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On Optimizing Wireless Mesh Networks: From Theoretical Capacity Analysis to Practical Algorithm Design

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Electrical Engineering (Communication Theory and Systems) by

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2008
The dissertation of Ping Zhou is approved, and it is acceptable in quality and form for publication on microfilm:

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Chair

University of California, San Diego

2008
To my beloved wife, Wenji,
and my parents.
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ABSTRACT OF THE DISSERTATION

On Optimizing Wireless Mesh Networks: From Theoretical Capacity Analysis to Practical Algorithm Design

by

Ping Zhou

Doctor of Philosophy in Electrical Engineering (Communication Theory and Systems)

University of California, San Diego, 2008

Professor Ramesh R. Rao, Chair

A wireless mesh network (WMN) is a hierarchical network consisting of mesh clients, mesh routers and gateways. Mesh routers constitute a wireless mesh backbone, to which mesh clients are connected, and gateways are chosen among mesh routers providing Internet access. In this dissertation, throughput performance of WMNs has been investigated from theoretical capacity analysis to practical algorithms’ design. For such a network with \( N_c \) randomly distributed mesh clients, \( N_r \) regularly placed mesh routers and \( N_g \) gateways, assuming that each mesh router can transmit at \( W \) bits/s, the per-client throughput capacity has been derived as a function of \( N_c, N_r, N_g \) and \( W \). The result
illustrates that, in order to achieve high capacity performance, mesh backbone network must be optimally designed. An innovative gateway placement algorithm is thus developed. It determines the location of a gateway based on a new performance metric. Therefore, given a certain number of gateways, the proposed gateway placement scheme provides a framework of maximizing the throughput of WMNs through proper placement of these gateways. Experimental results show that it constantly outperforms other schemes with a large margin. Algorithms on choosing the optimal number of mesh routers are also presented. Two problems are investigated. In Maximum Throughput Partition problem, the ideal throughput is achieved by optimally partitioning the network with a proper number of backbone nodes. In Maximum Throughput Partition with Hops’ number Constraint problem, a similar problem is studied but with constraint on the average number of hops in the backbone network. The results show that it is critical to find an appropriate size of the backbone network for a WMN, especially when the hops’ number constraint is imposed. The research findings and results can be used as guidelines for protocol design and deployment of WMNs.
I

Introduction

In this chapter, basic concepts of wireless mesh networks (WMNs) are introduced. WMNs are compared with other popular wireless networking technologies in various aspects. And state-of-the-art solutions of increasing throughput of multi-hop wireless networks are also presented.

1.1 Network Architecture and Unique Characteristics of WMNs

Most of today’s wireless networks can be categorized into the 3 network architectures defined as follows:

- **Ad hoc architecture**: all nodes are in a flat wireless network. Data packets are transmitted by multi-hop fashion.
- **Hybrid architecture**: hierarchical network architecture. A wired infrastructure network, comprising of base stations, is overlaid on a conventional ad hoc wireless network, comprising of user nodes. Data packets can be transmitted either purely by user nodes in multi-hop fashion or by making use of the wired infrastructure.
- **Mesh architecture**: hierarchical network architecture. A wireless infrastructure network, comprising of mesh routers, is overlaid on a conventional ad hoc
wireless network, comprising of mesh clients. Data packets are transmitted in multi-hop fashion by making use of the wireless infrastructure, which is called *wireless mesh backbone*.

Ad hoc architecture is commonly adopted in applications of wireless sensor networks and wireless personal area networks (WPANs), as shown in Figure 1.1. The biggest advantage of this architecture is that ad hoc wireless networks can be easily deployed with very low cost.

Many today’s commercial wireless networks, such as wireless local access networks (WLANs) and cellular networks are using hybrid architecture in its infrastructure mode. In such a mode, user nodes are connected to base stations as a star topology and data packets are transmitted to infrastructure network by one hop. Figure 1.2 shows the network architecture of a cellular network, which is an example of hybrid architecture. Such an architecture can significantly reduced the complexity of protocols at user end and its infrastructure network can be centrally controlled and scheduled for quality of service. Thus, wireless networks with hybrid architecture can be easily implemented and commercialized.
Figure 1.2: A cellular network: an example of wireless networks using hybrid architecture

With a wireless infrastructure, mesh architecture offers all the advantages of both ad hoc architecture and hybrid architecture. More specifically, a WMN is a two-tier wireless network. In the first tier, mesh routers self-configure an ad hoc wireless network as a wireless mesh backbone; gateways are chosen among mesh routers and are wired to Internet. In the second tier, mesh clients are connected to the mesh backbone for network access. Wireless mesh backbone can be rapidly deployed with minimal cost and provides a robust, efficient, reliable, and flexible system that supports the network access for mesh clients. Mesh backbone can also provide mesh clients with various services and resources through their gateway and bridging functions [1]. With infrastructure support, the complexity of communication protocols in mesh clients can be reduced significantly. All these advantages reinforce WMNs as a promising wireless technology for
numerous applications, e.g. broadband home networking, community and enterprise networking, public Internet access, and so on [2, 3]. Figure 1.3 shows a typical application of WMNs.

![Figure 1.3: A typical application of wireless mesh networks](image)

It is believed that in the near future most WMNs will be deployed in an infrastructure mode, where mesh clients are connected to their nearest mesh routers by one hop like a star topology while multi-hop transmissions are only allowed in mesh backbone. In what follows we will call such a WMN an *infrastructure WMN* [4].

Very recent achievements in wireless communications, such as advanced source and channel coding [5-7], MIMO [8-10], multiple-radio and multiple-channel technology [11, 12],
dramatically increase the channel capacity of wireless links, which becomes comparable to that of wired connections. Hence, WMNs are making their way into commercial use rapidly. Some companies are offering wireless mesh networking products [13-15], and industrial standards groups are also working actively on new specifications for wireless mesh networking [16-21].

Although mesh backbone in WMNs works in ad hoc architecture, its unique characteristics distinguish it from conventional ad hoc networks, i.e., generally mesh routers are static and powerful wireless devices deployed in deterministic locations without strict constraints of energy, computing power, and memory. Hence, traditional research topics in ad hoc networks such as link scheduling [22-25], routing [26-36], and topology control [25, 37-42] find in WMNs new challenges since these existing solutions do not apply directly or efficiently. More specifically, while previous research works in ad hoc networks put more focus on power-efficiency [43-47] and support of users’ mobility [48, 49], solutions in WMNs should emphasize more on protocol efficiency, performance and quality of service. Therefore, new scheduling algorithms [50-54] as well as routing algorithms [55-59] are developed for WMNs very recently. Many researchers claim that routing and scheduling should be considered jointly to achieve ideal performance of WMNs [11, 12, 60-64]. And some of the above works are developed with the consideration of multiple radios and multiple channels [11, 12, 56, 58, 59, 63].

1.2 Increasing Throughput of Multi-hop Wireless Networks

It is very well-known that the throughput capacity of multi-hop wireless networks degrades with the size of network [65-75]. Therefore, how to increase throughput performance of such networks is a very important research topic. Since WMN is a case of multi-hop wireless networks, it is desired to have a quick review of all major current solutions for obtaining capacity gain. In the following, mobility, network coding, and infrastructure approach are considered as three solutions to increase throughput of multi-hop wireless networks.
In general sense, mobility degrades network performance in many wireless paradigms. However, Grossglauser and Tse presented a novel approach to increasing the capacity of wireless ad hoc network by exploiting nodes mobility [76]. In their work, mobility comes into the picture by shuffling node locations, creating numerous instances when excellent channels between different nodes can be exploited. Hence, capacity gain is obtained from the realization of the maximal spatial transmission concurrency in each network snapshot. The result in [76] assumed loose delay constrains and infinite node buffers. With a tighter delay constraint, the maximum achievable throughput must decrease. The optimal tradeoff between throughput and delay are characterized and studied in [77-79].

Ahlswede, Cai, Li and Yeung introduced a novel idea, called network coding in [80]. Network coding refers to coding at the nodes within a network, which is different from the existing source coding schemes. In source coding, information is compressed only at the source node and decompressed only at the sink in the network. In the paradigm of network coding, a node within the network consists of a set of encoders which are dedicated to the individual output channels. Each of these encoders encodes the information received from all the input channels into codewords and sends them on the corresponding output channel. At a sink node, there is also a decoder which decodes the multicast information. They show that codes can also be used to improve throughput through networks. Capacity gain of network coding arises from the transmission efficiency of information in the networks. Follow-up research works on network coding can be found in [81-88].

The third solution is defined as infrastructure approach. One of the examples is hybrid networks [68-72]. As discussed in the last section, user nodes can communications with each other by making use of wired infrastructure. This, indeed, reduces the average path length of wireless communications and thus, increases the throughput capacity. In the next several chapters of this
dissertation, it is demonstrated that the throughput of a WMN can also benefit from its mesh backbone.

The rest of this dissertation is organized as follows. In chapter II, theoretical asymptotic capacity of infrastructure wireless mesh networks is derived and compared with that of ad hoc networks and hybrid networks. Chapter III and Chapter IV provide practical algorithms to optimize mesh backbone for throughput. More specifically, a gateway placement algorithm is given in Chapter III and optimizing the number of mesh routers is discussed in Chapter IV. This dissertation is concluded in Chapter V.
II

Capacity of Wireless Mesh Networks

2.1 Introduction

It is believed that in the near future most WMNs will be deployed in an infrastructure mode, where mesh clients are connected to their nearest mesh routers by one hop like a star topology while multi-hop transmissions are only allowed in mesh backbone. Multiple channels or radios can be used for mesh clients, mesh routers, and mesh gateways. In what follows we will call such a WMN an infrastructure WMN.

Many research problems still remain open in WMNs [4]. Among them, one of the most challenging research topics is to study the capacity of WMNs. The theoretical capacity of WMNs remains unknown, although a few theoretical results have been obtained for the capacity of other multi-hop wireless networks.

For random ad hoc networks, Gupta and Kumar [65] derived that the per-node throughput capacity was $\Theta(W/\sqrt{N_c \log N_c})$ bits/s, whereas, for arbitrary wireless networks, the aggregate transport capacity was proved to be $\Theta(W\sqrt{N_c})$ bits-m/s. Similar results were obtained for three dimensional wireless networks in [66]. So far, the definition of wireless network capacity and the methodologies of deriving such capacity given in [65, 66] have been extensively adopted by
researchers to study the capacity of multi-hop wireless networks. Kulkarni and Viswanath [67] obtained the same lower-bound on the capacity of random ad hoc networks as that in [65]. However, a different approach was followed in order to circumvent the complicated analytical procedures. They showed that the lower-bound of wireless network capacity could be derived if several properties of node locations were determined.

For mobile ad hoc networks, the throughput capacity was analyzed in [74]. It was shown that, by exploiting nodes’ mobility, the average throughput per source-destination pair could be kept constant even as the number of nodes increases. This was achieved through a two-hop-relaying strategy, and a sender transmitted packets to its receiver only when they were close enough to each other. As a result, the improved throughput was actually obtained at the cost of possibly excessive delays and buffer requirement.

The capacity of hybrid ad hoc networks was investigated in [68-72]. A hybrid ad hoc network consists of base stations and ad hoc nodes. All base stations are connected as a wired network, providing an infrastructure for ad hoc nodes. Communications between two ad hoc nodes are fulfilled via other ad hoc nodes in a multi-hop fashion or simply via the infrastructure. Liu et al. [68] studied the aggregate throughput capacity based on a specific hybrid network model in which $N_g$ base stations were regularly placed in the center of hexagonal cells and $N_c$ ad hoc nodes were placed at random. Two routing strategies were also considered to determine whether data were forwarded in a multi-hop fashion or through the infrastructure. Their results showed that the infrastructure network improved the capacity of ad hoc networks under the assumption that the number of base stations scales faster than a certain threshold. In the case of $N_g = \Theta(N_c)$, the per-node throughput capacity of hybrid networks was studied by Kozat and Tassiulas in [69]. Unlike the model in [68], the model in [69] assumed that both base stations and ad hoc nodes were distributed randomly. The throughput capacity was analyzed under two
connectivity conditions. Strong connectivity condition required all ad hoc nodes to be connected with each other without any support of base stations while weak connectivity condition only required the overall network topology graph to be connected. Their results illustrated that, under the strong connectivity condition, a per-node throughput capacity of $\Theta(W/\log N_c)$ bits/s could be achieved. However, even under the weak connectivity condition, $\Theta(1)$ bits/s could not be achieved. A more flexible model was considered in [70], where base stations were placed in any deterministic manner and ad hoc nodes were placed at random. The per-node throughput capacity of such a random hybrid ad hoc network was derived by identifying the number of base stations in three scaling regimes. More specifically, with infrastructure support, the per-node throughput capacity of hybrid ad hoc networks could be effectively improved, i.e., $\Theta(N_b W/ N_c)$, if the number of base stations scaled faster than $\sqrt{N_c/\log N_c}$ but slower than $N_c/\log N_c$. However, for whatever scheduling and routing schemes, the per-node throughput capacity of hybrid ad hoc networks could not exceed $\Theta(W/\log N_c)$ bits/s, even when the number of base stations was optimally chosen. Capacity bounds of hybrid ad hoc networks were also studied in [71, 72] using Physical Model instead of Protocol Model.

The above-mentioned theoretical results cannot be applied to WMNs, because of the two distinct features of WMNs: 1) multi-tier hierarchical network architecture; 2) support of both Internet access and internal networking. Compared with conventional ad hoc networks, WMNs are multi-tier hierarchical networks in which different types of communications exist in various nodes. For instance, communications between two mesh clients are different from those between two mesh routers or between gateways and mesh routers. Consequently, the throughput capacity of WMNs must be studied by considering all such different types of communications. Moreover, the capacity is not only depicted as a function of the number of client nodes, but also as a function of the number of routers and the number of gateways. In comparison with hybrid ad hoc
networks, WMNs use wireless links instead of wired lines to connect backbone networks. In the capacity analysis of hybrid ad hoc networks, communication links among backbone nodes are assumed to have unlimited capacity, and such communications do not cause interference on other communications, e.g., communications between two client nodes or communications between client nodes and backbone nodes. However, these assumptions are no longer valid in WMNs. For either conventional or hybrid ad hoc networks, capacity is defined based on the fact that traffic originates/terminates in the same network. However, this is not true for WMNs, as traffic can also go to or come from the Internet. Such a difference demands a new capacity analysis framework ranging from capacity definition to capacity derivation.

In this paper, the per-client throughput capacity of infrastructure WMNs is studied. To the best of our knowledge, this is the first complete analysis on the throughput capacity of WMNs. It provides clear answers to the following questions:

- How throughput capacity of infrastructure WMNs scales with the number of mesh clients, the number of mesh routers and the number of gateways?
- What is the critical number of mesh routers and gateways in asymptotic scaling that can achieve the optimal throughput performance?

Moreover, our analytical results reveal that, with a proper number of gateways and mesh routers, the throughput capacity of infrastructure WMNs has the same asymptotic scaling as that of hybrid ad hoc networks. In this sense, WMNs can achieve much better throughput performance than conventional ad hoc networks.

The results also imply that backbone optimization in WMNs is highly desired. Without enough number of gateways, the scalability of WMNs can be worse than that of conventional ad hoc networks: arbitrarily increasing the number of non-gateway mesh routers may deteriorate throughput capacity of WMNs.
The rest of this chapter is organized as follows. In Section 2.2, a typical WMN model is presented. The main results and analytical methodology are overviewed in Section 2.3, which is followed by detailed derivations of upper-bounds and lower-bounds on the per-client throughput capacity in Sections 2.4 and 2.5, respectively. The generality of the analytical model and implications of the derived theoretical results are discussed in Section 2.6. This chapter is concluded in Section 2.7.

2.2 System Model

2.2.1 Network Topology

In an infrastructure WMN as shown in Figure 2.1, $N_c$ mesh clients are assumed to be uniformly and independently distributed on a unit square $R = [0,1]^2$. $R$ is partitioned evenly into $(1/l_s)^2$ small cells $R_i^t = [0,l_s]^2$ ($i = 1...(1/l_s)^2$), and a mesh router is placed in the center of each cell. Let $N_r$ denote the number of mesh routers, then $N_r = (1/l_s)^2$. In what follows, we will limit our interests to the case of $N_i \leq N_c$ i.e., the number of mesh routers is smaller than that of mesh clients. Mesh routers constitute a wireless mesh backbone providing a wireless infrastructure for mesh clients. In each cell, mesh clients are connected to the mesh router like a star topology, i.e., no direct communication is available among mesh clients, and the mesh router works as a hub for mesh clients. Among all the mesh routers, there are $N_g$ routers wired to Internet, working as gateways. Obviously $N_g \leq N_r$, i.e., the number of gateways cannot exceed the number of mesh routers. Such a WMN is referred as an infrastructure WMN in [4], which will be typical network architecture in future WMN applications.
2.2.2 Network Communications

Each mesh client is a data source and a data destination. Unlike mesh clients, mesh routers are neither data source nor data destination; they only route and forward data for mesh clients. In this paper, the following definitions of communications will be frequently used:

- **Local communications**: it is referred as the communications between a mesh router and a mesh client, which consists of uplink local communications and downlink local communications;

- **Backbone communications**: it is referred as the communications between two mesh routers. It includes the communications between a gateway and a mesh router, which consists of uplink Backbone communications and downlink Backbone communications.

Figure 2.1: Network topology of an infrastructure WMN
The same channel or multiple channels can be used for different communication links. However, the analysis remains the same, as will be explained in Section 2.6.1. Thus, in the following analysis, the same channel is assumed to be used in the entire WMNs.

In infrastructure WMN, traffic can be classified into two types: external traffic for Internet accessing and internal traffic between two mesh clients. All external traffic will be routed to or from gateways by multi-hop fashion in backbone communications. Internal traffic can either be routed with the support of wired gateways or purely among mesh routers without gateway functionality, denoted as gateway internal traffic and non-gateway internal traffic, respectively.

For internal traffic, the \( i \)-th mesh client located at \( X_i \) chooses its destination as follows: it first randomly chooses a location \( Y_i \) such that \( X_i \) and \( Y_i \) are independently and uniformly distributed on \( R \), and then selects the destination \( X_j \) among \( X_1, \ldots, X_n \) that is closest to \( Y_i \). Similar destination selection model was also adopted in capacity analysis of conventional and hybrid ad hoc networks [65, 66, 68-70] and exhibited no locality [65].

2.2.3 Transmission Model

It is assumed that the transmission range of uplink local, downlink local, and backbone communications is \( r_u \), \( r_d \), and \( r_b \), respectively. There are relatively abundant bandwidth and no interference on transmissions either between Internet and a gateway or two gateways. A mesh router can transmit at \( W \) bits/s. In addition, it can receive packets from only one sender at a time and cannot transmit and receive packets simultaneously. The same constraint is imposed on mesh clients. To coordinate simultaneous transmissions in the same channel, the same Protocol Model as defined in [66] is applied, i.e., if a transmission from node \( S_i \) to \( S_j \) is successful, then the following conditions must be satisfied: 1) \( |S_i - S_j| \leq r_i \); 2) for every other transmitting node \( S_k \),
\[ |S_i - S_j| \geq (1 + \Delta)r_g \], where \( r_i \) and \( r_g \) correspond respectively to the transmission range of node \( S_i \) and \( S_g \) and \( \Delta \) is a fixed positive constant that represents a guard zone in the Protocol Model.

2.2.4 Definition of Throughput Capacity

In existing work of capacity analysis [65, 66, 68-70], throughput capacity is defined as asymptotic scaling of the per-node throughput. In addition, throughput is evaluated based on the fact that traffic is totally internal, i.e., originating/terminating in the same network. In this paper, we also use asymptotic scaling of the per-node throughput to define capacity. However, the definition needs to be enhanced to capture two types of traffic: internal traffic as in ad hoc networks and external traffic from/to the Internet.

**Definition 1:** *feasible throughput.* A throughput of \( \lambda(N_c, N_r, N_g) \) bits/s for each node is feasible, if by gateway placement, time-space scheduling, routing and bandwidth assignment schemes, and allowing buffering at intermediate nodes, every node can send or receive \( \lambda(N_c, N_r, N_g) \) bits/s on average to/from its chosen destination/source node. For internal traffic, the chosen node is located in the same network, while for external traffic, it represents a particular selected gateway for the Internet accessing.

**Definition 2:** *asymptotic scaling for the per-node throughput.* The asymptotic scaling of \( \lambda(N_c, N_r, N_g) \), denoted by \( \Lambda(\lambda) \), is of order \( \Theta(\lambda(N_c, N_r, N_g)) \) bits/s, if there are deterministic constants \( c \) and \( c' \) such that \( 0 < c < c' < \infty \) and

\[
\lim_{N_g \to \infty} \text{Prob}(\Lambda(\lambda) = c\lambda(N_c, N_r, N_g) \text{ is feasible}) = 1, \\
\liminf_{N_g \to \infty} \text{Prob}(\Lambda(\lambda) = c'\lambda(N_c, N_r, N_g) \text{ is feasible}) < 1.
\]

Considering heterogeneous nodes in WMNs, two types of feasible throughput need to be defined: \( \lambda_R \) denotes the per-router throughput in the backbone communications and \( \lambda_C \) denotes the
per-client throughput of the infrastructure WMN. For per-client throughput $\lambda_c$, its asymptotic scaling depends on composition of internal and external traffic. Given a composition ratio $x$ of internal and external traffic, a corresponding asymptotic scaling for the per-client throughput is denoted as $\Lambda_x(\lambda_c)$. Accordingly, $\Lambda_I(\lambda_c)$ denotes the asymptotic scaling for the per-client throughput when only internal traffic exists in WMNs, while $\Lambda_E(\lambda_c)$ denotes the asymptotic scaling for the per-client throughput when only external traffic is available.

Definition 3: throughput capacity. The throughput capacity of WMNs, denoted as $C(\lambda_c)$, is defined as the maximum asymptotic scaling for the per-client throughput among all possibilities of traffic composition in WMNs, i.e., $C(\lambda_c) = \max_x(\Lambda_x(\lambda_c))$. Therefore, the throughput capacity of WMNs represents the maximum per-client throughput that WMN architecture offers and it is independent of the composition ratio of internal and external traffic.

2.3 Overview of the Analysis

Four regimes are defined as follows:

Regime I): $N_r = O(N_c / \log N_c)$ and $N_g = O(\sqrt{N_r})$;

Regime II): $N_r = O(N_c / \log N_c)$ and $N_g = o(\sqrt{N_r})$;

Regime III): $N_r = \omega(N_c / \log N_c)$ and $N_g = O(\sqrt{N_r})$;

Regime VI): $N_r = \omega(N_c / \log N_c)$ and $N_g = o(\sqrt{N_r})$.

Main result. 1) For internal traffic, the per-client throughput of the WMN achieves the following asymptotic scaling:
2) For external traffic, the per-client throughput of the WMN achieves the following asymptotic scaling:

\[ \Lambda_i(\lambda_c) = \Theta\left(\frac{N_e W}{N_c}\right), \text{ Regime I} \]  
\[ \Lambda_i(\lambda_c) = \Theta\left(\frac{W}{\sqrt{N_r \log N_c}}\right), \text{ Regime II} \]  
\[ \Lambda_i(\lambda_c) = \Theta\left(\frac{W}{N_r \log N_c}\right), \text{ Regime III} \]  
\[ \Lambda_i(\lambda_c) = \Theta\left(\frac{N_e W}{N_r \log N_c}\right), \text{ Regime IV} \]  

(2.1)

3) The per-client throughput capacity of the infrastructure WMN is achieved as follows:

\[ C(\lambda_c) = \Theta\left(\frac{N_e W}{N_c}\right), \text{ Regime I} \]  
\[ C(\lambda_c) = \Theta\left(\frac{W}{\sqrt{N_r \log N_c}}\right), \text{ Regime II} \]  
\[ C(\lambda_c) = \Theta\left(\frac{W}{N_r \log N_c}\right), \text{ Regime III} \]  
\[ C(\lambda_c) = \Theta\left(\frac{N_e W}{N_r \log N_c}\right) \text{ and } \Omega\left(\frac{N_e W}{N_c}\right), \text{ Regime IV} \]  

(2.2)

(2.3)

In the following, the overall methodology of our analysis is described. It is assumed that \( N_r = o(1) \), i.e. the number of mesh routers scales faster than a constant. The special case that \( N_r \) is a constant is not considered, since the same result can be derived with a much simpler approach. In what follows, all external traffic is assumed to be from mesh clients to Internet in the uplink direction since there is no difference in all derivation procedures and capacity results if assumed differently, i.e., all external traffic is in the downlink direction or external traffic is in...
both directions; in addition, the phrase “with high probability,” abbreviated as \( whp \), is used to stand for “with probability approaching 1 as \( N_c \) goes to infinity.”

In the first step, upper-bounds on per-router throughput of non-gateway traffic and gateway traffic in backbone communications, denoted by \( \lambda_{R_{-I}} \) and \( \lambda_{R_{-II}} \), respectively, are derived, i.e., \( \lambda_{R_{-I}} \leq c_s W / \sqrt{N_r} \) and \( \lambda_{R_{-II}} \leq N_g W / (N_r - N_g) \). And the upper-bound on \( \lambda_{R_{-II}} \) is valid for both gateway internal traffic and external traffic.

In the second step, upper-bounds on per-client throughput of non-gateway internal traffic, gateway internal traffic and external traffic, denoted by \( \lambda_{C_{-I}} \), \( \lambda_{C_{-II}} \) and \( \lambda_{C_{-III}} \), respectively, are obtained. Firstly, the following two results are derived: 1) There exists at least one mesh router that is connected by no less than \( N_c / N_r \) mesh clients; 2) In internal traffic, there exists at least one mesh client \( whp \), which is selected as the destination by more than \( c_s \log N_c \) mesh clients. Therefore, By the fact that the aggregate throughput of uplink local communications in each cell cannot exceed per-router throughput capacity of backbone communications, \((\lambda_{C_{-I}} N_c) / N_r \leq \lambda_{R_{-I}} \), \((\lambda_{C_{-II}} N_c) / N_r \leq \lambda_{R_{-II}} \) and \((\lambda_{C_{-III}} N_c) / N_r \leq \lambda_{R_{-II}} \) are obtained. Similarly, the aggregate throughput of downlink local communications in each cell cannot exceed per-router throughput capacity of backbone communications either, i.e., \( c_s \log N_c \lambda_{C_{-I}} \leq \lambda_{R_{-I}} \) and \( c_s \log N_c \lambda_{C_{-II}} \leq \lambda_{R_{-II}} \). For internal traffic, upper-bound on throughput can be sharpened by taking the smaller upper-bound of the one achieved in uplink and the one achieved in downlink. Incorporating such inequalities and substituting upper bounds on \( \lambda_{R_{-I}} \) and \( \lambda_{R_{-II}} \), upper-bounds on \( \lambda_{C_{-I}} \), \( \lambda_{C_{-II}} \) and \( \lambda_{C_{-III}} \) are obtained as follows:
By discussing $N_g$, an upper-bound on per-client throughput for external traffic is obtained as follows:

\[
\hat{\lambda}_c = O\left(\frac{N_g W}{N_c}\right),
\]  

(2.4)

which is valid for all 4 regimes.

For internal traffic, considering that $\hat{\lambda}_c \leq \lambda_{c,J} + \lambda_{c,H}$ and identifying the two scaling regimes of $N_r$, $\lambda_c$ is upper-bounded as follows

\[
\lambda_c \leq \frac{c_r W}{N_c} \sqrt{N_r} + \frac{N_g W}{(N_r - N_g) N_c / N_r}, \text{if } N_r = O(N_c / \log N_c);
\]  

(2.5)

\[\lambda_c \leq \sqrt{\frac{c_r W}{N_c} \log N_c} + \frac{N_g W}{(N_r - N_g) c_r \log N_c}, \text{if } N_r = o(N_c / \log N_c).\]  

(2.6)

The two scaling regimes in (5) and (6) can be further split into four by identifying the two scaling regimes of $N_g$. With these four regimes, the upper-bound achieves the same asymptotic scaling as that given in (1).

In the third step, the above upper-bounds are proved to be tight by constructing feasible throughput with the same asymptotic scaling. For internal traffic, in the constructed traffic model, all traffic is non-gateway traffic when $N_g = O(\sqrt{N_r})$ and all traffic is gateway traffic when
A TDMA transmission scheduling scheme is employed. Via TDMA, \( W_1, W_2, W_3 \) are assigned to uplink local, backbone, and downlink local communications respectively, and \( \sum_{i=1}^{3} W_i = W \). By the scheduling scheme, for each type of communications, capacity degradation due to interference can be bounded by a constant factor.

Let \( N^S \) denote the maximum number of mesh clients falling into any specific cell and \( N^D \) denote the maximum number of mesh clients whose randomly chosen destination clients fall into any specific cell. Upper-bounds with the same asymptotic scaling are achieved on \( N^S \) and \( N^D \), i.e., \( O(N_c / N_r) \) if \( N_r = O(N_c / \log N_c) \); \( O(\log N_c) \) if \( N_r = o(N_c / \log N_c) \). Therefore, the aggregate throughput of uplink and downlink local communications, denoted as \( N^S \lambda_c \) and \( N^D \lambda_c \), are both upper-bounded.

When \( N_g = O(\sqrt[N_r]{N_c}) \), it is demonstrated that, if each client has \( \alpha \) packets to send, then each router is the source and destination of at most \( h = \max(\alpha N^S, \alpha N^D) \) packets in backbone communications. Therefore, \( h \) is upper-bounded as follows: \( h = O(\alpha N_c / N_r) \) if \( N_r = O(N_c / \log N_c) \); \( h = O(\alpha \log N_c) \) if \( N_r = o(N_c / \log N_c) \). In order to determine the time needed for sending these \( h \) packets in the backbone, wireless mesh backbone is mapped to mesh-connected array in parallel computing. With a well-studied routing scheme in parallel computing, \( h \) packets can reach destination cells from source cells within time \( T = O(h \sqrt[N_r]{N_c} / W_2) \). By \( \lambda_c = \alpha / T \), a feasible per-client throughput of backbone communications is achieved as follows:

\[
\lambda_c = \Omega\left(\frac{\sqrt[N_r]{N_c} W_2}{N_c}\right), \text{ Regime I}) ;
\lambda_c = \Omega\left(\frac{W_2}{\sqrt[N_r]{N_c} \log N_c}\right), \text{ Regime III}).
\]
When $N_g = o(N_r)$, gateways are placed such that there are at most $c_{16}N_r/N_g$ mesh routers connecting to any gateway. Assuming each non-gateway mesh routers can reach its gateway by one hop, each router can achieve a feasible throughput $(N_gW_2)/(c_{16}N_r)$ in the backbone communications. Dividing by $N^S$, a feasible per-client throughput of uplink backbone communications is achieved as follows:

$$\lambda_c = \Omega\left(\frac{N_gW_2}{N_c}\right), \text{ Regime II};$$

$$\lambda_c = \Omega\left(\frac{N_gW_2}{N_c \log N_c}\right), \text{ Regime IV}).$$

(2.8)

It is shown that the above throughput can also be accommodated by downlink backbone communications. Thus it is a feasible per-client throughput of backbone communications.

Finally, assignment schemes are specified for $W_1$, $W_2$, and $W_3$ such that 1) $W_2 = \Theta(W)$; 2) $W_1$ and $W_3$ are large enough to accommodate the per-client throughput in (2.7) and (2.8) in both uplink and downlink local communications. Since the feasible per-client throughput has the same asymptotic scaling as the upper-bound in all four regimes, the asymptotic scaling for the per-client throughput for internal traffic of the infrastructure WMN as shown in (2.1) is derived. For external traffic, the procedures to construct a feasible throughput is very similar as those for internal traffic when $N_g = o(N_r)$. A feasible per-client throughput is achieved as the first line in (2.8) for both Regime I) and II); and a feasible per-client throughput is achieved as the second line in (2.8) for both Regime III) and IV). Therefore, the asymptotic scaling for the per-client throughput for external traffic of the WMN as shown in (2.2) is derived.

By the results in (2.1) and (2.2), the throughput capacity result in (2.3) can be easily obtained by using the formula $C(\lambda_c) = \max(\Lambda_{\text{i}}(\lambda_c), \Lambda_{\text{e}}(\lambda_c))$ in each regime. The reason is given
as follows. Let $\lambda_c^I$ and $\lambda_c^E$ denote the maximum per-client throughput that can be achieved for internal and external traffic, respectively. Let $\lambda_c^x$ denote the maximum per-client throughput that can be achieved for a specific composition ratio $x$ of internal and external traffic. Then the following inequality must be satisfied: $\max_x(\lambda_c^x) \leq \lambda_c^I + \lambda_c^E$. Thus,

$$\max_x(\Lambda_x(\lambda_c)) \leq \Lambda_I(\lambda_c) + \Lambda_E(\lambda_c) = \max(\Lambda_I(\lambda_c), \Lambda_E(\lambda_c))$$

It is obvious that $\max_x(\Lambda_x(\lambda_c)) \geq \max(\Lambda_I(\lambda_c), \Lambda_E(\lambda_c))$. Therefore, we have obtained:

$$C(\lambda_c) = \max(\Lambda_I(\lambda_c), \Lambda_E(\lambda_c)) = \max_x(\Lambda_x(\lambda_c)).$$

In what follows, the above procedures of obtaining (2.3) from (2.1) and (2.2) will not be repeated. And efforts will focus on achieving upper bounds and lower bounds of the per-client throughput for internal and external traffic, respectively.

2.4 Upper-Bounds

2.4.1 Background Results

Lemma 1 gives the necessary and sufficient conditions for connectedness \textit{whp} when $N_c$ identical nodes are placed at random in a unit square. The result is proved by using the result on the longest edge of the Euclidean minimal spanning tree [89]. Lemma 2 provides two formulas directly from Chernoff Bound [90, 91], which will be very useful in this paper to estimate the actual number of mesh clients under certain conditions.

**Lemma 1.** Assume that $N_c$ nodes are distributed uniformly and independently on a unit square. If $\rho(N_c)$ denotes the common transmission range for each node, then the following two statements hold: 1) If $\rho(N_c) = c \sqrt{\log N_c / \pi N_c}$, then the communication graph is
connected whp where \( c_1 > 1 \). 2) If \( \rho(N_c) = c_2 \sqrt{\log N_c / \pi N_c} \), then the communication graph is disconnected with whp where \( 0 \leq c_2 < 1 \).

**Proof.** The critical transmission range for connectedness is equal to the longest edge of the Euclidean minimal spanning tree (MST). Assuming that \( N_c \) nodes are uniformly and independently distributed in a unit square and \( M(N_c) \) denotes the length of the longest MST edge, Penrose [89] has proven the following result for \( M(N_c) \):

\[
\lim_{N_c \to \infty} P[N_c \pi M^2(N_c) - \log N_c \leq \beta] = \exp(-e^{-\beta}),
\]

(2.9)

where \( \beta \) is an arbitrary real value. Let \( \rho(N_c) \) denote the common transmission range for each node. If \( \rho(N_c) = c_1 \sqrt{\log N_c / \pi N_c} \) (\( c_1 > 1 \)), then

\[
N_c \pi \rho^2(N_c) = c_1^2 \log N_c = \log N_c + (c_1^2 - 1) \log N_c.
\]

By (2.9),

\[
\lim_{N_c \to \infty} P[N_c \pi M^2(N_c) - \log N_c \leq c_1^2 \log N_c] = \lim_{N_c \to \infty} \exp(-e^{-\lim_{N_c \to \infty} (c_1^2 - 1) \log N_c}) = 1
\]

Note that \( \beta = \lim_{N_c \to \infty} (c_1^2 - 1) \log N_c = +\infty \) since \( c_1 > 1 \). Thus, the critical transmission range is no larger than \( c_1 \sqrt{\log N_c / \pi N_c} \) (\( c_1 > 1 \)) whp. The above result is equal to the statement that the communication graph is connected whp, if \( \rho(N_c) = c_1 \sqrt{\log N_c / \pi N_c} \) (\( c_1 > 1 \)).

If \( \rho(N_c) = c_2 \sqrt{\log N_c / \pi N_c} \) (\( c_2 < 1 \)), the following formula is adopted, which can be obtained directly from (2.9):
\[
\lim_{N_c \to \infty} P\left[ N_c \pi M^2(N_c) - \log N_c > \beta \right] = 1 - \exp(-e^{-\beta}).
\]

Similarly, it can be proved that the critical transmission range is larger than
\[
c_2 \sqrt{\log N_c / \pi N_c} \quad (c_2 < 1)
\]
whp, which is equal to the statement that the communication graph is disconnected whp if \( \rho(N_c) = c_2 \sqrt{\log N_c / \pi N_c} \) \( (c_2 < 1) \).

**Lemma 2.** Let \( N \) be a random variable with binomial distribution \( B(N_c,p) \).

\[
\text{For any } \delta \in [0,1], \ P(N \leq (1-\delta)N_c p) \leq e^{-N_c p \frac{\delta^2}{2}}; \tag{2.10}
\]

\[
\text{For any } \delta \geq 0, \ P(N \geq (1+\delta)N_c p) \leq e^{-N_c p [(1+\delta) \ln(1+\delta) - \delta]}. \tag{2.11}
\]

### 2.4.2 Upper-Bounds on Per-router Throughput in Backbone Communications

In this section, upper-bounds on per-router throughput are obtained for non-gateway traffic and gateway traffic, respectively.

Without gateway support, routing in the mesh backbone network is equal to routing in a flat ad hoc network with \( N_r \) nodes on a regular grid. Therefore, following the similar derivations in [65] for capacity bounds of arbitrary ad hoc networks, the aggregate transport capacity of non-gateway traffic in backbone communications \( \lambda_{r,i} N_r \overline{L} \) is bounded as follows:

\[
\lambda_{r,i} N_r \overline{L} \leq c_3 W \sqrt{N_r}, \tag{2.12}
\]

where \( \lambda_{r,i} \) is the per-router throughput of non-gateway traffic in backbone communications, \( \overline{L} \) is the average routing distance of a bit in backbone communications, \( c_3 \) is a constant. In the following, Lemma 3 shows that \( \overline{L} \) is lower bounded by a constant value.
Lemma 3. The average routing distance of a bit in backbone communications, denoted as $\bar{L}$, is at least $\Lambda(2) - o(1)$, where $\Lambda(2)$ is a constant representing the mean distance between two randomly picked points in a unit square [94].

Proof. $|\cdot|$ is denoted as the Euclidean distance between two points on the region $R$. The triangle inequality gives

$$|X_i - X_j| \geq |X_i - Y_i| - |Y_i - X_j|,$$

as shown in Figure 2.2, where $X_j$ denotes the location of the destination client closest to the independently and uniformly chosen point located at $Y_i$ that the source client located at $X_i$ chooses to communicate with.
By Lemma 1, each mesh client is at most \( c_1 \sqrt{\log N_c / \pi N_c} \) away from its closest mesh client. Since \( X_j \) denotes the closest mesh client to \( Y_i \), the randomly picked location \( Y_i \) must be at most \( c_1 \sqrt{\log N_c / \pi N_c} / 2 \) away from \( X_j \), i.e. \( |Y_i - X_j| \leq c_1 \sqrt{\log N_c / \pi N_c} / 2 \). Thus,

\[
|X_i - X_j| \geq |X_j - Y_i| - c_1 \frac{\log N_c}{\pi N_c}.
\] (2.13)

As shown in Figure 2.2, the source client and the destination client are at most \( \sqrt{2} l_s / 2 \) away from their connected mesh routers respectively, which are located in the center of the cells. Thus,

\[
L \geq |X_i - X_j| - \sqrt{2} l_s,
\] (2.14)

where \( L \) is a random variable denoting the Euclidean distance between the router in the source cell and the router in the destination cell when each bit is transmitted in backbone communications. Substituting \( l_s = 1 / \sqrt{N_c} \) into (2.14) and incorporating (2.13) and (2.14) yield

\[
L \geq |X_i - Y_j| - c_1 \frac{\log N_c}{\pi N_c} - \frac{\sqrt{2}}{\sqrt{N_c}}.
\] (2.15)

Taking expectation on both sides of (2.15) yields

\[
\overline{L} \geq E(|X_i - Y_j|) - o(1).
\]

Note that \( \sqrt{2} / \sqrt{N_c} = o(1) \) since \( N_c = o(1) \). \( E(|X_i - Y_j|) \) is the mean distance between two randomly picked points in a unit square. It has been solved by a well studied problem known as “hypercube line picking” [94]. The expected distance between two randomly
picked points in a unit \(d\)-dimensional hypercube is denoted as \(\Lambda(d)\). \(\Lambda(d)\) is a constant value when \(d\) is known, e.g. \(\Lambda(2) \approx 0.5214\). Therefore, \(\overline{L} \geq \Lambda(2) - o(1)\).

Thus, an upper-bound on \(\lambda_{r,i}\) is obtained as follows:

\[
\hat{\lambda}_{r,i} \leq \frac{c_4 W}{\sqrt{N_r}},
\]

(2.16)

Here \(c_4 = c_3 / (\Lambda(2) - o(1))\).

In the case that all traffic goes through gateways, a mesh router with gateway function can always achieve abundant throughput in backbone communications since it is wired to Internet and the other gateways. As indicated in the Section 2.2.4, per-node throughput capacity is not the maximum throughput of a particular node but the maximum one that every node can achieve. Thus, bounds on per-router throughput capacity will be calculated on mesh routers without gateway function, i.e., non-gateway mesh routers. In this case, the following fact is always true: the aggregate throughput of all non-gateway mesh routers cannot exceed the aggregate throughput of all gateways. Therefore, the following inequality is obtained if \(N_g \leq N_r / 2\):

\[
\lambda_{r-II} \leq \frac{N_r W}{N_r - N_g},
\]

(2.17)

where \(\lambda_{r-II}\) denotes the per-router throughput of gateway traffic in backbone communications. If \(N_r / 2 \leq N_g < N_r\), \(\lambda_{r-II}\) is obviously upper bounded by \(W\) since at least one mesh router must use the wireless channel to access to gateway. If \(N_g = N_r\), \(\lambda_{r-II}\) is unbounded since every mesh router becomes a gateway. (2.17) is valid for both gateway internal traffic and external traffic.
2.4.3 Upper-Bound on Per-client Throughput

In the first step, an upper-bound on per-client throughput is obtained by counting the constraints from local communications.

**Lemma 4.** There exists at least one mesh router that is connected by no less than $N_c / N_r$ mesh clients.

**Proof.** Lemma 4 can be easily proved by contradiction. Assuming all mesh routers are connected by less than $N_c / N_r$ mesh clients, then the total number of mesh clients in the network will be less than $N_c$.

**Lemma 5.** There exists at least one mesh client whp, which is selected as the destination by more than $c_5 \log N_c$ mesh clients in internal traffic transmissions, where $c_5$ is a constant.

**Proof.** Lemma 1 indicates that there exists at least one mesh client whp, denoted as $X_k$, at the distance of $\tilde{r}(N_c)$ away from all the other mesh clients if $\tilde{r}(N_c) = c_2 \sqrt{\log N_c / \pi N_c}$ ($0 \leq c_2 < 1$). Therefore, a mesh client $X_i$ will choose $X_k$ as its destination if the randomly chosen location $Y_i$ falls into the disk of radius $\tilde{r}(N_c)/2$ centered at $X_k$. Let $N_i$ be the random variable denoting the number of mesh clients that choose $X_k$ as its destination. Then $N_i$ must be larger or equal than a random variable with binomial distribution $B(N_c, \pi r^2(N_c)/4)$, which is denoted as $N_k'$. Thus, the following inequality always holds for any arbitrary function $F$: $P(N_k \leq F) \leq P(N_k' \leq F)$. Since $N_c \pi r^2(N_c)/4 = c_5^2 \log N_c / 4$, using Chernoff Bound in (2.10) yields
The above probability goes to zero as $N_c \to \infty$. Thus $N_k > (1 - \delta)c_2^2 \log N_c / 4$ whp, i.e., there are more than $c_3 \log n$ senders choosing $X_k$ as the receiver, where $c_3$ is a constant and $c_3 = (1 - \delta)c_2^2 / 4$ ($0 \leq \delta \leq 1$).

**Proposition 1.** 1) For internal traffic, the per-client throughput is upper-bounded, i.e.

$$\lambda_c = O\left(\frac{N_c W}{N_c}\right), \text{ if } N_c = O\left(\frac{N_r}{\log N_c}\right);$$

$$\lambda_c = O\left(\frac{W}{\log N_c}\right), \text{ if } N_r = o\left(\frac{N_c}{\log N_c}\right).$$

(2.18)

2) For external traffic, the per-client throughput is upper-bounded, i.e.

$$\lambda_c = O\left(\frac{N_c W}{N_c}\right).$$

(2.19)

**Proof.** In local communications, all mesh clients have to compete for the wireless channel to communicate with mesh routers. And the capacity of the wireless channel is $W$. From Lemma 4, the inequality $N_c \lambda_c / N_c \leq W$ must satisfy for both uplink and downlink local communications. Thus, (2.19) is obtained for external traffic. From Lemma 5, the inequality $c_4 \log N_c \lambda_c \leq W$ must satisfy for downlink local communications for internal traffic. For internal traffic, since all uplink traffic will go through downlink communications finally, upper-bound on throughput can be sharpened by taking the smaller upper-bound of the one achieved in uplink and the one achieved in downlink. Therefore, per-client throughput for internal traffic can be upper-bounded by $\min(N_c W / N_c, W / (c_4 \log N_c))$. Finally (2.18) can be obtained by identifying the number of mesh routers in two scaling regimes.
Proposition 1 gives a very loose upper-bound on per-client throughput since in most cases the wireless channel of capacity $W$ is not only shared by all transmissions in local communications but also by those in backbone communications.

In the second step, it will be shown that the aggregate throughput of local communications in each cell is upper-bounded by per-router throughput capacity of backbone communications. $\lambda_{c\rightarrow l}$, $\lambda_{c\rightarrow II}$ and $\lambda_{c\rightarrow III}$ are defined as per-client throughput of non-gateway internal traffic, gateway internal traffic and external traffic in the WMN, respectively.

**Proposition 2.** The per-client throughput of non-gateway internal traffic is upper-bounded, i.e.

\[
\begin{align*}
\lambda_{c\rightarrow l} &= O\left(\frac{\sqrt{N_r} W}{N_c}\right), \text{ if } N_r = O\left(\frac{N_c}{\log N_c}\right); \\
\lambda_{c\rightarrow l} &= O\left(\frac{W}{\sqrt{N_r \log N_c}}\right), \text{ if } N_r = o\left(\frac{N_c}{\log N_c}\right).
\end{align*}
\]  

(2.20)

**Proof.** In the infrastructure WMN, traffic from uplink local communications doesn’t go to backbone communications if a sender and its receiver happen to be in the same cell. It is shown as follows that this amount of traffic is negligible. Recall that in internal traffic the destination for each mesh client is independently chosen as the mesh client nearest to a randomly located point. The randomly located point is possibly outside the cell where the destination locates. By Lemma 1, it is at most $c_i \sqrt{\log N_c / \pi N_c} / 2$ away from the boundary of the cell. Therefore, the probability that a receiver is chosen in the same cell as its sender is smaller than $1/N_r^*$, which is given as follows:

\[
\frac{1}{N_r^*} = \left(\frac{1}{\sqrt{N_r}} + c_i \sqrt{\frac{\log N_c}{\pi N_c}}\right)^2
\]

(2.21)

Note that it scales as $o(1)$ since $N_r = o(1)$. Thus, by Lemma 4, there exists at least one cell whose aggregate throughput of non-gateway internal traffic going from uplink local
communications to backbone communications can be lower-bounded by 
\((1 - o(1))N_c \lambda_{c,I} / N_r\) \(\text{whp}\). In addition, it can not exceed per-router throughput capacity of 
non-gateway traffic in backbone communications, i.e.

\[ (1 - o(1)) \frac{N_c}{N_r} \lambda_{c,I} \leq \lambda_{r,I} \]  \hspace{1cm} (2.22)

Substituting (2.16) into (2.22) yields

\[ \lambda_{c,I} \leq \frac{c W \sqrt{N_c}}{N_r}. \] \hspace{1cm} (2.23)

In the infrastructure WMN, a mesh client receives all the traffic from its 
connected mesh router through downlink local communications. It will be shown as 
follows that most of the traffic sent by the mesh router to the mesh client is previously 
received from backbone communications, and this amount of traffic must be 
accommodated by the per-router throughput capacity of backbone communications.

The probability that a sender locates at the same cell as its receiver is \(1/N_r\). From 
Lemma 5, there exists at least one mesh client \(\text{whp}\), which is selected as the destination by 
more than \(c_s \log N_c\) mesh clients in internal traffic. Therefore, among all the senders, there 
are more than \((1 - 1/N_r)c_s \log N_c\) senders \(\text{whp}\) whose traffic must go through backbone 
communications and all such traffic is sent to the mesh client by its connected mesh router. 
Thus, the above traffic should be accommodated by the per-router throughput capacity of 
backbone communications, i.e.

\[ (1 - o(1))c_s \log N_c \lambda_{c,I} \leq \lambda_{r,I} \] \hspace{1cm} (2.24)

Note that \(1/N_r\) scales as \(o(1)\) since \(N_r = o(1)\). Substituting (2.16) into (2.24) yields
here \( c_6 \) is a constant and \( c_6 = c_4 / c_5 \).

Combining (2.23) and (2.25), \( \lambda_{c_{-I}} \) can be upper-bounded as follows:

\[
\lambda_{c_{-I}} \leq \min \left( \frac{c_4 W \sqrt{N_r}}{N_c}, \frac{c_6 W}{\sqrt{N_r} \log N_c} \right).
\]

Finally the result can be obtained by identifying the number of mesh routers in two scaling regimes.

Similarly, an upper-bound on the per-client throughput of gateway internal traffic is obtained as

\[
\lambda_{c_{-II}} \leq \min \left( \frac{N_g W}{(N_r - N_g) N_c / N_r}, \frac{N_g W}{(N_r - N_g) c_3 \log N_c} \right).
\]

By identifying the number of mesh routers in two scaling regimes, Proposition 3 is obtained as follows.

**Proposition 3.** If \( N_g < N_r \), then the per-client throughput of gateway internal traffic is upper-

\[
\begin{align*}
\lambda_{c_{-II}} &= O \left( \frac{N_g W}{(N_r - N_g) N_c / N_r} \right), & \text{if } N_r = O \left( \frac{N_c}{\log N_c} \right); \\
\lambda_{c_{-II}} &= O \left( \frac{N_g W}{(N_r - N_g) \log N_c} \right), & \text{if } N_r = \omega \left( \frac{N_c}{\log N_c} \right).
\end{align*}
\]

(2.26)

Similarly, an upper-bound on \( \lambda_{c_{-III}} \) is obtain in Proposition 4 by using the result from

Lemma 4. Note that Lemma 5 is not valid for external traffic.
**Proposition 4.** If \( N_g < N_r \), then the per-client throughput of external traffic is upper-bounded as follows:

\[
\lambda_{c_{-\text{III}}} = \Theta\left(\frac{N_g W}{(N_r - N_g)N_c / N_r}\right). \tag{2.27}
\]

Finally, upper bounds on the per-client throughput capacity for both internal and external traffic of the infrastructure WMN are achieved in Corollary 1.

**Corollary 1.** 1) The per-client throughput for internal traffic satisfies

\[
\lambda_c = \Theta\left(\frac{N_r W}{N_c}\right), \text{ Regime I};
\]

\[
\lambda_c = \Theta\left(\frac{N_r W}{\sqrt{N_r} \log N_c}\right), \text{ Regime II};
\]

\[
\lambda_c = \Theta\left(\frac{N_g W}{N_c \log N_c}\right), \text{ Regime III};
\]

\[
\lambda_c = \Theta\left(\frac{N_g W}{N_r \log N_r}\right), \text{ Regime IV}).
\]

2) The per-client throughput for external traffic satisfies

\[
\lambda_c = \Theta\left(\frac{N_r W}{N_c}\right). \tag{2.28}
\]

**Proof.** The per-client throughput for internal traffic of the WMN is obviously upper-bounded by the sum of the per-client throughput of non-gateway internal traffic and gateway internal traffic, i.e., \( \lambda_c \leq \lambda_{c_{-\text{I}}} + \lambda_{c_{-\text{II}}} \). Upper-bounds on \( \lambda_{c_{-\text{I}}} \) and \( \lambda_{c_{-\text{II}}} \) are obtained in Proposition 2 and 3. Therefore, the per-client throughput capacity can be upper-bounded. More specifically, if \( N_r = \Theta(N_c / \log N_c) \) and \( N_g < N_r \),

\[
\lambda_c \leq \frac{c_s W \sqrt{N_r}}{N_c} + \frac{N_g W}{(N_r - N_g)N_c / N_r}. \tag{2.30}
\]
By identifying the number of gateways in two scaling regimes, results in regime I) and II) in Corollary 1 can be achieved except the case when $N_g = N_r$. If all mesh routers are gateways, i.e., $N_g = N_r$, per-client throughput is only constrained by local communications. In this case, upper-bound achieved in Proposition 1 is adopted, i.e. $\lambda_c = O(N_r W / N_c) = O(N_g W / N_c)$, if $N_r = N_g$. Hence, result in regime II) is obtained for all the cases. Similarly, if $N_r = O(N_c / \log N_r)$ and $N_g < N_r$, 

$$\lambda_c \leq \frac{c_g W}{\sqrt{N_r \log N_c}} + \frac{N_g W}{(N_r - N_g)c_s \log N_c}$$

(2.31)

By identifying the number of gateways in two scaling regimes and incorporating corresponding result in Proposition 1, results in regime III) and VI) can be achieved. Finally, by the same discussion on $N_g$, (2.29) can be easily obtained from (2.19) and (2.27).

2.5 Constructive Lower-Bounds

In this section, feasible per-client throughput is achieved by construction, which has the same asymptotic scaling as the upper-bound displayed in Corollary 1. Hence the upper-bound is proved to be tight.

A specific traffic model is constructed for internal traffic as follows: 1) each mesh client sends $\alpha$ packets to its randomly chosen destination client at $\lambda_c$ bits/s; 2) when $N_g = O(\sqrt{N_r})$, all traffic will be routed in backbone purely by mesh routers without gateway support; 3) when $N_g = \omega(\sqrt{N_r})$, all traffic will be routed through gateways. For external traffic, the procedures to construct a feasible throughput have no difference from those for internal traffic when assuming
all traffic is gateway traffic. Hence, we will mainly focus on the case of internal traffic and the case of external traffic will be discussed only when necessarily.

Subsection 2.5.1 will obtain upper-bounds on the number of senders and receivers in any cell. Specific gateway placement, scheduling, routing will be discussed in subsection 2.5.2 and bandwidth assignment schemes will be proposed in subsection 2.5.3.

2.5.1 Local Communications in Each Cell

Let $N_i^S$ denotes the maximum number of mesh clients falling into any specific cell. Then the aggregated throughput of uplink local communications in each cell cannot exceed $\lambda_c N^S$ bits/s for both internal and external traffic. By Chernoff Bound, Lemma 6 shows that $N^S$ can be upper-bounded.

**Lemma 6.** Let $N_i^S$ be the random variable denoting the number of mesh clients falling into the $i$-th cell. And $N^S = \max_i(N_i^S)$. If $N_r = O(N_c / \log N_c)$, then $N^S < c_N N_c / N_r$ whp; If $N_r = o(N_c / \log N_c)$, then $N^S < c_0 \log N_c$ whp, where $c_N$, $c_0$ are both positive constants.

**Proof.** In the model a unit square is split into $N_c$ equivalent small squares and $N_c$ mesh clients are uniformly and independently distributed in the unit square. Therefore, $N_i^S$ is a random variable with binomial distribution $B(N_c,1/N_c)$. Using Chernoff Bound in (2.11) yields

$$P(N_i^S \geq (1+\delta)N_i^S/N_r) \leq e^{-N_r \frac{N_i^S ((1+\delta)\log(1+\delta)-\delta)}{}}$$

Let $\delta = 1$,
Applying the union bound gives the probability that any of the \(N_r\) cells has less than \(2N_c/N_r\) mesh clients:

\[
P(N_i^S < 2\frac{N_c}{N_r}, \forall i) = 1 - P(N_i^S \geq 2\frac{N_c}{N_r}, \exists i) \\
\geq 1 - N_r e^{-(2\log 2 - 1)\frac{N_c}{N_r}}.
\]

Assuming \(N_r \leq c_{10} N_c / \log N_c\) for sufficient large \(N_c\) yields

\[
P(N_i^S < 2\frac{N_c}{N_r}, \forall i) \geq 1 - c_{10} \frac{N_c}{\log N_c \cdot N_c^{2\log 2 - 1}}
\]

where \(c_{10}\) is a positive constant. Thus, for any positive constant \(c_{10} \leq 2 \log 2 - 1\),

\[
\lim_{N_r \to \infty} P(N_i^S < 2\frac{N_c}{N_r}, \forall i) = 1
\]

when \(N_r \leq c_{10} N_c / \log N_c\) for sufficient large \(N_c\), i.e. \(N_r = O(N_c / \log N_c)\).

If \(N_r = \Omega(N_c / \log N_c)\), there exists a positive constant \(c_{11}\) such that \(c_{11} \log N_c \geq 2N_c / N_r\) for sufficient large \(N_c\). It implies that the following inequality always holds for any \(N_r\), when \(N_r = \Omega(N_c / \log N_c)\):

\[
P(N_i^S < c_{11} \log N_c, \forall i) \geq P(N_i^S < 2\frac{N_c}{N_r}, \forall i).
\]

Thus,

\[
\lim_{N_r \to \infty} P(N_i^S < c_{11} \log N_c, \forall i) \geq \lim_{N_r \to \infty, N_r = \Omega(N_c / \log N_c)} P(N_i^S < 2\frac{N_c}{N_r}, \forall i).
\]
Indicated by (2.33), the right side of the above inequality equals to 1 for any positive constant $c_{10} \leq 2\log 2 - 1$. Thus,

$$
\lim_{N_r \to \infty} P(N_r^s < c_{11} \log N_r, \forall i) = 1
$$

when $N_r = o(N_c / \log N_c)$.

For internal traffic, let $N^D_i$ denotes the maximum number of mesh clients whose randomly chosen destination clients falling into any specific cell. Then the aggregated throughput of downlink local communications in each cell cannot exceed $\lambda_c N^D$ bits/s. In the following lemma 7 will show that $N^D$ can be upper-bounded.

**Lemma 7.** Let $N^D_i$ be the random variable denoting the number of mesh clients whose randomly chosen destination clients fall into the $i$-th cell. And $N^D = \max_i (N^D_i)$. If $N_r = O(N_c / \log N_c)$, then $N^D < c_{12} N_c / N_r$ whp; if $N_r = o(N_c / \log N_c)$, then $N^D < c_{13} \log N_c$ whp, where $c_{12}$, $c_{13}$ are both positive constants.

**Proof.** Recall the destination picking scheme defined in the model: a source client first pick a random location in the unit square; the destination is then chosen as the closest mesh client to the random point. In the proof of Proposition 2, it is known that the randomly picked point is possibly outside the cell where the destination client locates. But it is at most $c_{i} \sqrt{\log N_c / \pi N_c} / 2$ away from the boundary of the cell. A binomial distributed random variable $N^D_i$ with $B(N_c, 1/N_r)$ is constructed, where $1/N_r$ is given in (2.21).

It is easy to see that $N^D_i \geq N^D_i$ for any $i$. Thus, the following inequality always holds for any arbitrary function $F$:

$$
P(N^D_i < F, \forall i) \geq P(N^D_i < F, \forall i).
$$
The method obtaining the upper-bound on $N_i^0$ is not much different from that in the proof of Lemma 6. Thus we skip several steps and obtain a similar formula as (2.32):

$$P(N_i^0 < 2 \frac{N_c}{N_r}, \forall i) \geq P(N_i^0 < 2 \frac{N_c}{N_r}, \forall i) \geq 1 - N_r e^{-\frac{(2 \log 2 - 1) N_r}{N_c}}.$$ 

If $N_r \leq c_{14} N_c / \log N_c$ for sufficient large $N_c$, observing the equation in (2.21) yields the following equation for $1 / N_r$:

$$\frac{1}{N_r} = c_{15} + o\left(\frac{1}{N_r}\right),$$

where $c_{14}$ and $c_{15}$ are two positive constants and $c_{15} \geq 1$. Thus,

$$P(N_i^0 < 2 c_{15} \frac{N_c}{N_r} + o\left(\frac{N_c}{N_r}\right), \forall i) \geq 1 - N_r e^{-\frac{(2 \log 2 - 1) c_{15} N_c}{N_r} + o\left(\frac{N_c}{N_r}\right)} \geq 1 - c_{14} \frac{N_r}{\log N_c} e^{-\frac{(2 \log 2 - 1) c_{15} N_r}{N_c}} \geq 1 - c_{14} \frac{N_c}{\log N_c} e^{-\frac{(2 \log 2 - 1) c_{15} N_r}{N_c}}.$$ 

If $c_{14} \leq c_{15} (2 \log 2 - 1)$, then

$$\lim_{N_r \to \infty} P(N_i^0 < c_{12} \frac{N_c}{N_r}, \forall i) = 1.$$ 

when $N_r = O(N_c / \log N_c)$.

If $N_r = \Omega(N_c / \log N_c)$, there exists a positive constant $c_{13}$ such that $c_{13} \log N_c \geq c_{12} N_c / N_r$ for sufficient large $N_c$. Applying the similar method as that in the proof of Lemma 6 yields
\[
\lim_{{N_r \to \infty}} P(N_i^D < c_{14} \log N_c, \forall i) \geq \lim_{{N_r \to \infty}} \frac{N_i}{N_r} P(N_i^D \leq c_{12} \log N_c, \forall i) = 1,
\]

when \( c_{14} \leq c_{15}(2\log 2 - 1) \). Thus,

\[
\lim_{{N_r \to \infty}} P(N_i^D < c_{13} \log N_c, \forall i) = 1
\]

when \( N_r = o(N_c / \log N_c) \).

2.5.2 Feasible Per-client Throughput in Backbone Communications

In this section, it is shown that every mesh client is guaranteed to achieve a throughput in backbone communications. The following two cases are considered separately for internal traffic:

1) when \( N_g = O(\sqrt{N_r}) \); 2) when \( N_g = \omega(\sqrt{N_r}) \).

2.5.2.1 In the Case When \( N_g = O(\sqrt{N_r}) \)

A feasible per-client throughput is achieved by assuming all traffic is routed purely by mesh routers without gateway support. It is realized by a TDMA scheduling and a routing algorithm in parallel computing.

A TDMA scheduling scheme is applied and guarantees successful transmissions for all the nodes in the network. In the first step, separate time slots are assigned to different types of communications so that they do not interfere with each other, i.e., uplink local, backbone, and downlink local communications can transmit at \( W_1, W_2, \) and \( W_3 \) bits/s, respectively, subject to two constraints:

\[
\sum_{i=1}^{3} W_i = W \quad \text{and} \quad W_1 + W_3 \leq W',
\]

where each mesh router and mesh client can transmit at \( W \) bits/s and \( W' \) bits/s, respectively. In order to assign different values to \( W_1, W_2, \) and \( W_3 \), the time slot for uplink local, backbone, and downlink local communications must have different length.
In the second step, each type of communications divides its assigned time slot into small-slots, and each cell is assigned such a small-slot so that simultaneous transmissions can only be carried out in cells that have enough distance in between. As shown in Figure 2.4, simultaneous transmissions can only exist in cells that are \((k_i - 1)\) cells apart. Hence, in backbone communications, each mesh router can only have one small-slot every \(k_i^2\) small-slots. In local communications, the small-slot is further split into separate mini-slots. Assigned a different mini-slot, each mesh client is guaranteed to have successful transmission or reception.

In section 2.2.3, it is assumed that the transmission range of uplink local, downlink local, and backbone communications is \(r_u\), \(r_d\), and \(r_b\), respectively. By choosing these transmission ranges as the linear function of \(l_s\), i.e., \(r_s = c_s l_s\), Lemma 8 will show that \(k_i\) can be an integer depending only on \(\Delta\), here \(l_s\) denotes the side-length of a cell and \(c_s\) represents a different constant corresponding to a different transmission range. For example, \(r_u = r_d = \sqrt{2} l_s / 2\) and \(r_b = l_s\), which are large enough to guarantee the network topology of infrastructure WMNs. 

![Figure 2.3: TDMA Scheduling (here \(k_i = 5\)](image)
Lemma 8: \( k_i \) can be an integer depending only on \( \Delta \).

Proof: As shown in Figure 2.3, let two transmitting nodes \( S_i \) and \( S_k \) in two cells having \((k_i-1)\) cells apart from each other and let \( S_j \) be the receiver of \( S_i \). In the model, the location of the receiver \( S_j \) is either within the same cell as \( S_i \) in the case of local communications or in the center of the neighboring cells of \( S_i \) in the case of backbone communications. Thus, \( |S_k - S_j| \geq (k_i - 1)l_s \). According to the Protocol Model, if \( S_i \) and \( S_k \) can transmit simultaneously in the same channel, then \( |S_k - S_j| \geq (1 + \Delta)c_i l_s \) needs to be satisfied. Thus, it is required that \((k_i - 1)l_s \geq (1 + \Delta)c_i l_s \), i.e., \( k_i \geq c_i (1 + \Delta) + 1 \).

Choosing \( k_i = \lceil c_i (1 + \Delta) + 1 \rceil \) yields the result. \( \lceil x \rceil \) denotes the smallest integer larger than \( x \).

For simplicity, we choose \( k_i = \max \lceil c_i (1 + \Delta) + 1 \rceil \) so that \( k_i \) represents an integer of a constant value in the following of the paper. Lemma 8 implies that, for each type of communications, capacity degradation due to interference can be bounded by a constant factor, i.e., each cell is guaranteed to have \( W_1 / k_i^2 \) bits/s and \( W_3 / k_i^2 \) bits/s for uplink and downlink local communications, respectively; and each mesh router is guaranteed to have \( W_2 / k_i^2 \) bits/s for backbone communications.

By using a similar methodology introduced by Kulkarni and Viswanath in [67], a well-studied routing algorithm in parallel computing is adopted to achieve a feasible per-client throughput in backbone communications.

Proposition 5. The per-client throughput of backbone communications can achieve whp

\[
\lambda_c = \Omega\left(\frac{W_2}{\sqrt{N_c} \log N_c}\right), \text{ Regime III).}
\]

\[
\lambda_c = \Omega\left(\frac{W_2}{\sqrt{N_c} W_2}\right), \text{ Regime I).}
\]
Proof. In an \( l_M \times \ldots \times l_M \) mesh-connected array, \( l_M^d \) identical processors denoted as PUs are connected by a regular \( d \)-dimensional grid of bi-directional communication links. Each PU is directly connected to its nearest neighbors only. For \( h \times h \) routing each PU is the source and destination of at most \( h \) packets. If each PU can transmit at 1 packet/s, \( h \times h \) routing on \( d \)-dimensional meshes can be solved within \( hl_M/2 + o(hl_M) \) seconds when each PU has a buffer of \( h \) packets [92].

By considering mesh routers as PUs and assuming that \( r_p = l_s \), the mesh backbone is immediately mapped to the mesh-connected array in parallel computing. Here \( l_M = 1/l_s = \sqrt{N_r} \). In the assumption of internal traffic, each mesh client sends \( \alpha \) packets to its randomly chosen destination client at \( \lambda_c \) bits/s. By the discussion in Section 2.5.1 and Section 2.5.2, it is known that each mesh router is a source of at most \( \alpha N_s \) packets and a destination of at most \( \alpha N_d \) packets in backbone communications. \( N_s \) and \( N_d \) are upper-bounded in Lemma 6 and Lemma 7 respectively. Therefore, each mesh router is the source and destination of at most \( h \) packets in backbone communications. And \( h \) is indicated as follows:

\[
\begin{align*}
    h = \alpha c_{17} \frac{N_c}{N_r} \text{ packets whp, if } N_r = O\left( \frac{N_c}{\log N_c} \right), \\
    h = \alpha c_{18} \log N_c \text{ packets whp, if } N_r = o\left( \frac{N_c}{\log N_c} \right),
\end{align*}
\]

where \( c_{17} = \max(c_8, c_{12}) \) and \( c_{18} = \max(c_9, c_{13}) \).

Notice that there are still differences between the wireless mesh backbone network and the parallel computing network: 1) In the parallel computing network all the PUs can successfully transmit packets simultaneously since PUs are connected by wired links. However, by the transmission scheduling scheme, only one of the \( k_i^2 \) wireless
mesh routers can transmit packets successfully in one time slot; 2) In parallel computing each PU can simultaneously communicate with its 4 neighbors bi-directionally. However, in our mesh backbone network, transmission occurs from one mesh router to the other in one direction. Totally there is a factor of 8 here. Indicated by the above discussions, the solving time $T$ of $h \times h$ routing in our wireless mesh backbone network is lengthened by a constant factor of $8k_1^2$, which is bounded by the following inequality:

$$T \leq 8k_1^2 \cdot \left[ \frac{hl_M}{2} + o(hl_M) \right].$$  \hspace{1cm} (2.36)

Assume that each packet has $c_{19}$ bits ($c_{19} \geq 1$). Substituting $l_M = \sqrt{N_r}$ into (2.36) and normalizing $T$ by $(W_2/c_{19})$ yield

$$T' = \frac{c_{19}T}{W_2} \leq \frac{4k_1^2c_{19}h\sqrt{N_r}}{W_2}.$$  \hspace{1cm} (2.37)

Notice that $o(hl_M)$ can be neglected since the results will be expressed in the manner of asymptotic scaling. Substituting (2.35) into (2.37) yields

$$T' \leq \frac{4k_1^2\alpha c_1c_{19}N_c}{\sqrt{N_rW_2}} \text{whp, if } N_r = O\left(\frac{N_c}{\log N_c}\right);$$

$$T' \leq \frac{4k_1^2\alpha c_{is}c_{19}\log N_c\sqrt{N_r}}{W_2} \text{whp, if } N_r = o\left(\frac{N_c}{\log N_c}\right).$$

Since $\lambda_c = \alpha / T'$, the per-client throughput in backbone communications can achieve whp

$$\lambda_c \geq \frac{\sqrt{N_rW_2}}{4k_1^2c_1c_{19}N_c}, \text{ Regime I);}$$

$$\lambda_c \geq \frac{W_2}{4k_1^2c_{is}c_{19}\log N_c\sqrt{N_r}}, \text{ Regime III).}$$  \hspace{1cm} (2.38)

2.5.2.2 In the Case When $N_g = o(\sqrt{N_r})$
A specific gateway placement scheme is proposed at first. Similar transmission scheduling scheme as case 1) is then adopted. Finally a feasible per-client throughput in backbone communications is achieved by specifying a simple routing scheme.

$N_g$ gateways are placed in the network as shown in Figure 2.4. Let $\lfloor x \rfloor$ denote the largest integer smaller than $x$ and $\lceil x \rceil$ denote the smallest integer larger than $x$. First, the network is split evenly into $\left(\lfloor \sqrt{N_g} \rfloor\right)^2$ small squares and a gateway is placed on one of the mesh routers inside each small square. Then the rest $N_g - \left(\lfloor \sqrt{N_g} \rfloor\right)^2$ gateways are placed arbitrarily on the others mesh routers. With the above gateway placement scheme, there is at least one gateway in each small square of side-length $1/\lfloor \sqrt{N_g} \rfloor$. All the non-gateway mesh routers inside a specific small square are connected to the gateways by one-hop in the same square. If a mesh router is on
the boundary of different squares, it can connect to any gateway in the above squares. Lemma 9 shows that the number of mesh routers connected to any gateway can be upper-bounded.

**Lemma 9:** The number of mesh routers connected to any gateway cannot exceed \( c_{16} N_r / N_g \).

**Proof:** By the gateway placement scheme, any gateway is connected by at most \( \left\lfloor \sqrt{N_r} / \left\lfloor \sqrt{N_g} \right\rfloor \right\rfloor^2 \) mesh routers. And \( \left\lfloor \sqrt{N_r} / \left\lfloor \sqrt{N_g} \right\rfloor \right\rfloor^2 \) can be upper-bounded in the following:

\[
\left( \frac{\sqrt{N_r}}{\sqrt{N_g}} \right)^2 \leq \left( \frac{\sqrt{N_r}}{\sqrt{N_g} - 1} + 1 \right)^2
= \left( \frac{\sqrt{N_r}}{\sqrt{N_g} - 1} + \frac{\sqrt{N_g}}{\sqrt{N_r}} \right)^2 \frac{N_r}{N_g},
\]

where \( \sqrt{N_g} / (\sqrt{N_g} - 1) \) is a decreasing function of \( N_g \) and is upper-bounded by \( \sqrt{2} / (\sqrt{2} - 1) \) if \( N_g \geq 2 \); and \( \sqrt{N_g} / \sqrt{N_r} \) is upper-bounded by 1. When \( N_g = 1 \), there are \((N_r - 1)\) mesh routers connected to the only gateway. Thus, the number of mesh routers connected to any gateway cannot exceed \( c_{16} N_r / N_g \), here \( c_{16} = (\sqrt{2} / (\sqrt{2} - 1) + 1)^2 \).

A very similar TDMA scheduling scheme is applied. The only difference is that small-slots of backbone communication are first assigned to gateways instead of non-gateway mesh routers in the case 1). Each gateway is assigned such a small-slot so that simultaneous transmissions can only be carried out on gateways that have enough distance in between. By mapping the small square of side-length \( 1 / \left\lfloor \sqrt{N_g} \right\rfloor \) in Figure 2.3 to the cell in Figure 2.5, it can be shown with the similar procedures in Lemma 8 that in backbone communications, each gateway can only have one small-slot every \( k_i \) small-slots and \( k_i \) can be an integer depending only on \( \Delta \). The small-slot will be split equally for uplink and downlink transmissions in backbone.
Therefore, each gateway can accommodate $W_2/(2k_2^2)$ aggregate throughput in either uplink or downlink transmissions. Since a mesh router can access to its connected gateway by one-hop, each gateway services its connected mesh routers by a round-robin fashion.

It is shown in Proposition 6 that a feasible per-client throughput in backbone communications can be achieved whp in case 2).

**Proposition 6.** The per-client throughput of backbone communications can achieve whp

\[
\lambda_c = \Omega\left(\frac{N_g W_2}{N_c}\right), \text{Regime II};
\]

\[
\lambda_c = \Omega\left(\frac{N_g W_2}{N_r \log N_c}\right), \text{Regime IV}).
\]

**Proof.** Since the number of mesh routers per gateway is upper-bounded by Lemma 9, the per-router throughput can achieve at least $N_g W_2/(2c_{10} k_2^2 N_r)$ in the uplink transmissions of backbone communications. A feasible per-client throughput in the uplink transmissions of backbone communications is thus obtained as $N_g W_2/(2c_{10} k_2^2 N_r N^s)$. Substituting upper-bound on $N^s$ in Lemma 6, we obtain

\[
\lambda_c \geq \frac{N_g W_2}{2c_{10} c_6 k_2^2 N_r N_c}, \text{Regime II};
\]

\[
\lambda_c \geq \frac{N_g W_2}{2c_{10} c_6 k_2^2 N_r \log N_c}, \text{Regime IV}).
\]

In the following we will show that the per-client throughput in (2.40) can be accommodated in the downlink transmissions of backbone communications whp. It is equal to show the following inequality holds whp:

\[
\frac{W_2}{2k_2^2} \geq \frac{c_{10} N_r}{N_g} N_0 \lambda_c
\]
Substituting upper-bound on $N^D$ in Lemma 7 and combining with (2.40), we prove the above proposition.

Combining Proposition 5 and 6, a feasible per-client throughput of backbone communications is achieved by construction. Notice that it has the same asymptotic scaling in all 4 regimes as the upper-bound displayed in (1) of Corollary 1. The difference is this throughput is constrained by $W_2$, the bandwidth assigned to backbone communications.

2.5.3 Feasible Per-client Throughput of the WMN

It will be shown in this section that with proposed assignment schemes for $W_1$, $W_2$ and $W_3$, a feasible per-client throughput $\lambda_c^*$ can be achieved whp in all 3 types of communications.

2.5.3.1 In the Case When $N_g = O(\sqrt{N_r})$

When $N_g = O(\sqrt{N_r})$, the proposed assignment scheme is given as follows:

\[ W_1 = W_2/(4 \sqrt{N_r}) \text{ and } W_1 = W_3, \text{ subject to } \sum_{i=1}^{3} W_i = W \text{ and } W_1 + W_3 \leq W'. \]

Thus,

\[ W_2 = \frac{2 \sqrt{N_r} W}{2 \sqrt{N_r} + 1}; \]

\[ W_1 = W_3 = \frac{W}{2(2 \sqrt{N_r} + 1)}; \]

\[ W' \geq \frac{W}{2 \sqrt{N_r} + 1}. \]

Substituting (2.41) into (2.38) yields a feasible per-client throughput $\lambda_c^*$ in terms of $W$ in regime I) and III):
\[
\lambda_c^* = \frac{N \cdot W}{2k_1^2 c_{i7} c_{i9} N_r (2\sqrt{N_r} + 1)}, \text{ Regime I};
\]
\[
\lambda_c^* = \frac{W}{2k_1^2 c_{i8} c_{i9} \log N_r (2\sqrt{N_r} + 1)}, \text{ Regime III}).
\]

This is achieved \textit{whp} in backbone communications. It will be shown next that \(\lambda_c^*\) can be accommodated in both uplink and downlink local communications.

If each client transmits at \(\lambda_c^*\), then the aggregated uplink throughput in each cell can not exceed \(\lambda_c^* N^S\). Recall that \(N^S\) denotes the maximum number of mesh clients falling into any specific cell, which is upper-bounded in Lemma 6. Thus,

\[
\lambda_c^* N^S \leq \frac{c_{i7} W}{2k_1^2 c_{i7} c_{i9} (2\sqrt{N_r} + 1)} \text{ whp, Regime I;}
\]
\[
\lambda_c^* N^S \leq \frac{c_{i8} W}{2k_1^2 c_{i8} c_{i9} (2\sqrt{N_r} + 1)} \text{ whp, Regime III}).
\]

Recall that \(c_{i9}\) denotes the number of bits in each packet. Applying the fact that \(c_{i9} \geq 1\), \(c_{i7} = \max(c_{i8}, c_{i17})\), and \(c_{i8} = \max(c_{i9}, c_{i18})\) yields

\[
\lambda_c^* N^S \leq \frac{W}{2k_1^2 (2\sqrt{N_r} + 1)} \text{ whp, Regime I;}
\]
\[
\lambda_c^* N^S \leq \frac{W}{2k_1^2 (2\sqrt{N_r} + 1)} \text{ whp, Regime III}).
\]

Similarly, the aggregated downlink throughput in each cell can not exceed \(\lambda_c^* N^D\), and \(N^D\) has an upper-bound in Lemma 7. Thus,

\[
\lambda_c^* N^D \leq \frac{W}{2k_1^2 (2\sqrt{N_r} + 1)} \text{ whp, Regime I;}
\]
\[
\lambda_c^* N^D \leq \frac{W}{2k_1^2 (2\sqrt{N_r} + 1)} \text{ whp, Regime III}).
\]
Substituting (2.42) into (2.45) and (2.46) yields \( \lambda_c^* N^3 \leq W_i / k_i^2 \) and \( \lambda_c^* N^0 \leq W_j / k_j^2 \).

Note that with the given scheduling scheme, each cell can be guaranteed to have \( W_i / k_i^2 \) bits/s and \( W_j / k_j^2 \) bits/s for uplink and downlink local communications respectively. Therefore, \( \lambda_c^* \) can be accommodated in both uplink and downlink local communications.

2.5.3.2 In the Case When \( N_g = o(\sqrt{N_r}) \)

When \( N_g = o(\sqrt{N_r}) \), the proposed assignment scheme is given as follows:

\[ W_i = N_g W_2 / (2N_r) \] and \( W_i = W_3 \), subject to \( \sum_{i=1}^{3} W_i = W \) and \( W_i + W_j \leq W' \). Thus,

\[ W_2 = \frac{N_g W}{N_r + N_g} \]  

(2.47)

\[ W_1 = W_2 = \frac{N_g W}{2(N_r + N_g)} \]  

(2.48)

\[ W' \geq \frac{N_g W}{N_r + N_g} \]  

(2.49)

Substituting (2.47) into (2.40) yields a feasible per-client throughput \( \lambda_c^* \) in terms of \( W \) in regime II) and IV):

\[ \lambda_c^* = \frac{N_g N_r W}{2k_i^2 c_g c_{16} N_c (N_r + N_g)}, \text{ Regime II}; \]  

(2.50)

\[ \lambda_c^* = \frac{N_g W}{2k_i^2 c_g c_{16} \log N_c (N_r + N_g)}, \text{ Regime IV}). \]

This is achieved \( \text{whp} \) in backbone communications. By exactly the same procedures as those When \( N_g = O(\sqrt{N_r}) \), it can be shown that \( \lambda_c^* \) can also be accommodated in both uplink
and downlink local communications. Thus $\lambda_c^\ast$, which is displayed in (2.44) and (2.50), is proved to be a feasible per-client throughput for internal traffic of the infrastructure WMN.

It is important to note that the condition in (2.43) and (2.49) must be satisfied in order to obtain $\lambda_c^\ast$ as shown in (2.44) and (2.50). In other words, the channel capacity of mesh clients needs to be large enough so that it does not throttle the throughput capacity of the entire network. Since $\lambda_c^\ast$ displayed in (2.44) and (2.50) has the same asymptotic scaling in all four regimes as the upper-bound given in Corollary 1, main results in (2.1) are derived.

For external traffic, by following similar procedures as those for internal traffic when $N_g = \omega(\sqrt{N_r})$, a feasible per-client throughput is achieved with the same scaling as the first line in (2.50) for both Regime I) and II); and a feasible per-client throughput is achieved with the same scaling as the second line in (2.50) for both Regime III) and IV). Therefore, the asymptotic per-client throughput for external traffic of the infrastructure WMN is derived as shown in (2.2). It does not achieve closed form when $N_r = \omega(N_c/\log N_c)$, and this will subject to further works.

2.6 Discussions and Implications

2.6.1 Generality of Network Model

The network architecture of our network model has been justified in many applications. Firstly, by using infrastructure mode in local communications, the complexity of protocols and the requirement of computing capability can be maximally reduced on clients. Hence network users, i.e., mesh clients, can actually achieve much better throughput performance because of less protocol overhead and shorter in-node processing time. This is one of the most
important advantages that WMNs offer over other wireless networking solutions.

Secondly, by deploying backbone networks on regular-grid, better coverage and connectivity can be achieved as well as more efficient protocols, e.g. scheduling and routing schemes. The above claims were partially verified by Robinson and Knightly in their very recent measurement –parameterized simulations [93]. They found that regular-grid topology achieved significantly better coverage than hexagonal and random topologies, and moderate grid perturbations had only minimal impact on coverage area when practical deployment could not achieve strict regular structure. They also found that regular grid backbone network performed significantly better than randomly deployed one in terms of fair mesh capacity.

Although single channel is assumed in our analytical model, our derivations and results remain the same for nodes with multiple radios or multiple channels [11-12]. In those systems, $W$ is the combined wireless channel capacity of all radio interfaces on a mesh router. For example, if a router has two non-interfering radios which can transmit at $W_i$ and $W_{ii}$, respectively, then $W = W_i + W_{ii}$. In fact, derivations in section 2.5.3 indicate that maximum throughput performance is achieved in our analysis since a globally optimized bandwidth assignment scheme is applied to the concurrent radios or channels. Dedicating different radios or channels to local and backbone communications is an inefficient use of the resources. This has been verified by simulations in [93].

2.6.2 Proper Number of Mesh Routers and Gateways

As indicated in regime I) and III) of (3), $C(\lambda_c)$ is not a function of $N_g$. It implies that wired gateways cannot effectively increase the throughput capacity of the WMN if its number is smaller than a certain threshold. In this case, ideal throughput performance is obtained in section 2.5 by assuming all traffic is non-gateway traffic. If traffic is required to go through gateways, e.g., for Internet accessing, the throughput performance could be worse, as indicated in regime I)
and III) of (2.2). In this case, mesh routers with gateway function are bottlenecks throttling the throughput while non-gateway mesh routers are idle, resulting in waste of bandwidth resources. Therefore, to increase the throughput, solutions should be developed to reduce external traffic load. Possible approaches include: 1) installing cache servers in the network and connecting them to non-gateway mesh routers; 2) Enabling peer-to-peer communications on the backbone network and using cooperative downloading/uploading function for Internet accessing. In the first approach, external traffic is reduced without adding internal traffic, while the second approach reduces external traffic but also increases internal traffic.

When the number of gateways is larger than a certain threshold, i.e., \( N_g = o(\sqrt{N_r}) \), adding more gateways can effectively increase the throughput capacity of the WMN, as indicated in regime II) of (2.3). In this case, external and internal traffic can both achieve the throughput capacity by fully utilizing wired gateways, as indicated in regime II) of (2.1) and (2.2). If bandwidth to Internet, i.e. the number of gateways, is the major cost for certain applications, by considering \( N_g \) as cost, then per-cost per-user throughput capacity is a constant. Therefore, the infrastructure WMN, in the case of regime II), provides an ideal commercial model for the applications. When \( N_g = \Theta(N_c / \log N_c) \) and \( N_r = \Theta(N_c / \log N_c) \), the infrastructure WMN can achieve its best throughput as \( \Theta(W / \log N_c) \).

Adding mesh routers into the network can effectively increase the throughput performance, as indicated in regime I) of (2.3). However, arbitrarily increasing the number of non-gateway mesh routers may also result in throughput performance degradation, which is implied in the results in regime III) of (2.3). Explanations are given as follows. Adding mesh routers, on one hand, can benefit the throughput capacity by mitigating bottleneck effects in local communications. From Lemmas 6 and 7, when \( N_r = O(N_c / \log N_c) \), adding mesh routers can
linearly decrease the number of mesh clients per router in both uplink and downlink local communications and thus increase the throughput capacity by a factor of \( N_r \); but it can no longer decrease the number and benefit the throughput capacity when \( N_r = o(N_c / \log N_c) \). On the other hand, increasing the number of mesh routers deteriorates throughput capacity due to decreased per-router throughput in backbone communications, since there are more mesh routers contending for the wireless channel. According to (2.16), it decreases the throughput capacity by a factor of \( \sqrt{N_r} \). By considering the above two effects together, adding mesh routers can benefit the throughput capacity by a factor of \( \sqrt{N_r} \) when \( N_r = O(N_c / \log N_c) \); but it deteriorates throughput capacity by a factor of \( \sqrt{N_r} \) when \( N_r = o(N_c / \log N_c) \). Thus, the optimal number of mesh routers for the network is \( \Theta(N_c / \log N_c) \).

In order to avoid excessive mesh routers, dynamic management of mesh backbone network becomes a critical feature of infrastructure WMNs. For example, when the number of clients or traffic pattern keeps changing over time, some mesh routers need to be switched on or off in order to achieve optimal throughput performance.

2.6.3 Comparing with Conventional and Hybrid Ad Hoc Networks

Since all the previous capacity results of either conventional or hybrid ad hoc networks are achieved with only internal traffic in the networks, they are only compared with our main results for internal traffic shown in (2.1).

When gateways are not effective for throughput capacity, i.e., \( N_g = O(\sqrt{N_r}) \) in regime I) and III) of (2.1), the scalability of the infrastructure WMN is no better than that of random ad hoc networks. It is clear that in this case the per-client throughput capacity achieves its maximal value of \( \Theta(W / \sqrt{N_r \log N_c}) \) when the number of mesh routers scales as \( \Theta(N_c / \log N_c) \). It is interesting
to find that the capacity value is equal to the per-node throughput capacity of random ad hoc networks [65]. Explanations are given as follows. Considering a flat random ad hoc network with $N_s$ source nodes and $N_r$ relay nodes, Gupta and Kumar indicated in [65] that adding relay nodes cannot effectively increase the asymptotic throughput capacity when $N_r \leq N_s$. In this sense, the infrastructure WMN without effective gateway support provides a worse network architecture since it requires all traffic go through mesh routers. Hence, bottlenecks on mesh routers result in worse asymptotic throughput capacity especially when there are very few routers with respect to the number of clients.

However, the WMN can also achieve the same asymptotic throughput capacity as that of hybrid ad hoc networks if the network is optimally designed, i.e., by deploying optimal number of mesh routers and gateways and applying optimal schemes and protocols, such as gateway placement, routing, scheduling and bandwidth assignment schemes. More specifically, the capacity result in regime II) of (2.1) achieves the same asymptotic value as that of hybrid ad hoc networks in [70]. More interestingly, the capacity is achieved without requiring a large number of gateways. Actually $N_g$ only need to scale faster than $\sqrt{N_r}$.

The same asymptotic per-client throughput as that of hybrid ad hoc networks is also achieved by WMNs in regime II for Internet accessing. By considering an IEEE 802.11 WLAN as a special hybrid network operating in an infrastructure mode, the throughput capacity result of the infrastructure WMN implies a very promising future for 802.11 WLANs. For example, the current widely deployed 802.11 WLANs can be economically merged and expanded into WMNs by replacing some access points with wireless mesh routers with gateway functionality and all the other access points with cheap non-gateway mesh routers. The new WLAN mesh network will have much larger coverage with minimal throughput degradation.
2.7 Conclusion

In this chapter, asymptotic capacity of infrastructure WMNs was derived. The theoretical results suggest that, in order to achieve high throughput performance, a WMN needs to be deployed with an appropriate number of mesh routers and gateways. Asymptotically a WMN can achieve a comparable throughput capacity as a hybrid ad hoc network, making it a better wireless networking solution than both conventional and hybrid ad hoc networks. The implications of our theoretical results can be used as guidelines for protocol design and deployment of WMNs. As indicated in the main results of throughput capacity, the upper bound and the low bound in regime IV) do not have the same asymptotic scaling. However, the gap between the two bounds in this regime does not impact insights on the scaling law of the throughput capacity of WMNs. How to derive a close form for regime IV) is subject to future research.

2.8 Acknowledgement

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II

On Optimizing Gateway Placement for Throughput in Wireless Mesh Networks

3.1 Introduction

Gateway placement is one of the most challenging but important problem in the research of WMNs. There are some analogous works in wired or cellular networks. For example, a number of studies have been carried out to place web proxies or server replicas to optimize clients’ performance [95-97]. Another example is the base station placement problem in cellular networks [98-100]. However, when wireless links replace wired links and multi-hop communications replace single-hop communications, more comprehensive traffic modeling schemes are required to solve the backbone nodes placement problem in multi-hop wireless networks. More recently the Connected Disk Cover (CDC) problem was investigated by Srinivas and Modiano [101]. CDC problem focused on network connectivity of WMNs by deploying the minimum number of backbone nodes. Bejerano studied gateway placement in multi-hop wireless networks [102].
this work, network nodes were partitioned into minimal number of disjoint clusters that satisfied throughput and delay constraints.

Unlike all the above research work, in this paper we aim to develop a gateway placement algorithm for maximizing throughput of WMNs. To the best of our knowledge, little research work has been carried out along this direction. However, throughput is one of the most critical parameters that ensure the services of WMNs to meet the requirements of customers.

To develop the throughput-oriented gateway placement algorithm, we first derive a new performance metric called multi-hop traffic-flow weight (MTW) to take into account major factors that impact throughput of WMNs. Such factors include the number of mesh routers, mesh clients, and gateways as well as traffic demands from mesh clients, locations of gateways, and interference among gateways. Based on MTW, an iterative algorithm is proposed to determine the best location of a gateway. Each time a gateway is chosen to co-locate with the mesh router that has the highest MTW.

Since MTW is closely coupled with throughput calculation of WMNs, a throughput computation model needs to be derived. However, throughput analysis of wireless networks is an extremely challenging research topic. Throughput capacity of multi-hop wireless networks has been studied by many recent works. Gupta and Kumar [65, 66] derived the per-node throughput capacity for static ad hoc networks. The throughput capacity of mobile ad hoc networks was analyzed by Grossglauser and Tse [76]. The capacity of hybrid ad hoc networks was investigated in [68-70]. All such results of throughput analysis cannot be applied to WMNs, because the network architecture of WMNs is much different from either conventional ad hoc networks or hybrid ad hoc networks. Compared with conventional ad hoc networks, WMNs are hierarchical networks in which there exist different types of communications among various nodes. In comparison with hybrid ad hoc networks, WMNs use wireless links instead of wired lines to
connect backbone networks. In the throughput analysis of hybrid ad hoc networks, communication links among backbone nodes are assumed to have unlimited capacity, and such communications do not cause interference on other communications. However, these assumptions are no longer valid in WMNs. Furthermore, all the above throughput results are obtained as asymptotic value by assuming that the size of the network goes to infinity. Since real networks always have limited size, these asymptotic results provide very few information for practical network design.

Thus, in this paper a non-asymptotic analytical model is derived to calculate the throughput of WMNs. TDMA is assumed to schedule packet transmissions in mesh clients, mesh routers, and gateways. Two radio interfaces are assumed to be equipped on a mesh router so that it can communicate with a mesh client and a mesh router at the same time. Since gateways are the busiest routers in the network, an optimal TDMA scheme is first applied to all the gateways so that in each time slot simultaneous transmissions on gateways do not interfere with each other. In addition, the scheduling scheme guarantees that each gateway can be assigned a maximum number of time slots. Time slots assigned to a gateway are then split into separate small slots that are further assigned to all the associated mesh clients with this gateway. The number of small time slots assigned to each client is in proportion to the number of hops in the backbone that the client need to access to its gateway. In this way, a certain amount of throughput is virtually guaranteed in the backbone for each mesh client. Similarly, a virtual throughput can also be reserved in communications between a mesh router and a mesh client. Finally, the throughput of the WMN is obtained by choosing the smaller one of the above two throughputs.

By integrating the throughput computation model and MTW, the gateway placement algorithm can greatly enhance the throughput performance of WMNs. Simulations are carried out in this paper to compare the proposed scheme with other schemes such as random placement,
regular placement, and busiest router placement. Experimental results show that our gateway placement algorithm outperforms all these schemes with a large margin.

The rest of this chapter is organized as follows. In Section 3.2, a typical WMN model is described and two problems for optimal gateway placement are formulized. A gateway placement algorithm is proposed in Sections 3.3, while the throughput computation model needed by this algorithm is derived in Section 3.4. The numeric results are obtained in Section 3.5 to evaluate the performance of the proposed algorithm. This paper is concluded in Section 3.6.

3.2 System Model and Problem Formulation

3.2.1 Network Topology

A typical WMN model for Internet accessing is proposed as follows and is illustrated in Figure 3.1. $N_c$ mesh clients are assumed to be distributed on a square $R = [0, l]^2$. $R$ is partitioned evenly into $(l/l_s)^2$ small cells $R'_j = [0, l'_s]^2$ ($j = 1...((l/l_s)^2)$), and a mesh router is placed in the center of each cell. Let $N_r$ denote the number of mesh routers, then $N_r = (l/l_s)^2$. In what as follows, we will limit the case of interests to that where $1 < N_r \leq N_c$, i.e., there are more than one mesh routers and the number of mesh routers is smaller than that of mesh clients. Mesh routers constitute a wireless mesh backbone providing a wireless infrastructure for mesh clients. In each cell, mesh clients are connected to the mesh router like a star topology, i.e., no direct communication is available among mesh clients, and the mesh router works as a hub for mesh clients. Such a WMN is referred as an infrastructure WMN in [4], which will be very popular in future WMN applications. Among all the mesh routers, there are $N_g$ routers wired to Internet, working as gateways. It is obvious that $1 \leq N_g \leq N_r$, i.e., the number of gateways cannot exceed the number of mesh routers.
Each mesh client is a data source and a data destination. All mesh clients are equivalent such that they always have the same amount of packets to send or receive during a certain time. Unlike mesh clients, mesh routers are neither data source nor data destination; they only route and forward data for mesh clients. All traffic is assumed to go through gateways. Each mesh router is associated with its nearest gateway such that it relays packets to or from it. Assuming that the shortest path routing is applied, the nearest gateway of a mesh router is defined as the gateway that the mesh router can access to by the minimal number of hops. In the situation that a mesh router has more than one nearest gateways, the traffic load of the router will be equally shared by all its nearest gateways. A mesh client is said to be associated with a gateway if its connected router is associated with the gateway. Hence, all traffic load of a mesh client will also be equally shared by all its potentially associated gateways.

In this paper the following definitions of communications will be frequently used:
• **Local communications**: it is referred as the communications between a mesh router and a mesh client;

• **Backbone communications**: it is referred as the communications between two mesh routers, which includes the communications between a gateway and a mesh router;

• **Downlink communications**: it is referred as the communications from a gateway to a mesh client, in which a data packet is first relayed among mesh routers in backbone communications and is then sent by a mesh router to one of its connected mesh clients;

• **Uplink communications**: it is referred as the communications from a mesh client to a gateway, in which a data packet is sent in the exact reverse direction as described in downlink communications.

### 3.2.2 Transmission Model

Each mesh router is equipped with two radio interfaces such that it transmits at $W_1$ bits/s in backbone communications and it transmits at $W_2$ bits/s in local communications. Each mesh client transmits at $W_2$ bits/s in local communications. We assume that $W_1$ and $W_2$ are orthogonal so that local communications do not interfere with backbone communications. Note that mesh routers and mesh clients use the same radio interface in local communications. In addition, mesh routers can receive packets from only one sender at a time and cannot transmit and receive packets simultaneously. The same constraint is imposed on mesh clients.

In either local communications or backbone communications, simultaneous transmissions are coordinated by the Protocol Model as defined in [65], i.e., if a transmission from node $S_i$ to $S_j$ is
successful, then the following conditions must be satisfied: 1) $|S_i - S_j| \leq r_i$; 2) for every other transmitting node $S_k$, $|S_k - S_j| \geq (1 + \Delta)r_i$, where $r_i$ and $r_k$ correspond respectively to the transmission range of node $S_i$ and $S_k$ and $\Delta$ is a fixed positive constant that represents a guard zone in the Protocol Model.

### 3.2.3 Problem Formulation

**Problem 1:** Optimal gateway placement for maximizing aggregate throughput of WMNs, i.e. in the above WMN model, given $N_c$, $N_r$, $N_g$, $W_1$, $W_2$ and specific clients’ distribution, routers’ distribution, transmission, scheduling and routing protocols, $N_g$ gateways are chosen among $N_r$ mesh routers such that,

$$\sum_{i=1}^{N_g} TH(i, N_g)$$

is maximized, where $TH(i, N_g)$ denotes the per client throughput of the $i$th mesh client when $N_g$ gateways are deployed.

**Problem 2:** Optimal gateway placement for maximizing the worst case of per client throughput in the WMN, i.e. in the above WMN model, given $N_c$, $N_r$, $N_g$, $W_1$, $W_2$ and specific clients’ distribution, routers’ distribution, transmission, scheduling and routing protocols, $N_g$ gateways are chosen among $N_r$ mesh routers such that,

$$\min_{i=1}^{N_c} TH(i, N_g)$$

is maximized.
3.3 Multi-Hop Traffic-Flow Weight Gateway Placement Algorithm

Adding new gateways can increase throughput in backbone communications by effectively reducing the average number of hops each packet needs to access to gateways and reducing the traffic load on existing gateways. However, the above benefits may be dramatically mitigated by careless gateway placement since new gateways may also introduce more interference to existing gateways. Therefore, a good gateway placement algorithm can maximally relieve traffic load in the network but introduce minimal interference.

A good gateway placement algorithm should also be adaptive to the deployed number of gateways. A relative small number of deployed gateways means large number of hops a packet needs to access to gateways, in which case huge traffic load results from packets’ long distance traveling in the network. Therefore, geometry-balanced placement algorithms, e.g. regular placement, may achieve good results since they can effectively reduce the average number of hops. In the opposite case, when a relatively large number of gateways are planned to deploy, placing the gateways in the areas with the most traffic load may be simply the best solution.

In this section, an innovative gateway placement algorithm is introduced, which has all the above-mentioned benefits.

3.3.1 Adaptive multi-hop traffic-flow weight

A traffic-flow weight, denoted as $MTW(j)$, is calculated on the mesh router $R^j$, $j=1...N_r$. Each time a new gateway will be placed on the router with the highest weight. The weight computation is adaptive to the following factors: 1) the number of mesh routers and the number of gateways, i.e., $N_r$ and $N_g$; 2) traffic demands from mesh clients; 3) the location of
existing gateways in the network; 4) The interference from existing gateways. Factors 1) to 3) will be discussed in this sub-section and factor 4) will be presented in the next sub-section.

![Table](image)

**Figure 3.2: An example of multi-hop traffic-flow weight**

In the first step of the algorithm, a variable called weight of hops’ number, denoted as $W_{hop}$, is decided. $W_{hop}$ is a function of $N_r$ and $N_g$, and is given as follows:

$$W_{hop} = \text{round}\left(\frac{\sqrt{N_r}}{2\sqrt{N_g}}\right).$$  \hfill (3.1)

$W_{hop}$ can be considered as an estimation on the average number of hops that a packet needs to travel from a gateway to a mesh router.

In the second step, local traffic demand on each mesh router, denoted as $D(j), j = 1...N_r$, is calculated. $D(j)$ displays the traffic demand from all the mesh clients connected to $R^j$. In our WMN model, all mesh clients are equivalent. Therefore, the number of mesh clients connected to
$R'$ is used as $D(j)$. Figure 3.2 (a) shows an example of $D(j)$ when 200 mesh clients are uniformly distributed and 25 mesh routers are placed on a 5-by-5 regular grid.

In the third step, $MTW(j)$ is calculated with $D(j)$ and $W_{hop}$ as follows:

$$MTW(j) = (W_{hop} + 1) \times D(j)$$
$$+ W_{hop} \times \text{(traffic demand on all 1-hop neighbors of } R')$$
$$+(W_{hop} - 1) \times \text{(traffic demand on all 2-hop neighbors of } R')$$
$$+(W_{hop} - 2) \times \text{(traffic demand on all 3-hop neighbors of } R')$$
$$+...$$

Please note that negative items are not counted in the above formula. Figure 3.2 (b) demonstrates an example that how $D(j)$ and $W_{hop}$ are combined to affect gateway placement. Figure 3.2 (b) is an example of MTW, which is calculated using $D(j)$ as depicted in Figure 3.2 (a) and $W_{hop} = 3$.

From (3.1), we know that in this case $N_g = 1$. So there is only one gateway needed to be deployed and it will be placed in the center of the WMN.

With $MTW(j)$, the first gateway will be placed on the router with the highest weight. In the next step, $D(j), j = 1...N_r$, will be re-adjusted with $W_{hop}$. Assuming that the gateway is placed at $R'$, the traffic demand value of $R'$ and all its neighbors within $(W_{hop} - 1)$ hops away will be set as 0, and the value of $R'$’s $W_{hop}$ hops neighbors will be reduced to half. In this way, the other gateways are less likely to be placed in a location near the existing gateways. In the next subsection, interfere among gateways will also be counted in the computation of MTW.

3.3.2 Optimal sharing efficiency of gateways

It is assumed that two gateways interfere with each other if they are within the distance of $IntD$-hops in backbone communications. $IntD$ is defined as $Interfering Distance of gateways$. In
the first step, *table of interfering gateways* is constructed by the steps as follows: 1) each gateway appears as a single line in the table; 2) except the above lines, all the lines contain more than one gateways representing all possible combination such that in each line, any two gateways interfere with each other; 3) The line with more gateways always appears in the higher position in the table. For example, seven gateways are deployed on a 5-by-5 mesh backbone grid, as shown in Figure 3.3 (a) and its table of interfering gateways is displayed in Figure 3.3 (b), here $IntD = 2$.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.3 Obtaining the optimal sharing efficiency on gateways

In the second step, each gateway is assigned a percentage number in the procedures as follows: 1) initially all gateways are assigned with a value of 100%; 2) The table of interfering gateways is searched from the top line to the last line with more than one gateway at a speed of
one line per step; 3) In each step, all gateways in a specific line are split into 2 groups by threshold value of \((\frac{1}{\text{the number of gateways in the line}})\). The first group contains the gateways with larger value than the threshold value and the second group has the rest of the gateways in this line; 4) all gateways in the first group will be re-assigned a new percentage value calculated as follows:

\[
\frac{1 - \text{sum of all the percentage value in the second group}}{\text{the number of the gateways in the first group}}
\]

5) the procedures of 3) and 4) repeat until finish. In the example shown in Figure 3.3, gateway 3, 4, 5 and 7 are re-assigned a percentage value of 25% in the computation of the first line; gateway 2 is re-assigned a percentage value of 50% in the computation of the second line; gateway 2 and 6 are re-assigned a percentage value of 37.5% in the computation of the third line; gateway 1 is re-assigned a percentage value of 62.5% in the computation of the ninth line. The final results are shown in Figure 3.3 (c).

The optimal traffic scheduling scheme on gateways is constructed. In the scheme, time slots in backbone communications are assigned to all gateways such that successful simultaneous transmissions can be always carried out in each time slot. And each gateway can be guaranteed to have a number of time slots, which is equal to the total number of all time slots times the percentage value obtained in the previous step. Figure 3.3 (d) shows an example of such a TDMA scheme. The above percentage value assigned to a gateway is defined as the optimal sharing efficiency for the specific gateway, denoted as \(G_{\text{eff}}(k), \ k = 1, \ldots, N_g\).

Finally, adding a new gateway into the network with the presence of existing gateways will have the following procedures: 1) from previous steps, choosing the router with the highest weight as a potential location for gateway placement; 2) adding the potential location into the existing table of interfering gateways and re-constructing the table; 3) computing the sharing efficiency for the potential gateway location by the new table of interfering gateways; 4) Re-
adjusting the highest weight by timing the sharing efficiency, i.e. $MTW'(j) = MTW(j) \times G_{\text{eff}}(j)$;

5) if the new weight is still larger than the second highest weight, then place the gateway in the location. Otherwise, repeat the above steps from 2) to 5) until obtaining the location.

### 3.3.3 Other gateway placement algorithms

Table 3.1: An example for RGP on a 6-by-6 regular grid

<table>
<thead>
<tr>
<th>$N_g$</th>
<th>Gateway Placement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Choose the busiest router from the location of (3,3), (3,4), (4,3), (4,4)</td>
</tr>
<tr>
<td>2–4</td>
<td>Choose the $N_g$ busiest routers from the location of (2,2), (2,5), (5,2), (5,5)</td>
</tr>
<tr>
<td>5–7</td>
<td>Choose the first 4 gateways at the location of (2,2), (2,5), (5,2), (5,5) and</td>
</tr>
<tr>
<td></td>
<td>choose the rest on the other routers with the highest traffic demand</td>
</tr>
<tr>
<td>≥9</td>
<td>36 routers are split into 4 groups. In each group, any two routers are at least</td>
</tr>
<tr>
<td></td>
<td>2-hops away, e.g. (1,1), (1,3), (1,5), (3,2), (3,4), (3,6), (5,1), (5,3), (5,5)</td>
</tr>
<tr>
<td></td>
<td>are in one group. Choose the first gateway on the busiest router and choose the</td>
</tr>
<tr>
<td></td>
<td>rest 7 gateways on the next 7 busiest routers in the same group with the first</td>
</tr>
<tr>
<td></td>
<td>one.</td>
</tr>
<tr>
<td></td>
<td>36 routers are split into 4 groups as above. Choose the first gateway on the</td>
</tr>
<tr>
<td></td>
<td>busiest router, then choose the next 8 gateways on the other routers in the</td>
</tr>
<tr>
<td></td>
<td>same group with the first one and choose the rest on the other routers with</td>
</tr>
<tr>
<td></td>
<td>the highest traffic demand</td>
</tr>
</tbody>
</table>
The above proposed algorithm (MTWP) will be compared with the following three gateway placement algorithms:

- **Random Placement** (RDP): \( N_g \) gateways choose their placement location randomly on \( N_g \) mesh routers

- **Busiest Router Placement** (BRP): \( N_g \) gateways choose their placement location on the \( N_g \) mesh routers with the highest traffic demand defined by

\[
D(j), j = 1,...N_r
\]

- **Regular Placement** (RGP): as many as possible gateways are placed based on regular patterns and the rest of them choose their placement location on the same number of mesh routers with the highest traffic demand defined by

\[
D(j), j = 1,...N_r.
\]

Table 1 gives an example of RGP on a 6-by-6 regular grid.

### 3.4 Traffic Scheduling for Throughput Computation

In this section, TDMA schemes are applied for traffic scheduling. Based on these schemes, we provide a framework for throughput computation in WMNs.

The WMN model indicates that all wireless mesh routers contend for the same wireless channel of capacity \( W_1 \) in backbone communications and all mesh routers and mesh clients contend for the same wireless channel of capacity \( W_2 \) in local communications. Therefore, the throughput of the \( i \)th mesh client when \( N_g \) gateways are deployed, denoted as \( TH(i,N_g) \), is generally constrained by both \( W_1 \) and \( W_2 \). Since \( W_1 \) and \( W_2 \) are orthogonal, \( TH(i,N_g) \) can be obtained by computing the throughput constrained by \( W_1 \) and the throughput constrained by \( W_2 \) separately, i.e.,
\[ TH(i, N_g) = \min\{TH_{W_1}(i, N_g), TH_{W_2}(i)\}, \quad i = 1 \ldots N_c. \] (3.2)

Here \( TH_{W_1}(i, N_g) \) is defined as the throughput of the \( i \)th mesh client in backbone communications when there are \( N_g \) gateways in the WMN and \( TH_{W_2}(i) \) is defined as the throughput of the \( i \)th mesh client in local communications. Note that \( TH_{W_2}(i) \) is independent of \( N_g \) in the WMN model. (3.2) indicates that a feasible per client throughput can be achieved by taking the smaller one of \( TH_{W_1}(i, N_g) \) and \( TH_{W_2}(i) \).

Since both clients and routers cannot send and receive at the same time, \( W_1 \) and \( W_2 \) should be split for uplink and downlink communications respectively, i.e., \( c_1 W_1 \) and \( c_2 W_2 \) are assigned to downlink communications, and \((1-c_2)W_1\) and \((1-c_2)W_2\) are assigned to uplink communications, where \( c_1 \) and \( c_2 \) are some constants between 0 and 1. Generally, throughput of a mesh client should be obtained as the sum of uplink throughput and downlink throughput. Choosing the value of \( c_1 \) and \( c_2 \) requires knowledge on actual applications running on clients, which is beyond the objectives of this paper. It is assumed in the following of this paper that downlink traffic is dominant in the WMN. Therefore, most of \( W_1 \) and \( W_2 \) will be assigned to downlink communications and throughput is decided by downlink throughput, which is constrained by \( c_1 W_1 \) and \( c_2 W_2 \). This is not an uncommon case in today’s applications of WMNs, for instance, in the application of Internet accessing. Please note that the methodology proposed in this section can actually be used to obtain throughput of WMNs when uplink and downlink traffic both present, however, with the above simplified model, we can focus on the illustration of our main ideas without distraction from trivial discussions.

3.4.1 Throughput in backbone communications
Time slots in backbone communications are first assigned to gateways so that no gateways interfere with each other. The TDMA scheduling scheme on gateways is assumed to satisfy the following two conditions: 1) Time slots are assigned to each gateway as equally as possible; 2) Under the condition of 1), each gateway should have as much as possible time slots for successful transmissions. In section 3.3.2, an algorithm to obtain the sharing efficiency on all the gateways, denoted as $G_{\text{eff}}(k), k = 1...N_g$, is provided and is illustrated by an example, as depicted in Figure 3.3. In this algorithm, a traffic scheduling scheme satisfying the above two conditions is also constructed. In the scheme, the $k$th gateway can be guaranteed to have a number of time slots, which is equal to the total number of all time slots times $G_{\text{eff}}(k)$. Hence, the $k$th gateway is guaranteed to have an aggregate throughput of $G_{\text{eff}}(k) \times c_i W_i$ in backbone communications. By the TDMA scheme, interfering gateways share the same wireless channel while non-interfering gateways can transmit simultaneously.

In the next step, time slots of a gateway will be further split into small time slots to have the following two properties: 1) Each mesh client associated with the specific gateway should have separate small time slots for “interference free” transmissions; 2) Each of such mesh clients should achieve a common throughput in backbone communications, i.e., $TH_{W_i}(i_1, N_g) = TH_{W_i}(i_2, N_g)$, if...
mesh clients \(i_1\) and \(i_2\) are associated with the same gateway. It is assumed that a mesh router \(R^j\) has \(N_c(j)\) connected mesh clients and it is located \(N_{\text{hop}}(j)\) hops from its associated gateway. The second property requires that \(R^j\) be assigned \(N_c(j) \times N_{\text{hop}}(j)\) small time slots if there are no simultaneous transmissions along the way from the gateway to \(R^j\). Figure 3.4 shows that simultaneous transmissions can be scheduled, if \(R^j\) is more than \(\text{SRD}\)-hops away from its gateway. \(\text{SRD}\) is defined as the Slot Reuse Distance, for instance, \(\text{SRD} = 3\) in Figure 3.4. Therefore, the actual time slot that a \(R^j\)-connected mesh client need to meet the second property, denoted as \(N_{\text{hop}}'(j)\), has the following relationship with \(N_{\text{hop}}(j)\):

\[
N_{\text{hop}}'(j) = N_{\text{hop}}(j), \quad \text{if } N_{\text{hop}}(j) < \text{SRD};
\]

\[
N_{\text{hop}}'(j) = \text{SRD}, \quad \text{if } N_{\text{hop}}(j) \geq \text{SRD}.
\]

Hence, with the first property all mesh clients associated with a specific gateway require total \(\sum_j N_c(j) \times N_{\text{hop}}'(j)\) small time slots for “interference free” transmissions in backbone communications. With the consideration that a mesh router may have more than one potentially associated gateways, the \(k\)th gateway can guarantee the following per client throughput for all its associated mesh clients in backbone communications:

\[
TH_g(k) = \frac{G_{\text{eff}}(k) \times cW_1}{\left(N_c(j) \times N_{\text{hop}}'(j) \div N_g(j)\right)} , \quad (3.3)
\]

where \(N_g(j)\) denotes the number of potentially associated gateways with the mesh router \(R^j\).

Assuming the \(i\)th mesh client is connected with the mesh router \(R^j\), then the throughput of the \(i\)th mesh client in backbone communications is given as follows:
\[ TH_{W_c}(i, N_g) = \frac{\sum_{k=1}^{N_e(j)} TH_g(k)}{N_g(j)}. \] (3.4)

3.4.2 Throughput in local communications

A TDMA scheduling scheme is applied and guarantees successful transmissions in local communications.

Separate time slots are first assigned to different mesh routers so that simultaneous transmissions can only be carried out in cells that have enough distance in between, i.e., simultaneous transmissions can only exist in cells that are \((\sqrt{CRF} - 1)\) cells apart, where \(CRF\) is defined as \textit{Cell Reuse Factor}. Hence, in downlink communications, each mesh router can only have one slot every \(CRF\) time-slots, as depicted in Figure 3.5, here \(CRF = 4\).

The above slot is further split into separate small-slots. Assigned a different small-slot, each mesh client is guaranteed to obtain successful reception from its associated mesh router. Therefore,

\[ TH_{W_c}(i) = \frac{c_2 W_c}{CRF \times N_e(j)}, \quad i = 1...N_c. \] (3.5)

Note that with the above TDMA scheme, all the mesh clients associated with the same mesh router will have the same throughput in local communications, i.e., \(TH_{W_c}(i_1) = TH_{W_c}(i_2)\), if clients \(i_1\) and \(i_2\) are associated with the same mesh router.
3.4.3 Feasible throughput in WMN

Combining equations (3.2) ~ (3.5), a feasible throughput of the \( i \)th mesh client in the WMN can be obtained as follows:

\[
TH(i, N_g) = \min \left\{ \sum_{k=1}^{N_{g}(j)} \frac{G_{\text{eff}}(k) \cdot c_i W_i}{(N_c(j) \times N_{\text{hop}}(j) + N_g(j)) / N_g(j)} \times \frac{c_i W_i}{\text{CRF} \times N_c(j)} \right\},
\]

here \( i \)th mesh client is assumed to be connect with the mesh router \( R^j \).

When all mesh routers are chosen as gateways, i.e., \( N_g = N_r \), throughput of the \( i \)th mesh client is only constrained by local communications, i.e., \( TH(i, N_r) = TH_{\text{WTH}}(i) \). Therefore, an upper bound is obtained for the aggregate throughput:

\[
\sum_{j=1}^{N_r} TH(i, N_g) \leq \sum_{i=1}^{N_r} TH_{\text{WTH}}(i) = \frac{c_i W_i}{\text{CRF}} \times \sum_{j=1}^{N_r} u(j),
\]

where \( u(j) = 1 \), if \( R^j \) has at least one connected client; \( u(j) = 0 \), if \( R^j \) has no connected client. And an upper bound is also obtained for the worst case of per client throughput:
The above upper bounds are independent of $N_g$. Actually they are the maximal value that \( \sum_{i=1}^{N_i} TH(i, N_g) \) and \( \min_i TH(i, N_g) \) can achieve for any number of gateways.

### 3.5 Numeric Results and Discussion

Using the framework of throughput computation defined in section 3.4, throughput of this WMN is studied by simulations in this section. In all the simulations we assume $N_c = 200$, $N_r = 36$, and $l = 1000m$, i.e. there are 200 mesh clients distributed in a square region of $1000m \times 1000m$; the square is split evenly into 36 small square cells and a mesh router is placed in the center of each cell. In addition, we assume $CRF = 4$, and $SRD = 3$, and $IntD = 2$. A certain number of gateways will be placed on the top of the best-fit mesh routers based on a certain placement algorithm. Since mesh clients in all cases follow a random distribution, the results in all plots are obtained as an average over 200 iterations.
In the first case, we study the relationship between channel capacity of mesh routers and the number of gateways. We assume that all mesh clients are uniformly distributed and each of them can transmit at 10Mbps in downlink communications, i.e., $c_iW_2 = 10\text{Mbps}$. The aggregate throughput of the WMN versus the number of gateways is shown in Figure 3.6, where gateways are placed by the proposed MTWP algorithm and the channel capacity of mesh routers varies from 10Mbps to 25Mbps with an increment of 5Mbps. Our results confirms the fact that the number of gateways can be dramatically reduced by using more powerful mesh routers in the backbone, e.g. 6 gateways with mesh router transmitting at 25Mbps can achieve much better throughput performance than 15 gateways with mesh router transmitting at 10Mbps.
Figure 3.7: The comparison of the aggregate throughput with uniformly distributed mesh clients

Figure 3.8: The comparison of the worst case of per client throughput with uniformly distributed mesh clients
In the second case, as shown in Figure 3.7 and Figure 3.8, we compare throughput performance of 4 gateway placement algorithms in the WMN. We assume that all mesh clients are uniformly distributed and each mesh client and mesh router can transmit at 10\(Mbps\) and 20\(Mbps\), respectively. The results show that the proposed MTWP algorithm clearly outperforms the other algorithms in both the aggregate throughput and the worst case throughput. The regular placement algorithm achieves the second best results because it is a geometry-balanced algorithm which can effectively reduce the average distance between a gateway and its associated mesh routers.

Figure 3.9: The comparison of the aggregate throughput with unevenly distributed mesh clients
In the third case, as shown in Figure 3.9 and Figure 3.10, we compare throughput performance of 4 gateway placement algorithms when mesh clients are distributed unevenly in the network, as depicted in Figure 3.11. Please note that in each of the 9 regions in Figure 3.11, nodes are still uniformly distributed, however, nodes density is very different among the 9 regions. In this
case, MTWP algorithm outperforms the other 3 algorithms in every single case. Here we double the channel capacity of mesh clients assuming mesh clients and mesh routers can both transmit at 20Mbps. Otherwise, improvements by gateway placement algorithms may not be observed since very low throughput of local communications becomes the major constraint for throughput performance of the whole WMN, which results from very high nodes’ density in some regions.

In both the second and third cases, as shown in Figure 3.7-3.10, the MTWP algorithm has the biggest improvement on the throughput when the number of gateways is chosen from 5 to 8. An explanation is given as follows: with more than 4 gateways in a 6-by-6 grid backbone network, gateways start to interfere with each other. Comparing with the other 3 algorithms, MTWP algorithm has a unique mechanism to mitigate such interference among gateways. Thus, countering interference among gateways is very critical for a gateway placement algorithm.

![Figure 3.12: The comparison of the aggregate throughput per gateway with uniformly distributed mesh clients](image)
The number of gateways

The aggregate throughput per gateway (bps)

MTWP
RDP
BRP
RGP

Figure 3.13: The comparison of the aggregate throughput per gateway with unevenly distributed mesh clients

An important problem that WMN service providers face is the deployment cost involved in setting up the gateways. Thus, we define the performance metric to evaluate the cost of a gateway placement algorithm: the aggregate throughput per gateway. Corresponding to Figure 3.7, the gateway placement costs are reflected in Figure 3.12. These results indicate that there exist an optimal number of gateways that achieve best tradeoff between gateway cost and throughput. More importantly, it is illustrated that MTWP is the most cost-efficient scheme, since each gateway achieves the highest aggregate throughput. For unevenly-distributed mesh clients, results of throughput per gateway versus the number of gateways are shown in Figure 3.13. Again, the MTWP algorithm is the most cost-effective.

3.6 Conclusion

The problem of gateway placement in WMNs for enhancing throughput was investigated in this chapter. A gateway placement algorithm was firstly proposed based on multi-hop traffic weight. A non-asymptotic analytical model was also derived to determine the achieved throughput
by a gateway placement algorithm. Based on such a model, the performance of the proposed gateway placement algorithm was evaluated. Numerical results illustrated the proposed algorithm achieved much better performance than other schemes. It was also proved to be a cost-effective solution.

It should be noted that the MTWP algorithm proposed in this chapter did not consider the cross-optimization between the location of gateways and the throughput of WMNs. Thus, the throughput achieved by MTWP is not necessarily optimal and can be lower than the maximum throughput. Optimizing gateway placement together with the throughput computation model is our next research goal.

3.7 Acknowledgement

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IV

On Optimizing the Number of Mesh Routers for Throughput in Wireless Mesh Networks

4.1 Introduction

Throughput capacity of multi-hop wireless networks has been studied by many recent works. Gupta and Kumar [65] derived the per-node throughput capacity for static ad hoc networks. The throughput capacity of mobile ad hoc networks was analyzed by Grossglauser and Tse [76]. The capacity of hybrid ad hoc networks was investigated in [68-70]. And the theoretical capacity of infrastructure WMNs was derived in Chapter II. All the above works assumed 100% multiple access efficiency so that no practical MAC protocols have been considered. Silvester and Kleinrock [103] incorporated slotted ALOHA MAC protocol into their capacity analysis of multi-hop networks with regular structure. By analyzing the same MAC protocol, Liu and Haenggi [104] achieved the throughput of fading sensor networks with both regular and random topologies.
In most of above research, theoretical results have been obtained as asymptotic value by assuming that the size of the network goes to infinity. Since real networks always have limited size, these asymptotic results provide very little information for practical network design. In this paper, we propose a framework within which non-asymptotic throughput of WMNs can be obtained by computing several deterministic parameters. While node density was proved to be a critical factor for throughput performance of conventional ad hoc networks [105, 106], we demonstrate in this paper that under various conditions and constraints, ideal throughput of WMNs can be achieved by deploying a proper number of backbone nodes.

The rest of this chapter is organized as follows. In Section 4.2, a typical WMN model is described. The Maximum Throughput Partition (MTP) Problem and the Maximum Throughput Partition with Hops’ number Constraint (MTPHC) Problem are formulized in Section 4.3. MTP and MTPHC solutions are presented in Sections 4.4 and 4.5, respectively. The numeric results are obtained by simulations in Section 4.6. This paper is concluded in Section 4.7.

4.2 System Model

In a typical WMN, as shown in Figure 4.1, $n$ mesh clients are assumed to be uniformly and independently distributed on a square $R = [0, l]^2$. $R$ is partitioned evenly into $(l/l_s)^2$ small cells $R'_i=[0,l_s]^2$ ($i=1...(l/l_s)^2$), and a mesh router is placed in the center of each cell. Let $m$ denote the number of mesh routers, then $m=(l/l_s)^2$. In what as follows, we will limit the case of interests to that where $1 < m \leq n$. Mesh routers constitute a wireless mesh backbone providing a wireless infrastructure for mesh clients. In each cell, mesh clients are connected to the mesh router like a star topology, i.e., no direct communication is available among mesh clients, and the mesh router works as a hub for mesh clients. Such a WMN is referred as an infrastructure WMN in [4], which will be very popular in future WMN applications.
Each mesh client is a data source, which randomly chooses another mesh client as its destination. The cells where the source client and the destination client are located are defined as the source cell and the destination cell, respectively. Unlike mesh clients, mesh routers are neither data source nor data destination; they only route and forward data for mesh clients. Three types of communications are identified in the network: 1) Uplink Communications: a source client transmits data to its mesh router in the source cell; 2) Inter-cell Communications: mesh routers relay data in a multi-hop fashion from the source cell to the destination cell on the mesh backbone; 3) Downlink Communications: the mesh router in the destination cell transmits data to the destination client. Both uplink and downlink communications are also defined as intra-cell communications.

Each mesh router is equipped with two radio interfaces such that it transmits at $W_1$ bits/s in inter-cell communications and it transmits at $W_2$ bits/s in downlink communications. Each mesh client transmits at $W_1$ bits/s in uplink communications. We assume that $W_1$ and $W_2$ are orthogonal. Note that mesh routers and mesh clients use the same radio interface in intra-cell communications. And we split the bandwidth evenly for uplink and downlink communications,
respectively so that all three types of communications don’t interfere with each other. Since all uplink traffic will finally go to downlink in this model, such a bandwidth-split scheme is reasonable and it can be implemented easily by applying a simple TDMA scheme. In addition, mesh routers can receive packets from only one sender at a time and cannot transmit and receive packets simultaneously. The same constraint is imposed on mesh clients.

We assume that a transmission is successful if the signal-to-noise-and-interference ratio (SINR) is above a certain threshold $\Theta$. The SINR is given by $P_0 d^{-\alpha}/(N_0 + I)$, where $P_0$ denotes the transmit power, $d$ is the distance between the transmitter and the receiver, $\alpha$ is the path loss exponent, $N_0$ denotes the noise power, and $I$ is the interference power, which is the sum of the received power from all the undesired transmitters. In what as follows, we assume that $N_0 = 0$, i.e., in wireless networks, dominant interference is from other nodes instead of background noise.

We use the slotted ALOHA as the multi-access protocol and a shortest path routing protocol is used in inter-cell communications.

### 4.3 Problem Formulation

In the above WMN model, given $n$, $W_1$, $W_2$ and specific transmission, scheduling and routing protocols, per client throughput, denoted as $TP(m,r)$, is a function of the number of mesh routers $m$ and the common transmission range all mesh routers choose in inter-cell communications, denoted as $r$. Since $W_1$ and $W_2$ are orthogonal, $TP(m,r)$ can be obtained by computing $TP_{W_1}(m,r)$ and $TP_{W_2}(m)$ separately, where $TP_{W_1}(m,r)$ is feasible per client throughput in inter-cell communications and $TP_{W_2}(m)$ is feasible per client throughput in intra-cell communications. Obviously,
\[ TP(m, r) = \min \{ TP_{i_1}(m, r), TP_{i_2}(m) \}. \]

We now give a precise formulation for the two problems that will be addressed in this paper: (i) The Maximum Throughput Partition (MTP) Problem and (ii) The Maximum Throughput Partition with Hops’ number Constraint (MTPHC) Problem. In both the problems, we partition the network such that the worst case of per client throughput is maximized, which is denoted as \( \min TP(m, r) \). The worst case means that such throughput is obtained by considering the most heavily loaded nodes in the network. For example, in intra-cell communications, throughput is always calculated in the cell with the largest number of mesh clients and in inter-cell communications, throughput is always obtained on the routers in the center of the backbone network. Hence, the obtained throughput is a guaranteed per client throughput that can be achieved by every node in the WMN.

**Problem MTP**: In the WMN model, given \( n, W_1, W_2 \) and specific transmission, scheduling and routing protocols, the network region \( R \) is partitioned into \( m \) small cells such that,

\[
\min TP(m, r) = \min \{ \min TP_{i_1}(m, r), \min TP_{i_2}(m) \}, m = i^2 \text{and} 1 < i < \sqrt{n}, \tag{4.1}
\]

is maximized.

**Problem MTPHC**: In the above WMN model, given \( n, W_1, W_2 \) and specific transmission, scheduling and routing protocols, the network region \( R \) is partitioned into \( m \) small cells such that,

\[
\min TP(m, r) = \min \{ \min TP_{i_1}(m, r) \mid \bar{h} \leq c, \min TP_{i_2}(m) \}, m = i^2 \text{and} 1 < i < \sqrt{n}, \tag{4.1}
\]

is maximized, where \( \bar{h} \) denotes the average number of hops per bits in inter-cell communications and \( c \) is a constant.
4.4 MTP Solution

**Proposition 1**: Given $n$, $W_1$ and specific transmission, scheduling and routing protocols, the worst case of feasible per client throughput in inter-cell communications can be obtained as follows:

$$
\min TP_{W_1}(m,r) = \frac{W_1E_1(m,r)}{\overline{h}(m,r)\overline{N}_{\max}(n,m)}, m = i^2 \text{ and } 1 < i < \sqrt{n},
$$

(4.2)

where $E_1(m,r)$ denotes resource sharing efficiency of inter-cell communications, $\overline{h}(m,r)$ denotes average number of hops per bit in inter-cell communications, $\overline{N}_{\max}(n,m)$ denotes the expected maximal number of mesh clients in any cell.

**Proposition 2**: Given $n$, $W_2$ and specific transmission and scheduling protocols, the worst case of feasible per client throughput in intra-cell communications can be obtained as follows:

$$
\min TP_{W_2}(m) = \frac{W_2E_2(n,m)}{2\overline{N}_{\max}(n,m)}, m = i^2 \text{ and } 1 < i < \sqrt{n},
$$

(4.3)

where $E_2(n,m)$ denotes resource sharing efficiency of intra-cell communications.

**Corollary 1**: Given $n$, $W_1$, $W_2$ and specific transmission, scheduling and routing protocols, the worst case of per client throughput of WMNs can be obtained as follows:

$$
\min TP(m,r) = \min \left\{ \frac{W_1E_1(m,r)}{\overline{h}(m,r)\overline{N}_{\max}(n,m)}, \frac{W_2E_2(n,m)}{2\overline{N}_{\max}(n,m)} \right\}, m = i^2 \text{ and } 1 < i < \sqrt{n}.
$$

(4.4)

In what as follows we will give the solution of computing $E_1(m,r)$, $E_2(n,m)$, $\overline{h}(m,r)$, and $\overline{N}_{\max}(n,m)$.
4.4.1 Sharing efficiency of inter-cell communications

The sharing efficiency here is equivalent to the multiple access efficiency of slotted ALOHA protocol in the regular grid backbone network. A similar case has been well studied in [103], in which one can find the detailed proof of proposition 3.

**Proposition 3**: In the slotted ALOHA backbone network, assuming that nodes transmit at equal power levels with probability $p$, the success probability of a transmission given a desired transmitter-receiver distance $r$ and $(m-2)$ other nodes at distances $d_i$ ($i = 1, \ldots, m-2$) is

$$
P_s(m, r, p) = \prod_{i=1}^{m-2} \left( 1 - \frac{\Theta p}{(d_i / r)^\alpha + \Theta} \right),
$$

In the above formula, $d_i$ is easy to get since all mesh routers are placed regularly in the backbone network. Recall that $E_i(m, r)$ will be obtained by assuming the receiver is located at the center of the backbone network. Therefore, $E_i(m, r, p)$ is given by

$$
E_i(m, r, p) = p(1 - p)P_s(m, r, p),
$$

(4.5)
since $p$ is the probability that a source router transmits and $1-p$ is the probability that its 1-hop receiver does not transmit in the same timeslot.

Finally, $E_i(m, r)$ can be obtained by

$$
E_i(m, r) = \max_{p = p_{\text{opt}}} \{ E_i(m, r, p) \},
$$
i.e., we can always obtain the optimal transmission probability. Figure 4.2 and Table 4.1 give examples of $E_i(m, r)$ from simulations. In the examples, the minimum transmission range is adopted in inter-cell communications, i.e., $r = l / \sqrt{m}$.
Figure 4.2: The sharing efficiency of inter-cell communications $E_i(m, p)$ for a regular grid backbone network, with $\Theta = 2, \alpha = 4, r = l/\sqrt{m}$.

Table 4.1: Numeric results of $E_i(m, r)$, with $\Theta = 2, \alpha = 4, r = l/\sqrt{m}$.

<table>
<thead>
<tr>
<th>$m$</th>
<th>4</th>
<th>9</th>
<th>16</th>
<th>25</th>
<th>36</th>
<th>49</th>
<th>64</th>
<th>81</th>
<th>100</th>
<th>121</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_i$</td>
<td>0.154</td>
<td>0.079</td>
<td>0.071</td>
<td>0.064</td>
<td>0.062</td>
<td>0.060</td>
<td>0.060</td>
<td>0.059</td>
<td>0.058</td>
<td>0.058</td>
</tr>
<tr>
<td>$m$</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
<td>256</td>
<td>289</td>
<td>324</td>
<td>361</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>$E_i$</td>
<td>0.058</td>
<td>0.058</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
</tr>
</tbody>
</table>

4.4.2 Sharing efficiency of intra-cell communications

In uplink communications, transmissions are scheduled by a TDMA-slotted ALOHA combined scheme. As shown in Figure 4.3, first, a TDMA scheduling scheme guarantees that each cell can get a time slot for transmission every 4 time slots such that there is no transmission occurring simultaneously in its 8 neighboring cells. Second, in the specific time slot, all nodes in the cell content for the uplink communications using slotted ALOHA. Therefore,
Figure 4.3: A TDMA scheduling scheme in intra-cell communications

\[ E_2(n,m) = \frac{1}{4}(1 - \frac{1}{N_{\text{max}}})^{N_{\text{max}} - 1} \approx \frac{1}{4e} \]

### 4.4.3 Average number of hops

The average number of hops can be obtained by

\[ \overline{h}(m,r) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{m} h_{ij}(m,r)}{m(m-1)} \]

where \( h_{ij}(m,r) \) denotes the number of hops from node \( i \) to node \( j \) using shortest path routing.

Table 4.2 displays examples of \( \overline{h}(m,r) \) from simulations.

<table>
<thead>
<tr>
<th>( m )</th>
<th>16</th>
<th>36</th>
<th>64</th>
<th>100</th>
<th>144</th>
<th>196</th>
<th>256</th>
<th>324</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{h}(m,r = l/\sqrt{m}) )</td>
<td>2.67</td>
<td>4.00</td>
<td>5.33</td>
<td>6.67</td>
<td>8.00</td>
<td>9.33</td>
<td>10.67</td>
<td>12.00</td>
<td>13.33</td>
</tr>
<tr>
<td>( \overline{h}(m,r = \sqrt{2l}/\sqrt{m}) )</td>
<td>1.90</td>
<td>2.82</td>
<td>3.75</td>
<td>4.68</td>
<td>5.61</td>
<td>6.54</td>
<td>7.48</td>
<td>8.41</td>
<td>9.34</td>
</tr>
</tbody>
</table>
4.4.4 Expected maximum number of mesh clients in any cell

Let $N_j(n,m)$ be the random variable denoting the number of mesh clients falling into the $j$-th cell. The expected maximum number of mesh clients in any cell $\bar{N}_{\text{max}}(n,m)$ is defined as follows:

$$
\bar{N}_{\text{max}}(n,m) = E[\max_j (N_j(n,m))], 1 \leq j \leq m.
$$

$\bar{N}_{\text{max}}(n,m)$ is used to compute throughput in the most heavily loaded cell. It is different from the mean number of client in each cell, which is $n/m$. Figure 4.4 displays both the numeric results of $\bar{N}_{\text{max}}(n,m)$ and $n/m$ by simulations when there are 400 mesh clients in the network.

![Figure 4.4: Numeric results of $\bar{N}_{\text{max}}(n,m)$ and $n/m$, with $n = 400$.](image-url)
4.5 MTPHC Solution

In the next section, our simulation results will show that for MTP Problem, optimal throughput can always be achieved by using the shortest transmission range in inter-cell communications. The shortest transmission range is the minimal transmission range that keeps the backbone network fully connected. However, in order to meet the latency requirement, sometimes longer transmission range must be adopted to reduce the number of hops, which results in much lower sharing efficiency of inter-cell communications. Therefore, lower throughput is expected. In such cases, optimal network partition will be very different from the cases without hops’ number constraint.

MTPHC problem can be solved by computing MTP results with a specific transmission range, which makes the average number of hops in inter-cell communications satisfy the requirement, i.e. \( \bar{h}(m,r) \leq c \). The algorithm is as shown in Table 4.3.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MTPHC Framework</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: for all ( m ) do</td>
<td></td>
</tr>
<tr>
<td>2: Choose the minimum transmission range for inter-cell communications, i.e. ( r = l / \sqrt{m} ).</td>
<td></td>
</tr>
<tr>
<td>3: if ( \bar{h}(m,r) \leq c ) then calculate the MTP results indicated by (1) and (4).</td>
<td></td>
</tr>
<tr>
<td>4: else increase ( r ) to the next level and go back to step 3.</td>
<td></td>
</tr>
<tr>
<td>5: return the optimal partition number and throughput.</td>
<td></td>
</tr>
</tbody>
</table>
4.6 Simulation Results

In this section, we calculate the numeric results by simulations. For both problems, we assume $W_1 = 54 \text{Mbps}$, $W_2 = 11 \text{Mbps}$, $\Theta = 2$, $\alpha = 4$, and $l = 1000m$.

For MTP problem, Figure 4.5 shows the per client throughput by varying the number of mesh routers. For each $n$, throughput of WMN is decided by 2 curves. Here flat curves indicate feasible per client throughput in inter-cell communications while steep ones indicate feasible per client throughput in intra-cell communications. They are obtained by (4.2) and (4.3), respectively. As demonstrated in (4.4), per client throughput of the WMN is thus achieved by always taking the smaller one of the two throughputs. With 300 randomly distributed mesh clients, the optimal worst case of per client throughput has been achieved as 58.1Kbits/s by partitioning the network into 10by10 grid, i.e. deploying 100 mesh routers. Similarly, in the case of $n = 400$ and $n = 500$, the optimal throughput is 48.5Kbits/s and 42.2Kbits/s by deploying 144 and 196 mesh routers, respectively. Therefore, more mesh clients need the backbone comprising of more mesh routers to achieve the best throughput performance. Optimal throughput is always achieved by using the minimum transmission range in inter-cell communications. In our model, it is equal to $l/\sqrt{m}$.

When the number of mesh routers is small, bottleneck is from intra-cell communications. Large ratio of clients’ number to routers’ number results in small throughput. Thus, in this case, adding mesh routers in the backbone network can effectively increase the throughput. When the number of mesh routers is large, per client throughput is throttled by inter-cell communications. Adding mesh routers has two side effects. Benefits from decreasing the ratio of clients’ number to routers’ number will be mitigated by more incurred interference among routers. By always using the minimum transmission range, such interference is translated into longer routing path that degrades the throughput. In this case, adding more backbone nodes does not necessarily increase
the throughput. Therefore, deploying an appropriate number of backbone nodes in WMNs is very important not only for saving the cost but also for better performance.

In MTPHC problem, the optimal throughput is evaluated by imposing constraint on the average number of hops in inter-cell communications. Figure 4.6 shows the per client throughput by varying the number of mesh routers under the condition of $\bar{h}(m, r) \leq c$. With a loose constraint, e.g. $\bar{h}(m, r) \leq 10$, in this case, the optimal throughput is 42.2Kbits/s by deploying 196 mesh routers when assuming there are 500 mesh clients in the network. This is the same result as that in MTP problem. However, with tighter constraints, the results are very different from the results of MTP problem. The optimal throughput is 38.0Kbits/s and 29.7Kbits/s by deploying 81 and 64 mesh routers, respectively when $\bar{h}(m, r) \leq 6$ and $\bar{h}(m, r) \leq 4$. So, the trade-off between throughput and the number of hops has been observed. Figure 4.6 also demonstrates that adding excessive backbone nodes may dramatically degrade throughput performance in MTPHC problem. Therefore, when considering hops’ number constraint, it is much more critical to find a proper size of the backbone network for a WMN.

The results of MTPHC problem provide us another finding. In the case that we have less than optimal number of mesh routers, the idea always using the minimum transmission range may not be ideal. For example, in Figure 4.6, if the backbone network has equal or less than 64 mesh routers, it is better to adopt a larger transmission range, which is equal to $\sqrt{2l} / \sqrt{m}$. Comparing with using the minimum transmission range, i.e., $r = l / \sqrt{m}$, the ideal one can achieve the same optimal throughput but less hops in the backbone network. Hence, MTPHC solution can also calculate the ideal transmission range when the backbone has less-than-optimal number of mesh routers.
Figure 4.5: Numeric results of MTP Problem

Figure 4.6: Numeric results of MTPHC Problem, with $n=500$. 
4.7 Conclusion

In this chapter, a framework has been proposed to achieve non-asymptotic per client throughput of WMNs. We show that throughput of WMNs is constrained by both inter-cell and intra-cell communications. Deploying a proper number of backbone nodes can effectively balance the traffic load between these communications. Therefore, optimal throughput is obtained. When less-than-optimal number of backbone nodes is deployed, the ideal transmission range of backbone nodes can be also achieved by our solution. More sophisticated transmission, scheduling and routing schemes will be adopted in the future work.

4.8 Acknowledgement

This chapter, in full, is a reprint of the material as it appears in Ping Zhou, B. S. Manoj, and Ramesh R. Rao, "On Optimizing Non-Asymptotic Throughput of Wireless Mesh Networks", to appear in 5th Annual IEEE Consumer Communications & Networking Conference (CCNC 2008), Las Vegas, NV, Jan. 2008. The dissertation author was the primary researcher and author of this paper.
V

Conclusion

WMN is a very promising wireless networking technology in the near future. In this dissertation, throughput performance of WMNs has been investigated from theoretical capacity analysis to practical algorithms’ design.

The asymptotic throughput capacity of infrastructure WMNs is studied in Chapter II. For such a network with $N_c$ randomly distributed mesh clients, $N_r$ regularly placed mesh routers and $N_g$ gateways, assuming that each mesh router can transmit at $W$ bits/s, the per-client throughput capacity has been derived as a function of $N_c$, $N_r$, $N_g$ and $W$. The result illustrates that, in order to achieve high capacity performance, the number of mesh routers and the number of gateways must be properly chosen. It also reveals that an infrastructure WMN can achieve the same asymptotic throughput capacity as that of a hybrid network by choosing only a small number of mesh routers as gateways.

The above theoretical results indicate that in order to achieve ideal throughput performance, mesh backbone of WMNs must be optimally designed. An innovative gateway placement scheme is proposed for WMNs in Chapter III. It determines the location of a gateway based on a new performance metric called multi-hop traffic-flow weight (MTW). The MTW computation takes into account many factors that impact the throughput of WMNs, i.e., the
number of mesh routers, the number of mesh clients, the number of gateways, traffic demand from mesh clients, locations of gateways, and possible interference among gateways. Thus, given a certain number of gateways, the proposed gateway placement scheme provides a framework of maximizing the throughput of WMNs through proper placement of these gateways. The throughput of WMNs is calculated based on a TDMA scheduling scheme in this paper. However, other methods of throughput computation can be applied to the same framework of gateway placement. The performance of the proposed gateway placement scheme is evaluated through simulations. Experimental results show that it constantly outperforms other schemes with a large margin.

Algorithms on choosing the optimal number of mesh routers are presented in Chapter IV. Two problems are investigated. In Maximum Throughput Partition (MTP) problem, the ideal throughput is achieved by optimally partitioning the network with a proper number of backbone nodes. In Maximum Throughput Partition with Hops’ number Constraint (MTPHC) problem, a similar problem is studied but with constraint on the average number of hops in the backbone network. The results show that it is critical to find an appropriate size of the backbone network for a WMN, especially when the hops’ number constraint is imposed. Our solution of MTPHC problem can be also used to obtain the ideal transmission range when less-than-optimal number of backbone nodes is deployed. Comparing with the minimum transmission range, the ideal one can achieve the same optimal throughput but effectively reduce the average number of hops in mesh backbone communications.

Some of the above-mentioned works can be further investigated in the future. In the gateway placement algorithm, mesh routers are assumed to be placed on a regular grid. Although regular placement of mesh routers has many benefits [93], such a placement is hardly realized in practical deployment. Therefore, in order to verify the effectiveness of the proposed gateway
placement algorithms, more placement patterns of mesh routers should be studied, e.g. a certain level perturbation from ideal grid placement, random placement and so on. The research of gateway placement can also be extended with more theoretical analysis since current work doesn’t demonstrate the proposed algorithm as an optimal solution or tell how to achieve the optimal one.
Bibliography


