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Young Kun Bae (Ph. D. thesis)

August 1982
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CROSS SECTIONS FOR BALMER-ALPHA EXCITATION IN
HEAVY PARTICLE COLLISIONS*

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Ph.D. Thesis

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ABSTRACT

Doppler shifted and unshifted Balmer-alpha radiation has been observed in the absolute sense for energetic \(H^+, H_2^+\) and \(H_3^+\) ions incident on molecular hydrogen by the method of decay inside the target within the energy range of 20 keV to 150 keV. Most of the measurements were based on single-collision conditions, but a simple thick-target experiment has been tried for the case of dissociative excitation of the target molecules by H atoms.

The Balmer-alpha radiation emitted by hydrogen and deuterium beams has been used as a diagnostic method of neutral beam parameters. One important neutral beam parameter is the species mix between \(H^+, H_2^+\) and \(H_3^+\) ion currents produced by the ion source and accelerator. This species mix can be resolved by analysis of the Balmer-alpha radiation if the beam is observed along an off normal axis with sufficient spectral resolution to separate the Doppler shifted radiation components from each other. An impediment to this approach to measuring the ion species is that some of the required cross sections have not been measured. This
is the motivation for the presented experimental work.

A home made monochromator gave enough optical throughput and spectral resolution for separation of the Doppler shifted lines from the unshifted lines. By selectively varying the target pressure and the distance of travel into the target prior to the observation region, excitation cross sections for three different angular momentum states (3s, 3p and 3d) have been determined. Combinations of a linear polarizer and a half-wave plate were used for polarization measurement. Separation of the individual Zeeman levels has been tried for the 3p state from the information obtained from the polarization. Theoretical estimates of the cascading corrections have been applied in the case of both thin and thick targets. The intensity development equations for thick targets also have been derived.

Cross sections for 3s production show general agreement with previous measurements, while those for 3p and 3d differ by as much as a factor of two. Target dissociative excitation cross sections show good agreement with previous measurements except those measured by Williams, et al..
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CHAPTER 1
Introduction

There has been considerable effort to derive useful energy from fusion reactions for many years. One of the most advanced and prospective technologies for a practical fusion reactor utilizes magnetic confinement which contains the fusion plasma with strong magnetic fields. To achieve a plasma with the necessary high temperature the usual reactors require auxiliary plasma heating, for which a promising technique is injecting of energetic neutral deuterium beams into the plasma.\(^{(1)}\) These pass through the confining magnetic field and get trapped in the confinement region by either charge exchange or ionization collision processes. Thereafter the beam energy is shared with the target plasma by Coulomb collisions. A brief explanation of the idea is shown in Fig. 1-1; \(D^{+}, D_{2}^{+}\) and etc. ions are formed in the ion source. After acceleration they are neutralized in the gas neutralizer. Usually this neutralization process is extremely complicated since it involves multiple atomic and molecular collisions. The beam will be composed of only neutral particles after it goes through the gas neutralizer and sweeping magnet which eliminate charged particles from the beam.

To develop a multi-mega watt neutral atom beam, it is required for the beam to travel over a distance of several meters with a minimum loss. Therefore, it is vital to be
able to define and measure both the beam steering direction and beam angular divergence. It is also important to be able to measure the relative quantities of the different species in the mixed ion beam extracted from the ion source. The resulting neutral beam before injecting into the plasma is composed of \( \text{D}^0 \) with full energy \( E \), which comes from \( \text{D}^+ \), \( \text{D}^0 \) with \( E/2 \) and \( \text{D}_2^0 \) with \( E \) from \( \text{D}_2^+ \), \( \text{D}^0 \) with \( E/3 \) and \( \text{D}_2^0 \) with \( 2E/3 \) from \( \text{D}_3^+ \) and \( \text{D}^0 \) with \( E/10 \) from \( \text{D}_2 \text{O}^+ \). This is shown in Fig. 1-2 schematically.

In general the effective charge exchange and ionization cross sections depend on the beam energy. Hence, the mean free path lengths of the neutrals are dependent on both beam energy and beam species. Moreover in these neutral beams the peak power density may approach 30 kW/cm\(^2\) which is at least one order of magnitude higher than can be deposited on even cooled conventional diagnostic probes. Therefore, it has been long desired to develop alternative non-disturbing diagnostic methods. One of the most successful methods is the optical spectroscopic beam diagnostics\(^{(1a)}\). The spectrum of Balmer-alpha radiation emitted by fast deuterium neutrals is observed inside the neutralizer along an optic axis at an angle to the neutral beam axis. The radiation is Doppler shifted sufficiently to resolve the three dominant energy components of the neutral beam. The typical spectrum of the radiation is shown in Fig. 1-3.

To make a quantitative assessment of such a spectrum,
Upper: Fabry-Parot Interferogram of Beam
Lower: that of calibration H-alpha source

TYPICAL SPECTRAL PROFILE FROM NEUTRALIZER
XBL 829-11771

Fig. 1-3
many kinds of cross sections for light emission are needed, some of which are unknown. The reason for the lack of knowledge is the inherently low signal to noise ratios normally experienced in the relevant experiments.

A number of authors have measured relevant cross sections for producing n=3 state hydrogen atoms by hydrogenic molecular ions and protons incident on molecular hydrogen. Hughes and co-workers (1b)-(11) have reported measurements which are often utilized in on-going research related to fusion. Thomas and co-workers (12)-(13) and other authors have reported, (14)-(25) discussed and extended Hughes' works. A common characteristic of most of their experiments is that energetic ions and/or atoms were passed through a gas target cell, and the radiation of the fast emerging neutral atoms was observed down stream in a reduced pressure region. Thomas' work (13) was an exception. For Balmer-alpha this allowed a separation of the s, p and d states and a measurement of both excitation cross section and respective lifetimes.

In the work presented herein, the observation of the emitted light was within the target cell itself, and the Doppler shift was used for the separation of the radiation from the fast beam and that from the target gas. Furthermore, in this work it was possible to move the aperture so that the distance the beam traveled before observation as well as the gas pressure and beam energy could be varied as needed. The specific apparatus and method will be discussed
in the following chapters.

More specifically, the reactions

\[ H^+ + H_2 \rightarrow H^+ + H + H^*(n=3 \text{ to } 2) \] (1-1)

and

\[ H^0 + H_2 \rightarrow H^0 + H + H^*(n=3 \text{ to } 2) \] (1-2)

have been studied for \( H^+ \) beam incident on a thick hydrogen target by using species development equations. (26)

The charge changing reaction leaving the fast atom in an excited state

\[ H^+ + H_2 \rightarrow H^*(3s,3p,3d) + (H^0) \] (1-3)

has been studied with \( H^+ \) ions and thin hydrogen gas targets.

The individual cross sections of Zeeman levels of the \( 3p \text{ to } 2s \) transition were studied by observation of the polarization which gave correction factors for total cross section. In many cases it was difficult to measure polarizations due to the low level of polarization involved.

The reactions

\[ H_2^+ + H_2 \rightarrow \left[ H_2^+ \right] + H + H^*(n=3 \text{ to } 2) \] (1-4)

and

\[ H_3^+ + H_2 \rightarrow H^*(3s,3p,3d) + H^+ + (H_2) \] (1-5)

and

\[ H_3^+ + H_2 \rightarrow \left[ H_3^+ \right] + H + H^*(n=3 \text{ to } 2) \] (1-6)
and

\[ \text{etc.,} \]

\[ H_3^+ + H_2 \rightarrow H^*(3s,3p,3d) + (H_2^+) + (H_2) \quad (1-7) \]

\[ \text{etc.} \]

have been studied for \( H_2^+ \) and \( H_3^+ \) beams on thin hydrogen targets.

Only the cross sections of \( H^+ \) impact on \( H_2 \) had been known when this work started. Those measurements show good agreement with the present results. Recently Williams\(^{(28)}\) et al. have measured the cross sections of the target dissociation, formation of the fast 3s state and the sum of 3p and 3d states for \( H^+, H_2^+ \) and \( H_3^+ \) impact on \( H_2 \) up to 100 keV, some of which show agreement with the present results. No exact theoretical estimates of these cross sections have been made due to the lack of knowledge of molecular collisions. For the purpose of rough comparison the theoretical values of the cross sections for \( H^+ \) impact on \( H \) atoms have been used. Because at high energy the bonding of molecules could be negligible, these values should be close to half of the value of \( H^+ \) impact on \( H_2 \). It turned out this argument was right in many cases. The present results show very good agreement with these values too. The detailed comparison is given in Chapter 5.

Thick-target experiments have been studied only for the dissociative excitation of target molecules for \( H^+ \) impact on \( H_2 \). The problems for other thick target experiments are the difficulties of the separation of an excited state hydrogen atom from the ground state hydrogen atom. The
second difficulty is that there are too many unknown parameters going together resulting in statistically poor confidence. The third difficulty was that the cross sections to be measured couldn't be independent measurements. Rather they were dependent on the knowledge of other cross sections measured in some cases by others. In that sense the thick-target experiments are not good for confident measurements. Hence, the target dissociation of H₂ molecules by H atoms was experimentally studied using both thin and thick targets, but the theoretical description for other thick target systems is given in Appendix B.

Originally only relative measurements were planned because in the real application absolute values aren't needed. But seldom do absolute values of the cross sections measured by different authors agree with each other. Normalization requirements and conflicting published results suggested the need for new absolute measurements leading to the absolute calibration of the light detecting system. An advantage of this calibration procedure came from the usage of a home made monochromator which gave a large acceptance solid angle and an accurate wave-length response spectrum. Another advantage came from the usage of a one-to-one image technique which made it possible to cancel all geometric factors as explained in Chapter 4.A.
A. General Atomic Physics

In this section a brief survey of theoretical models and calculation methods for the excitation in heavy particle collisions will be presented. The phenomenon of interest in this work is the formation of excited states either by charge transfer or by dissociation. Relatively few theoretical calculations have been developed for those phenomena, because of the difficulty in the calculation of more than three body interactions. For the charge transfer calculation there have been three lines of approach in general; the Born approximation, the quantal impulse approximation and the binary encounter approximation. Although there is no exact definition, what is called low or high velocity is determined by the velocity of the first orbit electron in the Bohr atom; i.e. \( v_0 = 2.19 \times 10^8 \text{ cm/sec.} \) For a collision between two systems at high energy the systems don't have enough time to interact strongly with each other, therefore the molecular effect can be negligible. In that case the Born approximation gives the simplest theoretical calculation. If the two systems are labeled A and B, then the Hamiltonian can be written as

\[
H = ( T_R + H_A + H_B ) + V \quad (2-1)
\]

where \( T_R = -\left( \frac{\hbar^2}{2\mu} \right) \nabla^2_R \) and \( H_A \) and \( H_B \) are Hamilton-
ians for the isolated systems A and B in a reference frame with respect to the center of mass of each system. \( \vec{R} \) is the relative separation between the two centers of mass and \( V \) is the interaction potential between two systems. The differential cross section for a scattering of the system A by the system B accompanied by a transition of the internal structure of A and B from the initial state \( \psi_I \) to the final state \( \psi_F \) is given by (29a)

\[
\frac{d\sigma}{d\Omega} = \left( \frac{\mu}{2\pi} \right)^{\frac{1}{2}} \left( \frac{K_F}{K_I} \right) |M_{FI}|^2
\]  

(2-2)

where \( \vec{W}_I \) and \( \vec{W}_F \) are the initial and final relative momenta of the two systems and \( M_{FI} \) is

\[
M_{FI} = \langle e^{i\vec{K}_F \cdot \vec{R}} \psi_F | V | e^{i\vec{K}_I \cdot \vec{R}} \psi_I \rangle
\]

(2-3)

But for the case of rearrangement collisions like

\[
A + B \rightarrow C + D
\]

(2-4)

the Hamiltonian is decomposed in two different ways

\[
H = T_I + H_A + H_B + V_I
\]

\[
H = T_F + H_C + H_D + V_F
\]

(2-5)

But there is an ambiguity as to whether to use \( V_I \) or \( V_F \) as
V in the matrix element $M_{FI}$. This is called the post/prior discrepancy.

Earlier theoretical work applying this method to the electron capture by a proton incident on a hydrogen atom was undertaken by numerous authors (30)-(34). Oppenheimer demonstrated that at high incident velocity charge capture occurs almost entirely to s-state and that the probability of capture into a state of principal quantum number $n$ varies as $n^{-3}$. This is the famous $n^{-3}$ law for charge capture cross sections.

In order to take into account multiple collisions of the charge exchanged electron which are important in the transfer process, a rigorous expression is given by (35)

\[
\frac{d\sigma}{d\Omega} = \left(\frac{2\pi\mu}{\hbar^2}\right)^2 \frac{|K_F|}{|K_I|} |R_{FI}^+|^2
\]

where

\[
R_{FI}^+ = \langle e^{i\hat{K}_F \cdot \hat{R}} \psi_F | V_F | e^{i\hat{K}_I \cdot \hat{R}} \psi_I >
\]

\[
+ \lim_{\epsilon \to 0^+} \langle e^{i\hat{K}_F \cdot \hat{R}} \psi_F | V_F | G_{\epsilon^+} V_I e^{i\hat{K}_I \cdot \hat{R}} \psi_I >
\]

and

\[
G_{\epsilon^+} = \left( E - H^+ + i\epsilon \right)^{-1}
\]

This is the so-called quantal impulse approximation (36).

In the binary encounter approximation (37), the atomic electrons and the nucleus of each system are considered to act as independent scattering centers. In other words the mutual interaction between components in the same system are neglected during the collision process.
All the theoretical results for the charge capture of a proton incident on the hydrogen atom are compared with experimental results for protons incident on molecular hydrogens, under the assumption that a hydrogen molecule acts like two isolated hydrogen atoms. But it has been shown\(^{(38)}\) that this is not exactly true. It has been shown\(^{(39)}\) that a previously overlooked backscattering contribution to the cross section becomes dominant in the high energy limit, making the cross section depend on energy as \(E^{-3}\) instead of \(E^{-6}\).

Very few theoretical calculations have been developed for the dissociative excitation of the molecular beam. The detailed channel of molecular dissociation process will be followed. For \(H_2\) target gas dissociation three possible mechanisms are

\[
\begin{align*}
\text{H}^+ + \text{H}_2 & \rightarrow (\frac{\text{H}^+}{m}) + \text{H}^* + \text{H}^+ & \text{(2-7)} \\
\text{H}^+_m + \text{H}_2 & \rightarrow \left( \frac{\text{H}^+_m}{m} \right) + \text{H}^* + \text{H}^+ + \text{e} & \text{(2-8)} \\
\text{H}^+_m + \text{H}_2 & \rightarrow \left( \frac{\text{H}^+_m}{m} \right) + \text{H}^* + \text{H}^* & \text{(2-9)}
\end{align*}
\]

where \(m = 1, 2\) and \(3\).

For the high energy proton the first and second processes can be ruled out by Keene's\(^{(40)}\) failure to observe an appreciable number of slow protons in the study of protons impact on \(H_2\). Even for \(m=2\) and \(m=3\) cases the third process would seem to be dominant since Afrosimov\(^{(41)}\) has measured
secondary slow proton production by these ions impact on
H₂ and found this to be small.

From the same argument as before the possible mechanisms
for the projectile dissociation can be written as

\[
\begin{align*}
H_2^+ + H_2 & \rightarrow H^* + H^* + (H_2^+) \\
H_2^+ + H_2 & \rightarrow H^* + H^* + (H_2^+) \\
H_3^+ + H_2 & \rightarrow (H_2^+) + H^* + (H_2^+) \\
H_3^+ + H_2 & \rightarrow (H_2^+) + H^* + (H_2^+)
\end{align*}
\]
B. Life-Time of the Excited States and Intensity Development Equations.

In order to calculate the life-time of a certain excited state one should determine transition probabilities between that state and other states, i.e. the matrix elements of the electric dipole moment between two states.

For the n=3 state there are three different angular momentum states, 3s, 3p and 3d and the effective life-time of the state i is given by

\[ \frac{1}{\tau_i} = \sum_j A_{ij} \]

where \( \tau_i \) is the effective life time of the state i and \( A_{ij} \) is the transition probability between the higher state i and the lower state j. This leads to \( \tau_{3s} = 16.0 \times 10^{-8} \) sec, \( \tau_{3p} = 0.54 \times 10^{-8} \) sec, and \( \tau_{3d} = 1.56 \times 10^{-8} \) sec. (41a)

The light intensity equations along the beam direction are needed to evaluate the excitation cross sections from the experimental data. Under the single collision condition without cascading of the higher state into the state i, the differential equation for the number density of the projectile particles in the excited state i, \( N_{bi} \), is given by

\[ \frac{dN_{bi}}{dt} = (N_{a} \sigma_{j}^{*} v) N_{b} - \left( \frac{1}{\tau_{i}} \right) N_{bi} \]

(2-15)
where $N_b$; number density of the projectile,
$N_a$; number density of the target,
$\sigma_i^*$; excitation cross section into the state $i$,
v; velocity of the projectile in the lab. frame.

Under the assumption that there is no collision in the pre-target region and setting the time as $t=0$ when the projectile goes through the entrance aperture, the boundary condition for that equation is that at $t=0$, $N_b = 0$. Therefore

$$N_b = K_i \left( 1 - e^{-x/vT_i} \right) \quad (2-16)$$

where $K_i = N_a \sigma_i^* v N_b T_i$, and $x=vt$.

With the geometry of the system like that shown in Fig. 2-1, the total number of photons $J_{ij}$ emitted from the shaded section is given by

$$J_{ij} = \int L+\ell \ A N_b A_{ij} \ dx$$
$$= (I_b/e) N_a \ell T_i A_{ij} \sigma_i^* h_i(L) \quad (2-17)$$

where $I_b = e A N_b v$; the current of the beam,
$e$; charge of the electron,
$A$; cross sectional area of the beam,
and $h_i(L) = (1 - (vT_i/\ell)(1 - e^{-\ell/vT_i}) e^{-L/vT_i})$.

For $L \gg \ell$, $h_i(L) \approx g_i(L) = (1 - e^{-L/vT_i})$.

Therefore, the expected signal $S(L)$ from the photon detection system is given by.
\[ S(L) = \sum_i \left( \frac{\Omega}{4\pi} \right) T_Q J_{ij}(L) \]  

(2-18)

where \( \Omega \); observational solid angle

\( T \); transmittance of the optical system

\( Q \); quantum efficiency of the photon detector.

Finally \( S(L) \) can be expressed as

\[ S(L) = D \left( \frac{I_b}{e} \right) N_a \ell \sum_i A_{ij} \tau_i g_i(L) \]  

(2-19)

where \( D \) is the detection efficiency defined as

\[ D = \left( \frac{\Omega}{4\pi} \right) Q T \]  

(2-20)

So far the cascading effect has been neglected; including it changes equation (2-15) to

\[ \frac{dN_{bi}}{dt} = \left( \frac{I_b}{e} \right) \sigma_i^* N_a - \left( \frac{1}{\tau_i} \right) N_{bi} + \sum_{k>i} A_{ki} N_{bk} \]  

(2-21)

This is a 1st order self consistent differential equation.

Since the excitation cross section follows \( n^{-3} \) law and \( A_{ki} \) becomes smaller for the higher \( k \) the magnitude of the self-consistent term should be small. Hence, a perturbation method can be applied to solve this equation. The solution including the first order term can be written as

\[ N_{bi} = N_{bi}^0 + N_{bi}^1 \]  

(2-22)
then the equation for the zeroth order is given by

\[
\frac{dN_{bi}^0}{dt} = \left( I_b/e \right) N_a \sigma_i^* - \left( 1/\tau_i \right) N_{bi}^0
\]  
(2-23)

which has the same form with equation (2-15). Therefore the zeroth order solution can be written as

\[
N_{bi}^0 = K_i g_i(x)
\]  
(2-24)

The equation for the first order term is then

\[
\frac{dN_{bi}^1}{dt} = - \left( 1/\tau_i \right) N_{bi}^1 + \sum_{k>1} A_{ki} N_{bi}^0
\]  
(2-25)

In order to solve this equation one starts with a homogeneous part of the solution

\[
N_{bi}^{1h} = \alpha_i e^{-t/\tau_i}
\]  
(2-26)

and a particular part of the solution

\[
N_{bi}^{1p} = \sum_{k>1} \left( \beta_k A_{ki} K_k \left( 1 - e^{-t/\tau_k} \right) + \gamma_k \right)
\]  
(2-27)

Boundary conditions lead to

\[
\beta_k = \tau_k \tau_i / ( \tau_k - \tau_i )
\]
\[ \gamma_k = K_k A_{ki} \tau_i^2 / (\tau_k - \tau_i) \]
\[ \alpha_i = - \sum_{k > i} \gamma_k \]

The combination of these constants leads to a final form as

\[ N_{bi} = \tau_i \sum_{k > i} A_{ki} K_k \left( \frac{\tau_k g_k(x) - \tau_i g_i(x)}{\tau_k - \tau_i} \right) \] (2-28)

Only the single collision system has been treated to this point, but the multiple collision system will be presented also. For the real experiment the simplest multiple collision system is the case of the target molecule dissociative excitation by protons. The equations for other cases have been developed and they will be presented in Appendix B.

Consider the two component system including \( H^+ \) and \( H^0 \) and neglecting \( H^- \). The relative density of \( H^0 \) and \( H^+ \) in the projectile beam can be denoted by \( y_0 \) and \( y_+ \). Neglecting the excited state effect one can write the equation for \( y_0 \) and \( y_+ \) as

\[ y_0 + y_+ = 1 \] (2-29)

and

\[ \frac{dy_0}{d\pi} = - \sigma_{01} y_0 + \sigma_{10} y_+ \] (2-30)

where \( \pi = N_{bx} \) and \( \sigma_{10} \) and \( \sigma_{01} \) are charge exchange cross sections. The boundary condition is given at \( \pi = 0 \) where \( y_0 = 0 \) and \( y_+ = 1 \). Then the solutions are
\[ y_+ = F_{1\infty} + F_{0\infty} e^{-\sigma_T} \]  
(2-31)

and

\[ y_0 = F_{0\infty} \left( 1 - e^{-\sigma_T} \right) \]  
(2-32)

where \( \sigma_T = \sigma_{10} + \sigma_{01} \), \( F_{0\infty} = \sigma_{10}/\sigma_T \) and \( F_{1\infty} = \sigma_{10}/\sigma_T \).

The light intensity from the target, \( J \) is given by

\[ J = \int_{L+\ell}^{L} N_a A v N_b \left( \sigma_+ y_+ + \sigma_0 y_0 \right) \, dx \]  
(2-33)

where \( \sigma_+ \) and \( \sigma_0 \) are dissociative excitation cross sections of \( \text{H}_2 \) by \( \text{H}^+ \) and \( \text{H}^0 \) beam.

For \( L \gg \ell \) this can be simplified as

\[ J = N_a A v N_b \left( \sigma_+ \left( F_{1\infty} + F_{0\infty} e^{-\sigma_T} N_a L \right) \right) + \sigma_0 \left( 1 - e^{-\sigma_T} N_a L \right) \cdot \ell \]  
(2-34)
C. Polarization

Studying the interaction between the beam and target, one can assume the system to be azimuthally symmetric due to the cylindrical geometry of the beam. Assigning the beam axis as the $z$-axis and observing along the axis which has an angle $\theta$ with the $z$-axis, $I_{\perp}(\theta)$ is defined as the local relative intensity of photons which have their electric fields normal to the $z$-axis and $I_{\parallel}(\theta)$ as the intensity of photons with the electric fields parallel to the $z$-axis as shown in Fig. 2-2. With this geometry, the polarization at an angle $\theta$ with the $z$-axis can be specified as

$$P(\theta) = \frac{I_{\parallel}(\theta) - I_{\perp}(\theta)}{I_{\parallel}(\theta) + I_{\perp}(\theta)} \tag{2-35}$$

To derive $P(\theta)$ in terms of cross sections one starts with the quantum mechanical form of dipole radiation under the assumption that the dipole process is dominant in the radiative transition. The equation for the dipole radiation is given by (42)

$$I(\Omega, \hat{e}_j) = n'(e^2/2\pi c^3)\omega^4 |\hat{e}_j \cdot <N'|\hat{F}|N>|^2$$

$\text{------------------- (2-36)}$ 

where $I(\Omega, \hat{e}_j)$ is the intensity of radiation in the direction of $\Omega$ which has the electric field along the unit vector $\hat{e}_j$ and $|N\rangle$ is the ket for the higher energy quantum state and $|N'\rangle$ is that for lower energy quantum state,
where \( n' \) represents the density of the excited state \( N \), \( \hat{r} \) the length and direction of the individual dipole, and \( \omega \) the angular frequency of the radiation. Since the radial part of the dipole transition doesn't contribute to the polarization only the angular part is of importance. From this knowledge the previous equation can be simplified to

\[
I(\Omega, \hat{e}_j) = \sum_{N,N'} K \sigma_N |\hat{e}_j \cdot <N'|\hat{r}|N>|^2 (2-37)
\]

where

\[
K = \frac{e^2 \omega^4}{2\pi c^3 f(x)}
\]

and \( \sigma_N \) is the cross section for the state \( |N> \) and \( f(x) \) is a function of the travel distance of the beam into the target under the single collision condition.

The equations for \( I_{\perp}(\theta) \) and \( I_{\parallel}(\theta) \) are

\[
I_{\perp}(\theta) = \sum_{N,N'} K \sigma_N |\hat{x} \cdot <N'|\hat{r}|N>|^2 (2-38)
\]

\[
I_{\parallel}(\theta) = \sum_{N,N'} K \sigma_N |(-\cos \theta \hat{y} + \sin \theta \hat{z}) \cdot <N'|\hat{r}|N>|^2 (2-39)
\]

But

\[
\begin{align*}
\hat{x} \cdot \hat{r} &= -r \sqrt{4\pi/3} (Y_{11} + Y_{1-1})/\sqrt{2} \\
\hat{y} \cdot \hat{r} &= -r \sqrt{4\pi/3} (Y_{11} - Y_{1-1})/\sqrt{2} \\
\hat{z} \cdot \hat{r} &= r \sqrt{4\pi/3} Y_{10}
\end{align*}
\]

The quantum number \( N \) represents a set of quantum numbers.
\( (n, l, m) \) without spin consideration, where \( n \) is the principal quantum number, \( l \) the angular quantum number and \( m \) the magnetic quantum number. Then in the wave function representation \( \langle N' \mid \hat{x} \cdot \hat{r} \mid N \rangle \) can be written as

\[
\langle N' \mid \hat{x} \cdot \hat{r} \mid N \rangle = \int \! d^3r \quad \psi_{n' l' m'} \left( \hat{x} \cdot \hat{r} \right) \psi_{nlm}
\]

\[
= \int \! dr \ r^2 \ R_{n'l'} R_n \ \int \! d\Omega \ Y_{l'm'} \left( \hat{x} \cdot \hat{r} \right) Y_{lm}
\]

\[
= g \ < Y_{l'm'} \mid \hat{x} \cdot \hat{r} \mid Y_{lm} >
\]

where \( \psi \) is the wave function, \( R \) is the radial part of that and

\[
g = \sqrt{4\pi / 3} \ \int \! dr \ r^2 \ R_{n'l'} R_{nlm}
\]

Therefore

\[
I_\perp(\theta) = K' \sum_{N,N'} \sigma_N / 2 \ | \langle Y_{l'm'} | Y_{11} + Y_{1-1} | Y_{lm} \rangle |^2
\]

\[
I_\parallel(\theta) = K' \sum_{N,N'} \sigma_N |\cos \theta < Y_{l'm'} | Y_{11} + \frac{Y_{1-1}}{\sqrt{2}} i | Y_{lm} \rangle + \sin \theta < Y_{l'm'} | Y_{10} | Y_{lm} \rangle |^2
\]

where \( K' = K \ g^2 \).

For the \( 3s \rightarrow 2p \) transition

\[
I_\perp(\theta) = K' \sum_{N,N'} \sigma_N = I_\parallel(\theta)
\]

Thus the \( 3s \rightarrow 2p \) transition is not polarized. For the \( 3p \rightarrow 2s \) transition one considers the \( m=+1, m=0 \) and \( m=-1 \) states
separately. Because of the symmetry between clockwise and counter-clockwise rotations, there should be no difference between \( m=+1 \) and \( m=-1 \) states. Defining their cross sections as \( \sigma_{1+} \), \( \sigma_{10} \) and \( \sigma_{1-} \) and setting \( \sigma_{1+} = \sigma_{1-} = \sigma_{11} \), then

\[
\begin{align*}
I_{\perp}(\theta) &= K' \sigma_{11} \\
I_{\parallel}(\theta) &= K' \left( \cos^2 \theta \sigma_{11} + \sin^2 \theta \sigma_{10} \right)
\end{align*}
\]

Therefore the polarization is given by

\[
P(\theta) = \frac{\sin^2 \theta (\sigma_{10} - \sigma_{11})}{\sigma_{10} \sin^2 \theta + \sigma_{11} (1 + \cos^2 \theta)}
\]  

(2-46)

For the 3d + 2p transition the situation is more complicated. Defining \( \sigma_{22} \), \( \sigma_{21} \) and \( \sigma_{20} \) as the cross sections for \( m=2,1 \) and \( 0 \) states and by the same symmetry argument as before the equations for \( I_{\parallel} \) and \( I_{\perp} \) are

\[
\begin{align*}
I_{\parallel}(\theta) &= K' \sum_{m, m'} \sigma_{2m} |(\cos \theta / \sqrt{2}) < Y_{1m} | Y_{11} - Y_{1-1} | Y_{2m} > + \sin \theta < Y_{1m} | Y_{10} | Y_{2m} > |^2 \\
I_{\perp}(\theta) &= \frac{K'}{2} \sum_{m, m'} \sigma_{2m} |< Y_{1m} | Y_{11} + Y_{1-1} | Y_{2m} >|^2
\end{align*}
\]

(2-47)

Table (2-1) has been prepared for these values. Then the final equations defining the intensities are
| $m$ | $m'$ | $40\pi < Y_{1m}, |Y_{11}| Y_{2m}>$ | $40\pi < Y_{1m}, |Y_{1-1}| Y_{2m}>$ | $40\pi < Y_{1m}, |Y_{10}| Y_{2m}>$ |
|-----|-----|-----------------|-----------------|-----------------|
| 2   | 1   | 12              | 0               | 0               |
| 1   | 0   | 6               | 0               | 0               |
| 0   | 1   | 0               | 2               | 0               |
| 0   | 0   | 0               | 0               | 8               |
| 0   | -1  | 2               | 0               | 0               |
| -1  | 0   | 0               | 6               | 0               |
| -2  | -1  | 0               | 12              | 0               |

**TABLE (2-1)**
\[ I_\perp(\theta) = 2 K' \left( 6 \sigma_{22} + 3 \sigma_{21} + \sigma_{20} \right) \]

\[ I_{II}(\theta) = I_\perp(\theta) \cos^2 \theta + 8 K' \sigma_{20} \sin^2 \theta \]

Finally the polarization is given by

\[ P(\theta) = \frac{\sin^2 \theta \left( \sigma_{20} - \left( 2 \sigma_{22} + \sigma_{21} \right) \right)}{(2 + 3 \sin^2 \theta) \left( \frac{\sigma_{20}}{3} + (1 + \cos^2 \theta)(2 \sigma_{22} + \sigma_{21}) \right)} \]

\[ \text{(2-50)} \]
CHAPTER 3
Apparatus

A. Monochromator and Other Optics

To increase the detection efficiency a Littrow type monochromator (43) was designed and constructed in this laboratory. The basic structure of this type monochromator was very simple. It had two slits, one large grating with some adjustment equipments, one large achromatic lens and a 90° 1''x1'' prism. A more detailed description is presented in Appendix A.

The optics were designed to give the largest observation solid angle. Two lenses with 10 cm focal length and 4 cm diameter were used to give a one-to-one target image on the slit area of the monochromator. One lens was mounted on a Varian flange which had an optical window, with a 10 cm distance from the target. Between these two lenses was positioned a linear polarizer and a half wave plate. Since the monochromator had the greater sensitivity to the horizontally polarized light, in the measurement of the vertical component the polarization of light was rotated by 90° to give it a horizontal polarization. The physical idea of this arrangement is described below.

Consider the geometry shown in Fig. 3-1. With Jones matrix notation (44), the initially vertically polarized light is represented by \( (1, 1)^t \) and the half wave plate as
If the fast and slow axis of the half wave plate is set as shown in Fig. 3-1, through the half wave plate the state of polarization is transformed according to the following equation and the result of which is horizontally polarized light.

\[
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 \\
1
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
-1
\end{bmatrix}
\]  

(3-2)

Practically there is a certain attenuation factor due to the finite transmittance of the half wave plate. To measure horizontally polarized light, the fast and slow axis should be set parallel to the slit and the beam direction. In this geometry the incident light is represented as \((1, 0)^t\) and the half wave plate as

\[
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\]  

(3-3)

Therefore, the final result is given by

\[
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\]  

(3-4)

i.e. there is no rotation effect at all. In practice the relative transmittances of the horizontal axis and vertical
axis for the optical system were needed and measured with an unpolarized white light source.

After the monochromator one 5 cm focal length lens was used for one-to-one imaging of the exit slit image onto the window of the photomultiplier tube.
B. Ion Source and Accelerator

The electrostatic accelerator used in this research was a 150 kV TNC (Texas Nuclear Corporation) equipped with an R.F. ion source. It had extendable high voltage capability with a SF$_6$ gas system, up to 250 kV. Due to the design of this accelerator no usable beam current was obtainable below 10 kV. For lower energy beam a number of stacks for electrostatic acceleration were used for electrostatic focusing by the use of clippleads. The lower the beam energy was the more clippleads were used. By that way the beam current was increased.

The ion source bottle was a typical commercial R.F. type made of quartz. The position of the R.F. rings was adjusted to give maximum current. With a proton beam, an aluminium canal was used to produce maximum proton species. With H$_2^+$ and H$_3^+$ beam a nickel canal was used. For H$_2^+$ beam a low stack gas pressure ($\sim$3x10$^{-6}$ Torr) and for H$_3^+$ beam a relatively high stack gas pressure ($\sim$8x10$^{-6}$ Torr) gave the best results.

After acceleration the beam was focused by two electrostatic quadrupole lenses oriented at 90° to each other. The focused beam entered a steering plate section with two vertical sets of steering plates. The beam then entered a bending magnet, was momentum analyzed, and was bent at an angle of approximately 15°. The purity of the beam was ensured by using the Hall-probe signal to select the correct mass.
ions to be sent to the target chamber. The schematic dia-
gram of the whole system is given in Fig. 3-2.

The beam then passed through the first collimator of the
gas cell which had a 4 inch diameter diffusion pump used
for pumping out. Then the beam entered the gas target whose
length was defined being from the observation region to the
movable 1/16 inch diameter aperture system. The beam then
impinged on the pyroelectric detector.
35 kV ACCELERATOR

DIFFUSION PUMP

QUADRUPOLE FOCUS LENS

STEERING PLATES

MOMENTUM ANALYZING MAGNET

ZERO DEGREE FARADAY CUP

GAS CELL (DIFFUSION PUMP)

PHOTOMULTIPLIER

MONOCHROMATOR

IONIZATION GAUGE

FARADAY CUP / PYROELECTRIC DETECTOR

MOVABLE APERTURE

COLLIMATOR

TITANIUM PUMPS

XBL 829-11775

FIG. 3-2
C. Target System

The target chamber was basically made of a 34 inch length 1\(\frac{1}{2}\) inch diameter stainless steel pipe. As shown in Fig. 3-3 there were two additionally attached 1\(\frac{1}{2}\) inch diameter stainless steel pipe sections with angles 54.8° and 90° to the main pipe section. The 54.8° angled section was designed for two purposes. One was to give sufficient Doppler shift of light for separation of the radiation of fast particles from that of slow particles; the other was to simplify the polarization correction factor for cross sections. The 90° angled section was used for pumping out the residual gas and pressure measurement of the main target chamber. This section was connected to an ionization gauge, a capacitance manometer (Barocel) and a 4 inch diameter diffusion pump through a hand operated valve and 3/8 inch aperture. The ionization gauge was used for monitoring the zero point drifting of the Barocel and approximate residual background gas pressure measurements. The 4 inch diameter diffusion pump was used to reduce the back-ground pressure and to help to pump out during outgassing. The residual gas pressure was normally less than 2\(\times\)10\(^{-6}\) Torr during the experimental procedure. The Barocel was calibrated by an oil manometer before usage. The optical window which was connected to 54.8° angled section was made of fused silica.

To provide minimum light reflection, every surface inside
the target chamber was blackened with Aquadag (micro-graphite in aqueous base) dissolved in ethyl alcohol.

The movable aperture system had two o-rings to minimize loss of gas from the target chamber. Pumping out was through a hole at the center of the aperture system. The schematic is presented in Fig. 3-4.
FIG. 3-3
D. Detector

a. Photomultiplier Tube

A photomultiplier tube (EMI 9862A) was used for photon detection and the statistics was based on counting. The main limitation of the photon detection system came from the dark current due to thermionic electrons emitted by the photocathode, and poor quantum efficiency in the red light region. To decrease the dark current the photomultiplier tube was cooled to $-25^\circ C$ and only a small center part of the photocathode (1 cm diameter) was used. The high voltage for the photocathode was then selected for the best signal-to-noise ratio and pulse-height distribution. About 5% quantum efficiency and about 5 count/sec dark count allowed about 100 photon resolution of the photomultiplier tube in a second.

The pulse shapes from the pre-amp and the main-amp are shown in Fig. 3-5. The typical pulse height distribution is also shown in Fig. 3-6. From the pulse rise time of the pre-amp (~50 $\mu$sec) a nonlinear effect of the photon detection system can be expected for count rates of more than $10^4$ count/sec. This was experimentally verified in Chapter 4-A.

b. Pyroelectric Detector / Faraday Cup

The incident beam current was measured with a pyroelectric detector connected to lock-in amplifier (Princeton Applied Research) model number H-R-8 and was normalized to the
SIGNAL FROM PRE-AMP

XBB 829-8375
SIGNAL FROM MAIN AMP

Fig. 3-5
TYPICAL PULSE HEIGHT SPECTRUM

NUMBER OF PULSES

PULSE HEIGHT

XBB 829-8374

FIG. 3-6
Faraday cup signal. The schematic diagram of this is shown in Fig.3-7. The characteristics of the pyroelectric detector is known fairly well\(^{(45)}\) and a brief explanation is given below. The detector consisted of a 2.5 cm diameter polarized lead zirconate titanate ceramic disk which was silvered on both sides to provide electrical contact. The contact resistance between the surface and the electric contactor was confirmed to be much less than 1 ohm with an ohm-meter. The ion beam was chopped by electrostatic plates and an alternating signal was obtained from the detector. This had two advantages, number one was that the dark current of the photomultiplier tube could be discriminated from the real signal, and number two was that it simplified the amplification and interpretation of the signal and permitted an increased signal to noise ratio by measuring the signal with a phase-sensitive amplifier. Fig. 3-8 shows triggering signal used for the phase sensitive detection. Often in the measurement of low power signal the acoustical and electrical noise problems occur, but with phase sensitive detection these were negligible. The typical signal from the lock-in amplifier is shown in Fig. 3-9 and the detailed electronics are shown in Fig. 3-10. Since it was desired that the cross sections should be absolute, normalization of the pyroelectric detector signal was required. The Faraday cup was constructed with the pyroelectric crystal as an integral part and the secondary electron loss was prevented by the use of magnetic suppression.
TRIGGERING SIGNAL

ON OFF

BEAM

SIGNAL + NOISE

NOISE

FIG. 3-8
RAW SIGNAL FROM PYROELECTRIC CRYSTAL

XBB 829-8377

UPPER: SIGNAL FROM LOCK IN AMP
LOWER: TRIGGERING SIGNAL FOR BEAM

FIG. 3-9
Data Acquisition System

- PA: Pre Amplifier
- PG: Pulse Generator
- PMT: Photo-Multiplier Tube
- SCA: Single Channel Analyzer
- PHA: Pulse Height Analyzer
- PD: Pyroelectric Detector
- IFC: Inter-Face Controller
- CI: Counter-Integrator
- PSA: Phase Sensitive Amplifier
- DCI: Digital Current Integrator
- HV: High Voltage
- SP: Steering Plate
The Faraday cup was connected directly to an Ortec 439 Digital Current Integrator which was also used for measurement of the signal from the pyroelectric detector. Ortec 770 Counters and 771 Timer-Counters were used for counting and timing.

To measure $K$ which was defined as the ratio of Faraday cup signal to pyroelectric detector signal the gas target density was reduced to zero and the signal from the Faraday cup was integrated during 20 to 30 seconds. The signal from the pyroelectric detector was then integrated during the same amount of time as before. After one measurement, $K$ was then measured in the reverse way. The accuracy of $K$ depended on the stability of the current. In the normal case the standard error was less than a few percent. In all measurements the power of the incident ion beam exceeded the levels where one could expect problems.
CHAPTER 4
Procedure and Data Analysis

A. Absolute Optical Calibration

Absolute optical calibration requires calibration of the sensitivity of the total detection system to the best known standard photon source. The best known standard photon source is a standard tungsten ribbon lamp. Since the photon detection system used in this work was extremely sensitive, neutral filters were required for reducing the light intensity. Attention was also given to the linearity of the detection system. This is shown in Fig. 4-1 indicating the range of response for which linearity could be expected. Every measurement was limited to this range.

The equation for the relation between the known emissivity of the tungsten standard lamp and the detection efficiency of the system can be developed as follows. Define \( E_t \) as the number of photons which come from unit area of the tungsten filament within a unit solid angle and unit wavelength per second. Then the number of counts per second, \( S \), from the detector due to the standard lamp is

\[
S = D \left( 4\pi E_t A_m \Delta \lambda \right) \tag{4-1}
\]

where \( A_m \) denotes the area of the tungsten filament to be seen by the monochromator (the slit area of the monochro-
mator because a one-to-one image onto the slit area was used). $\Delta \lambda$ is the resolution of the monochromator and $D$ is the detection efficiency of the system which includes the detection solid angle and the quantum efficiency of the photon detector. Therefore $D$ is given by

$$D = \frac{S}{4\pi E_t A_m \Delta \lambda} \quad (4-2)$$

The next process is to relate the excitation cross section to the number of counts per second from the detector. At equilibrium (i.e. without considering the distance dependency of the emission into the target) the total number of photons per second, $N_p$, come from a length $\ell$ and density $N_a$ of the target due to the current $I_b$ of the beam is given by

$$N_p = N_a \sigma \left( \frac{I_b}{e} \right) \ell \quad (4-3)$$

where $e$ is the charge of an electron and $\sigma$ is the excitation cross section. On the other hand the number of counts per second, $N_s$, from the photon detector due to $N_p$ is

$$N_s = D N_p \quad (4-4)$$

Therefore, from the equations (4-2), (4-3) and (4-4) the excitation cross section can be expressed as
From this equation parameters which are needed for absolute optical calibration can be determined.

To ensure the monochromator slit was completely covered by the image of the tungsten strip, a 4.5±0.1 mm length center portion of the slit was used. By that way $A_m$ was given by $\ell \times (4.5\pm0.1)$ mm$^2$ and the width of the slit $\ell$ canceled out automatically in the equation (4-5). $\Delta \lambda$ was given by either the spectral line from the Doppler unshifted target emission, or that from $H_2$ discharge lamp. The scale unit of the monochromator was calibrated by Ne and $H_2$ discharge lamps. The spectrum from Ne lamp is shown in Fig. 4-2.

$S$ required care in its measurement. Using neutral filters and with the detector in its mode, it was determined that the detector was linear to $10^4$ count/sec. Above that rate, piling up of the pulses due to a pulse rise time of the order of 0.1 msec caused nonlinearity as shown in Fig. 4-1. $S$ was measured with a known current for the tungsten ribbon filament lamp and with a combination of neutral density filters to give an allowable count rate. The current of the standard lamp was then decreased to give a reasonable count rate such as 100 count/sec. This was integrated more than 100 second to give a less than 1 % standard counting error. From the above procedure the relative ratio between those amounts of filament current was developed with less than 1 %
FIG. 4-2

RELATIVE INTENSITY

CHANNEL NUMBER

XBL 829-11781
standard error. The neutral filter was then changed to give maximum allowable count rate and this rate was determined with less than 1% standard error. Again the lamp current was decreased to give a reasonable count rate. By repeating these procedures the lamp emissivity versus the filament current was developed without the knowledge of transmission characteristics of neutral filters. This is shown in Fig. 4-3. At the smallest current, the lamp can be used without any filter. So far these procedures were performed at a fixed wavelength. The detection efficiency at other wavelengths was also required. Fig. 4-4 shows the wavelength response of the detection system to be constant within a few percents for the wavelengths of interest. Therefore, the detection efficiency could be set to be constant with a few percent standard error.
RELATIVE INTENSITY

STANDARD LAMP CALIBRATION AT 6588 Å

VOLTAGE OF STANDARD LAMP

FIG. 4-3
RELATIVE INTENSITY

SPETRAL RESPONSE OF THE OPTICAL SYSTEM

FIG. 4-4
B. Beam Tuning

After pumping down of the entire system the accelerator was turned on and the desired beam energy was set by the accelerator H.V. power supply. Then the palladium leak was tuned on to give the desired amount of hydrogen gas into the R.F. bottle which was checked by the ion gauge in the stack section of the accelerator. Initially the beam was tuned by letting it run straight down the accelerator and into a Faraday cup placed on the zero degree axis of the accelerator just after the momentum analyzing magnet. Without any collimation the current was typically $10^{-4}$A with moderate ion source extraction voltage and stack gas pressure. After a brief warm up time (normally about 20 minutes) all power suppliers and palladium leak were stable enough to give a few percent fluctuation in the beam intensity on a moderate time scale. Then the bending magnet was energized and set to the value needed to bend the desired energy and species beam into the beam line. After passing through one collimator (100 mil diameter) and one entrance aperture of the gas target cell, the beam hit the pyroelectric detector/Faraday cup in the gas target. The maximum beam current was obtained by adjusting steering plates, electrostatic quadrupole, the gas pressure inside the R.F. bottle, the extraction voltage and the focus voltage of the ion source. The amount of beam depended on the energy and species. The typical current on the
the final pyroelectric detector / Faraday cup inside the target cell was the order of \( \frac{1}{2} \mu \text{A} \). The beam was stable during at least several minutes after tuning.

C. Development of Spectral Profile

After the beam was stabilized the beam was chopped by the use of a square wave potential applied to the steering plate. Then the calibration factor for the pyroelectric detector was measured as it was described in Chapter 3-D-b. Since the Doppler shifted line and the unshifted line had different spectral profiles, a well developed spectral profile was needed for separation of the Doppler shifted line from the unshifted line and the determination of a needed correction factor. The difference between those two spectral profiles resulted from an angular broadening effect.

The Doppler shift of the light which comes from the particle which moves with the velocity \( v \) can be described by

\[
\lambda = \gamma \lambda_0 \left( 1 - \beta \cos \theta \right)
\]  

(4-6)

where \( \lambda_0 \) is the unshifted wave-length, \( \beta = v/c \), \( c \) the velocity of the light and \( \gamma = 1/\sqrt{1 - \beta^2} \). For a fixed \( \beta \ll 1 \) the angular broadening is given by

\[
\Delta \lambda_A = \lambda_0 \beta \sin \theta \Delta \theta
\]  

(4-7)
For the Doppler unshifted light, there are only instrumental and thermal broadening effects. For the Doppler shifted line, there are angular broadening effects as well as instrumental broadening effect. As shown in the Appendix A, the broadening of the monochromator was about 10 Å (full width at half maximum); and the thermal broadening is given below. According to the Franck-Condon principle when there is a collisional break up of the target molecule due to the energetic beam, the average kinetic energy of daughter particles is about 3 eV (46). If the kinetic energies of the particles are in a Gaussian distribution the broadening of the light which comes from the target is given by (47)

$$\Delta \lambda_{th} = \frac{2}{c} \left( \frac{2 \ln 2 k_B T}{m} \right)^{1/2}$$  (4-8)

where $k_B$ is the Boltzmann constant, $T$ is the temperature of the particles and $m$ is the mass of the particle. For $k_B T \approx 3$ eV, $\Delta \lambda_{th}$ is about 0.8 Å. Since the individual broadening effect behaves like an independent vector component the total broadening of the unshifted light is

$$\Delta \lambda_{Total} = \sqrt{\Delta \lambda_I^2 + \Delta \lambda_{th}^2} = 10 \text{ Å}$$  (4-9)

where the other broadening effects are negligible and this will be shown in Appendix A.

For the Doppler shifted light as discussed earlier, there is an observational angular broadening effect. Therefore, the total broadening effect is given by
\[ \Delta \lambda_T = \sqrt{\Delta \lambda_I^2 + \Delta \lambda^2 + \Delta \lambda_A^2} \]  \hspace{1cm} (4-10)

Fig. 4-5 shows the typical spectral line profile.

Since intensity measurements were normally made by setting the monochromator at the peak of the line, corrections were necessary based on the shape of the line. For the unshifted line which remained at a constant setting of the monochromator, this correction could be included in the absolute calibration. The correction for the shifted line was the ratio of the counting rate at the peak of the line to integrated counting rate over the breadth of the line. It was typically 10 to 50%.

D. Linearity Test

Linearity of signal with pressure provides a test of the system for all single collision effects. Both single and multiple collision effects are linear with respect to beam current. Frequently, during the course of an experiment, linearity tests were applied for assurance that the instrumentation was performing properly. Typical result of linear response with the current is shown in Fig. 4-6; that of linear response with the target pressure is shown is Fig. 4-7.
RELATIVE INTENSITY

EMISSION FROM TARGET

90 keV H⁺

EMISSION FROM BEAM

CHANNEL NUMBER
XBL 829-11784

FIG. 4-5
FIG. 4-6

90 keV $H^+$
Target Length = 5.0 cm

RELATIVE INTENSITY

PRESSURE IN MICRON

XBL 829-11785
RELATIVE INTENSITY

TARGET EMISSION

CURRENT OF BEAM
XBL 829-11786

FIG. 4-7
E. Data Analysis

All data were taken and analyzed using a HP 9845B (Hewlett Packard Co.) desk top computer. For the determination of the calibration factor $K$ of the pyroelectric detector, a computer software program was used. Standard deviation was determined as errors for the $K$ value and a typical integration time for a $K$ measurement was 20 seconds. Soon after the determination of a $K$ value, the spectral line shape was determined for separation of the Doppler shifted line from the unshifted line and measurement of a line broadening factor. The program LINE was used for this purpose. This was composed of two parts. The first part was for the automatic data taking and storing of data. The second part was for the least square curve fitting of the spectral line shape. This curve fitting program had the basic structure for least square fitting of arbitrary combination of Gaussian curves. But for the unshifted line, the line shape was strictly triangular as shown in Chapter 4-C and the Gaussian curve fitting was invalid. Since one of the main broadening factors for the Doppler shifted line was an angular broadening which was asymmetrical, the Doppler shifted line was a slightly asymmetrical Gaussian curve. This curve fitting and other least square curve fitting used here after, were mainly based on the program, PISA$^{(48)}$, except some minor modifications. Since the line shape itself was independent of the target
pressure, a high pressure target was used which decreased integration time to obtain sufficient signal to noise ratio (normally about 30 seconds) and less than 1% standard counting error, was 20 seconds at the peak of the line shape. As mentioned earlier the beam was chopped to allow for measurement of the real signal plus the background signal and the background signal alone. The quantities measured were

\[ T_{on} \; ; \; \text{time when the beam was on}, \]
\[ T_{off} \; ; \; \text{time when the beam was off}, \]
\[ C_{on} \; ; \; \text{total counting signal from the photomultiplier tube when the beam was on}, \]
\[ C_{off} \; ; \; \text{total counting signal from the photomultiplier tube when the beam was off} \]
\[ I_p \; ; \; \text{total signal from the pyroelectric detector when the beam was on}, \]
\[ K \; ; \; \text{the ratio of counting signal from Faraday cup to that of pyroelectric detector which gave the amount of current in } 10^{-8} \text{ A units.} \]

Then the signal/sec/10^{-8} A is given by

\[
S = \frac{C_{on}/T_{on} - C_{off}/T_{off}}{I_p K/T_{on}} \quad (4-11)
\]
The curve fitting formula for the line was

\[ y = a e^{-\frac{(x - b)^2}{c}} \]  \hspace{1cm} (4-12)

In the thin target measurements, it was necessary to confirm that the measurements satisfied the single collision condition. To do that the curve for light intensity versus target density was developed. The signal formula was the same as before, and the curve fitting formula was

\[ y = a + bx \]  \hspace{1cm} (4-13)

where "a" was confirmed to be near zero.

After measuring the signal intensity versus the distance of penetration of the beam into the gas target, the sum of data were computer analyzed using the program PISA. Every data point was weighted by a weighting factor \( w_i \) which is given by (49)

\[ w_i = \frac{1/\sigma_i^2}{1/N \sum_{i} (1/\sigma_i^2)} \]  \hspace{1cm} (4-14)

where \( N \) is number of data points and \( \sigma_i^2 \) is the estimated variance of each data point which was given by the quadrature of the sum of the related uncertainties. The uncertainties came from counting statistics, pressure and length measurements of the target. The error from the counting statistics can be expressed as
The uncertainty of the pressure measurement was mainly from the zero point drift of the capacitance manometer and given approximately as 0.1 micron per 200 second measurement time at most. The uncertainty from the measurement of the distance through which the beam went was 0.1 cm at most. These effects were considered when the collection of data were analyzed.

A curve fitting formula for separation of the 3s, 3p and 3d state cross sections was

\[ y = \sum_i a_i \left( 1 - e^{-x/\tau_i} \right) \]  

(4-16)

where \( v \) is the beam velocity and \( \tau_i \) is the effective lifetime of each excited state. Fig. 4-8 shows the typical result of this curve fitting. For the thick target measurement of the target dissociative excitation in the case of \( H^+ \) impact on \( H_2 \) the curve fitting formula was given by

\[ y = a + b e^{-\sigma_T x} \]  

(4-17)

where \( \sigma_T = \sigma_{10} + \sigma_{01} \), and \( \sigma_{10} \) and \( \sigma_{01} \) are charge exchange cross section for \( H^+ \) and \( H^0 \) as defined earlier.
The typical result of this curve fitting is shown in Fig. 4-9.

So far no polarization correction has been discussed. If the observation apparatus is set to observe emission from the target at an angle $\theta$ to the beam direction as shown in Fig. 4-10, then the correction factor for the anisotropy in emission is given by (50)

$$
\gamma = \frac{(3 - P)(C + 1)(\alpha + 1)}{3(1 - \cos^2 \theta)(C\alpha + 1)}
$$

(4-18)

where $P$ is the polarization of the emission and $C$ is the ratio of the emission intensity polarized parallel to the beam direction to that polarized perpendicular to the beam direction. And $\alpha$ is the ratio of the instrumental sensitivity parallel to the beam direction to that polarized perpendicular to the beam direction. For $\theta = 54.76^0$ the first factor in the above equation is 1. Since in this experiment $1/\alpha = 0$, therefore $\gamma$ is simply given by

$$
\gamma = \frac{1 + 1/C}{2} = \frac{1 + I_\|/I_\perp}{2}
$$

(4-19)

where $I_\|$ and $I_\perp$ were defined earlier in Chapter 2.
\[ \frac{K_{II}}{K_{I}} = a \]

\[ \frac{l_{II}}{l_{I}} = C \]

**FIG. 4-10**
CHAPTER 5
Results and Discussion

A. Results

The final results and error estimations are presented in Table 5-1 --- 5-5 and Fig. 5-1 --- 5-17. The detailed methods of data analysis have been given in Chapter 4, and the error analysis is given in this section. In general there are two classes of errors. The one is the class of systematic errors such as those which result from limited accuracy of calibrated equipment, from the impurity of prepared materials and from pre-estimated values used in the experiment. The other is the class of the random errors, such as those result from the statistical fluctuations of counting finite numbers of events and from small unpredictable drifts in the instrumental sensitivities and zeros.

The important quantities from which resulted systematic errors were the beam and target impurity, the uncertainties in measurement of the target density, the beam current and the beam energy, the uncertainty in calibration of the photon detector and the uncertainty in pre-estimated values of the life times of the excited quantum states. The beam purity was confirmed by a magnetic mass analyzer with less than 1% error. The purity of the target gas was limited by the impurity of the target gas itself and by residual back-
ground gas in the target chamber. The purity of the target gas had been assured by the manufacturer and the impurity level should not exceed 1%. Since the minimum target pressure used in this experiment was about 0.2 mTorr where residual background pressure was maintained less than 2 μTorr the error from this should not exceed 1% because the cross sections for H₂ gas is almost the same as that of the impurity gas. The uncertainty in the measurement of the beam current was due to two factors. The one was the uncertainty in normalization of the pyroelectric signal to the Faraday cup signal. The other was the limitation of the accuracy in the measurement of the Faraday cup current. The former uncertainty was kept to be about 1%. The accuracy of the Faraday cup current was established by checking the calibration of the digital current integrator. The test was carried out with a standard current source and gave less than 1% error. The calibration of the capacitance manometer was checked by comparing it to an oil-manometer. The oil-manometer and the capacitance manometer agreed to within 5% which allowed up to 7% maximum uncertainty in the pressure measurement. The accuracy of the beam energy was confirmed by the terminal voltage of the accelerator which was calibrated with a known standard meter, and was found to be accurate to within 5%.

The calibration of the detection sensitivity of the photon detection system including optics employed a tungsten strip filament lamp as a standard source of light emission.
The emissivity of the standard lamp at a specified filament current (35 A) was given by the manufacturer; and had an accuracy with a 5% maximum error. In the calibration procedure the other working tungsten strip lamp was used. Therefore the normalization of this working lamp to the standard lamp resulted in 10% maximum uncertainty. The development of the lamp emissivity versus the filament current characteristics which was used in the calibration procedure resulted in 3% maximum error. In the measurement of the resolution $\Delta \lambda$ and the slit length there are 5% and 2% uncertainties. The uncertainty in the wavelength response of the detector system was maximum 3%. The effect of the uncertain velocities and lifetimes had been studied by J.C. Ford et al. (13), and typically a 1% change in those produced changes in the 3s, 3p and 3d cross sections of, respectively, 0.5, 0.5 and 1.5 %.

For the purpose of final data interpretation there is another way of classification of errors. Conventionally the cross sections are described as functions of the energy, therefore, the cross section curves have the uncertainty in the structure of the curves themselves and in the absolute magnitudes of the whole curves. The class of errors which varies the internal structure of the cross section curve is defined as a relative uncertainty. And the class of errors which varies the absolute magnitudes of the whole curves is defined as an absolute uncertainty. The absolute uncertainty
presented here, was determined by the possible errors in beam
energy, pressure, current and detection sensitivity measure-
ment, etc. The total absolute uncertainty was given by taking
the root of the sum of the squares of these errors and found to
be 17% at the most.

The main sources of the random errors were limited
counting statistics and zero point drift of the capacitance
manometer during the measurement procedure. The random
effects resulted by counting statistics was kept to be the
order of 1%. Even the total random error during this ex-
periment was kept small, there might still be large random
error in the derived cross sections for the 3p and 3d state
excitations of the projectile. This was due to either rela-
tively small magnitudes of those or/and branching ratio. This
effect was counted during the establishment of the relative
uncertainties of the cross sections.
$H^+ + H_2$ Collision, Excitation Cross Sections in $10^{-18} \text{cm}^2$

<table>
<thead>
<tr>
<th>Energy in keV</th>
<th>Slow n=3</th>
<th>Fast 3s</th>
<th>Fast 3p</th>
<th>Fast 3d</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$5.01 \pm 0.11$</td>
<td>$4.29 \pm 0.26$</td>
<td>$10.7 \pm 1.1$</td>
<td>$1.06 \pm 0.13$</td>
</tr>
<tr>
<td>30</td>
<td>$3.70 \pm 0.15$</td>
<td>$6.60 \pm 0.41$</td>
<td>$7.9 \pm 1.7$</td>
<td>$0.81 \pm 0.20$</td>
</tr>
<tr>
<td>40</td>
<td>$2.68 \pm 0.05$</td>
<td>$5.47 \pm 0.33$</td>
<td>$7.0 \pm 1.4$</td>
<td>$0.33 \pm 0.16$</td>
</tr>
<tr>
<td>50</td>
<td>$2.42 \pm 0.04$</td>
<td>$4.77 \pm 0.29$</td>
<td>$4.0 \pm 1.2$</td>
<td>$0.34 \pm 0.14$</td>
</tr>
<tr>
<td>65</td>
<td>$1.81 \pm 0.03$</td>
<td>$3.00 \pm 0.18$</td>
<td>$1.12 \pm 0.75$</td>
<td>$0.29 \pm 0.09$</td>
</tr>
<tr>
<td>80</td>
<td>$1.65 \pm 0.03$</td>
<td>$2.23 \pm 0.14$</td>
<td>$\text{-----}$</td>
<td>$\text{-----}$</td>
</tr>
<tr>
<td>100</td>
<td>$1.33 \pm 0.02$</td>
<td>$1.13 \pm 0.07$</td>
<td>$0.39 \pm 0.28$</td>
<td>$0.133 \pm 0.034$</td>
</tr>
<tr>
<td>125</td>
<td>$1.18 \pm 0.02$</td>
<td>$0.693 \pm 0.042$</td>
<td>$0.31 \pm 0.17$</td>
<td>$0.0266 \pm 0.0070$</td>
</tr>
<tr>
<td>150</td>
<td>$1.04 \pm 0.01$</td>
<td>$0.239 \pm 0.015$</td>
<td>$\text{-----}$</td>
<td>$\text{-----}$</td>
</tr>
</tbody>
</table>

Table 5-1
**H\(^+\) + H\(_2\) Collision**

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Polarization in %</th>
<th>Cross Sections in 10(^{-18})cm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3p</td>
<td>3d</td>
</tr>
<tr>
<td>20</td>
<td>25±10</td>
<td>-8±12</td>
</tr>
<tr>
<td>30</td>
<td>9±21</td>
<td>-2±24</td>
</tr>
<tr>
<td>40</td>
<td>7±20</td>
<td>-1±50</td>
</tr>
</tbody>
</table>

Table 5-2
$H_2^+ + H_2$ Collision, Excitation Cross Sections in $10^{-18}\text{cm}^2$

<table>
<thead>
<tr>
<th>Energy in keV</th>
<th>Slow n=3</th>
<th>Fast 3s</th>
<th>Fast 3p</th>
<th>Fast 3d</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3.81±0.08</td>
<td>4.64±0.47</td>
<td>5.9±1.4</td>
<td>3.75±0.30</td>
</tr>
<tr>
<td>30</td>
<td>4.95±0.18</td>
<td>4.94±4.94</td>
<td>7.6±1.6</td>
<td>3.86±0.31</td>
</tr>
<tr>
<td>40</td>
<td>5.12±0.20</td>
<td>4.81±0.44</td>
<td>8.1±1.7</td>
<td>4.01±0.32</td>
</tr>
<tr>
<td>50</td>
<td>4.98±0.18</td>
<td>5.51±0.55</td>
<td>7.9±1.8</td>
<td>4.20±0.34</td>
</tr>
<tr>
<td>65</td>
<td>4.87±0.15</td>
<td>5.59±0.56</td>
<td>10.8±2.2</td>
<td>3.80±0.31</td>
</tr>
<tr>
<td>80</td>
<td>4.12±0.23</td>
<td>5.51±0.55</td>
<td>11.2±1.9</td>
<td>3.26±0.26</td>
</tr>
<tr>
<td>100</td>
<td>3.52±0.08</td>
<td>4.24±0.60</td>
<td>7.4±1.4</td>
<td>3.52±0.28</td>
</tr>
<tr>
<td>125</td>
<td>3.01±0.07</td>
<td>3.71±0.34</td>
<td>6.3±1.2</td>
<td>3.29±0.26</td>
</tr>
<tr>
<td>150</td>
<td>2.23±0.10</td>
<td>2.72±0.25</td>
<td>6.4±1.1</td>
<td>2.71±0.22</td>
</tr>
</tbody>
</table>

Table 5-3
$H_3^+ + H_2$ Collision, Excitation Cross Sections in $10^{-18} \text{cm}^2$

<table>
<thead>
<tr>
<th>Energy in keV</th>
<th>Slow n=3</th>
<th>Fast 3s</th>
<th>Fast 3p</th>
<th>Fast 3d</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2.50±0.05</td>
<td>3.73±0.26</td>
<td>2.30±1.06</td>
<td>3.22±0.26</td>
</tr>
<tr>
<td>30</td>
<td>3.61±0.12</td>
<td>5.03±0.35</td>
<td>3.95±1.31</td>
<td>3.31±0.27</td>
</tr>
<tr>
<td>40</td>
<td>4.37±0.11</td>
<td>6.18±0.43</td>
<td>9.10±2.28</td>
<td>4.32±0.35</td>
</tr>
<tr>
<td>50</td>
<td>4.96±0.15</td>
<td>6.66±0.47</td>
<td>7.14±1.72</td>
<td>4.48±0.36</td>
</tr>
<tr>
<td>65</td>
<td>4.95±0.10</td>
<td>8.00±0.56</td>
<td>7.69±2.08</td>
<td>4.05±0.33</td>
</tr>
<tr>
<td>80</td>
<td>5.02±0.12</td>
<td>7.20±0.51</td>
<td>14.6±3.6</td>
<td>3.24±0.26</td>
</tr>
<tr>
<td>100</td>
<td>5.38±0.14</td>
<td>7.47±0.53</td>
<td>11.2±2.8</td>
<td>3.56±0.29</td>
</tr>
<tr>
<td>125</td>
<td>5.03±0.08</td>
<td>7.58±0.53</td>
<td>9.5±2.4</td>
<td>3.79±0.31</td>
</tr>
<tr>
<td>150</td>
<td>3.84±0.10</td>
<td>6.97±0.49</td>
<td>12.3±3.1</td>
<td>2.12±0.17</td>
</tr>
</tbody>
</table>

Table 5-4
$H^0 + H_2$ Collision  
Production of Slow $n=3$ state H-atoms

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Cross Sections ($10^{-18} \text{cm}^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2.16±0.31</td>
</tr>
<tr>
<td>25</td>
<td>1.69±0.24</td>
</tr>
<tr>
<td>35</td>
<td>1.21±0.17</td>
</tr>
<tr>
<td>50</td>
<td>0.91±0.14</td>
</tr>
<tr>
<td>75</td>
<td>0.69±0.13</td>
</tr>
</tbody>
</table>

Table 5-5
\[ \text{PRESENT} \]
\[ \text{FORD} \]
\[ \text{WILLIAMS} \]
\[ \text{HUGHES} \]
$H^+ + H_2 \rightarrow H^*(3p) + [H_2^+]$

Fig. 5-3
\[ \text{BATES}\]

\[ \text{PRESENT}\]

\[ \text{HUGHES}\]

\[ \text{FORD}\]

\[ H^+ + H_2 \rightarrow H^*(3d) + [H_2^+] \]

Fig. 5-4

BEAM ENERGY in keV

XBL 829-11793
\[ H^+ + H_2 \]

Present

\[ \sigma_p + c_d \]

Williams

Hughes

Fig. 5-5

XBL 829-11794
Fig. 5-7

\[ \text{BEAM ENERGY in keV} \]

\[ 10^{-17} \text{cm}^2 \]

\[ \text{PRESENT} \]

\[ \text{O HUGHES} \]

\[ \triangle \text{WILLIAMS} \]

\[ \square \text{FORD} \]

\[ \frac{H^+}{H_2} + H_2 \rightarrow H^*(3s) + [H] + [H_2]^+ \]

XBL 829-11796
Fig. 5-8

\[ \text{BEAM ENERGY in keV} \]

\[ 10^{-17} \text{cm}^2 \]

\[ 10^{-18} \]

\[ \text{\( \bullet \): PRESENT} \]

\[ \text{\( \times \): ANDREEV} \]

\[ \text{H}_2^+ + \text{H}_2 \rightarrow \text{H}^*(3p) + \text{H}^+ + [\text{H}_2] \]
\[ \text{H}_2^+ + \text{H}_2 \rightarrow \text{H}^*(3d) + \text{H}^+ + [\text{H}_2] \]

\[10^{-17}\text{cm}^2\]

\[10^{-16}\text{cm}^2\]

20 30 40 50 60 70 80 90 100 110 120 130 140 150

BEAM ENERGY in keV

\(\text{XBL 829-11798}\)

Fig. 5-9
Fig. 5-10

\[ \frac{H^+}{H_2} + H_2 \]

\[ 0.118 \sigma_p + \sigma_d \]

BEAM ENERGY in keV

XBL 829-11799
\[
\begin{align*}
\text{Fig. 5-11} & \\
\text{XBL 829-11800} &
\end{align*}
\]
\[ \text{PRESENT} \]
\[ \text{HUGHES} \]
\[ \text{WILLIAMS} \]

\[ \begin{align*}
\text{H}_3^+ + \text{H}_2 \rightarrow & \text{H}^*(3s) + \left[\text{H}_2^+\right] + \left[\text{H}_2\right]
\end{align*} \]
\[ \text{BEAM ENERGY in keV} \]

\[ 0^{-17} \text{cm}^2 \]

\[ 0^{-18} \text{cm}^2 \]

\[ H_3^+ + H_2 \rightarrow H^*(3p) + [H_2^+] + [H_2] \]

Fig. 5-13
\[
H_3^+ + H_2 \rightarrow H^*(3d) + [H_2^+] + [H_2]
\]
\[ H_3^+ + H_2 \]

\[ 0.118 \sigma_p + \sigma_d \]

Fig. 5-15

XBL 829-11804
B. Discussion

a. H⁺ case

There are three published sets of measurement, by Hughes (4), Ford (14) and Williams (29) for the case of H⁺ incident on H₂. For 3s excitation cross sections (Fig. 5-2) Hughes', Thomas', and the present data agree completely with each other. The data by Williams show slightly larger cross sections within the energy range of 20 keV to 60 keV. The 3s state has the larger cross sections and therefore is the easiest to measure. There is poor agreement in the case of the 3p cross sections (Fig. 5-3) between Hughes' and the present results; on the other hand there is good agreement between Ford' and the present results. For the 3d cross sections (Fig. 5-4) Hughes' and the present results agree with each other generally, but Ford' results are a factor 10 smaller. Bates (32) and Mapleton (34) calculated theoretically the 3s, 3p and 3d charge capture cross sections of H⁺ incident on H atom. Although there is no exact verification, in the high energy collision a target hydrogen molecule acts like two isolated hydrogen atoms except at very high energy where the backscattering contribution to the cross section becomes dominant (36). Theoretical values of 3s, 3p and 3d charge capture cross sections by H⁺ agree well with the present results.

For target dissociative excitation cross sections the present results show excellent agreement with Hughes'
although his maximum absolute uncertainty was 40%. The results by Ford are a factor 2 smaller than Hughes' and the present results. The results by Williams are about a factor 2 larger than Hughes' and the present results.

b. \( H_2^+ \) case

There are two published results for the case of target dissociative excitation of \( H_2^+ \) incident on \( H_2 \) (Fig. 5-5) by Hughes(5) and Williams(29). Hughes' and the present results show good agreement with each other whereas the results by Williams are about a factor 2 larger than those two cross sections. Ford measured 3s and 3d dissociative excitation cross sections(16) (Fig. 5-6, Fig. 5-8) of the \( H_2^+ \) beam at the higher energies covered in this experiment, which show good agreement in the case of 3s cross section, but show poor agreement in the case of 3d cross section. The 3s cross sections by Hughes and Williams show good agreement with the present results. The 3p cross section by Andreev(51) (Fig. 5-7) show good agreement with the present results within the experimental error range. Polarization was not observable within the experimental error range.

c. \( H_3^+ \) case

There are also two published results for the case of target dissociative excitation of \( H_3^+ \) incident on \( H_2 \) (Fig. 5-11) by Hughes(5) and Williams(29). Hughes' and present
results show general agreement but the results of Williams are in poor agreement. Since the Doppler shifted cross sections by Hughes were measured in two different experimental geometries, some systematic fluctuation might be expected. This might explain the reason that the unshifted cross sections by Hughes are slightly larger than the present results; on the other hand, the Doppler shifted cross sections by Hughes are slightly smaller than the present ones. The 3s cross sections (Fig. 5-12) by Hughes and Williams show general agreement with the present results. There were no available 3p and 3d cross sections by other authors. Polarization effects were not observable within the experimental error range.
CHAPTER 6
Conclusions

There are two different methods of the excitation cross section measurement. The first one is the method of decay inside the target and the other one is that of decay outside the target. To date almost all experiments have been performed with the latter method. The conventional method of separation of different life-time states has been used in those experiment. But for short life-time states it is impossible to develop intensity decay curves due to the finite resolution in the length measurement. Numerical simulation was performed to verify this method. For the n=3 state excitation the 3p excitation cross section showed at least 100% less than the given value although there was 1% fluctuation in the decay curve. Also it was not possible to get the cross sections for H$_2^+$ and H$_3^+$ particles because of their slowness. On the other hand the measurements of those cross sections were possible by the method of the decay inside the target which was used in this experiment. Therefore, the main advantage of this method is that one can measure short life-time state excitation cross sections. The main difficulty of this method was that the single-collision condition should be confirmed for every step because the target thickness was changed in every step. Another advantage was that it was possible to get the information of the target simult-
aneously. One could extend this method to the thick target experiment, but the statistical fluctuation didn't allow any convincing measurement. Another difficulty came from the fact that the excited state particles of the beam could not be distinguished from the ground state particles. For the atomic hydrogen experiment one can estimate the amount of the excited state atoms the $n^{-3}$ law which is an empirical law. But this gives another unavoidable uncertainty.

In general two different types of optical system have been used. One type used a combination of filters and the other type used a monochromator. Because of small optical throughput of monochromator the former method was frequently used for Balmer- alpha excitation experiment. In the use of filters the transmittance versus wavelength should be developed with extreme care. And in procedure of calibration it is necessary to develop the relation between line radiation and continuous radiation. It was found that the calibration of the monochromator system was easier than that of a filter combination system.

There are two immediately possible experiments without any modification of this experiment. These are the case of $H_2O^+$ and $H^0$ impact on $H_2$. Since a $H_2O^+$ particle gives only 1/18 energy of the total beam energy to the hydrogen atom the expected Doppler shift of the radiation from the projectile should be small. If $D_2O^+$ is used instead of $H_2O^+$
more Doppler shift can be achieved. It is possible to get more resolution of the monochromator by modifying the slit width, but separate calibration factors are required. The ion source for H$_2$O$^+$ is now being developed and the magnet mass analyzer shows enough resolution to separate O$^+$, OH$^+$, H$_2$O$^+$ and H$_3$O$^+$ as shown in Fig. 6-1.

In the case of H$^0$ impact on H$_2$ additional electrostatic plates and a gas cell neutralizer are needed. The difficulty to perform this experiment mainly lies on the amount of beam. For the practical application the cross section of the dissociative excitation of H$^0$ impact on H$_2$ is also needed. Up to date no experiment on H$^0$ impact on H$_2$ has been tried, simply because it was nearly impossible to get a pure H$_2^0$ beam. No practical method to pure H$_2^0$ beams has been suggested.
CURRENT ON THE TARGET FARADAY CUP

CURRENT OF MAGNETIC MASS ANALYZER

XBL 829-11806 104
CHAPTER 7

References


2. R.H. Hughes, S. Lin and L.L. Hatfield, Phy. Rev. 130, 2348 (1963)


33. J.D. Jackson, H. Shiff, Phy. Rev. 89, 359 (1953)
34. R.A. Mapleton, J. Phy. B. 1, 529 (1968)
40. J.P. Keene, Phil. Mag. 40, 369 (1949)
43. M. Born and E. Wolf, Principles of Optics, New York, Macmillan (1964)
47. H.E. White, Introduction to Atomic Spectra, McGrow-Hill, New York (1934)
48. W.W. Hicks, LBL Report-2470 (1973)
51. E.P. Andreev, V.A. Ankudinov and S.V. Bobashov,

A. Discussion of the monochromator

In this section the detailed broadening effects of the monochromator used in this work are presented. As shown in Fig. 8-1 the monochromator was designed to give large optical throughput (f number was 3.7) and was composed of a 30 cm focal length and 8.2 cm diameter achromatic lens, a 1200 line ruling per mm grating, one 1" x 1" 90° prism and a plain red filter. The slit heights of both entrance and exit slit were 1 cm and the slit widths were set to be 1.2 mm. The plain red filter was used to cut down the overlapping blue spectrum from the third order diffraction because in order to give maximum refractance the monochromator was set to give the second order diffraction of the grating. The adjustment of the monochromator was done with a white light source and a He-Ne laser. The wavelength calibration of the monochromator was done with H₂, Na and Ne discharge lamps.

The first factor of the instrumental broadening was due to the natural grating resolution. Consider the case of the monochromatic light with wavelength \( \lambda \) incident on the entrance slit. Assume that the light with wavelength \( \lambda \) which passes through the center of the entrance slit and is
incident on the grating at the corresponding angle $\alpha$ and is diffracted into an angle $\beta$, also passes through the center of the exit slit. Then $\alpha$ and $\beta$ must satisfy the usual grating equation such as

$$\frac{d}{\lambda} \left( \sin \alpha + \sin \beta \right) = m \quad (8-1)$$

where $d$ is the spacing between rulings on the grating and $m$ is the order of the grating maxima. The dispersion $D$ which gives the displacement at the exit slit corresponding to a unit change in the wavelength $\lambda$ is given by

$$D = \frac{dy}{d\lambda} = F \frac{d\beta}{d\lambda} = \frac{Fm}{d \cos \beta} \quad (8-2)$$

where $F$ is the focal length of the achromatic lens. The intensity $I(x,y)$ at a position $y$ (or for the diffracted angle $\beta'$) of the exit slit due to unit intensity of light with wavelength $\lambda$ incident on the entrance slit at a position $x$ corresponding incident angle $\alpha'$ is given by

$$I(x,y) = \frac{d}{\lambda} \left( \sin \alpha' - \sin \alpha + \sin \beta' - \sin \beta \right)$$

$$= \frac{d}{\lambda F} \left( x \cos \alpha + y \cos \beta \right) \quad (8-3)$$

From this equation a full width at half maximum intensity is given by
\[ w = \frac{0.88 \lambda D}{m N} = \frac{0.88 \lambda F}{d N \cos \beta} \] (8-4)

With the equations (8-2) and (8-4) the broadening due to the natural grating resolution is given by

\[ \Delta \lambda_G = \frac{d \cos \beta}{m F} w = \frac{0.88}{m N} \lambda \] (8-5)

where \( N \) is the total number of rulings on the grating. In this experiment \( \Delta \lambda_G \) was much less than \( 1 \) Å.

The main broadening effect was due to the finite slit width, where \( w \) in the previous equation was given by a full value of the slit width. From this equation the broadening due to the finite slit width is given by

\[ \Delta \lambda_s = \frac{d \cos \beta}{m F} s \] (8-6)

where \( s \) is the slit width and \( \sin \beta = \lambda / d \). In this experiment \( F = 30 \) cm, \( m = 2 \), \( s = 0.12 \) cm, \( d = 1/12000 \) cm and \( \cos \beta \approx 0.6 \), therefore, \( \Delta \lambda_s \) was about \( 10 \) Å.

There were other broadening effects due to curvature of the image of the entrance slit at the exit slit, etc. But they were quite negligible. The total instrumental broadening was given by the square root of the square sum of the individual broadening effect. This is

\[ \Delta \lambda_I = \sqrt{\Delta \lambda_s^2 + \Delta \lambda_G^2 + \Delta \lambda_c^2 + \ldots} \approx 10 \) Å (8-7)
B. Comments on the Thick Target Experiments

To date there have been no multiple collision experiments. The reason is that in order to do that one must handle a very complicated situation and deduce cross sections from indirect measurements. But for real applications it is necessary to deal with such a thick target system. Therefore, the description (analytically or numerically) of that system is strictly required. In this section the intensity evolution of the excited state atoms in the multiple collision system is presented with consideration of the cascading effect. The local intensity of the excited state atoms is determined by the competition between the life time decay effect, the collisional decay (or excitation) effect and the excitation from the primary and secondary particles. Since the cascading term could be treated self-consistently, it was treated by perturbation method in this calculation.

Consider the radiation which comes from the fast particles in the case of $H^+$ incident on $H_2$ gas. Let the parameters be

- $v$; velocity of the fast particle,
- $\tau_i$; life time of the excited state $i$,
- $N_a$; target density,
- $N_b$; density of the projectile,
- $x$; travel distance of the beam from the entrance aperture,
$Q_{ij}$; cross section for collisional transition between the state $i$ and $j$,

$Q^0_i$; excitation cross section from the ground state,

$Q^+_i$; electron capture cross section of $H^+$ into the excited state $i$,

$Q^{2+}_i$; dissociative excitation cross section of $H^+_2$ into $H$ atom of the state $i$,

$Q^{3+}_i$; dissociative excitation cross section of $H^+_3$ into $H$ atom of the state $i$.

Then the excited state density $N_{bi}$ follows the equation

$$\frac{dN_{bi}}{dx} = -\frac{1}{\nu \tau_i} N_{bi} - N_a \left( \sum_{j \neq i} Q_{ij} \right) N_{bi}$$

$$+ N_a N_b \left( Q^0_i y_0 + Q^+_i y_+ \right) \quad (8-8)$$

The first term represents the lifetime decay of the excited state $i$, the second term represents collisional transition from the state $i$ to the state $j$, the third term is for the excitation of $H^0$ particle into the state $i$ and the fourth term is for the charge capture of $H^+$ particle into the state $i$. The previous equation can be written again as

$$\frac{dN_{bi}}{dx} = -\alpha_i N_{bi} + (y_0 Q^0_i + y_+ Q^+_i) N_a N_b \quad (8-9)$$
\[ \alpha_i = \frac{1}{\nu \tau_i} + N_a \sum_{i \neq j} Q_{ij} \]

The solution of this equation with the boundary conditions is given by

\[ N_{bi} = \frac{N_a N_b}{\alpha_i} \left( F_{00} Q_i^0 + F_{10} Q_i^+ \right) \left( 1 - e^{-\alpha_i x} \right) \]
\[ + \frac{N_a N_b}{\alpha_i - \beta} F_{00} \left( Q_i^+ - Q_i^0 \right) \left(e^{-\beta x} - e^{-\alpha_i x} \right) \quad (8-10) \]

where \( \beta = N_a \left( \sigma_{10} + \sigma_{01} \right) \), \( F_{00} = \sigma_{10} / \left( \sigma_{10} + \sigma_{01} \right) \)
and \( F_{10} = \sigma_{01} / \left( \sigma_{10} + \sigma_{01} \right) \).

The total radiation strength from the portion \( \ell \) of the beam is given by

\[ J = \sum_i \int_{L}^{L+\ell} A N_{bi} A_{ij} \, dx \quad (8-11) \]

for \( L \gg \ell \), \( J \) can be written as

\[ J = A \ell \sum_i A_{ij} N_{bi} \quad (8-12) \]

The equation which include cascading effect can be written as

\[ \frac{dN_{bi}}{dx} = -\alpha_i N_{bi} + ( \gamma_0 Q_i^0 + \gamma_+ Q_i^+ ) N_a N_b \]
\[ + \frac{1}{v} \sum_{k > i} \frac{A_{ki} N_{bk}}{\omega_{ki}} \quad (8-13) \]
where \( A_{ki} = A_{ki} + Q_{ki} N_a v \).

In this way the solution can be developed like

\[
N_{bi} = N_{bi}^0 + N_{bi}^1
\]

(8-14)

and the equation for the first order perturbation term is

\[
\frac{dN_{bi}^1}{dx} = -\alpha_i N_{bi}^1 + \frac{1}{v} \sum_{k>i} A_{ki}^1 N_{bk}^0
\]

(8-15)

In general including \( H_2^+ \) and \( H_3^+ \) case the master equation is given by

\[
\frac{dN_{bi}}{dx} = -\alpha_i N_{bi} + \alpha_a N_a N_b Y_i^q + \frac{1}{v} \sum_{k>i} A_{ki} N_{bk}
\]

(8-16)

where \( Y_i^q \) is given by

\[\begin{align*}
H^+ \text{ case:} & \quad Y_i^1 = \gamma_0 Q_i^0 + \gamma_+ Q_i^+ \\
H_2^+ \text{ case:} & \quad Y_i^2 = \gamma_20 Q_i^{20} + \gamma_2^+ Q_i^{2+} + Y_i^1 \\
H_3^+ \text{ case:} & \quad Y_i^3 = \gamma_3^+ Q_i^{3+} + Y_i^2
\end{align*}\]

(8-17)

where \( Q_i^{20} \) is the dissociative excitation cross section of \( H_2^0 \) into \( H \) atom of the state \( i \). The solutions for \( \gamma_0, \gamma_+ \), etc. are derived by Kim(26) analytically. Using the same notations the parameters are given by
\[ S_{1}^{(2)} = \frac{1}{2} ( C_{01}^{21} + C_{00}^{21} ) + C_{01}^{22} (10) , \]

\[ \gamma_{0}' = 0 \]

\[ \gamma_{1}^{(2)} = \frac{1}{2} ( S_{1} + S_{2} (1) \sqrt{( S_{1} - S_{2} )} + C_{10}^{22} C_{01}^{22} ) \]

\[ \gamma_{3} = \frac{1}{3} ( C_{11}^{31} + C_{10}^{31} ) + \frac{2}{3} ( C_{11}^{32} + C_{10}^{32} ) \]

\[ \varepsilon = C_{10}^{11} + C_{01}^{11} = \beta N_{a} \]

and for cyclic i, j and k (for (i,j,k) = (1,2,3))

\[ G_{i} = (-1)^{i-1} \frac{C_{10}^{32} C_{10}^{22} - C_{10}^{32} ( \gamma_{i}' - S_{2} )}{( \gamma_{i} - \gamma_{j} ) ( \gamma_{i} - \gamma_{k} )} \]

(8-18)

\[ H_{i} = - \frac{C_{11}^{32} C_{10}^{22} - C_{01}^{32} ( \gamma_{i}' - S_{2} )}{( \gamma_{i} - \gamma_{j} ) ( \gamma_{i} - \gamma_{k} )} \]

and

\[ K_{1}^{(2)} = \frac{G_{1}^{(2)} ( C_{00}^{21} - 2C_{10}^{11} ) + H_{1}^{(2)} ( C_{10}^{21} - 2C_{10}^{11} )}{\varepsilon - \gamma_{1}'^{(2)}} \]

(8-19)

\[ K_{3} = \frac{G_{3} ( C_{00}^{21} - 2C_{10}^{11} ) + H_{3} ( C_{10}^{21} - 2C_{10}^{11} ) + C_{11}^{31} - 3C_{10}^{11}}{\varepsilon - \gamma_{3}'} \]

and for cyclic (i,j)=(1,2)
\[ C_i = \frac{c_{22} c_{02}}{\gamma_i - \gamma_j} \frac{1}{s_2 - \gamma_i} \]  

\[ D_i = \frac{(s_2 - \gamma_j') (c_{21}^{10} - 2c_{21}^{11}) - c_{22} (c_{20}^{20} - 2c_{10}^{10})}{(\gamma_i - \gamma_j') (\varepsilon - \gamma_i')} \]  

where \( C \)'s are cross sections multiplied by \( N_a \) and are shown in Fig. 8-2 and Fig. 8-3.

Then the solutions are given by

**\( H^+ \) case**;  
\[ y_0 = F_0 \infty (1 - e^{-\varepsilon x}) \]  
\[ y_+ = 1 - y_0 \]

**\( H_2^+ \) case**;  
\[ y_{2+} = \frac{2}{\Sigma_{m=1}^\infty} C_m e^{-\gamma_m' x} \]

\[ y_{20} = \Sigma_{m=1}^\infty E_m e^{-\gamma_m' x} \]

\[ y_0 = 2F_0 \infty (1 - e^{-\varepsilon x}) + \Sigma_{m=1}^\infty D_m (e^{-\gamma_m' x} - e^{-\varepsilon x}) \]

\[ y_+ = 2 - y_0 - 2( y_{20} + y_{2+} ) \]

**\( H_3^+ \) case**;  
\[ y_{3+} = e^{-\gamma_3' x} \]

\[ y_{20} = \Sigma_{m=1}^3 G_m e^{-\gamma_m' x} \]
Fig. 8-2
\[ y_{2+} = \sum_{m=1}^{3} H_m e^{-\gamma_m x} \]

\[ y_0 = 3F_0 (1 - e^{-\varepsilon x}) + \sum_{m=1}^{3} K_m (e^{-\gamma_m x} - e^{-\varepsilon x}) \]

\[ y_+ = 3 - y_0 - 2y_{20} - 2y_{2+} - 3y_{3+} \]

In the case of \( H^+ \), let \( \gamma_1 = \varepsilon \) and let

\[
 f^0_i = \frac{3 N_a N_b}{\alpha_i} ( F_{0\infty} Q_i^0 - F_{1\infty} Q_i^+) \\
 f^1_i = \frac{N_a N_b}{\gamma_1 - \alpha_i} ( Q_i^0 - Q_i^+ ) F_{0\infty}
\]

then the solution is given by

\[
 N_{b_i}^0 = \sum_{m=0}^{1} f^m_i ( e^{-\gamma_m x} - e^{-\alpha_i x} ) \\
 N_{b_i}^1 = \frac{1}{v} \sum_{k \geq 1} A_{i}^k ( \frac{1}{\alpha_i - \gamma_m} \sum_{m=0}^{1} f^m_k ( e^{-\gamma_m x} - e^{-\alpha_i x} ) \\
 + \frac{1}{\alpha_i - \alpha_k} ( e^{-\alpha_i x} - e^{-\alpha_k x} ) )
\]

In the case of \( H_2^+ \), let \( \gamma_3 = \varepsilon \) and for \( j = 1, 2 \) let \( \gamma_j = \gamma_j^i \)

\[
 F_i^j = \frac{N_a N_b}{\alpha_i - \gamma_j} ( (Q_i^{20} - 2Q_i^+) E_j + (Q_i^{2+} - 2Q_i^+) C_j )
\]
$F_i^3 = \frac{N_a N_b}{\gamma_3 - \alpha_i} ( Q_i^0 - Q_i^+ ) ( 2F_{0\infty} + \sum_{m=1}^{2} D_m )$

then the solution is given by

$N_{bi}^0 = \sum_{m=0}^{3} F_i^m ( e^{-\gamma_m x} - e^{-\alpha_i x} )$

$N_{bi}^1 = \frac{1}{v} \sum A_{ik} \left( \sum_{m=0}^{3} \frac{F_k^m}{\alpha_i - \gamma_m} + \sum_{m=0}^{3} \frac{F_k^m}{\alpha_i - \alpha_k} \right) ( e^{-\alpha_i x} - e^{-\alpha_k x} )$
$$N_{bi}^1 = \frac{1}{v} \sum_{k>i} A_{ik} \left( \sum_{m=0}^{4} \frac{F_k^m}{\alpha_i - \gamma_m} \right) \left( e^{-\gamma_m x} - e^{-\alpha_i x} \right)$$

$$+ \sum_{m=0}^{4} \frac{F_k^m}{\alpha_i - \alpha_k} \left( e^{-\alpha_i x} - e^{-\alpha_k x} \right)$$
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