real and virtual baryon states indicated in Fig. 5 we study the problem in the DA only. This is no restriction as compared to FMS.

The vertices have in general a momentum structure. With FMS we shall ignore this for the strong vertex but not for the weak vertices $X_i$ ($i = 1, 2, 3$). The reason for insisting on this dependence is that most models for non-leptonic decays depend very sensitively on the hyperon nucleon mass difference. For example an effective interaction $\bar{\Lambda} \gamma_\alpha (1 + \gamma_5) p \partial_\alpha \pi$ gives an $\alpha_\Lambda \sim 0.9$ but would give $\alpha_\Lambda = 0$ if we neglect $m_\Lambda - m_p$. Such mass difference effects are not fully exhibited for constant $X_i$, as we shall see.

The $\Delta T = 1/2$ rule implies

$$X_3 = X_2 - X_1$$

(6.2)

and relates the transition matrices $\eta\eta$ as follows.

$$\eta^- = \eta^+ - \eta^0 \sqrt{2}.$$ 

(6.3)

Put

$$X_i = a_i + b_i \gamma_5,$$

$$a_i = A_i + i \gamma q C_i,$$ 

$$b_i = B_i + i \gamma q D_i,$$ 

(6.4)

where $A-D$ are functions of the 4-momentum transfer $-q^2$. Apart from common factors we have then
\[ m^+ = \gamma_5 \frac{s^+}{M - 1} - \frac{\eta^+}{M + 1}, \quad (6.5) \]
\[ \sqrt{2} m^0 = \gamma_5 \frac{s^0}{M - 1} - \frac{\eta^0}{M + 1}, \quad (6.6) \]
\[ m^- = \gamma_5 \frac{s^+ - s^0}{M - 1} - \frac{\eta^+ - \eta^0}{M + 1}, \quad (6.7) \]

\[ \xi^+ = a_1(M^2) - \epsilon a_2(l) - M c_1(M^2) + \epsilon c_2(l), \quad (6.8) \]
\[ \eta^+ = b_1(M^2) + \epsilon b_2(l) - M d_1(M^2) + \epsilon d_1(l), \quad (6.9) \]
\[ \xi^0 = a_1(M^2) - \epsilon a_1(l) - M c_1(M^2) + \epsilon c_1(l), \quad (6.10) \]
\[ \eta^0 = b_1(M^2) + \epsilon b_1(l) - M d_1(M^2) + \epsilon d_1(l), \quad (6.11) \]
\[ \epsilon = G/G_1. \quad (6.12) \]

The nucleon mass has been put = 1, M is the \( \Sigma \)-mass.

So far we have only used the DA for the strong vertices. We ask what happens if we let the weak ones share this symmetry. This establishes a relation between \( X_1 \) and \( X_2 \) but this relation depends on the \( \Delta I \)-structure of the weak interactions.

Consider first a general \( H^{(0)} \)-interaction. This shares with the strong interactions the doublet charge symmetry property. Thus \((\Sigma^+ \mid p) = (\Sigma^0 \mid n)\), that is
\[ X_1 = X_2 \quad (\Delta I = 0). \quad (6.13) \]
For $H^{(1)}$ the situation is more complex as can be seen with reference to the three kinds of $\Delta I = 1$ couplings of Eqs. (3.21-23). One finds

for $j s, \rho s'$ ($\Delta I = 1$) : $X_1 = -X_2$ \hspace{1cm} (6.14)

for $j s'$ ($\Delta I = 1$) : no simple relation.

A further investigation of $j s'$ did not reveal any chance reason why $X_1 = \pm X_2$ might be valid for this case. We exclude $j s'$ from the following, not because there is any argument against this interaction but because we have nothing to say about it.

Equations (6.8-11) now give

for $\Delta I = 0$ : \[ \xi^+ = \xi^o = A_1(M^2) - \epsilon A_1(1) - M C_1(M^2) + \epsilon C_1(1), \] \hspace{1cm} (6.15)
\[ \eta^+ = \eta^o = B_1(M^2) + \epsilon B_1(1) - M D_1(M^2) + \epsilon D_1(1); \] \hspace{1cm} (6.16)

for $\Delta I = 1$ (not $j s'$) :

\[ \xi^+ = A_1(M^2) + \epsilon A_1(1) - M C_1(M^2) - \epsilon C_1(1), \] \hspace{1cm} (6.17)
\[ \eta^+ = B_1(M^2) - \epsilon B_1(1) - M D_1(M^2) - \epsilon D_1(1), \] \hspace{1cm} (6.18)

while $\xi^o, \eta^o$ are still given by Eqs. (6.10-11).
1) Consider the special case

\[
\begin{align*}
\text{All } C, D &= 0 \\
\text{All } A, B &\text{ independent of } q^2 \\
\epsilon &= \pm 1
\end{align*}
\]

(6.19)

This last restriction is just the one to $G^\pm$ symmetry. It follows now from Eqs. (6.5-12) that $\eta^0$ is pure s(p) wave for $\epsilon = +1(-1)$, in either case in contradiction with the large asymmetry observed in $\Sigma_0^+$. Equation (6.19) is one of the assumptions of FMS. Hence these authors need further decay mechanisms, for which they choose the $K_\pi$ graphs discussed in Section 6(a).

2) In the FMS treatment the contributions to the $\Sigma^-$ graph of Fig. 5 are zero. It can now easily be traced back how this comes about. FMS assume that the weak vertex has the form $a_Y + b_Y \gamma_5$ where $a_Y$, $b_Y$ are constants and are taken to be the same for all graphs. In the present language, FMS assume that Eq. (6.13) is true, that is, at least for baryon contributions they make the assumption of a pure $\Delta I = 0$ interaction. To this there is of course no objection. It is important to note, however, that it is therefore implicitly contained in the work of FMS that not all weak vertices can be iterates of a pure $(I = 1, \Delta S = 0) \times (I = 1/2, |\Delta S| = 1)$ coupling, in accordance with the more general reasoning of previous sections.

3) To obtain a parity pure answer for $\eta^+$, FMS proceed by using the same $\Delta I = 0$ vertex as was mentioned before. We know from the general discussion that, if $R^{(0)}$ conserves parity then $\eta^+$ does the same. Here FMS follow a different course which is probably somewhat more restrictive. On the one hand they do not assume that $R^{(0)}$ conserves parity, on the other they
assume Eq. (6.19) to be true. This indeed leads to a P-conserving \( \gamma_+ \), namely s(p) wave for \( \epsilon = +1(-1) \), see Eqs. (6.15-16) and (6.19).

4) Thus the FMS model is equivalent to an effective \( H^{(1)} \) of the type \( K_\pi \) and an effective \( H^{(0)} \) of the type \( N_2 N_1 \).

5) It is instructive to note that there are other solutions to Eqs. (6.5-12) which give the desired properties of \( \gamma_\pm \). These are obtained by dropping Eq. (6.19).

Example: Take an effective \( H^{(0)} \) with \( A_1 = B_1 = C_1 = 0 \) and a nonvanishing constant \( D_1 \), denoted by \( D_1^{(0)} \). Take further an effective \( H^{(1)} \) with \( B_1 = C_1 = D_1 = 0 \) and a nonvanishing and constant \( A_1 \), denoted by \( A_1^{(1)} \). Put \( \epsilon = -1 \). Then we obtain from Eqs. (6.15-18).

\[
\gamma^+ = D_1^{(0)}, \quad \gamma^- = -2A_1^{(1)}\gamma_5(M-1)^{-1}.
\]

Hence without using \( K_\pi \) we have obtained a nonvanishing P-conserving \( \gamma_\pm \) with the required opposite parities. Note moreover that the rates for \( \Sigma^+ \) and \( \Sigma^- \) are of the desired same order of magnitude if \( D_1^{(0)} \neq A_1^{(1)} \). In fact these rates equal each other for \( A_1^{(1)} = 1, 3D_1^{(0)} \).

Equation (6.20) is given as an example, not as a proposal. It shows that even if single pole dominance is correct it is not at all obvious what one should conclude from that.

Moreover Eq. (6.20) provides a second example of a situation where the DA is enough for \( H^{(1)} \). Indeed, under the conditions stated to obtain Eq. (6.20) the contributions of \( H_1^{(1)} \) to \( \Sigma^+ \) vanish, see Eqs. (6.17-18). Hence there is once more no need for the additional argument referred to in Section 2(b).
7. CONCLUDING REMARKS

The previous sections have dealt with the structure of weak non-leptonic interactions in a purely descriptive way. The question arises, if the $\Delta I = 0, 1$ rule is correct could one see why these interactions consist of two parts, $H(0)$ and $H(1)$?

We would like to present a speculation on this question. The strong interactions (2.1) and (5.2) which make up the DA can be written as

$$ j \pi + (\bar{s}K + s\bar{K}) \quad (7.1) $$

where $j$ denotes the isotopic structure of the $\pi$-field source. Assume that the $\pi(K)$ fields have small $K(\pi)$ components,

$$ \pi \rightarrow \pi + \Omega_1 K, \quad K = (K^+, K^-, K_1^0) \quad (7.2) $$

$$ \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \rightarrow \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} + \Omega_2 \begin{pmatrix} \pi^+ \\ 2^{-1/2} \pi^0 \end{pmatrix}, $$

where $\Omega_1, \Omega_2$ may be operators proportional to a weak coupling constant, but are independent of $T, I$ or $K$. Then

$$ j \sim \pi \quad \text{generates} \quad j \sim \Omega_1 K; \quad (7.3) $$

$$ \bar{s}K + s\bar{K} \quad \text{generates} \quad s^+ \Omega_2 \pi^+ + s^- \Omega_2 \pi^- + \frac{1}{\sqrt{2}} (s^0 + s^0) \Omega_2 \pi^0. $$

These couplings are both of the type $js$, hence $H^{(1)}$ see Eq. (3.21). We might say that the Eqs. (7.2) generate a "schizon scheme without schizons".

We can generate $H^{(0)}$ by

\begin{align}
N_1 &\rightarrow N_1 + \Omega_{12} N_2 & N_3 &\rightarrow N_3 + \Omega_{34} N_4 \\
N_2 &\rightarrow N_2 + \Omega_{21} N_1 & N_4 &\rightarrow N_4 + \Omega_{43} N_3 .
\end{align}

(7.4)

In this way

\[ j \pi \text{ generates } t \pi , \]

which is a \( j t \) coupling, hence \( H^{(0)} \) see Eq. (3.18). Also

\[ s^0 K^0 + s^0 \overline{K}^0 \text{ generates } \rho(K^0 + \overline{K}^0) \]

(7.6)

which is also an \( H^{(0)} \) see Eq. (3.16). Finally

\[ s^- K^- + s^+ K^+ \text{ generates } \]

\[ \begin{cases} \overline{N}_1 N_4 K^+ + \text{ h.c.} \\
\overline{N}_2 N_3 K^+ + \text{ h.c.} \end{cases} \]

(7.7)

These are both \( H^{(0)} \), \( |\Delta s| = 1 \) but of a type not considered before.

Equation (7.7) is a bona fide \( \Delta T = 1/2 \) coupling. Equation (7.8) can be written as \( \overline{N}(\rho^+ K^+ + \rho^- K^-)N \), see Eq. (3.7). Hence it is \( \Delta \bar{I} = 0 \) but a mixture of \( \Delta K = 1/2, 3/2 \) and therefore a mixture of \( \Delta T = 1/2, 3/2 \).

Nevertheless the DA does not allow the reaction

\[ K^+ \rightarrow \pi^+ + \pi^0 \]

(7.9)

to take place via this coupling, as the reaction has \( \Delta \bar{I} = 2 \), the coupling \( \Delta \bar{I} = 0 \). Put differently, Eq. (7.8) allows

\[ K^+ \rightarrow \Sigma^+ + \overline{\Sigma}^0 , \]
but these particles cannot recombine in the DA to \( 2\pi \) as they belong to different doublets.

If the DA gets \textbf{broken} however, \( K^+_{\pi^2} \) can take place. In this sense we may perhaps be justified to call this non-electromagnetic \( \Delta T = \frac{3}{2} \) effect a "small" effect. A dimensionless parameter which characterizes the DA is \( \delta = (M_\Sigma - M_\Lambda)/M_\Lambda \). In the DA, \( \delta = 0 \), its actual value is \( \delta \approx 0.067 \). If we consider \( \delta \) as a measure for the amplitude ratio \( K^+_{\pi^2}/K^0_{\pi^2} \) then the ratio of rates would be \( \sim \delta^2 = 0.005 \), a suggestive order of magnitude.

For the present we shall not pursue such arguments any further and in particular leave open the question of the parity structure of \( H(0) \) and \( H(1) \).

The common orders of magnitude of all weak processes suggests a common generic mechanism of leptonic and non-leptonic decay couplings. If the latter are generated by the strong interactions, one may ask if the same should not be true of the leptonic decays. I do not think that such a question can be answered in a theory which does not account for the law of baryon conservation.

Finally we summarize what is general, what is special about the conclusions obtained in I, where the particular choice \( H(1) = js \), \( H(0) = jt \) was studied.

In I we obtained parity clash for \( G^+ \)-symmetry. In this paper we have shown that this is also possible for \( G^- \)-invariance, while at the same time Eq. (2.15) holds true.\(^{16}\)

In I we explored the consequence of two further assumptions, namely

(a) The S-violating baryon currents are either all pure \( V \) (i.e. \( \gamma^\lambda \)) or pure \( A \) (i.e. \( \gamma^\lambda \gamma_5 \)).

(b) The same S-violating currents intervene in both leptonic and non-leptonic decays with \( |\Delta S| = 1 \).
It was shown in I that (a) implies odd $P(\Xi)$. This conclusion is not specific for the $(js, jt)$ coupling scheme. It was also shown in I that (a) and (b) imply the occurrence of leptonic $\Delta T = 3/2$ transitions. This last conclusion is more specifically true only if $H^{(0)}$ contains $jt$.

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3. The most recent results are given by Cork, Kerth, Wenzel, Cronin and Cool, University of California Report UCRL-9127.


5. A. Pais, "On Symmetries Shared by Weak and Strong Interactions," Il Nuovo Cimento (in the press). This paper will be referred to as I.

6. See for example I, Eq. (15).

7. \[ H = g \left( 2^{1/2} \bar{n} \gamma_\lambda \Sigma^+ \partial_\lambda \pi^+ + \bar{p} \gamma_\lambda \Sigma^+ \partial_\lambda \pi^0 \right) \]

   \[ + g_2 \left( 2^{1/2} \bar{n} \gamma_\lambda \gamma_5 \Sigma^- \partial_\lambda \pi^- + \bar{p} \gamma_\lambda \gamma_5 \Sigma^+ \partial_\lambda \pi^0 \right) \]


   To avoid misunderstanding, it should be stressed that Bludman discusses such interactions as effective rather than as primitive couplings.

8. See reference 2. Also A. Pais, Phys. Rev. 112, 624 (1958), Section II. Section V of this last paper contains a formal basis for the \((I, K)\)-quantum numbers in terms of the orthogonal 4-group.

9. G. Feldman, P. Matthews and A. Salam, "The Non-Leptonic Decay Modes of Hyperons," preprint. We refer to this paper as FMS.


13. See reference 5, Eq. (12). $G^\pm$ symmetry implies $G_1 = \pm G = G_4$. Here we do not always need to state the relative magnitude of $G_4$. It is possible but unnecessary to consider more general phase factors in Eqs. (2.9-10).


15. See reference 5, Eqs. (32) and (33).

16. In I we considered an instance (called the first possibility) where Eq. (2.15) holds true even though $\alpha_+ = \alpha_- = 0$ is not explicitly realized. This is the case for $G^-$ symmetry and pure js-coupling. This shows that if Eq. (2.15) were to be in agreement with experiment, it would not be possible to conclude with definiteness that the weak interactions are of the form $H^{(0)} + H^{(1)}$. Nevertheless it is gratifying that this form of $H$ leaves no ambiguity in the value of this relative helicity.

17. For simplicity we put $\alpha_4 = \alpha_1$.


19. See reference 2, Eq. (12).

20. Similar to the way small non-static electric (magnetic) phenomena accompany almost pure static magnetic (electric) phenomena.
FIGURE CAPTIONS

Fig. 1. Examples of graphs which contribute to $\Theta$ and to $\tau$ decay. $\Delta$, $\Box$, 0 denote effective strong $K$, strong $\pi$, weak vertices, respectively.

Fig. 2. $K$-contributions which conserve parity if $H(0)$, $H(1)$ violate parity (Wolfenstein).

Fig. 3. Contributions to $K^+ \rightarrow np$ ($\Delta I = 1$) and to $K^0 \rightarrow nn$ ($\Delta I = 0$).

Fig. 4. $K$-contributions which generally violate parity if $H(0)$, $H(1)$ violate parity.

Fig. 5. FMS graphs for transitions via a single baryon.
Fig. 1
Fig. 2
Fig. 3

\[ \text{MU-21969} \]
Fig. 4
MU-21966

Fig. 5
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