Title
Baryon number as the fourth color

Permalink
https://escholarship.org/uc/item/9wh1w9kn

Journal
Physical Review D - Particles, Fields, Gravitation and Cosmology, 92(5)

ISSN
1550-7998

Authors
Fornal, B
Rajaraman, A
Tait, TMP

Publication Date
2015-09-17

DOI
10.1103/PhysRevD.92.055022

Peer reviewed
Baryon Number as the Fourth Color

Bartosz Formal, Arvind Rajaraman, and Tim M. P. Tait

Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA

(Dated: June 23, 2015)

We propose an extension of the Standard Model in which baryon number is promoted to be part of a non-Abelian gauge symmetry at high energies. Specifically, we consider the gauge group \( SU(4) \times SU(2)_L \times U(1)_X \), where the \( SU(4) \) unifies baryon number and color. This symmetry is spontaneously broken down to the Standard Model gauge group at a scale which can be as low as a few TeV. The \( SU(4) \) structure implies that each SM quark comes along with an uncolored quark partner, the lightest of which is stabilized by the generalized baryon number symmetry and can play the role of dark matter. We explore circumstances under which one can realize a model of asymmetric dark matter whose relic abundance is connected to the observed baryon asymmetry, and discuss unique signatures that can be searched for at the LHC.

I. INTRODUCTION

The Standard Model (SM) provides an extremely successful description of the world. However, despite its indisputable virtues, many facts hint that it is just a low-energy description of a more fundamental theory.

In particular, the SM by itself does not contain a good dark matter candidate within its spectrum, nor does it offer an acceptable explanation for the observed baryon asymmetry in the universe. Although there exists a possibility that dark matter might be a secluded particle only connected to the SM via gravity, it remains certain that a successful baryogenesis mechanism will require an extension of the SM, at least at high energies. Indeed, one of the peculiar mysteries of baryogenesis is the fact that it involves an interaction which violates conservation of baryon number [1], and yet searches for proton decay require that the interactions mediating proton decay be extraordinarily suppressed. Given the observational evidence that the energy densities of dark matter and of baryons in the Universe are similar, it is tempting to hypothesize that the two mysteries may be related in some way. Models of asymmetric dark matter attempt to engineer this connection by hypothesizing that the dark matter has its own particle-antiparticle asymmetry which is connected in some way to the one observed in baryons [2,7].

In this paper we construct a model of asymmetric dark matter which arises as a by-product of promoting baryon number (together with color) into a non-Abelian gauge symmetry. The SM gauge group is extended to \( SU(4) \times SU(2)_L \times U(1)_X \), in which color is unified with the gauged baryon number symmetry into the single gauge group \( SU(4) \). The model contains a viable dark matter candidate which is the fourth component of one of the \( SU(4) \) quadruplets also containing the SM quarks. We explore the possibility of a successful mechanism for baryogenesis and discuss the properties of the dark matter candidate, whose mass is fixed through the connection between the dark matter relic density and the baryon asymmetry. We also note that the nonrenormalizable proton decay operators are at dimension seven, so the model does not require a huge desert above the electroweak scale.

The layout of the paper is the following: In Section II we present the particle content of the theory, list the possible interactions and discuss the symmetry breaking pattern. In Section III we show how the baryogenesis mechanism works in our model and analyze the dark matter candidate. In Section IV we discuss collider signatures and constraints on the model. The conclusions are presented in Section V.

II. MODEL

Our model is built on an extension of the SM gauge group:

\[
SU(4) \times SU(2)_L \times U(1)_X,
\]

where \( X \) is a linear combination of hypercharge and the diagonal generator of \( SU(4) \), and \( SU(3)_C \) is contained as a subgroup of \( SU(4) \). The SM quarks of each family are promoted to quadruplets of \( SU(4) \),

\[
\begin{pmatrix}
Q^e_i \ 
\tilde{Q}^b_i \ 
\tilde{Q}^g_i \\
Q^\ell_i \\
\tilde{Q}^u_i \\
\tilde{Q}^d_i
\end{pmatrix} \quad \begin{pmatrix}
u^e_i \ 
\tilde{u}^b_i \ 
\tilde{u}^g_i \\
\tilde{d}^e_i \\
\tilde{d}^b_i \ 
\tilde{d}^g_i
\end{pmatrix}
\]

where \( rbg \) are the values of the color index and the \( i \) are the \( SU(2) \) indices on the quark doublet. We have suppressed a generation index running from 1 to 3 on each field. As a consequence of embedding \( SU(3)_C \) into \( SU(4) \), each quark acquires a color singlet “quark partner”, denoted by \( \tilde{Q}_i \), \( \tilde{u} \), and \( \tilde{d} \), respectively. The SM leptons remain relatively unchanged (with \( X = Y \)) and, in addition, we introduce \( SU(4) \) singlet fields \( \tilde{Q}_{iR} \), \( \tilde{u}^\ell_i \), and \( \tilde{d}^\ell_i \), which after \( SU(4) \) breaking will marry the \( \tilde{Q}_{iL} \), \( \tilde{u}_R \), and \( \tilde{d}_R \):

\[
\begin{pmatrix}
\tilde{Q}_{iR} \\
\tilde{u}^\ell_i \\
\tilde{d}^\ell_i \\
\tilde{l}_{iL}
\end{pmatrix} \quad \begin{pmatrix}
\tilde{e}_R 
\end{pmatrix}.
\]

From here on we will suppress the \( SU(2) \) indices. This simple matter content successfully includes the SM fermions, and is anomaly free. As we will see below, it also contains an

1 A model based on \( SU(4) \) with the quark partners identified as the SM leptons (as occurs in Pati-Salam unification [3]) was proposed in Ref. [7].
The gauge covariant derivative has the form:

\[ \partial_{\mu} + ig_{A} G_{\mu} A^{\mathbf{A}} + ig_{3} W_{\mu}^{a} A^{a} + ig_{X} X_{\mu} X \],

where \( A^{\mathbf{A}} \), \( A^{a} \), and \( X \) are the \( SU(4) \), \( SU(2)_{L} \), and \( U(1)_{X} \) generators, respectively. The \( g_{A}^{ab} \) and \( g_{X} \) are Yukawa couplings generating masses only for the quark partner fields through the \( SU(4) \) and \( SU(2) \) spontaneous symmetry breaking, respectively. It is generically true for phenomenologically viable parameters that \( Y^{\Phi} \gg y(H) \) and \( Y^{\Phi} \gg y'(H) \), so the quark partners are to good approximation vector-like under the electroweak symmetry.

### A. Gauge Bosons

The potential for the quadruplet Higgs induces a vacuum expectation value (VEV)

\[ \langle \Phi \rangle = \frac{1}{\sqrt{2}} (0 \ 0 \ 0 \ V)^{T}, \quad (6) \]

and mediate interactions involving one ordinary quark together with its uncolored partner. The last gauge boson, corresponding to the generator \( T_{3}^{15} = \text{diag}(1,1,1,-3)/(2\sqrt{6}) \) (which generates baryon number), mixes with the \( U(1)_{X} \) boson to produce a massless state corresponding to the SM hypercharge interaction.

\[ Y = X + \frac{\sqrt{2}}{3} T_{15} = X + \frac{1}{6} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, \quad (8) \]

and a massive state \( Z' \),

\[ m_{Z'} = \frac{1}{2} \sqrt{g_{X}^{2} + \frac{3}{2} g_{A}^{2}} V, \quad (9) \]

which couples to pairs of quarks, quark partners, and leptons.

It is convenient to reparametrize the gauge couplings in terms of the hypercharge coupling \( g_{Y} \) and a mixing angle \( \theta_{4} \),

\[ g_{Y} = \frac{g_{X} g_{4}}{\sqrt{\frac{2}{3} g_{X}^{2} + \frac{3}{2} g_{A}^{2}}}, \quad \sin \theta_{4} = \frac{g_{X}}{\sqrt{\frac{2}{3} g_{X}^{2} + \frac{3}{2} g_{A}^{2}}}. \quad (10) \]
Note that at the scale $V$, the $SU(4)$ coupling $g_4$ matches onto the strong coupling constant $g_3$, and thus the measured value of $g_3$ determines $\theta_4$ for a given $V$. For $V \sim \text{TeV}$, $\sin \theta_4 \approx 0.28$. The mass eigenstates are:

$$
\begin{pmatrix}
Z'_\mu \\
B_\mu
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_4 & -\sin \theta_4 \\
\sin \theta_4 & \cos \theta_4
\end{pmatrix}
\begin{pmatrix}
G^5_{\mu} \\
X^1_{\mu}
\end{pmatrix},
$$

and the coupling of the $Z'$ to a pair of fermions with $T^{15}$ and hypercharge $Y$ can be written as,

$$
\frac{g_Y}{\sin \theta_4 \cos \theta_4} \left(-\sqrt{\frac{2}{3}} T^{15} + Y \sin^2 \theta_4 \right).
$$

This symmetry insures that the lightest of the quark partner fields is stable, and that all of the heavier ones will decay down to the lightest one. In that sense it is a continuous analogue to the $R$-parity often invoked in supersymmetric theories of dark matter, though in the present context it is a natural consequence of the original $SU(4)$ gauge symmetry together with the embedding of the SM quarks into its framework.\(^2\)

### C. Higher Dimensional Operators

We conclude the discussion of this section by tabulating a few of the most relevant nonrenormalizable interactions consistent with the gauge symmetries and leading to violations of the global symmetries.

At dimension five, there are operators involving the $SU(4)$-breaking Higgs which violate $U(1)_X$ and lepton number, leading to fast decay of the dark matter:

$$
\frac{1}{\Lambda_5^2} \left[ e_{abcd} c_1 \tilde{Q}_L \tilde{Q}_L H e_R + e_{2} \tilde{u}_R \tilde{\Phi} (H e L_L) + c_3 \tilde{d}_R \tilde{H}^* l_L \right],
$$

where $\Lambda_5$ characterizes the scale at which the new physics operates and the $c_i$ are complex dimensionless coefficients describing their relative weighting. These operators may be forbidden by imposing a $Z_2$ symmetry, and we will not consider them further.

Moving on to dimension six, there is a pair of operators which violate both $U(1)_X$ and conventional baryon number:

$$
\frac{1}{\Lambda_6^4} e_{abcda} \left[ c_4 \tilde{u}_R \tilde{u}_R \tilde{d}_R \tilde{d}_R + c_5 (\tilde{Q}_L e \tilde{Q}_L^T) (\tilde{Q}_L^T e \tilde{Q}_L^T) \right],
$$

which are much more difficult to forbid based on a discrete symmetry. They ultimately lead to the decay of the dark matter. Given the astrophysical bound on the dark matter lifetime of $\tau_{DM} \gtrsim 4.5 \times 10^{18}$ s \([12]\), a viable theory containing dark matter of mass $\sim \text{GeV}$ will result provided (assuming the $c_i$ are order one):

$$
\Lambda_6 \gtrsim 5 \times 10^{10} \text{ GeV}.
$$

As we will see below, these operators can also be useful to catalyze an asymmetry among both the baryons and in the dark matter sector.

The most relevant operators mediating proton decay are dimension seven and involve the $SU(4)$ Higgs field:

$$
\frac{1}{\Lambda_7^5} e_{abcda} \left[ c_6 \tilde{u}_R \tilde{u}_R \tilde{d}_R \tilde{d}_R e_R + c_7 (\tilde{Q}_L e \tilde{Q}_L^T) (\tilde{Q}_L^T e \tilde{Q}_L^T) \right].
$$

The experimental limit on the scale $\Lambda_7$ comes from nucleon decay experiments and is approximately given by:

$$
\Lambda_7 \gtrsim 10^{12} \text{ GeV}.
$$

\(^2\) For a model in which the stability of dark matter can be connected to lepton number, see e.g. \([10, 11]\).
III. BARYOGENESIS AND ASYMMETRIC DARK MATTER

The structure of $SU(4)$ enforces a connection between the dark matter number and conventional baryon number. Indeed, $SU(4)$ implies that at short distances they are precisely the same global symmetry. However, the dynamics of $SU(4)$ per se does not result in an asymmetry between particles and antiparticles in either sector. One option to generate an asymmetry is to invoke the dimension-six operators of Eq. (16), which fall out of equilibrium at high temperatures of order $\Lambda_0 \gtrsim 5 \times 10^{10}$ GeV and generically violate both $C$ and $CP$.

One could construct a specific UV completion of these operators and study the conditions on parameters through which a large enough asymmetry is generated. However, we choose to remain agnostic about the nature of the UV completion and take for granted that there exist choices of parameters such that their freeze-out process can generate a nonzero asymmetry. Given the high scale at which they act, these choices are unlikely to lead to unique predictions for low energy observables which would be measurable by current generation experiments.

The structure of the interaction involves three ordinary quarks and one quark partner, and thus the dimension-six interactions create an initial asymmetry in baryon number, $\Delta B_i$, which balances the asymmetry in the dark matter number, $\Delta \chi$,

$$\Delta B_i = -\Delta \chi \, .$$

The $SU(4)$ interactions, including its instantons, will maintain this relation. After $SU(4)$ breaking, the most important remnant will be the usual QCD instantons which preserve equilibrium between chiralities of the light quarks.

As the Universe evolves, the initial $B$ will be processed into a mixture of $B$ and $L$ by the $SU(2)$ instantons [13], resulting in a modest washout of the baryon asymmetry (until the electroweak phase transition renders them impotent). If the masses of the electroweak charged quark partners are somewhat above the electroweak scale, the equilibrium conditions essentially boil down to the SM ones [13], leading to a final baryon asymmetry:

$$\Delta B_f = -\frac{28}{79} \Delta \chi \, .$$

There will be very small changes (on the order of a percent) to the coefficient if some of the electroweak charged quark partners are lighter than the electroweak scale.

Given this prediction for the relation between the asymmetries in the baryonic and dark sectors (and assuming that the symmetric component of the dark matter efficiently annihilates away), the mass of the dark matter resulting in the observed ratio of baryonic to dark matter [14] can be inferred:

$$m_{\tilde{u}_1} = \frac{\Delta B_f}{\Delta \chi} \frac{\Omega_{DM}}{\Omega_B} m_{\text{proton}} \simeq 1.75 \text{ GeV} \, .$$

This mass may be engineered through small ($\lesssim 10^{-3}$) Yukawa couplings for the quark partner playing the role of dark matter.

FIG. 1: Representative diagram for dark matter annihilation.

A. Annihilation of Symmetric Component

A generic challenge for models of asymmetric dark matter is to realize a large enough annihilation cross section such that the symmetric component will efficiently annihilate away [5, 15]. In the $SU(4)$ model, where the characteristic scale of the bulk of the new physics is $\gtrsim$ TeV, this typically requires additional ingredients.

It would be economical to explore the choice of a very small quartic coupling $\lambda_4$, such that the $SU(4)$ Higgs mode is light enough to be produced in $\tilde{u}^c_1$ annihilations. This option could work, but will require fine-tuning to realize the light $\tilde{u}^c_1$ without simultaneously suppressing the coupling to the Higgs.

Perhaps a more appealing option is to introduce an additional light scalar particle $\psi$ which couples to the dark matter,

$$\mathcal{L}_\psi = g \bar{\psi} \tilde{u}_1^c \tilde{u}_1^c \, .$$

Such a scalar could arise as the fourth component of an additional $SU(4)$ quadruplet with $X = 1/2$, or could be a complete gauge singlet which mixes with such a state to pick up coupling to the dark matter. This interaction allows for the annihilation via $\tilde{u}_1^c \tilde{u}_1^c \rightarrow \psi \psi$, provided that $\psi$ is lighter than $\tilde{u}_1^c$ (see Fig. 1). The resulting cross section is velocity suppressed. Writing the thermally averaged annihilation cross section as $\langle \sigma_A \bar{v} \rangle = \sigma_0 T/m_{\tilde{u}_1}$, the energy density of relic dark matter particles in the present epoch is [16, 17]:

$$\Omega_{\tilde{u}_1} h^2 \simeq \left( \frac{1.75 \times 10^{-10}}{\text{GeV}^2} \right) \frac{1}{\sigma_0 \sqrt{g_\ast}} \left( \frac{m_{\tilde{u}_1}}{T_f} \right)^2 \, ,$$

where $T_f$ is the freeze-out temperature and $g_\ast$ is the corresponding number of relativistic degrees of freedom. For a scalar of mass $m_\psi \approx 1$ GeV efficient annihilation of the symmetric component requires a relatively modest coupling:

$$g \gtrsim 0.05 \, .$$

Variations are possible, such as introducing a cubic interaction for $\psi$ or lifting the velocity suppression by choosing $\psi$ to be a pseudoscalar.

---

3 One could also imagine a more standard symmetric dark matter scenario for a Yukawa coupling of order one. This would imply a dark matter mass at the $SU(4)$ breaking scale. Although the connection to baryogenesis is not clear in this case, it is an option still worth considering since it opens up new annihilation channels through intermediate $SU(4)$ and electroweak gauge bosons, and such a model can be probed by direct detection experiments.
IV. LHC PHENOMENOLOGY

Provided the scale of $SU(4)$ breaking $V$ is low enough, there are signatures related to the additional quark partners and gauge bosons at the Large Hadron Collider (LHC). In particular, the existence of an electrically neutral $Z'$ with decays into leptons furnishes the strongest current bound on $V$.

In the narrow width approximation, the cross section for producing a $Z'$ decaying to $f \bar{f}$ at a hadron collider (see Fig. 2) can be written as

$$
\sigma(pp \to Z' \to f \bar{f}) \simeq \frac{\pi}{48 \cdot 8} \sum_{q = u, d} c^f_q \cdot w_q(s, m_{Z'}^2) ,
$$

where the coefficients $c^f_q$ are given by:

$$
c^f_q = (g_q^R)^2 + (g_q^L)^2 \cdot Br(Z' \to f \bar{f})
$$

with $g_q^R$ and $g_q^L$ being the couplings of the $Z'$ to right- and left-handed quarks $q$, respectively, and $Br(Z' \to f \bar{f})$ denoting the branching ratio into $f \bar{f}$. The functions $w_q(s, m_{Z'}^2)$ contain the parton distribution functions (PDFs) for the proton at the $Z'$ mass scale.

From Eq. (12) the partial $Z'$ decay width into charged leptons is given by:

$$
\Gamma(Z' \to e^+ e^-) \simeq \frac{g_Y^2}{24 \pi} \left( \frac{5}{4} \tan^2 \theta_4 \right) m_{Z'} .
$$

The total width of the $Z'$ depends to some degree on the masses of the quark partners. In the limit where all of the decays are possible, the total width becomes:

$$
\Gamma_{\text{total}}(Z') \simeq \frac{g_Y^2}{24 \pi} \left( \frac{4 + 15 \tan^4 \theta_4}{\tan^2 \theta_4} \right) m_{Z'} .
$$

and the coefficients $c^e_q$ are:

$$
c^e_{d/u} \simeq g_Y^2 \left( \frac{2 \pm 6 \tan^2 \theta_4 + 9 \tan^4 \theta_4}{36 \tan^2 \theta_4} \right) \frac{\Gamma(Z' \to e^+ e^-)}{\Gamma_{\text{total}}(Z')} .
$$

Numerically, this corresponds to

$$
c^e_d \simeq 2.1 \times 10^{-4} ,
$$

$$
c^e_u \simeq 1.3 \times 10^{-4} ,
$$

for which one can translate the null results from searches for narrow dilepton resonances [19] [20] into a bound on the $Z'$ mass of

$$
m_{Z'} \gtrsim 2.0 \text{ TeV} .
$$

or in terms of the $SU(4)$ breaking scale,

$$
V \gtrsim 3.1 \text{ TeV} .
$$

The quark partners have somewhat more model-dependent signatures. They typically decay through the (off-shell) colored gauge bosons into two quarks and the dark matter, and thus each one produces a signature typical of a gluino in the Minimal Supersymmetric Standard Model. However, they are uncolored and do not have large production cross sections from gluon fusion, and thus will be much more weakly constrained than gluinos. Longer cascade decays through intermediary quark partners are also possible. The quark partners can be produced via:

- Drell-Yan-like production because of their electroweak couplings, through an off-shell $\gamma/Z$ or $W^\pm$;

- together with a quark through the decay of one of the colored gauge bosons; or

- in pairs through a $Z'$ decay (see Fig. 3).

These production modes typically produce more kinematic structure than is typical of gluino production (for example, a $Z'$ decay will tend to produce particles with energy of the order $m_{Z'}/2$). We leave a detailed recasting of the bounds from searches for jets plus missing momentum for future work.

V. CONCLUSIONS

We have constructed a novel extension of the Standard Model in which color is unified with baryon number into a single $SU(4)$ gauge group. The theory contains the minimal number of new degrees of freedom consistent with the enlarged gauge symmetry. The scale of $SU(4)$ breaking, at which the symmetry of the theory reduces to that of the Standard Model, can be as low as a few TeV and all new matter fields are vector-like with respect to the Standard Model gauge group and satisfy current LHC constraints. The structure of the $SU(4)$ demands that the quarks of the Standard Model each come with a partner field that is also a fermion, uncolored, and with electric charge $\pm 1$ or zero. The lightest...
of these states can be neutral (and largely a SM gauge singlet) and is suitable to play the role of dark matter.

The quark partners typically have an unbroken $U(1)$ symmetry which is connected by $SU(4)$ to ordinary baryon number, suggesting that there may be a deep connection between the apparent stabilities of dark matter and the proton. It is possible to engineer a situation such that the dark matter is asymmetric, in which case the lightest quark partner should have a mass around 1.75 GeV and be to good approximation a gauge singlet. Collider signatures include a rather classic quark partners themselves, which have a signature similar to perhaps with exotic decays into the quark partners), and the mass around $1\text{ metric}, in which case the lightest quark partner should have a possible to engineer a situation such that the dark matter is asymmetric, and thus have correspondingly weaker constraints.

Acknowledgments

The authors are very grateful to Yuri Shirman for an illuminating discussion about $SU(4)$ instantons, and are also pleased to acknowledge conversations with Mark Wise. This research is supported in part by NSF grant PHY-1316792. TMPT is also pleased to acknowledge support by the University of California, Irvine through a Chancellor’s Fellowship.


