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ON THE EMERGENCE OF A $U(2) \times U(2)$ GAUGE THEORY FOR LOW ENERGY MESONS

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Abstract.

We show how the non-linear sigma model, which describes the dynamics of pions, could generate an effective $U(2) \times U(2)$ gauge theory. The $\rho, \omega$ and $A, D$ vector mesons are identified as the massive gauge bosons.

Several relations are derived, that agree well with experimental data, but we do not find the $A\rho\pi$ coupling, which requires understanding of the mechanism that makes the $A$ and $\rho$ dynamical.

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The dynamics of massless pions ($\pi$) is described by the Lagrangian of the non-linear sigma model

$$L = (f^\pi_\pi^2/4) \text{Tr} \, a \, \mu \, a^\mu U^\dagger,$$

where $f^\pi_\pi = 93$ MeV is the pion decay constant and $U \equiv \exp(2i \pi^a T^a/f^\pi_\pi) \in \text{SU}(2)$ with $T^a$ the generators of the SU(2) algebra normalised to $[T^a, T^b] = i/2 \delta^{ab}$. The chiral SU(2)$_L \times$ SU(2)$_R$ symmetry acts as $U \rightarrow g_L U g_R^\dagger$. Recently it was argued$^{1,2,3}$ that if one splits $U = L^\dagger R$ the spurious U(2) gauge symmetry ($L \rightarrow H(x)L, R \rightarrow H(x)R, H(x) \in U(2)$) might become dynamical, i.e. the underlying theory of QCD generates in addition to (1) also the kinetic terms for the gauge bosons. These gauge bosons, which are massive through the Higgs mechanism, can be identified successfully with the $\rho$ and $\omega$ vector mesons. Going one step further one could try$^{4,5}$ to get a U(2) $\times$ U(2) gauge theory by introducing even more variables. Taking$^4$ $U = L^\dagger MR$ there is U(2) $\times$ U(2) invariance under $L \rightarrow H(x)L, R \rightarrow G(x)R, M \rightarrow H(x)MG(x)^\dagger$. In terms of the variables $L$, $M$, and $R$ there are four possible terms in the Lagrangian with three arbitrary coefficients (an overall factor of a classical Lagrangian is irrelevant). In this Letter we introduce the variables differently and obtain a Lagrangian with only a single parameter $a$. The relations that arise from our Lagrangian agree for $a \approx 2$ with the experimental data. But we do not obtain everything: the mass ratio of the $A$ and $\rho$ and the $\rho A\pi$ coupling are not calculable at this level. The explanation of these last two facts seems to require a detailed knowledge of the mechanism that makes these gauge bosons dynamical in the first place.
As mentioned above the splitting of the SU(2) variable \( U \) into \( L^+R \) led to the composite gauge fields of the \( \rho \) and \( \omega \), which are half the bosons of the desired \( U(2) \times U(2) \) gauge theory. We take our clue from the simple observation that Hermitian conjugation of \( U \) exchanges the chiral \( SU(2)_L \) and \( SU(2)_R \) groups, so that \( U \) and \( U^+ \) transform differently. Hence we split \( U \) and \( U^+ \) separately

\[
U = L^+R, \quad U^+ = \tilde{R}^+\tilde{L}, \tag{2}
\]

with \( L, R \in U(2) \), but with equal determinant, and similarly for \( \tilde{L}, \tilde{R} \). Note that the condition \( UU^+ = 1 \) leaves us with only three independent variables.

The symmetry \([U(2) \times U(2)]_{\text{local}} \times [SU(2)_L \times SU(2)_R]_{\text{global}}\) acts as follows

\[
\begin{pmatrix} L \\ R \end{pmatrix} \rightarrow H(x) \begin{pmatrix} L^\dagger g_L^+ \\ R^\dagger g_R^+ \end{pmatrix}, \quad \begin{pmatrix} \tilde{L} \\ \tilde{R} \end{pmatrix} \rightarrow G(x) \begin{pmatrix} \tilde{L}^\dagger g_L^+ \\ \tilde{R}^\dagger g_R^+ \end{pmatrix}. \tag{3}
\]

We introduce the \( U(2) \) gauge fields \( S_\mu = S_\mu^\alpha T^\alpha \) and \( T_\mu = T_\mu^\alpha T^\alpha \) taking values in the Lie algebra, which has the generators \( T^\alpha, \alpha = 0,1,2,3 \), normalised to \([T^\alpha, T^\beta] = \frac{1}{2} \delta^{\alpha\beta} \). Defining the covariant derivatives \( D_\mu \equiv (\partial_\mu - i S_\mu) \) and \( \tilde{D}_\mu \equiv (\partial_\mu - i T_\mu) \), we can write four invariant terms for the Lagrangian

\[
L = (\frac{f^2}{4}) \text{Tr}(D_\mu L^+ \pm D_\mu R R^+) \tag{4}
\]

and similarly for \( \tilde{L}_\pm \) in terms of \( \tilde{L}, \tilde{R} \) and \( \tilde{D}_\mu \). The most general Lagrangian is thus

\[
L = n(L_+ + aL^+) + \tilde{n}(\tilde{L}_+ + a\tilde{L}^+) \tag{5}
\]
where \( n, a, \tilde{n} \) and \( \tilde{a} \) are arbitrary coefficients. Now we note that the change of variables \( U \to U^\dagger \) results in the following exchanges: \( L \to \tilde{R}, R \to \tilde{L}, S_\mu \to T_\mu \) and \( L^\pm_\mu \to \tilde{T}^\pm_\mu \). This implies for (4) that \( n = \tilde{n} \) and \( a = \tilde{a} \), so that apart from the normalisation factor \( n \), only a single parameter \( a \) remains, whose precise value is determined by the underlying fundamental theory (QCD).

Let us write \( S_\mu = V_\mu - A_\mu \) and \( T_\mu = V_\mu + A_\mu \). Since we expect that QCD generates all possible terms in the effective Lagrangian we assume that in addition to (5), which up to now is equivalent to (1), kinetic terms for \( V_\mu \) and \( A_\mu \) appear also. In the symmetric gauge \( L_\dagger = \tilde{L}_\dagger = R = \tilde{R} = \xi \equiv \exp(i\alpha T^3/f_\pi) \) the Lagrangian is then

\[
\begin{align*}
L_{\text{eff}} & = \left( f^2/4 \right) \text{Tr} \left[ (a_\mu \xi^+ \xi^\dagger - a_\mu \xi \xi^\dagger)^2 \right] \\
& + a (a_\mu \xi^+ \xi^\dagger + a_\mu \xi \xi^\dagger - 2iV_\mu)^2 + a (2iA_\mu)^2 \\
& - \frac{1}{2g_V^2} \text{Tr} V_\mu V^{\mu\nu} - \frac{1}{2g_A^2} \text{Tr} A_\mu A^{\mu\nu}.
\end{align*}
\]

(6)

where we have taken \( n = 1/2 \) in order to get the standard kinetic term for the pions and where \( V_\mu \equiv a_\mu V - a_\nu V - i[V_\mu, V_\nu] \) is the field strength. Since the gauge group is \( U(2) \times U(2) \) there is no reason why the weights of the two kinetic terms generated should be equal and \( g_A \neq g_V \) in general, see our Ref. 3 for further discussion. In fact we know that the axial anomaly in QCD may have important consequences non-perturbatively. Electromagnetism can be incorporated in (6) by replacing \( a_\mu \rightarrow a_\mu + e' B_\mu \xi T^3 \) and adding the kinetic term for the \( B_\mu \) field. From (6) we have, after rescaling \( A_\mu \rightarrow g_A A_\mu \) and \( V_\mu \rightarrow g_V V_\mu \), the following relations
\[ m_{\rho^\pm}^2 = m_{\omega}^2 = (a/2)2g_V^2f_\pi^2 \quad , \]  
\[ (7a) \]

\[ m_A^2 = m_D^2 = (a/2)2g_V^2f_\pi^2 \quad , \]  
\[ (7b) \]

\[ g_{\rho\pi\pi} = (a/2)g_V \quad , \]  
\[ (7c) \]

\[ g_{B\pi\pi} = (a/2 - 1)e' \quad , \]  
\[ (7d) \]

\[ g_{\rho^0} = 2g_{\rho\pi\pi}f_\pi^2 \quad , \]  
\[ (7e) \]

where we have already identified the gauge bosons with the vector mesons \( \rho(770), \omega(783) \) for one \( U(2) \) factor and the axial vector mesons \( A(1270), D(1285) \) for the other. Furthermore there is a small mass difference between \( \rho^\pm \) and \( \rho^0 \), see Ref. 1.

For the value \( a \approx 2 \) all these "predictions" agree quite well with the experimental data;\(^7\) for example (7d) shows the vector dominance of the electromagnetic form factor of the pions and the \( \rho \)-photon mixing constant \( g_{\rho^0} \) satisfies the \( a \)-independent relation (7e); for further details and references see Ref. 4. As mentioned above, we cannot hope to calculate the ratio \( g_A/g_V \) and we have to assume that QCD gives, in addition to the value \( a \approx 2 \), a ratio \( g_A/g_V \approx 1.7 \) so that (7ab) agree with the experimental value\(^7\) for \( m_A/m_\rho \).\(^*\) Note that at this level (two derivatives in (4)) we do not get coupling of the \( A \) with the pions. This is as it should be since \( A \) decays predominantly in \( \rho\pi \), but this coupling we did not expect to find, because the \( \rho \) and \( A \) belong to different factors of our gauge group.\(^*\) Comparing our results with those of Ref. 4 we can say that we have fewer relations (7) than they, but all of ours are correct, whereas they had a problematic
one for their parameters $a = b = c = 2$, namely $g = 0.75 g_V$. Our Lagrangian (6) agrees with theirs (in the gauge $L^+ = R = \xi, M = 1$) for a particular choice of their parameters $a = c, b = 0$. It may seem somewhat strange that we ended up with a Lagrangian with a single parameter compared to theirs with three, while the only difference was a formal one of how to introduce the new variables. The reason that this formal difference can result in different physics is that in the two cases the black box for making the gauge bosons dynamic is asked to perform different operations: in our case it should make gauge bosons out of $L, R, \bar{L}, \bar{R}$, and in theirs out of $L, M, R$. We cannot argue from first principles what set the black box would prefer, but the successful and simpler results from our splitting method indicate that the $L, R, \bar{L}, \bar{R}$ are the better building blocks.

In conclusion, we can say that the hidden gauge symmetry of the non-linear sigma model may give rise to a $U(2) \times U(2)$ gauge theory with the $\rho, \omega$ and $A, D$ as gauge bosons, but a better understanding of the binding mechanism is needed to explain some facts otherwise unaccounted for, e.g. the mass ratio of $A$ and $\rho$ and the $A\rho\pi$ coupling. Still we consider it remarkable that the non-linear sigma model of pions leads us rather successfully to properties of baryons and vector mesons.

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Footnote on p. 4

* Hung\(^5\) claims to derive the mass ratio \(m_A^2/m_\rho^2 = 2\), but in our opinion this results from the ad hoc choice of \(b = 1\) in the Lagrangian he uses

\[(2-b)L_A + bL_A' + 2L_V\] (in his notation), which would give \(m_A^2/m_\rho^2 = 2/b\) in general. Also he claims to find a \(\rho A\pi\) coupling by having in \(L_A\) a term

\[ig_V \mu (\xi_L \xi_L^{-1} - \xi_R \xi_R^{-1})\], but obviously this vanishes, and furthermore his treatment of gauge invariance is obscure.
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