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EPR: WHAT HAS IT TAUGHT US?

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I. INTRODUCTION

This symposium commemorating the fiftieth anniversary of the paper of Einstein, Podolsky, and Rosen is a fitting place to review what that work and its sequels have taught us. *Prima facie*, the EPR paper appears to have been exceedingly counter-productive for the following reasons:

1. The work was quickly rebutted by Bohr, and this rebuttal was apparently accepted by most workers in the field.

2. Scientists who adopted the position advocated by Bohr have produced, in the intervening fifty years, a marvelous body of useful theory, whereas those following the course suggested by EPR have produced nothing of any certified practical value.

3. It has been shown by Bell that the conclusion reached by EPR is incompatible with their assumptions.

In view of this negative half-a-century record of achievement we must ask whether we have any reason to believe that anything of practical value will ever come from the paper that we celebrate here today.

I believe we do. Chemists [1-3] and physicists [4-7] have recently begun to examine the behavior of quantum mechanical systems that are very small, yet large enough to influence their environment in ways that appreciably modify their own behavior, *vis-à-vis* the behavior they would have if isolated. Because these systems are neither small enough to be treated as isolated (or as residing in a classically described environment) between preparation and detection, nor large enough to be treated classically, they do not conform to the format demanded by the Copenhagen interpretation. Indeed, the behavior of these systems depends on ontological considerations that were irrelevant in the situations covered by the Copenhagen interpretation, and that were systematically ignored in that interpretation. Scientists now face the task of enlarging the scope of quantum theory to cover these new situations, and comparing the empirical consequences of various ontological assumptions.

In this task scientists can be guided by a certain heritage of work generated by EPR, namely the fact that any theory that generates predictions that accord with the predictions of quantum theory in certain rudimentary EPR-type situations must violate EPR-locality (EPR-locality is a precise form of the EPR assumption that
a process of measurement carried out in one spacetime region cannot, in any way, disturb anything in a spatially separated region.) This result that EPR locality must be violated follows from arguments that are similar to those of Bell, but that depend upon no explicit or implicit assumptions akin to hidden variables or counterfactual definiteness. (Counterfactual definiteness is the notion that unperformed experiments have definite, albeit unknown, results.)

The way in which counterfactual definiteness and hidden-variables enters into the works of EPR, Bell, and others, will first be examined. Then it will be explained in detail how these assumptions can be avoided. Only by eliminating such assumptions can we show that EPR-locality itself must fail.

2. HISTORICAL BACKGROUND

Counterfactual definiteness (CFD) is the condition that there exists, in physical reality, in association with experiments that are not actually performed, definite values that represent the results that would be obtained if the experiments were performed. This condition plays a central role in the argument of EPR, and in all the works that rest upon it. A short review of these works, with emphasis on the role of CFD, will provide both a motivation and framework for formulating the EPR idea of locality in a way that does not depend upon CFD (or hidden variables, or physical realism) as a precondition, and does not entail CFD (or hidden variables, or physical realism) as a consequence, but that allows one to conclude, by means of a Bell-type argument, that no EPR-local theory can reproduce certain rudimentary predictions of quantum theory. This result is similar to Bell's theorem but much stronger, since it does not depend upon any explicit or implicit assumption of hidden variables or physical realism.

The EPR argument [8] has the form

EPR Assumptions $\Rightarrow$ CFD

CFD $\Rightarrow$ Incompleteness of Q.M.

That is, the argument shows, on the basis of some assumptions that will be discussed later, that certain noncommuting observables must have, in physical reality, simultaneously well-defined values, even though the corresponding measurements cannot both be performed. Quantum mechanics provides no description of these simultaneous values. Hence quantum-mechanical description of physical reality must be incomplete.

Bell [9] uses the argument of EPR to justify CFD, and then argues that

CFD $\Rightarrow$ DHV (Deterministic Hidden Variables)

and

DHV + LOC $\Rightarrow$ Q.M. is False

The word "deterministic" normally carries a temporal connotation: the state at one time determines the state at other times. The original EPR argument did not involve the idea of temporal development. On the other hand, Bell's paraphrasing of the EPR argument does introduce temporal relations. His key statement is:

"Since we can predict in advance the result of measuring any chosen component of $\vec{a}_2$, by previously measuring the same component of $\vec{a}_1$, it follows that the result of any such measurement must be predetermined".

This injection of temporal relationships is questionable. The problem is that the measurements of $\vec{a}_1$ and $\vec{a}_2$ take place in spatially separated spacetime regions, $R_1$ and $R_2$, and hence, according to the ideas of the theory of relativity, neither measurement is "previous" to the other. Thus the conclusion of determination seems unwarranted, at least by the argument given.

Actually, however, the notion of temporal development plays no role in Bell's subsequent considerations, which are based on the assumption that if $E_1$ and $E_2$ are two two-valued variables that represent the choice of experiment in regions $R_1$ and $R_2$, respectively, and if $\lambda$ is an appropriate set of hidden variables, then there are two functions, $r_1(E_1, E_2, \lambda)$ and $r_2(E_1, E_2, \lambda)$, that represent the results that would appear in regions $R_1$ and $R_2$, respectively, under conditions $(E_1, E_2, \lambda)$. No reference to "earlier times" is made, or needed. Thus DHV might be called "determining hidden variables", in the sense that the variables $(E_1, E_2, \lambda)$ determine the "results" of both the performed and unperformed measurement, where the "result" of an unperformed measurement is the result that the measurement would have, by virtue of CFD, if the experiment were performed.

For each fixed $\lambda$ the pair of functions $r_1(E_1, E_2, \lambda)$ and $r_2(E_1, E_2, \lambda)$ determine, for each quartet of possible measurements $(E_1, E_2)$, a corresponding quartet of results $(r_1, r_2)$. The locality condition LOC is the condition that, for every such quartet, the result $r_1$ must be independent of the choice $E_2$, and the result $r_2$ must be independent of the choice $E_1$. 
Bell's proof that DHV + LOC is incompatible with quantum mechanics is based on an averaging over an arbitrarily weighted ensemble of values of $\lambda$. However, by considering the results $r_1$ and $r_2$ to be the results of the full experiment, involving $n$ pairs of correlated particles, one can avoid altogether the introduction of DHV, and obtain directly the incompatibility of Q.M. with CFD + LOC [10].

Bell [11,12] and Clauser-Horne [13] consider also theories in which hidden variables determine, not the "results" themselves, as they do in DHV theories, but merely the probabilities of these results. The locality property is then taken to be the factorization property

$$P(r_1, r_2 | E_1, E_2, \lambda) = P(r_1 | E_1, \lambda)P(r_2 | E_2, \lambda).$$

This factorization property does indeed hold for the "probabilities" obtained by averaging over any ensemble of DHV quartets that satisfy LOC. However, the apparent generality obtained by passing to these stochastic HV theories is not real, for two reasons: First, any set of probabilities that satisfy the factorization property can be constructed, to arbitrary accuracy, by averaging over an ensemble of DHV quartets that satisfy LOC [14]. Hence any condition on probabilities that holds for every local stochastic HV theory holds also in every local DHV theory, and vice versa. Second, in order to obtain the strict correlation results of quantum theory that are the basis of the EPR argument one must use probability distributions that have zero variance [15] (zero dispersion). That is, one must pass over to the DHV subcase.

Clauser and Shimony [16], have described the conclusions they believe should be drawn from the works of EPR, Bell, and themselves. They assert that:

"The conclusions are philosophically startling: either one must totally abandon the realistic philosophy of most working scientists, or dramatically revise our concept of spacetime."

They define this realistic philosophy to be the view "according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone".

To arrive at this conclusion they follow essentially the argument of Bell: First the EPR argument is used to justify CFD, which justifies in turn the introduction of hidden variables. The hidden-variable description is then restricted by LOC, or the equivalent factorization condition. This leads to predictions that are incompatible with some rudimentary predictions of quantum theory.

The foundation of this argument is the argument of EPR, which is analyzed by Clauser and Shimony. A key ingredient is the famous EPR criterion of physical reality:

"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Clauser and Shimony recall how quantum theory allows one to predict with certainty the value of $p_2$ of system $s_2$ in $R_2$ by measuring the value of $p_1$ of $s_1$ in $R_1$. Alternatively, one can predict with certainty the value of $x_2$ of $s_2$ in $R_2$ by measuring the value of $x_1$ of $s_1$ in $R_1$. EPR go on to conclude that quantum theory is incomplete.

Clauser and Shimony then assert that:

"In our opinion the reasoning of EPR is impeccable, once an ambiguity in the phrase 'can predict', [which occurs in the EPR criterion of physical reality] is removed. In a narrow sense one can predict the value of a quantity only when an experimental arrangement is chosen for determining the value of that quantity. In a broad sense one can predict the value of a quantity if it is possible to choose an experimental arrangement for determining it. If the narrow sense is accepted then the argument of EPR clearly does not go through since the experimental arrangements for measuring the position and momentum of a particle are incompatible. From the standpoint of physical realism the broad sense of 'can predict' is the appropriate one, since from that viewpoint, one conceives a physical system to have a definite set of properties independently of their being observed, but which may of course be explored at the option of the experimenter. In the situation envisaged by EPR one can predict, in the broad sense, both $x_2$ and $p_2$. Hence if this sense of the ambiguous phrase is adopted then their argument does go through."

This Clauser-Shimony version of the EPR argument has two serious flaws:
1. The EPR criterion of physical reality seems reasonable if 'can predict' means that one actually has in hand all the information needed to make the prediction. But it becomes unreasonable if the broad sense of 'can predict' is adopted. For in that case a more precise statement of the "criterion", in some instances in which it must be used, would be: "If, without in any way disturbing a system, we would be able – if we were to perform an experiment that is in principle impossible to perform – to predict with certainty (i.e., with probability unity) the value of a physical quantity, then there is an element of physical reality corresponding to this physical quantity".

This criterion lacks the plausibility that the EPR version enjoyed.

2. Since one is trying to establish CFD (i.e., that some unperformed experiments have definite "results"), it begs the question to say: "From the standpoint of physical realism the broad sense of 'can predict' is the appropriate one, since from that viewpoint, one conceives a physical system to have a definite set of properties independently of their being observed". For this effectively interprets physical realism as counterfactual definiteness.

The EPR argument itself [8] is essentially different from the version described by Clauser and Shimony. EPR adhere strictly to a narrow sense of "can predict":

"Thus by measuring either A or B we are in a position to predict with certainty, and without in any way disturbing the second system, either the value of the quantity P (that is $p_4$) or value of the quantity Q (that is $q_6$). In accordance with our criterion of reality, in the first case we must consider the quantity P as being an element of reality, in the second case the quantity Q is an element of reality."

EPR then complete their argument for CFD (in region $R_2$) by insisting that the reality of quantities pertaining to $s_2$ in $R_2$ cannot depend on the process of measurement carried out on $s_1$ in $R_1$, which is spatially separated from $R_2$.

The EPR argument appears to be flawless. However, as eventually shown by Bell [9], it leads to consequences that are incompatible with its premises. Thus some of these premises must be incorrect. These premises are:

1. The validity of some rudimentary predictions of quantum theory;
2. The criterion of physical reality;
3. The locality condition that no potential physical reality pertaining to system $S_1$ in $R_2$ can in any way be disturbed by, or depend upon, a process of measurement carried out on $s_1$ in $R_1$, if $R_1$ is spatially separated from $R_2$.

The potential importance of the EPR-Bell argument is the established failure of some of its premises. These premises involve the metaphysical idea of "physical reality". Introduction of this idea into the premises was necessary in the context of the original EPR argument, whose aim was to prove incomplete the quantum mechanical description of physical reality. But this involvement with metaphysics renders the knowledge of the failure of some of the EPR premises of no practical value to orthodox scientists.

The question thus arises whether the EPR premises can be replaced by ones that can be useful to orthodox scientists, in the sense that knowledge of their failure could be of practical value to scientists who reject CFD, hidden variables, and "physical realism" (in the CFD sense employed by Clauser and Shimony).

The purpose of the development to be described next is to replace the two EPR premises that involve locality and physical reality by a single premise that does not involve "physical reality", but that expresses directly the EPR idea of locality in a way such that its failure imposes an unambiguous mathematical restriction on physical theories themselves, rather than on metaphysical ideas about "physical reality". This replacement also weakens the assumptions in a way such that they neither depend upon as preconditions, nor entail as consequences, the acceptance of CFD, hidden variables, or "physical realism". In this way the argument and its conclusions are brought within the realm of orthodox quantum thinking.

3. COUNTERFACTUAL CONDITIONS AS CONSTRAINTS ON PHYSICAL THEORIES

The potential utility in science of the knowledge of the failure of the EPR conception of locality lies in the restriction that this knowledge imposes on the structure of adequate physical theories. In this connection I mention the following points:

1. The analysis of quantum theory made by EPR is based explicitly upon the simultaneous consideration of mutually incompatible experimental situations. As a consequence, any faithful representation of the EPR conception of locality must involve counterfactuals.
2. My introduction, in the early seventies, of "counterfactual definiteness" stressed the importance of counterfactuals in the context of generalizations of Bell's theorem.

3. In spite of the points 1 and 2 just mentioned I assert that "counterfactual definiteness" is not an appropriate basis for a satisfactory generalization of Bell's theorem. This is because "counterfactual definiteness" entails a correspondence that associates with each of the never-performed experiments $E$ a definite (i.e., unique) result $R(E)$ that can be considered to be the result that definitely would have occurred if the never-performed experiment $E$ had been performed. In a deterministic context it might be reasonable to posit the existence of such a function $R(E)$. But in an indeterministic context the positing of such a function is completely unreasonable and unacceptable. Yet a satisfactory generalization of Bell's theorem must cover the nondeterministic case. Hence it must not posit "counterfactual definiteness", or any conditions that entail it.

4. To deal in a satisfactory way with the counterfactual aspect of the EPR conception of locality one must first recognize that counterfactuals are, per se, murky. A counterfactual statement acquires clear meaning only through explicit or implicit reference to theoretical assumptions about how the world operates. The rules of usage, and meanings, of the word "would" are rooted in implicit theoretical assumptions.

5. Theories about the world can be simpler than world itself. Indeed, no scientist pretends today that man possesses a theory that corresponds exactly to the actual world. And many despair of the hope that such a theory can ever be achieved. On the other hand, traditional physical theories are richer than reality in the sense that they cover not merely the one unique actual world, but rather vast collections of possible worlds. The utility and importance of physical theories rests on the fact that they cover hypothetical situations. For these theories are used to predict consequences of alternative possible courses of action, in order to promote choices that lead to desired ends. Thus the fact that physical theories deal with hypothetical situations can hardly be regarded as an unfortunate flaw. Rather it is an essential virtue. One should neither expect nor wish to rid physical theories of their hypothetical character.

6. The hypothetical character of physical theories provides the basis for meaningful counterfactual statements. For example, if the charge on a test particle were to be doubled, but everything else were left unchanged, then the acceleration of that particle would be doubled. This result follows from the general properties of classical physical theory, without regard to the specifics of any actual case.

7. Physical theories can be regarded as restrictions on the set of conceivable possibilities. (In statistical theories this "exclusion" is not absolute, and some caveats are required.) The way to make counterfactual assertions clear is to display explicitly their theoretical roots by expressing them as the restrictions imposed by theories of some specified class upon the set of all conceivable possibilities. Thus in our example of the test particle we can say: If under condition 1 the acceleration of a certain test particle would be $a_1$, then under condition 2, which differs from condition 1 only by the doubling of the charge on this test particle, the acceleration of the test particle would, according to theories of a certain class $C$ that includes classical physics, be $2a_1$.

8. This procedure demystifies counterfactual statements. One is not asked to puzzle over what could possibly be meant by a statement about something that does not exist, and could never exist. Rather, one is instructed to transcribe the counterfactual statement into restrictions imposed upon the set of all conceivable possibilities by theories of a specified class.

9. This linkage of counterfactuals to classes of physical theories meshes perfectly with purpose of the study of the EPR conception of locality. That purpose is to provide researchers with information that may be useful in their search for physical theories of greater scope. Thus the whole purpose is to elucidate the structural characteristics of adequate physical theories.

4. EPR-LOCALITY

The EPR-Bell experiment under consideration here has been described in detail elsewhere in the literature [17]. There is a choice between two experiments in region $R_1$, and a choice between two experiments in region $R_2$. Thus there are, in all, four alternative possible experiments, which are labelled $a$, $b$, $c$, and $d$. There are $n$ pairs of spin-$1/2$ particles, produced earlier in a singlet state. One particle from each pair enters a Stern-Gerlach apparatus in $R_1$, and the other enters the Stern-Gerlach apparatus in $R_2$. Each Stern-Gerlach device deflects the particle that enters it either up or down. Thus each of the four alternative possible experiments has $4^n$
conceivable possible results. The \( 4^n \) conceivable possible results of experiment \( a \) are labelled by a set of \( 4^n \) symbols \( \lambda_a \), where \( \lambda \) runs from one to \( 4^n \), etc.

The function \( r_1(\lambda_a) \) specifies the \( R_1 \)-region part of the conceivable possible result \( \lambda_a \). The result \( r_1 \) is represented by a sequence of \( n \) numbers each of which is either +1 or -1. Similarly, the function \( r_2(\lambda_a) \) specifies the \( R_2 \)-region part of the conceivable possible result \( \lambda_a \). The pair of functions \( (r_1(\lambda_a), r_2(\lambda_a)) \) gives a one-to-one mapping from the set of \( 4^n \) values of \( \lambda_a \) to the set of \( 2^n \times 2^n = 4^n \) pairs \( (r_1, r_2) \).

The similar functions \( (r_1(\lambda_a)) \), etc. are defined analogously.

In our example of the test particle that was allowed to take two alternative possible values of its charge, the theoretical constraints of classical physics connected the acceleration that would occur under these two alternative possible experimental situations. The format of the restriction imposed by this theoretical constraint was:

If under condition 1 the acceleration would be \( a_1 \), then under condition 2 the acceleration would be \( 2a_1 \).

This format would apply even to a situation in which the electric field at the point in question, and hence the acceleration, was controlled by some purely stochastic process (e.g. by a radioactive decay), and hence was not determined beforehand.

The condition stated above imposes a constraint in the space consisting of pairs of conceivable accelerations, where the first member of the pair is a conceivable possible acceleration under condition 1, and the second member is a conceivable possible acceleration under condition 2. If the pairs are written \( (a_1, a_2) \) then the constraint is \( a_2 = 2a_1 \); all theoretically allowed possibilities have the form \( (a_1, 2a_1) \).

The EPR locality condition is expressed in terms of exactly analogous constraints. If the labelling of the experiments is such that: \( a \) and \( b \) are the same in region \( R_1 \), \( a \) and \( c \) are the same in region \( R_2 \); \( b \) and \( d \) are the same in region \( R_3 \); and \( c \) and \( d \) are the same in region \( R_4 \), then the EPR locality conditions are:

1. If under condition \( a \) the phenomena appearing would be \( \lambda_a \), then under condition \( b \) the phenomena appearing would be a \( \lambda_b \) satisfying
   \[ r_1(\lambda_a) = r_1(\lambda_b) \quad (1a) \]

2. If under condition \( a \) phenomena appearing would be \( \lambda_a \), then under condition \( c \) the phenomena appearing would be a \( \lambda_c \) satisfying
   \[ r_2(\lambda_a) = r_2(\lambda_c) \quad (1b) \]

3. If under condition \( b \) the phenomena appearing would be \( \lambda_b \), then under condition \( d \) the phenomena appearing would be a \( \lambda_d \) satisfying
   \[ r_2(\lambda_b) = r_2(\lambda_d) \quad (1c) \]

4. If under condition \( c \) the phenomena appearing would be \( \lambda_c \), then under condition \( d \) the phenomena appearing would be a \( \lambda_d \) satisfying
   \[ r_1(\lambda_c) = r_1(\lambda_d) \quad (1d) \]

There are also four other conditions, which are obtained by exchanging, in each of the four conditions stated above, the roles of the two experiments to which it refers.

These conditions allow the result of any performed or unperformed experiment to be indefinite, in the sense demanded by orthodox quantum thinking. There never appears, either in this statement of the EPR conditions, or in any of the arguments that follow from it, any functions \( r_i(E_1, E_2) \) that specify what the result would be in region \( i \) under condition \( (E_1, E_2) \).

The first condition, \((1a)\), imposes a constraint in the space consisting of all possible pairs \( (\lambda_a, \lambda_b) \). The second condition, \((1b)\), imposes a constraint in the space consisting of all possible pairs \( (\lambda_a, \lambda_c) \). The third condition, \((1c)\), imposes a constraint in the space consisting of all possible pairs \( (\lambda_b, \lambda_d) \). And the fourth condition, \((1d)\), imposes a constraint in the space consisting of all possible pairs \( (\lambda_c, \lambda_d) \).

Thus all four conditions act in the space of all possible (i.e., conceivable) quartets \( (\lambda_a, \lambda_b, \lambda_c, \lambda_d) \). Each condition reduces by a factor of \( 2^n \) the set of allowed possibilities. Thus the four conditions together reduce the full set of \( 4^{4n} \) conceivable quartets to a theoretically allowed set of \( 2^{4n} \) quartets. Each of these theoretically allowed quartets is a local quartet: it is a quartet \( (\lambda_a, \lambda_b, \lambda_c, \lambda_d) \) that satisfies all four equations \((1)\).

The EPR locality condition, like quantum theory itself, does not specify which experiment is actually chosen, or what the result would actually be in any given situation: it does not operate in the realm of the actual. It deals rather with the structure of the theoretically allowed possibilities, within the framework of all conceivable possibilities. It imposes relationships that connect the possibilities for
what can happen under alternative possible conditions, without specifying or defining what would happen in any situation.

Because quantum theory and the EPR locality condition operate in the same theoretical realm their compatibility can be examined. In both cases the choice of experiment is a free variable, denoted by \( s \), which can take any one of the four values \( a, b, c, \) or \( d \). Quantum theory assigns a unique probability \( P(\lambda_s) \) to each of the conceivable possible results \( \lambda_s \).

One can define a correlation function \( z(\lambda_s) \) with the following properties [14]:

1. For each \( \lambda_s \) the value of \( z(\lambda_s) \) is a rational number in the closed interval \([-1,+1]\).

2. For each \( s \) there is a closed subset \( X_s \) of the set \([-1,+1]\) such that for any \( \epsilon > 0 \) there is an \( N(\epsilon) > 0 \) such that for all \( n > N(\epsilon) \)

\[
\sum_{\lambda_s \in X_s} P(\lambda_s) < \epsilon,
\]

where \( \wedge(X_s) \) is the set of \( \lambda_s \) such that \( z(\lambda_s) \) lies in \( X_s \). That is, for each \( s \), the probability that \( z \) lies in \( X_s \) can be made arbitrarily small by taking \( n \) sufficiently large.

3. For each local quartet \( (\lambda_a, \lambda_b, \lambda_c, \lambda_d) \) there is at least one value of \( s \) such that if \( \lambda_s \) is a member of the quartet then \( z(\lambda_s) \) lies in \( X_s \). That is, at least one member of every local quartet has, according to quantum theory, a probability that can be made smaller than any arbitrarily small number \( \epsilon > 0 \) by taking \( n \) sufficiently large.

For each \( s \) a conceivable result \( \lambda_s \) will be said to be "allowed by quantum theory" if \( z(\lambda_s) \) lies in the complement of \( X_s \), and to be "disallowed by quantum theory" if \( z(\lambda_s) \) lies in \( X_s \). The word "disallowed" signifies that for each \( s \) the total probability of all of the "disallowed" conceivable possibilities can be made smaller than any \( \epsilon > 0 \) by taking \( n \) sufficiently large.

To exhibit the conflict between EPR locality and the predictions of quantum theory let the conceivable results disallowed by quantum theory be excluded as prohibitively unlikely. Then the only remaining conceivable possibilities are those allowed by quantum theory.

The EPR locality conditions are to be implemented as a set of mathematical conditions imposed on the set of conceivable possibilities. It is assumed that under any condition some result must appear. The result that appears in any single experiment is indefinite, in the sense of quantum theory. But the eight EPR conditions of "nondisturbance" limit the possibilities for what can appear by linking each possibility for what can appear under any one condition to a limited set of possibilities for what can appear under each of the other three conditions.

Each local quartet represents a solution of the eight EPR locality conditions. If any member of the quartet is excluded from the set of allowed possibilities then this quartet no longer represents an allowed solution of the eight conditions. For example, if \( \lambda'_d \) is excluded from the set of allowed possibilities then one cannot use \( \lambda'_d \) as a solution of the requirement in (1c) that "then under condition \( d \) the phenomena appearing would be a \( \lambda_d \) satisfying ...". For \( \lambda'_d \) is not allowed to appear. But according to the argument given above every local quartet has at least one element that is disallowed by quantum theory. Consequently, none of the local quartets provides an allowed (by quantum mechanics) solution to the EPR conditions.

Clauser and Shimony [16] have remarked that one might look for solutions outside the realm of quartets. In particular, if one were to allow the disturbances \( D_1 \) and \( D_2 \) associated with the changes of experiments in \( R_1 \) and \( R_2 \), respectively, to be noncommuting, then the change from experiment \( a \) to experiment \( d \) via the sequence \( a \rightarrow b \rightarrow d \) could lead to a result different from that obtained via the sequence \( a \rightarrow c \rightarrow d \). This would take one outside the realm of quartets.

This noncommutability is indeed a mathematical possibility. But our aim is to formulate a condition that represents the intuitive locality idea of EPR. This idea is that if one restricts attention to the regions \( R_1 \) and \( R_2 \) then the disturbance \( D_1 \) arising from the change of experiment in \( R_1 \) is confined to \( R_1 \), and the disturbance \( D_2 \) arising from the change of experiment in \( R_2 \) is confined to \( R_2 \). Under these conditions the disturbances \( D_1 \) and \( D_2 \) commute. But then the two sequences lead to the same condition \( d \), and system closes on the quartets.

This completes the proof of the incompatibility of EPR locality with some rudimentary predictions of quantum theory.

The EPR condition is a strong condition. A requirement that it must hold in a physical theory would be a very strong requirement. However, a much weaker requirement is imposed by the following definition: A theory is EPR-local if and only if it is compatible with the EPR locality condition. This definition does not
demand that the theory actually impose the EPR locality condition, but merely that the theory not forbid the possibility that this condition could be imposed. In terms of this definition what has been shown is that any theory that reproduces some rudimentary predictions of quantum theory is EPR-nonlocal.

A discussion of some potential uses of this result is given in Ref. [17].

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